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A Simulation of High Energy Cosmic Ray Propagation I

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Abstract

We simulate high energy cosmic ray propagation of the energy region $10^{14.5}$-10^{18} eV in the inter steller circumstances. In conclusion, the diffusion process by turbulent magnetic fields is classified into several region by ratio of the gyro-radius and the scale of turbulence. When the ratio becomes larger then $10^{-0.5}$, the analysis with the assumption of point scattering can be applied with the mean free path E^2. However, when the ratio is smaller than $10^{-0.5}$, we need a more complicated analysis or simulation. Assuming the turbulence scale of magnetic fields of the Galaxy is 10-30pc and the mean magnetic fields strength is 3 micro gauss, the energy of cosmic ray with that gyro-radius is about $10^{16.5}$ eV.

1 Introduction

Many authors have studied cosmic ray propagation in magnetic fields(1). Especially, Gleeson and Axford had solved a diffusion equation with the assumption that cosmic ray are scattered by scattering centers. The recent observation of Galactic magnetic field(2) shows the average field strength is about 3 micro gauss and the turbulent fields are about 1.5 micro gauss. The gyro-radius of cosmic ray in the energy region, in which we are interested, becomes 0.1-300pc. The random magnetic fields cannot deflect the cosmic rays so hardly, but an amount of small deflection may cause a large scattering angle as a result. We know that, in some cases, we can sum up the effects in terms of point scattering and mean-free-path(mfp). However, we are not sure this is the case in the cosmic ray propagation in the turbulent magnetic fields, especially when the gyro-radius of the cosmic ray becomes near to the scale of irregularities. The assumption of point scattering can be an over simplification in the study of cosmic ray propagation in that energy region. It should be checked by the numerical calculation or simulation.

In this paper, we present a Monte Carlo simulation of cosmic ray propagation in the Galactic space and show a result in terms of diffusion tensor. Gleeson and Axford showed the diffusion tensor have three independent element ($K_{//}, K_{\perp}, K_T$). The analytic form of these element as a function of mean free path are given by

$$K_{//} = \frac{1}{3}\lambda c \quad , \tag{1}$$

$$K = \frac{K_{//}}{1 + (\omega \frac{\lambda}{c})^2} \quad \text{and} \tag{2}$$

$$K_T = \frac{(\omega \frac{\lambda}{c})\, K_{//}}{1 + (\omega \frac{\lambda}{c})^2} \quad , \tag{3}$$

where λ is the mfp and ω is the gyro frequency of cosmic rays in that magnetic field. When we leave from the idea of point scattering of cosmic ray, we should not use mfp at the beginning of the study. Rather it should be given as the result of diffusion process. In our simulation, we calculate the two diagonal elements $K_{//}$ and $K_{\perp}$. We check the consistency comparing these formula. When the consistency is maintained, we try to calculate the mfp from the formula.

2 Generation of random magnetic fields

It seems to be a difficulty in the simulation with irregular magnetic fields to make the flux line conserved. This difficulty is avoided in our case by generating the vector potentials by random numbers. The steps to generate the random magnetic fields are following. First we embed a lattice in the simulation space, whose lattice constant is chosen as the scale of the irregularity of the magnetic field. We generate the vector potential $A=(A_x,A_Y,A_Z)$ for each lattice point with the probability given by

$$\exp(-A_i/t)\ dA_i \quad (i=x,y,z), \tag{4}$$

where t is a parameter to adjust the irregular magnetic fields strength. The vector potential in the point other than the lattice points are given by linear interpolation. The irregular magnetic fields are given by

$$\vec{DH} = \mathrm{rot}(\ \vec{A}\ ), \tag{5}$$

with the random vector potential A. The total fields are given as the sum of irregular magnetic field and average field. When we wish to simulate the turbulent magnetic fields with some spectrum like the Kolmogorov's one, we prepare a series of lattices with different lattice constants. And sum up the irregular magnetic fields with the weight given by the spectrum.

The motion of the cosmic ray particles is determined by equation of motion;

$$m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v} \times \vec{H}). \tag{6}$$

However, in our simulation we neglect the effects from $\vec{E}$. In other words, we assumed the magnetic fields is statical. This assumption can be justified because the velocity of cosmic ray is essentially light velocity, but the velocity of the magneto-hydrodynamical waves are around 10^{-3} times of light velocity in the interstellar circumstances. This also means that in our simulation, we neglect the effect of Fermi type acceleration or the diffusion process in energy axis.

