ONSET OF SOLAR FLARES AS PREDICTED BY TWO-DIMENSIONAL MHD-MODELS OF QUIESCENT PROMINENCES

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1. Introduction. It is well known the close connection between the sudden disappearance ("disparition brusque") of the quiescent prominences and the two-ribbon flares (see e.g. [6]). During this dynamic phase the prominence ascends rapidly (typically with a velocity about 100 Km/sec) and disappears. In another later stage is observed material falling back into the chromosphere. The impact of this downfalling matter on the chromosphere produces the $H\alpha$-brigthening, which shows the symmetric double pattern. The occurrence of the "disparition brusque" is thought to be a consequence of a plasma instability of magnetohydrostatic structures (see e.g. [11]). Two-dimensional MHD-models for quiescent prominences have been worked out since the fifties. They describe the prominence in magnetohydrostatic equilibrium under the action of Lorentz forces, gas pressure gradients and the gravitational force. However, the stability properties of most of these models are not yet determined. We analyze by means of the MHD-energy principle [2] the stability properties of four prominence models. We show that all considered models undergo instabilities for parameters outside of the observed range at quiescent prominences. We consider the possibility that such instabilities in the flare parameter range may indicate just the onset of a flare.

2. Equilibrium and Stability Theory. We define a coordinate system with x-axis along prominence, y-axis perpendicular to the prominence sheet and z-axis vertical (opposed direction of the gravity acceleration). We take into account only plasma structures, which are independent on x. In a two-dimensional theory the magnetic field can be expressed as:

$$\underline{B} = \nabla A(Y,Z) \times \underline{e}_x + B_x(Y,Z)\, \underline{e}_x \qquad (1)$$

where A is the x-component of a vector potential. The equilibrium condition, which A must satisfy reads:

$$\Delta A = -4\pi \frac{\partial \Pi(A,\phi)}{\partial \phi} \quad , \quad \Pi(A,\phi) = P(A,\phi) + \frac{1}{8\pi} B_x^2(A) \qquad (2)$$

where P is the plasma pressure, the external gravitational field and $\rho = -\partial\Pi/\partial\phi = -\partial P/\partial\phi$ the mass density. Any two-dimensional prominence model corresponds to a particular choice of the functions $P(A,\phi)$, $Bx(A)$ and the boundary conditions. In order to analyze the stability properties of prominence models we use the MHD-energy principle of Bernstein et al. (1958), according to which the stability of an equilibrium configuration is determined by the behaviour of the potential energy functional $\delta W(\xi, \xi^*)$ resulting from a perturbation $\xi(\underline{r},t)$ to the system. For the two-dimensional equilibrium class the functional $\delta W(\xi, \xi^*)$ may be written in the form:

$$\delta W(\xi,\xi^*) = \frac{1}{2}\int\Big\{\frac{1}{4\pi}|\nabla_\perp a|^2 - \frac{\partial^2 \Pi}{\partial A^2}|a|^2 + \frac{1}{4\pi}|\underline{B}_\perp \cdot \nabla_\perp \xi_x - B_x \nabla_\perp \cdot \xi_\perp|^2 + K^2\Big[\frac{1}{4\pi}|B_x \xi_\perp - \xi_x \underline{B}_\perp|^2 +$$
$$+ \gamma P |\xi_x|^2\Big] + \frac{1}{4\pi}|\underline{B}_\perp \cdot \nabla_\perp \xi_x - B_x \nabla_\perp \cdot \xi_\perp|^2 + \gamma P |\nabla_\perp \xi_\perp|^2 - 2\mathrm{Re}\big[\varrho(\nabla_\perp \xi_\perp)\xi_\perp^* \nabla_\perp \phi\big] - \frac{\partial \varrho}{\partial \phi}|\xi_\perp \nabla_\perp \phi|^2 + 2k\,\mathrm{Im}\Big[\frac{1}{4\pi}$$
$$(\nabla_\perp a \times \underline{e}_x)\cdot(B_x \xi_\perp^* - \xi_x^* \underline{B}_\perp) - \frac{\Delta A}{4\pi}\xi_x a^* - \gamma P \xi_x (\nabla_\perp \xi_\perp^*) + \varrho \xi_x (\xi_\perp^* \cdot \nabla_\perp \phi) - \frac{B_x}{8\pi}\Delta A (\xi_\perp \times \xi_\perp^*)\,\underline{e}_x\Big]\Big\}\, d^3 r \qquad (3)$$

where $\nabla_\perp = \underline{e}_y \frac{\partial}{\partial y} + \underline{e}_z \frac{\partial}{\partial z}$, $a = -\xi_\perp \nabla_\perp A$, $\underline{B}_\perp = \nabla_\perp A \times \underline{e}_x$. We have supposed complex three-dimensional displacements of the following generalized form:

$$\xi(\underline{r},t) = \xi(\underline{r})\, e^{i\omega t} = \big[\xi_x(y,z)\underline{e}_x + \xi_\perp(y,z)\big]\, e^{ikx + i\omega t} \qquad (4)$$

It is assumed periodic boundary conditions along the prominence axis (x-direction)and $\xi = 0$ on the edge of the plasma region in the y,z-plane. The x-integration in equation (3) is to be carry out over one period. If δW is positive for all displacements which satisfy the boundary conditions, then the equilibrium is stable. The MHD-energy principle is a necessary and sufficient criterion for stability. The stability problem reduces therefore to analize the sign of the minimum of δW. It is interesting to note that to the variational problem which implies the minimisation of δW, is associated the Euler-Lagrange equation:

$$-\varrho\, \omega^2 \xi(\underline{r}) = F(\xi(\underline{r})) \qquad (5)$$

where $\underline{F}$ is a self-adjoint differential operator with time independent coefficients (see e.g. [1]). We use the normalization constraint: $\frac{1}{2}\int \varrho |\xi|^2 d^3 r = 1$ and obtain the spectrum of eigenvalues further one has $\omega^2_{min} = \mathrm{Min}\,\delta W(\xi, \xi^*)$

