INTERSTELLAR TURBULENCE, RANDOM DENSITY VARIATIONS, AND SCINTILLATION MEASUREMENTS

J. C. Higdon
Jet Propulsion Laboratory, 169-327
California Institute of Technology
Pasadena, CA 91109

1. Introduction. The presence of random electron variations suggests that the ionized interstellar medium is turbulent. In the interstellar plasma the presence of power spectra of such variations extending to spatial scales much less than a Coulomb mean free path, λ_c, is required by analyses of measurements of scintillation and angular broadening of pulsar radio signals (Armstrong et al., 1981). The existence of corresponding variations in magnetic field strength could efficiently scatter cosmic rays and thus constrain cosmic-ray propagation. Unfortunately both the origin of the electron density variations and mechanisms by which these variations couple to fluctuations in magnetic field strength are unknown. In Higdon (1984) I conjectured that the small-scale density variations are generated by the convective distortion of initially large-scale isobaric entropy structures in the turbulent interstellar plasma. The following is an investigation of the spectra of turbulent entropy structures, velocity, and magnetic fields at small spatial scales. The modifier small is employed here to characterize length scales much less than the dimension, L, containing the bulk of the turbulent energy.

2. Isobaric Entropy Structures. Isobaric entropy structures do not propagate and are stationary in the plasma rest frame. Two classes of isobaric entropy structures are identified in magnetogasdynamic fluids: tangential pressure balances and entropy waves (Jeffrey and Taniuti 1964). In the simplest case tangential pressure balances are coupled variations in temperature and density aligned in planes transverse to the direction of local mean magnetic field; $T_t/T_o=-N_t/N_o$, where N_t, N_o, T_t, and T_o are respectively a density variation, mean density, temperature variation, and mean temperature. Tangential pressure balances can possess gradients of wavenumber, K_t, such that $K_t\lambda_c>>1$; entropy waves and propagating magnetoacoustic waves with similar gradients are unstable and disperse on very short time scales (Barnes 1971).

The equation for the evolution of isobaric entropy structures in an electron-proton plasma can be well approximated from Braginskii (1965), for the temperature component, by

$$\partial T/\partial \tau + \underline{u}.\nabla T = DT/D\tau = \kappa_t \nabla_t^2 T + \kappa_p \nabla_p^2 T \qquad (1)$$

where τ is time; $\underline{u}$ is the hydrodynamic velocity; κ_p and κ_t are thermal diffusivities respectively parallel and transverse to the direction of the mean magnetic field, $\underline{Bo}$; and the components of the spatial gradient respectively parallel and transverse to $\underline{Bo}$ are ∇_p and ∇_t. Equation (1) assumes that an arbitrary entropy structure is a mixture of tangential pressure balances and entropy waves. Equation (1) shows that plasma entropy is conserved in the absence of thermal conductivity. Random convection produced by the inertial term of equation (1), $\underline{u}.\nabla T$, distorts the shape of a fluid volume and consequently leads to a statistical increase in the entropy gradient; the increase in the entropy gradient is halted ultimately at very small scales by the diffusive effect of thermal conductivity (e.g., Batchelor 1959).

When buoyancy forces are small, the reaction of isobaric entropy variations on the turbulent velocity and magnetic fields is negligible. When an additional constraint is met, that the dynamic turbulent components do not generate significant intensities of magnetoacoustic waves, the evolution of the turbulent velocity field as well as the magnetic field can be modeled as solenoidal. These fundamental simplifications can be shown to apply to the small-scale structures of the interstellar flows investigated by Higdon (1984).

3. Solenoidal Velocity Field. The nature of the velocity components that convect the passive entropy structures was investigated by Higdon (1984). At small scales equipartition existed between kinetic and magnetic energies. These solenoidal variations were strongly nonlinear and thus could not be interpreted, as in some models of interstellar turbulence, as weakly-interacting hydromagnetic waves. Small turbulent variations were modeled as statistically homogeneous, stationary, and nonhelical. Three-dimensional isotropy was not assumed; the presence of an approximately uniform mean magnetic field required that the turbulent variations be rotationally isotropic about $\underline{Bo}$ (Montgomery 1982). For such cases I developed a model of anisotropic turbulence, that was applicable at small scales, based on a study of Montgomery (1982) which showed that the presence of a strong mean magnetic field suppressed the generation of variations parallel to $\underline{Bo}$. I employed a model of Fyfe, Joyce and Montgomery (1977) of two-dimensional isotropic magnetohydrodynamic turbulence to approximate the evolution of the dominant transverse velocity components in the inertial range

$$F(K_t) = C_1 \varepsilon^{2/3}/K_t^{5/3} \qquad 1/L \ll K_t \ll K_{du} \tag{2}$$

$$\langle \underline{u}_t^2 \rangle = \int F(K_t)dK_t$$

where $\langle \underline{u}_t^2 \rangle$ is the mean-square turbulent velocity transverse to $\underline{Bo}$, C_1 is a constant of order unity, and ε is the energy transfer rate per unit mass. The inertial range terminates at K_{du} where viscous damping becomes important. In these planes transverse to $\underline{Bo}$ I showed that the viscosity coefficients are dramatically reduced compared to the viscosity found in the absence of mean fields when Braginskii (1965) transport theory is employed. Thus the energy-conserving inertial range extends to spatial scales significantly smaller than a Coulomb mean free path.

4. Spectrum of Entropy Structures. Distortions produced by turbulent convection parallel to the mean field are relatively weak. Employing Braginskii (1965) transport theory to model the anisotropic thermal diffusivities, it can be shown that inertial forces in this direction dominate the diffusive action of thermal conductivity only at large scales in the model flows of Higdon (1984). Only for $KL<10$, are the entropy flows parallel to $\underline{Bo}$ controlled by inertial forces. Thus at small scales the distorting effect of random turbulent convection acts primarily on the evolution of tangential pressure balances in planes transverse to $\underline{Bo}$.

