DIFFUSION OF STRONGLY MAGNETIZED COSMIC RAY PARTICLES IN A TURBULENT MEDIUM

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CR propagation in a turbulent medium is usually considered in the diffusion approximation. The methods of the derivation of a corresponding diffusion equation for different cases are proposed, for example, in the reviews /I,2/ and in the literature cited there. Here we obtain the diffusion equation for strongly magnetized particles in the general form and discuss the influence of a large-scale random magnetic field on CR propagation in interstellar medium.

CR diffusion equation. CR are assumed to propagate in a medium with a regular field $\vec{H}$ and an ensemble of random MHD waves. The energy density of waves on scales smaller than the free path l of CR particles is small. We use the collision integral of the general form which describes interaction between relativistic particles and waves in the quasilinear approximation /3/. Wave polarization is described by the polarization density matrix

$$S_{mn}^{(\alpha)}(\vec{k}) = \left(|\mathcal{E}_L^{\alpha}|^2 + |\mathcal{E}_R^{\alpha}|^2\right)^{-1} \cdot \begin{pmatrix} |\mathcal{E}_L^{\alpha}|^2 & \mathcal{E}_L^{\alpha}\mathcal{E}_R^{\alpha *} \\ \mathcal{E}_R^{\alpha}\mathcal{E}_L^{\alpha *} & |\mathcal{E}_R^{\alpha}|^2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1+\sigma^{\alpha} & \sigma_1^{\alpha} - i\sigma_2^{\alpha} \\ \sigma_1^{\alpha}+i\sigma_2^{\alpha} & 1-\sigma^{\alpha} \end{pmatrix}, \qquad (I)$$

Here $\mathcal{E}_{L,R}^{\alpha}(\vec{k}) = \mathcal{E}_x^{\alpha}(\vec{k}) \pm i\,\mathcal{E}_y^{\alpha}(\vec{k})$ $(k_z > 0)$ are the Fourier-components of the electric field of waves, the z-axis is directed along $\vec{H}$; the index α characterizes the type of wave (there exist Alfven, fast and slow magnetosonic waves). Energy densities of a random magnetic field of waves for transverse and longitudinal components with respect to $\vec{H}$ are equal to $M_{\perp}^{\alpha}(\vec{k})$ and $M_{\parallel}^{\alpha}(\vec{k})$, respectively. The interaction between relativistic particles and waves is of resonant character - particles scatter under the condition

$$\omega^{\alpha}(\vec{k}) - k_{\parallel}v_{\parallel} = s\,\omega_H\,, \quad s = 0, \pm 1, \ldots \qquad (2)$$

Taking into account smallness of the phase velocity of waves as compared with the particle velocities $v \approx c$, it is convenient to introduce the resonant value of the wavenumber

$$k_{res} = \frac{\omega_H}{v_{\parallel}} = \frac{e|Z|H}{pc|\mu|} = (r_H \cdot |\mu|)^{-1}. \qquad (3)$$

Here eZ, p, r_H are charge, momentum and gyroradius of a particle, $\mu = p_z/p$.

The effective frequencies of relativistic particle scattering in a turbulent medium are given by the relations (ψ is the azimuthal angle for $\vec{k}_{\perp}$):

$$\nu_{\pm}^{\alpha}(\vec{k},s) = \frac{4\pi^2}{H^2} k_{res}\,\omega_H \Big\{ (1-\delta_{s0}) \cdot \delta(k_{\parallel} - s k_{res}) M_{\perp}^{\alpha}(\vec{k}) \cdot \big[J_{s+1}^2 \cdot (1+\sigma^{\alpha}(\vec{k}) \cdot \mathrm{sign}(Z\mu)) + J_{s-1}^2 \cdot (1-\sigma^{\alpha}(\vec{k}) \cdot \mathrm{sign}(Z\mu)) + 2 J_{s+1} \cdot J_{s-1} \cdot \sigma_1^{\alpha}(\vec{k}) \cdot \cos 2\psi \big] + \delta_{s0} \cdot \delta\Big(k_{\parallel} - k_{\perp}\frac{\sqrt{v_a^2+v_s^2}}{v \cdot |\mu|}\Big) \cdot J_1^2 \, \frac{M_{\parallel}^{\alpha}(\vec{k})}{2} \left(\frac{k_{\parallel}}{k_{\perp}}\right)^2 \Big\}; \qquad (4)$$

$J_s \equiv J_s\left(\frac{k_{\perp}v_{\perp}}{\omega_H}\right)$. is the Bessel function.

