TEMPORAL EVOLUTION OF AN ENERGETIC ELECTRON POPULATION IN AN INHOMOGENEOUS MEDIUM. APPLICATION TO SOLAR HARD X-RAY BURSTS.

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1. Introduction. Energetic electrons accelerated during solar flares can be studied through the hard X-ray emission they produce when interacting with the solar ambient atmosphere. In the case of the non thermal hard X-ray emission, the instantaneous X-ray flux emitted at one point of the atmosphere is related to the instantaneous fast electron spectrum at that point. A hard X-ray source model then requires the understanding of the evolution in space and time of the fast particle distribution. The physical processes involved here are energy losses due to Coulomb collisions and pitch angle scattering due to both collisions and magnetic field gradients.

The evolution of the distribution is properly described by a Fokker-Planck equation (1) which has been solved numerically for steady state or impulsive (δ function) electron injections (2,3). However, its application in cases where the electron collisional lifetime and injection duration are of similar magnitudes has not yet been considered. Such cases are relevant for long-duration events where electrons are injected over a finite period. As a first approximation, a simpler mathematical approach is the use of a first order, non dispersive continuity equation in phase space, taking into account mean rates of change of the phase space variables. Such an equation relates the number of electrons of a specified energy and mean pitch angle at a given point to an arbitrary source function. The angular distribution of the electrons is not correctly described in this treatment, except in a mean sense. However, it includes pitch angle scattering adequatly for purposes of hard X-ray spatial distribution calculations and it has the great advantage of giving analytic, time dependant solutions for arbitrary source functions and ambient density structures. In this contribution, the main properties of the analytic solutions are presented for simple situations in order to illustrate the potential use of such calculations in the interpretation of coronal propagation of energetic electrons and of hard X-ray spatial distribution.

2. Basic characteristics. Energetic electrons are injected into an inhomogeneous, plane parallel atmosphere defined by a density $n(z)$ and a magnetic field $B(z)$ where z is the depth from some arbitrary point. $B(z)$ is assumed to be purely in the z-direction and the electron pitch-angle θ is the angle between the z-axis and the electron velocity ($\mu = \cos\theta$ is positive, resp. negative for an electron moving downwards, resp. upwards.) Electrons are injected for $t \geq 0$ at a rate $q(E,t,z,\mu)$ (number of electrons injected per second between z and $z + dz$, with energies between E and $E + dE$ and with $\cos\theta$ between μ and $\mu + d\mu$). The electron population evolves in the medium through different processes such as energy losses (mean rate dE/dt)

and scattering (mean rate $d\mu/dt$). The continuity equation is then given by :

$$\frac{\partial N(E,t,z,\mu)}{\partial t} + \frac{\partial}{\partial E}\left[N(E,t,z,\mu)\frac{dE}{dt}\right] + \frac{\partial}{\partial z}\left[N(E,t,z,\mu)\frac{dz}{dt}\right] + \frac{\partial}{\partial \mu}\left[N(E,t,z,\mu)\frac{d\mu}{dt}\right] = q(E,t,z,\mu) \qquad \text{(I)}$$

where dE/dt is the energy loss-rate through electron-electron Coulomb collisions (4) and $d\mu/dt$ is the sum of pitch angle scattering due to Coulomb collisions (5) and to magnetic field gradient (adiabatic invariance of the magnetic moment). Equation I has been solved analytically. The solutions and their physical interpretation are discussed in (6) and (7). The most critical approximation which consists to omit the velocity dispersion due to Coulomb collisions has also been discussed in (6). It has been shown that for mildly relativistic electrons (initial energy below 200 keV), the mean behaviour of the electrons is fairly well described by the analytic treatment.