3 Simulation

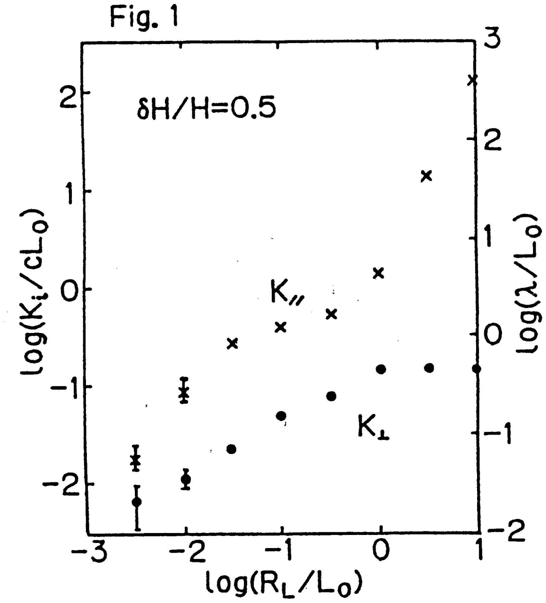
Our simulation is carried out as follows. We assumed that the average magnetic field(H_0) is aline with z-axis, and the average strength of the magnetic field is 3 micro gauss and that of the turbulent magnetic fields are 1.5 micro gauss in average. (We note that what is really meaningful is the ratio $\langle DH\rangle/H_0$, which is 1/2 in our case. The absolute strength is needed when we translate the gyro-radius to the energy in the relation to the scale of the turbulence in relation to the (maximum) turbulent scale ($=L_0$)). We first put a number of cosmic ray particles at the origin and follow the motion numerically determined by the equation. The step time for solving (6) numerically is chosen as 1/10 of the gyro frequency and is changed with gyro frequency. This step time is chosen so that the error caused by it is small in our numerical calculation and that the simulation is executed within the reasonable cpu-time. Taking the statistics of the position of cosmic ray in terms of $\rho_{\perp}=\sqrt{\langle x^2+y^2\rangle}$ and $\rho_{//}=\sqrt{\langle z^2\rangle}$, we can calculate

$K_\perp$ from the time dependence of $\sigma_\perp$ and $K_{//}$ from the time dependence of $\sigma_{//}$. The random magnetic field is regenerated every 250 step to accelerate the conversion of diffusion constants to average value.

4 Results

In fig.1, we show a result of our simulation in the form of dimension-less quantity; $K_i/(c \times L_0)$, where i stands for // and $\perp$, c right velocity and L_0 for maximum turbulent scale. The simulation of fig.1 is carried out by assuming that the magnetic fields' turbulence follows the Kolmogorov spectrum. The crosses and close ones show $K_{//}$ and $K_\perp$ respectively. We also made a simulation by assuming of the only one turbulent scale. We found a little difference between them. In both cases, however, we can say that the qualitative tendencies are the same. Therefore the farther simulation is executed assuming the only one scale in turbulence of magnetic field.

Fig. 1



The whole region can be divided into several regions by the diffusion tensor dependences on the R_L. When R_L is in the region between $10^{-1.5}xL_0$ and $10^{-0.5}xL_0$, the ratio $K_{//}/K_\perp$ is almost constant and both are proportional to $R_L^{0.3-0.4}$. When R_L becomes lager than $10^{-0.5}xL_0$, $K_{//}$ is proportional to R_L^2 and $K_\perp$ is almost constant. We also gave a scale in fig.1 to read the mfp directly from $K_{//}$ using (1). We find in the region where the R_L is larger than $10^{-0.5}xL_0$, (2) gives consistent relation between $K_\perp$ and $K_{//}$ assuming the mfp. However, the relation (2) is not fold for R_L smaller than $10^{-0.5}xL_0$. We may see that there is an another region, where the radius is smaller than $10^{-1.5}xL_0$, the slope of $K_{//}$ dependences on the R_L changes to steeper($\sim R_L^1$). However, the ambiguity of the calculation becomes large in this region and the reliability of simulation may be small.