Note that out of the three Stokes parameters $\mathcal{Z}$, $\mathcal{Z}_1$, $\mathcal{Z}_2$ equations (4), (6) involve $\mathcal{Z}$ and $\mathcal{Z}_1$, whereas for an axially symmetric wave distribution in the k-space they involve only $\mathcal{Z}$. All the quantities referring to waves are defined in a co-moving reference frame.

In the weak turbulence approximation used here, $\nu_\mu^d \ll \omega_H$. Actually, in interstellar medium for particle energies of I GeV, $\nu_\mu \sim$ IO^{-6} ω_H. The latter inequality means that particles are strongly magnetized and CR diffusion is locally one-dimensional, i.e. it proceeds only along the magnetic field. This fact allows us to pass over to the drift approximation and to average the kinetic equation (including the quasilinear collision integral) over fast particle rotation. The weak inhomogeneity of the regular magnetic field and motion of the medium with a nonrelativistic velocity $\vec{u} = c(\vec{\mathcal{E}}_0 \times \vec{H}) \cdot H^{-2}$ are taken into account in the drift approximation (here $\mathcal{E}_0$ is a regular electric field in a laboratory frame of reference, whereas in a co-moving frame of reference, in the approximation of infinite conductivity an electric field is equal to zero).

In the time interval $\Delta t \gg \nu^{-1}$ and on the scale $\Delta z \gg \ell \sim v\nu^{-1}$ the distribution function is close to isotropic. The isotropic part of the CR distribution function $f(t,\vec{r},p)$ obeys the diffusion equation, which in this case has the form

$$\frac{\partial f}{\partial t} - \nabla_i D_{ij} \nabla_j f + \left(\vec{u} + \frac{1}{3p^2}\frac{\partial}{\partial p}(p^3 \vec{u}_w)\right)\nabla f - \nabla(\vec{u}+\vec{u}_w)\frac{p}{3}\frac{\partial f}{\partial p} - \frac{1}{p^2}\frac{\partial}{\partial p} p^2 D_{pp}\frac{\partial f}{\partial p} = 0. \qquad (5)$$

Here $D_{ij} = D_\parallel h_i h_j$, $\vec{u}_w = u_w \vec{h}$, $\vec{h} = \vec{H}/H$, the velocity u_w, the diffusion coefficients along the field $D_\parallel$ and by the absolute value of the momentum are given by

$$D_\parallel = \frac{v^2}{4}\int_{-1}^{1} d\mu\,(1-\mu^2)\left(\Sigma \nu_\mu^d(\vec{k},s)\right)^{-1}; \quad u_w = \frac{3}{4}\int_{-1}^{1} d\mu\,(1-\mu^2)\left(\Sigma v_f^d \nu_\mu^d\right)\left(\Sigma \nu_\mu^d\right)^{-1}; \qquad (6)$$

$$D_{pp} = p^2 \int_{-1}^{1} d\mu\,\frac{1-\mu^2}{4}\left(\Sigma \nu_\mu^d \left(\frac{v_f^d}{v}\right)^2 - \left(\Sigma \nu_\mu^d\right)\left(\Sigma \frac{v_f}{v}\nu_\mu^d\right)\right); \quad v_f^d(\vec{k}) \equiv \frac{\omega^d(\vec{k})}{k_\parallel}, \ \Sigma \equiv \sum_{d,s,\vec{k}}.$$

The CR diffusion equation in the form (5) was first derived for a simplified collision integral in the paper /4/ (see also /2/).

<u>Diffusion in a medium with a rangom diffusion tensor</u>. The field H has been considered above to be weakly inhomogeneous and regular on the scales of the order of l. In interstellar medium $\delta H^2/H^2 \lesssim$ 0.I for l ~ I pc. Strong field fluctuations $\delta H/H \gtrsim$ I are observed for L ~IOO pc >> l. On this scale the field H should be considered random. There appears the problem of particle diffusion in a medium with a random diffusion tensor.