3. Stability Results for Prominence Models. We have evaluated the energy principle for four prominence models: Menzel, (M), [10]; Dungey, (D), [3]; Kippenhahn and Schlüter, (KS), [7]; Lerche and Low, (LL), [9]. One obtains generally stability statements by two procedures: (a) analytically, by manipulating the energy functional δW to recognize a definite sign and so to infer about the stability properties, (b) numerically, by carring out the minimisation of δW with the aid of a computer code. Our analytical results concerning to these models are reported elsewhere [4]. We obtain global stability for the KS-model in case of arbitrary 3D-displacements [5]; for the other models the stability statements are restrictive to special

classes of displacements (2D, long wavelengths, etc.). In order to obtain more general results concerning to broader class of displacements we have developed a numerical code based on the finite element-method. This procedure provides, besides qualitative stability statements, the largest growth rate in case of instability and the frequency of the fundamental oscillations of stable systems. Our code was tested successfully by applying it to simple systems, whose dynamic properties are well known (e.g. Alfvén waves, current sheets and sound waves in a constant gravitational field). We obtain numerically a stable behaviour for the four models in the parameter range of the observations of typical quiescent prominences ($T=7.10^3\,{}^\circ K$, $n_e=5.10^{10} cm^{-3}$, $B=5G$, thickeness $Y=5.10^3$ Km, height $Z=1.5-5.10^4$ Km). All models describe horizontal large-scale oscillations with periods between 16 and 80 min (see Table 1). Reported data indicate that quiescent prominences undergo actually horizontal oscillations from and towards the perturbation, which is originated in solar flares. The observed periods range from 6 to 80 min. [2], [8], [1].

Table 1. Periods of horizontal oscillations in quiescent prominences.

Model	Period (min)
M	40
D	55-80
KS	16
LL	17-50

Unstable behaviour is found only out of the observed parameter range. When one considers typical parameters of flares (e.g. $T \sim 10^7\,{}^\circ K$, $n_e \sim 10^{11} cm^{-3}$, $B \sim 50$ G) one obtains $\beta = \frac{8\pi n_e kT}{B^2} \gtrsim 1$. Just for these parameters we get instability only in case of LL model. The other models continue stable around these parameters. Figure 1 shows function of $\alpha \equiv \beta/2$ (we use dimensionless variables, so that ω^2_{min} is normalized by g/h, where $g=2.74 \times 10^4 cm/sec^2$, $h=kT/mg$ is the density scale height and m is the proton mass). We have separated the different physical effects (electromagnetic, compressional and gravitational parts) in the energy functional $\delta W(\xi_{min}, \xi^*_{min})$, so that it is possible to infer about the nature of the instability. For the situation illustrated on Fig. 1 we find that the instability is driven mainly by electromagnetic forces. Gravitation provides also an instabilizing effect. In opposition to this, the compression has continually a stabilizing effect. Such a gravitational-electromagnetic mixed instability has a typical growth rate $\Gamma = \sqrt{g/h}\,|\omega_{min}|/2\pi \sim 4.77\ 10^{-5} sec^{-1}$, i. e. a growth time $\tau \sim 5.8$ h. On the other hand, the impulsive phase of a flare elapses within few minutes, so that the found

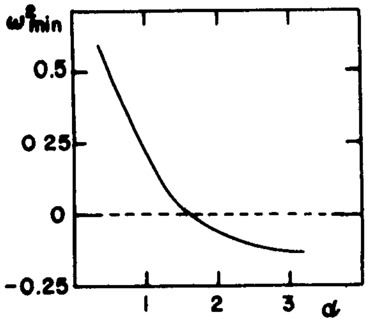

Fig.1. LL-Model: minimum eigenvalue ω^2_{min} as function of $\alpha=\beta/2$.

growth time may explain rather the evolution of the flare ribbons and the associated loop system which last several hours. The cause of the parameter shift in the quiescent prominence may reside in an external perturbation in form of a shock wave generated by a distant flare or in an internal perturbation in form of a newly emerging flux in the same active region. Our results can be considered as preliminaries because the studied two-dimensional models are still very simple to describe the complexity of quiescent prominences. However, further applications of our stability method to more realistic models give the possibility of a better description of the impulsive phase of solar flares.

References.

[1] Bashkirtsev et al.: 1983, Solar Phys., 82, 443.

[2] Bernstein, I. B. et al.: 1958, Proc. Roy. Soc., A244,17.

[3] Dungey, J. H.: 1953, MNRAS, 113, 180.

[4] Galindo Trejo, J.: 1985, to be published.

[5] Galindo Trejo, J. and Schindler, K.: 1984, Astrophys. J., 277, 422.

[6] Hyder, C.: 1967, Solar Phys., 2, 49.

[7] Kippenhahn, R. and Schlüter, A.: 1957, Z. f. Astrophys., 43, 36.

[8] Kleczek, J. and Kuperus, M.: 1969, Solar Phys., 6, 72.

[9] Lerche, I. and Low, B. C.: 1980, Solar Phys., 67, 229.

[10] Menzel, D. H.: 1951, Proc. Conf. on Dynamics of Ionised Media, London.

[11] Priest, E. R.: 1983, Solar Phys., 86, 33

[12] Ramsey, H. E. and Smith, S. F.: 1966, Astron. J., 71, 197.