We also executed simulations with the condition $\langle DH\rangle/H_0 = 1$ and 1/4 assuming of the only one (maximum) turbulent scale. These results are shown in fig.2 and in fig.3. The simbols are same with fig.1. The whole region is divided into three regions at the R_L; $10^{-1.5}xL_0$ and $10^{-0.5}xL_0$. The dependences of $K_{//}$ and $K_\perp$ on R_L are the same with the case of $\langle DH\rangle/H_0$ = 1/2. However, there is a quantitative difference in the value of $K_{//}$, $K_\perp$ and in the ratio $K_{//}/K_\perp$. They are rather rapidly varying functions of the $\langle DH\rangle$.

5 Summary and Discussions

If we assume that the turbulent scale(L_0) is 30pc and the average magnetic field strength is 3 micro gauss, the energy of the cosmic ray with the gyro-radius $10^{-0.5}xL_0$ is $10^{16.5}$eV for the protons. For the higher energy cosmic rays, the assumption of point scattering in the study of its propagation in Galactic space is a fairly good approximation with the mfp$\sim E^2$. However the facter in front of the power of E is a rapidly varying

Fig. 2

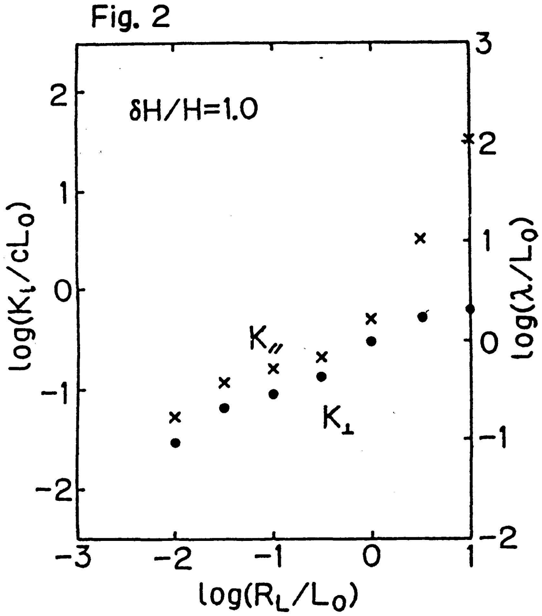
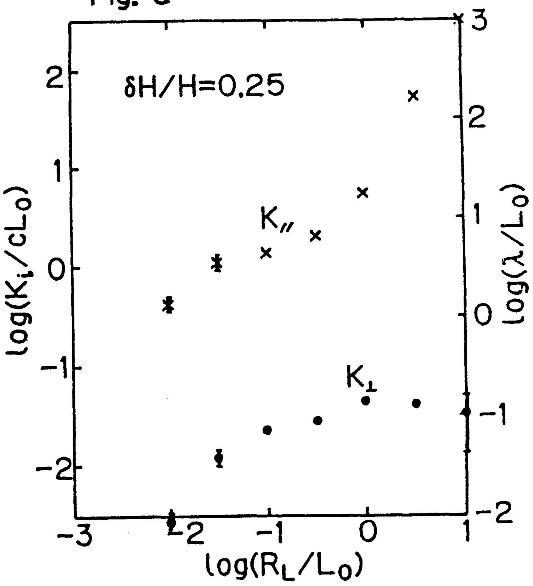


Fig. 3



function of ⟨DH⟩. We note, however, that this is a result of the accumulation of small deflection by the magnetic fields and the facter should be obtained by different method; for example simulation. For the lower energy cosmic rays than $10^{16.5}$ eV, the diffusion constants do not follow the formula (1)-(3). To study the cosmic ray propagation in this region, we need a simulation or more sophisticated analytic methods. It is noted that in this region the ratio of $K_{//}$ and $K_{\perp}$ is almost constant. If we assume the average strength of turbulent magnetic field is 1.5 micro gauss, the ratio is about 10. For the smaller energy cosmic ray than $10^{15.5}$ eV, the result of the simulation show a relatively large difference between different random number series. This fact might indicate that the other mechanism than the diffusion process like convectionshold be considred in the study of the propagation of cosmic ray with this energy in the Galaxy.

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