THE ROLE OF COSMIC RAYS IN MAGNETIC HYDRODYNAMICS OF INTERSTELLAR MEDIUM

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Cosmic ray (CR) propagation in the Galaxy and generally in the cosmic plasma is usually considered in the duffusion approximation. The diffusion is regarded to result from CR scattering due to their interaction with a magnetic and an electric field. In most cases the fields are assumed to be given. Meanwhile, in the Galaxy the CR energy density $w_{cr} \sim 1$ eV/cm, i.e. it is comparable with the energy densities of the magnetic field and turbulent motions in the interstellar gas. Therefore, for the Galaxy and for a number of other objects it becomes necessary to take into account the influence of CR on the gas dynamics and on the magnetic fields in this gas; see /I,2/. The simplest way to do this is to use the hydrodynamic approximation, but this is possible only on scales greatly exceeding the CR free path Λ before scattering and only for times larger than $\Lambda/v \approx \Lambda/c$. One should thus obtain corresponding MHD equations and establish the limits of their applicability.

What does the CR pressure affect? The mean force with which CR act on the gas is usually written as $\langle \vec{F} \rangle = -\nabla P_{cr}$, where P_{cr} is the CR pressure. But what is the pressure in our case where particles do not collide? It is clear that the force with which CR act on the background plasma is equal to $\vec{F} = c^{-1}(\vec{J}_g \times \vec{H}_{cr})$, where $\vec{J}_g$ is the current in plasma and $\vec{H}_{cr} \equiv \vec{B}_{cr}$ is a magnetic field created by the CR current $\vec{J}_{cr}$. For quasistationary fields

$$\langle \vec{F} \rangle = -\frac{1}{c}\langle \vec{J}_{cr} \times \vec{H}_g \rangle = -\frac{1}{c}\langle \vec{J}_{cr} \rangle \times \langle \vec{H}_g \rangle - \frac{1}{c}\langle \delta\vec{J}_{cr} \times \delta\vec{H}_g \rangle. \quad (I)$$

Here we have integrated over an ensemble of random magnetic fields $\vec{H}_g = \langle \vec{H}_g \rangle + \delta \vec{H}_g$, $\langle \delta \vec{H}_g \rangle = 0$ and, accordingly, $\vec{J}_{cr} = \langle \vec{J} \rangle_{cr} + \delta \vec{J}_{cr}$, $\langle \delta \vec{J}_{cr} \rangle = 0$. To calculate $\vec{J}_{cr}$, one should know the CR distribution function $f(t, \vec{r}, \vec{p})$. It obeys the equation (we assume that the electric field $\vec{\mathcal{E}} = 0$).

$$\frac{\partial f}{\partial t} + (\vec{v}\nabla)f + \frac{e}{c}(\vec{v} \times \vec{H}_g)\frac{\partial f}{\partial \vec{p}} = 0. \quad (2)$$

Assuming $f = \langle f \rangle + \delta f$, $\langle \delta f \rangle = 0$ and using the standard procedure of the quasilinear approximation, we find equations for $\langle f \rangle$ and δf. Integrating the equation for δf in the approximation of "nonmagnetized" CR with a charge Ze (this means that in calculation of the interaction between a particle and a single magnetic field inhomogeneity, particle motion may be considered rectilinear, i.e. the gyroradius $r_H \gg L$, where L is the correlation scale of the field $\delta \vec{H}_g$), we obtain

$$\delta f(t,\vec{r},\vec{p}) = \frac{Ze}{c}\int_{-\infty}^{t} dt_0\, \delta\vec{H}_g(\vec{r}-\vec{v}(t-t_0))\left(\vec{v}\times\frac{\partial}{\partial\vec{p}}\right)\langle f(t,\vec{r},\vec{p})\rangle. \quad (3)$$

In the diffusion approximation $\langle f\rangle$ has the form (for more details see /3/).

$$\langle f(t,\vec{r},\vec{p})\rangle = f_0 - \frac{3v_i D_{ij}}{v^2}\nabla_j f_0\,,\quad N=\int d^3p\, f_0\,,\quad w_{cr}=\int d^3p\, E f_0\,, \quad (4)$$

where $f_0(t,\vec{r},p)$ is an isotropic part of the function $\langle f\rangle$, N is the total CR concentration, the diffusion tensor is equal to (here $\vec{h} = \langle\vec{H}_g\rangle / \langle H_g\rangle$)

$$D_{ij} = v\Lambda\left[3\left(1+\Lambda^2/r_H^2\right)\right]^{-1}\left[\delta_{ij}+\left(\frac{\Lambda}{r_H}\right)^2 h_i h_j + \frac{Z}{|Z|}\frac{\Lambda}{r_H}e_{ijm}h_m\right]. \quad (5)$$