3. Evolution of the energetic electron population in some specific cases. We will examine two extreme cases where either pitch angle scattering due to the magnetic field gradient or to Coulomb collisions is negligible. In both cases, a non thermal electron distribution is continuously injected at an arbitrary depth $z = 0$ in a stratified medium with a density scale height H [$n(z) = n_o\, e^{z/H}$] and a magnetic field $B(z)$. We then compute :

$$N(E,t,z) = \int_0^1 d\mu\, N(E,t,z,\mu) \text{ for } E < 160 \text{ keV} \qquad \text{(II)}$$

For simplicity, $q(E,t,z,\mu)$ is chosen as :

$$\left.\begin{aligned} & q(E,t,z,\mu) = S_o\, E^{-\gamma}\, F(t)\, G(z)\, H(\mu) \\ & \text{where : } F(t) = t\,(2t_o - t) \text{ for } 0 \leqslant t \leqslant 2t_o \\ & \qquad\qquad\quad = 0 \qquad\qquad \text{elsewhere} \\ & G(z) = \delta(z) \text{ where } \delta(z) \text{ is the Dirac delta function} \end{aligned}\right] \qquad \text{(III)}$$

Two extreme cases are considered for $H(\mu)$: a beam distribution $[\delta(\mu-\mu_o)]$ and an isotropic one $[H(\mu) = 1 \text{ for } \mu > 0]$.

Figure 1 shows electron spectra as a function of depth at $t = t_o$ (maximum of the injection) and at $t = 2t_o$ (end of the injection) for both a beamed injection and an isotropic one, when the magnetic field is assumed to be uniform.

Figure 2 is similar to figure 1, in the case where $B(z)$ is given by : (e.g. 8) :

$$B(z) = \begin{cases} B_o\left[1 + \dfrac{z^2}{z_T^2}(R_m - 1)\right] & z < z_T \\ B_o\, R_m & z \geqslant z_T \end{cases}$$

where R_m is the mirror ratio at $z = z_T$.

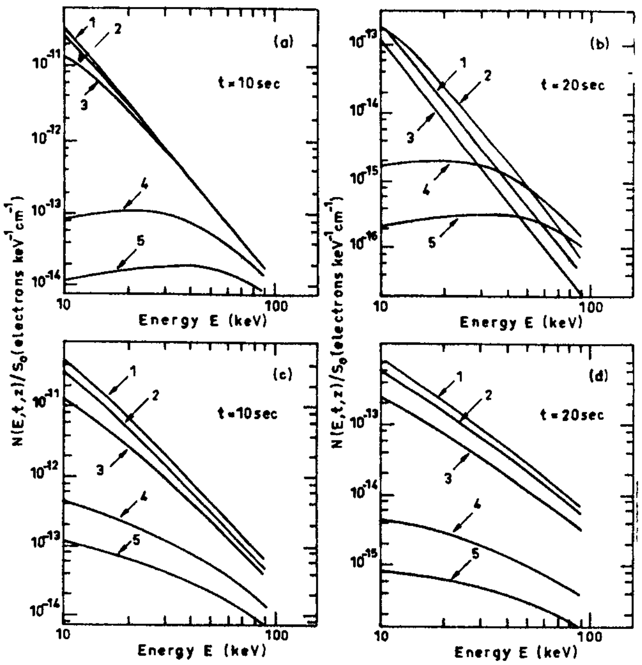

Fig. 1 :
Evolution with depth and time of $N(E,t,z)/S_o$ when the pitch angle scattering is due to collisions alone. Figures 2a and 2b correspond to a beamed injection (μ_o = 0.5). Curves 1, 2, 3, 4, 5 correspond respectively to z = 5 10^7cm, 10^8cm, 2 10^8cm, 5 10^8cm and 6 10^8cm.
The chosen parameters are : γ = 3, t_o = 10 sec, n_o = $10^{10}cm^{-3}$ at the injection point (z = 0).

For both cases, H is assumed to be 10^8cm. The general behaviour of electron spectra with depth is a progressive hardening. However, there are differences between beamed or isotropic injections or between the different scattering processes.