The evolution of entropy structures at small scales, KL>>10, is straightforward since the turbulent convection of entropy structures parallel to Bo can be disregarded. In this case small-scale entropy structures, as well as velocity fields, can be modeled as superpositions of two-dimensional isotropic structures. These random fields can be decomposed into single-component Fourier spectra, which for tangential pressure balances are,

$$\int G(K_t)dK_t = \langle T_t^2\rangle, \qquad \int P(K_t)dK_t = (N_o/T_o)^2 \int G(K_t)dK_t = \langle N_t^2\rangle \qquad (3)$$

The statistical increase in the entropy gradient produced by random convection can be interpreted as a transfer among the different Fourier components of the tangential pressure balances; $G(K_t)$ and $P(K_t)$ produced at $K_t \approx 1/L$ are transferred efficiently to large wavenumbers by the action of turbulent convection (e.g. Batchelor 1959). The large-scale entropy structures affect the properties of the small-scale tangential balances primarily through the rate of transfer of random tangential balances, χ. This transfer rate can be determined from the parameters of the large-scale turbulent variations. In the convective range, where dissipation by thermal conductivity and viscosity are negligible, dimensional analysis suggests that $G(K_t)$ can only depend on χ, K_t, and the properties of the small-scale, two-dimensional velocity field. (As shown in equation (2) the velocity field depends only on ε and K_t.) Employing Kolmogorov dimensional arguments,

$$G(K_t) = C_2\chi/[\varepsilon^{1/3}K_t^{5/3}] \qquad 10/L \ll K_t \ll K_{dT} \qquad (4)$$

$$P(K_t) = (N_o/T_o)^2 C_2\chi/[\varepsilon^{1/3}K_t^{5/3}]$$

where C_2 is a dimensionless constant of order unity.

The convective range terminates at K_{dT} where thermal diffusivity becomes important. Employing Braginskii (1965) transport theory for the dispersion of tangential pressure balances, it can be shown that the convective range for these variations extends to spatial scales much smaller than a Coulomb mean free path. This results because thermal conductivity, like viscosity, is dramatically reduced transverse to Bo compared to the conductivity found in the absence of mean magnetic fields.

Figure 1 shows the density spectrum determined by Armstrong et al. (1981) compared to a model density spectrum of tangential pressure balances in which equation (4) was used to calculate the properties of the convective range. The parameters for transport coefficients, magnetic, and velocity fields were taken from Higdon (1984) for model interstellar flows situated in cloud-debris HII regions. The mean-square density of the tangential pressure balances, $\langle N_t^2\rangle$, was considered to be a free parameter; comparison with the observations required that $\langle N_t^2\rangle/N_o^2 = 0.16$. Such a random density component is consistent with models of intense turbulent flows in heterogeneous thermal media, if the random entropy component is predominantly tangential pressure balances.

5. Conclusion. A model for the turbulent origin of the random density variations observed in the interstellar medium has been constructed. This model does not employ small-scale magnetoacoustic waves which dissi-

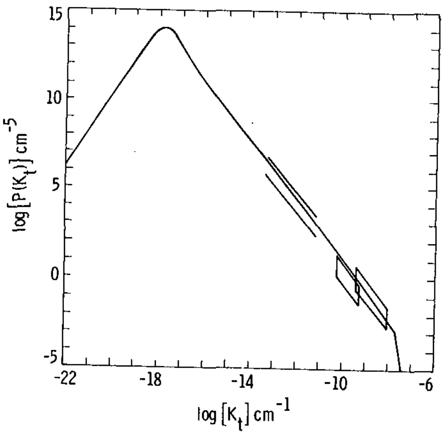

Figure 1. Comparison between the model spectrum (single line) and the spectrum determined from analyses of measurements of scintillation (parallel lines) and angular broadening (boxes) by Armstrong et al (1981).

pate very efficiently in interstellar plasmas. The model has three major components. First random turbulent distortions of initially large-scale isobaric entropy structures generate broad spectra of entropy, temperature and density variations. Second, due to the presence of a strong mean magnetic field, Bo, the small-scale velocity variations are concentrated in planes transverse to the direction of Bo. Third, in these directions the relevant dissipation processes, thermal conductivity and viscosity, are dramatically reduced compared to the values found in the absence of mean fields. Thus transverse to Bo the convective range of the isobaric entropy variations extends to very small spatial scales before the diffusive action of thermal conductivity becomes significant. Work in progress suggests that this model is applicable to analyses of random variations in the interplanetary medium, where fluctuations in density, velocity, and magnetic field, as well as relativistic particle scattering can be measured in detail.

6. Acknowledgements. This research was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. The author is a NAS/NRC Senior Resident Research Associate.

References

Armstrong, J. W. et al. 1981, Nature, 291, 561.
Barnes, A. 1971, J.G.R., 76, 7522.
Batchelor, G. K. 1959, J.F.M., 5, 113.
Braginskii, S. I. 1965, Rev. Plasma Phys., 1, ed. M. A. Leontovich (New York: Consultants Bureau), 205.
Fyfe, D., Joyce, G., and Montgomery, D. 1977, J. Plasma Phys., 17, 317.
Higdon, J. C. 1984, Ap. J., 285, 109.
Jeffrey, A., and Taniuti, T. 1964, Non-Linear Wave Propagation (New York: Academic Press), 176.
Montgomery, D. 1982, Physica Scripta, T2, 83.