The free path along the field $\langle\vec{H}_g\rangle$ is

$$\Lambda = 3c^2p^2\left[2Z^2e^2L\langle\delta H_g^2\rangle\int_0^\infty dy\,\Psi(y)\right]^{-1}. \quad (6)$$

We assume that

$$\langle\delta H_i(\vec{r}_1)\cdot\delta H_j(\vec{r}_2)\rangle = \frac{\langle\delta H_g^2\rangle}{3}\left[\Psi\left(\frac{x}{L}\right)\delta_{ij} - \Psi_1\left(\frac{x}{L}\right)\frac{x_i x_j}{x^2}\right], \quad (7)$$

where $\vec{x} = \vec{x}_1 - \vec{x}_2$, the functions Ψ and Ψ_1 are related by the condition $\nabla\delta\vec{H}_g = 0$.

Using formulas (2)-(7), we find

$$\langle\vec{F}\rangle = -\nabla P_{cr}\,,\quad P_{cr} = \frac{1}{3}\int_0^\infty d^3p\, pv f_0(t,\vec{r},p), \quad (8)$$

which is the answer to our question. Note that for $\Lambda \gg r_H$ the main contribution into the total force $\langle\vec{F}\rangle$ in the direction longitudinal with respect to the regular field is made by the fluctuation interaction $-\frac{1}{c}\langle\delta\vec{J}_{cr}\times\delta\vec{H}_g\rangle$ /4/, and in the perpendicular direction - by the interaction $-\frac{1}{c}\langle\vec{J}_{cr}\rangle\times$ $\times\langle\vec{H}_g\rangle$ (more precisely, by its part connected with the Hall current of CR). We have, in fact, extended the problems of momentum exchange between a relativistic charged particle and a "magnetic cloud" (see /5/) to the case of CR propagation in a turbulent medium.

<u>MHD equations for interstellar medium with an account of the action of CR.</u> We proceed from the hydrodynamic description of back-ground plasma motions and from the description of CR by means of the kinetic equation. The equation of back-ground plasma motion in the MHD approximation has the form

$$\rho\frac{d\vec{u}}{dt} = -\nabla P_g + \frac{1}{c}(\vec{J}_g\times\vec{H}) - ZeN\vec{\mathcal{E}}, \quad (9)$$

here $\vec{H} = \vec{H}_g + \vec{H}_{cr}$, $\vec{\mathcal{E}} = -\frac{\vec{u}}{c}\times\vec{H}$ is an electric field in a medium. We assume that in plasma there exists an excessive charge density $-ZeN$ which compensates the CR charge.

The averaged kinetic equation for CR now has the form

$$\frac{\partial\langle f\rangle}{\partial t} + (\vec{v}\nabla)\langle f\rangle + Ze\langle\vec{\mathcal{E}}\frac{\partial f}{\partial\vec{p}}\rangle + Ze\left(\frac{\vec{v}}{c}\times\langle\vec{H}\rangle\right)\frac{\partial\langle f\rangle}{\partial\vec{p}} + Ze\langle\left(\frac{\vec{v}}{c}\times\delta H\right)\frac{\partial\delta f}{\partial\vec{p}}\rangle = 0 \quad (10)$$

In the diffusion approximation with an account of the motion of the medium the distribution function $\langle f \rangle$ differs from (4) by an additional term $-\frac{3(\vec{v}\vec{u})}{v^2} p \frac{\partial f_0}{\partial p}$. Multiplying (I0) by $\vec{p}$ and integrating over d^3p, we have

$$\frac{w_{cr}+P_{cr}}{c^2}\frac{\partial \vec{u}}{\partial t} - \nabla P_{cr} - Ze\langle \vec{\mathcal{E}} N\rangle - \frac{1}{c}\left(\langle \vec{J}_{cr}\rangle \times \langle \vec{H}\rangle\right) - \frac{1}{c}\langle \delta\vec{J}_{cr} \times \delta\vec{H}\rangle = 0. \quad (II)$$

Here we elimitate terms quadratic in the small parameters u/v, Λ/R (R is the characteristic scale of the change in the CR concentration). The quantities w_{cr} and P_{cr} are defined by formulae (4), (8). Note that in hydrodynamics the density w of the internal gas density and the gas pressure p are defined in the frame of reference moving at a mean mass velocity $\vec{u}$ of the medium. In the case of CR it is more convenient to use a laboratory frame of reference. If the quadratic terms $(u/v)^2$ are neglected, both the definitions of the quantities w and p are equivalent.