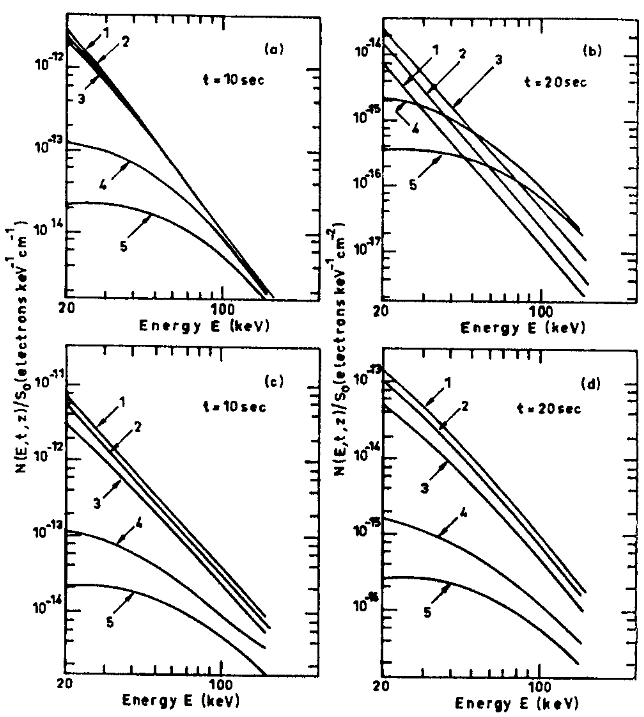

Fig. 2:
Same as figure 1 when scattering is due to a varying magnetic field with R_m = 2 and z_T = 3 10^9cm.

<u>Pitch angle scattering due to collisions</u> : For a beamed injection, there are "humps" in the spectra at large depths, especially at the maximum of the injection. At t = 2 t_o, spectra are softer and "humps"

are less pronounced because of the combined effects of the temporal behaviour of the injection and of the electron propagation and energy losses. For an isotropic injection, the hardening of the spectrum is less important and "humps" are no longer visible. At $t = 2t_o$, contrary to the case of a beam, the spectra are harder than for $t = t_o$ at each depth. However, "humps" may develop in that case after the end of the injection. These differences are due to the presence here of electrons with large pitch angles. The results presented here are similar to the ones obtained with steady-state treatments which also predict "humps" in the electron spectra at large depths for both a beamed (8) and an isotropic injection (2). The present treatment generalizes then previous results and allows moreover to study the time appearance of "humps" at each depth. Such a behaviour ("humps") indicates the breakdown of the purely collisional treatment of the evolution of the electron population.

Pitch angle scattering due to magnetic field gradients : The hardening of the electron spectra is smaller with depth and "humps" do not develop during the injection, even for a beam. These differences are due to the combined effects of the scattering process and of the magnetic mirroring. An increase in the magnetic field gradient still enhances this effect and harder spectra are generally obtained at least at low depths.

Calculations made for an extended injection region lead to results similar to the ones presented here for large depths as compared to the source extent. Of course, for both cases, at a given depth, the electron spectrum, as well as the hardness difference between different depths, strongly depend on the injection height.

4. Discussion and Conclusions. The model presented here allows to study the temporal, spatial and spectral evolution of non thermal electrons injected continuously in an inhomogeneous medium and to estimate the X-ray flux produced at each depth. This evolution depends on the characteristics of the electron injection and of the ambient medium. In these conditións, various evolutions with depth of electron spectra may be obtained. This is consistent with stereoscopic observations of partially occulted X-ray flares. Indeed, for coronally occulted events with similar occulting heights, different flux ratios and spectral hardness differences between occulted (observed by the instrument detecting the higher part of the flare) and unocculted fluxes are observed (9). Finally, the present calculations can provide a powerful and convenient framework for the interpretation of spatially resolved hard X-ray observations and the understanding of electron coronal propagation towards the interplanetary medium.

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