Summing up equation (II) and equation (9) averaged over the field fluctuations, we derive

$$\left(\frac{w_{cr}+P_{cr}}{c^2}+\rho\right)\frac{\partial \vec{u}}{\partial t} + \rho(\vec{u}\nabla)\vec{u} = -\nabla P_g - \nabla P_{cr} + \frac{1}{c}\left(\langle \vec{J}_g\rangle + \langle \vec{J}_{cr}\rangle\right)\times\langle \vec{H}\rangle + \frac{1}{c}\langle(\delta\vec{J}_g + \delta\vec{J}_{cr})\times\delta\vec{H}\rangle. \quad (12)$$

In the quasistationary approximation the regular field $\langle \vec{H}\rangle$ is determined from the equation

$$\mathrm{rot}\,\langle \vec{H}\rangle = \frac{4\pi}{c}\left(\langle \vec{J}_{cr}\rangle + \langle \vec{J}_g\rangle\right). \quad (I3)$$

Substituting (I3) into (12) and neglecting the small terms $\delta H \ll H$, $(w_{cr} + p_{cr}) \ll \rho c^2$, we have

$$\rho\frac{d\vec{u}}{dt} = -\nabla P_g - \nabla P_{cr} + \frac{1}{4\pi}\left(\mathrm{rot}\,\langle \vec{H}\rangle\right)\times\langle \vec{H}\rangle. \quad (I4)$$

We find the equation for w_{cr} by multiplying (I0) by E and integrating over d^3p

$$\frac{\partial w_{cr}}{\partial t} - \nabla \hat{D}\nabla w_{cr} + (\vec{u}\nabla)w_{cr} + (w_{cr}+P_{cr})\nabla\vec{u} = 0. \quad (I5)$$

The same result can be obtained simpler, by using the equation for f_0 in the diffusion approximation: $\frac{\partial f_0}{\partial t} - \nabla_i D_{ij}\nabla_j f_0 + (\vec{u}\nabla)f_0 - \frac{\nabla\vec{u}}{3}p\frac{\partial f_0}{\partial p} = 0.$

The rest of the MHD equation, except (I4), (I5) are obvious:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho\vec{u}) = 0, \quad \frac{\partial \langle \vec{H}\rangle}{\partial t} = \mathrm{rot}\,(\vec{u}\times\langle \vec{H}\rangle), \quad \nabla\langle \vec{H}\rangle\, 0. \quad (I6)$$

Equations (I4)-(I6) form a system of MHD equations for interstellar medium with an account of the action of CR. They must be supplemented with the equation of state for the background plasma and with the relations connecting p_{cr} and w_{cr} (see /6/): For ultrarelativistic CR $p_{cr} = w_{cr}/3$.

In the approximation used here, CR are an additional gas component with a large internal energy, but with a negligible mass density. The small-scale random field $\delta H_g \ll H$, which provides CR scattering, enters only the tensor $\hat{D}$. The field δH_{cr} induced by the fluctuations δf must be small:

$\delta\vec{H}_{cr} \ll \delta\vec{H}_g$, otherwise CR scattering cannot be considered in the test particle approximation. This condition is fulfilled if (δ_{cr} is the degree of CR anisotropy)

$$P_{cr} \cdot \delta_{cr} \ll H^2/4\pi . \qquad (I7)$$

is is assumed that in the system there are neither kinetic instabilities induced by CR (see /7/) nor a statistical acceleration of particles.

Various dissipative processes may be included into the equations if necessary. Note that CR scattering leads to viscosity and to large-scale conductivity (along the field $\langle \vec{H} \rangle$)

$$\eta_{cr\parallel} = \frac{2}{15}\int d^3p\, \Lambda p f_0 \; ; \; 6_{cr\parallel} = \frac{4(Ze)^2}{3}\int d^3p\, \frac{\Lambda}{p} f_0 . \qquad (I8)$$

For examples of the solution of the above MHD equations see /2,8–I0/).

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