Assuming the tensor fluctuations $D_{ij}(\vec{r})$ to be weak, one can obtain in the quasilinear approximation the equation for the distribution function $\langle f \rangle$ averaged over the ensemble of fluctuations $\delta D_{ij}(t,\vec{r})$ (we put $u = u_w = 0$):

$$\frac{\partial\langle f\rangle}{\partial t} - \nabla_i \langle D_{ij}\rangle \nabla_j \langle f\rangle + \hat{K}\langle f\rangle = 0, \qquad (7)$$

where $\hat{K}\langle f\rangle = \nabla_i \int \frac{d^3\kappa}{(2\pi)^3} \int_0^\infty d\tau\, U_{ijmn}(\vec{\kappa},\tau)[\kappa_j + i\nabla_j] e^{-\tau\kappa_\alpha\kappa_\beta\langle D_{\alpha\beta}\rangle}\cdot$ (8)

$\cdot [\kappa_m + i\nabla_m]\nabla_n \langle f(t-\tau,\vec{r})\rangle .$

The correlator of the Fourier-components of the tensor δD_{ij} is assumed to be

$$\langle \delta\tilde{D}_{ij}(t,\vec{\kappa}_1)\,\delta\tilde{D}_{mn}(t_0,\vec{\kappa}_2)\rangle = (2\pi)^3\delta^3(\vec{\kappa}_1+\vec{\kappa}_2)\,U_{ijmn}(\vec{\kappa},t-t_0) \quad (9)$$

The integral-differential equation (7) is reduced to the diffusion equation if the correlation time for δD is small: $\tau_c \ll L^2/\langle D\rangle$. In this case the term $\hat{K}\langle f\rangle$ in eq. (7) can be disregarded, and the effective diffusion tensor for $\langle f\rangle$ is equal to $D_{ij\,eff} = \langle D_{ij}\rangle$.

In the case of static fluctuations, more precisely for $\tau_c \gg L^2/\langle D\rangle$, we obtain from (8) in a first approximation

$$D_{in\,eff} = \langle D_{in}\rangle - \int \frac{d^3\kappa}{(2\pi)^3}\, \frac{\kappa_j\kappa_m V_{ijmn}}{\kappa_\alpha\kappa_\beta\langle D_{\alpha\beta}\rangle}, \quad \left(U_{ijmn}(\vec{\kappa},0) = V_{ijmn}(\vec{\kappa})\right). \quad (10)$$

The second term in (10) has the order $(\delta D)^2/D$ and is almost always small for $\delta D \ll D$. But for the case $D_{ij} = D_\parallel h_i h_j$ formula (10) leads to a zero diffusion coefficient across the regular homogeneous field $\langle \vec{H}\rangle$:

$$D_{\perp\,eff} = (\delta_{ij} - \langle h_i\rangle\langle h_j\rangle)\cdot\frac{1}{2}\, D_{ij\,eff} = 0. \quad (11)$$

This result apparently suggests a compound diffusion in the system /5,6/, i.e. an imposition of two independent wanderings: that of a magnetic field line and of a particle moving along this line. The summary transverse displacement of particles is here $r_\perp \propto t^{1/4}$ (under a usual diffusion $r_\perp \propto t^{1/2}$). The compound diffusion is violated if the local diffusion coefficient $D_\perp \neq 0$. The standard perturbation theory (10) does not, evidently, give a correct result for an anomalously small $D_\perp \ll D_\parallel (\delta H/H)^4$. This is just the case with CR in interstellar medium, where $D_\perp \sim r_H^2 v^2/D_\parallel \sim 10^{-12} D_\parallel$, and $\delta H/H \sim 1$. The problem has not yet been strictly solved (see the Discussion in /6-8/). Phenomenologically, diffusion across a regular field occurs due to spreading of random field lines which were initially at a distance r_H from one another /9,10/. If correlation between lines vanishes at a distance S_c, one can use (8) with the correlation function

$$U_{ijmn}(\vec{\kappa},\tau) = V_{ijmn}(\vec{\kappa})\cdot\frac{8}{\pi^2}\cdot\sum_{n=0}^{\infty}(2n+1)^{-2}\exp\left(-(2n+1)^2\,\frac{\pi^2 D_\parallel\cdot|\tau|}{4S_c^2}\right). \quad (12)$$

For a power-law spectrum of random field inhomogeneities $\delta\tilde{H}^2(k) \propto k^{-4+\varepsilon}$, $(\varepsilon > 0)$ the quantity $S_c(k) \approx L(\delta H/H)^{-2}(kL)^{1-\varepsilon}$ /11/, which gives

$$D_{\perp\,eff} \sim 0.2\left(\frac{\delta H}{H}\right)^4\cdot D_\parallel . \quad (13)$$

We obtain a different expression if the spectrum has one main scale L, see /IO, II/.

The estimate (I3) with an account of $\delta H/H \sim I$ leads to the conclusion that on the average the CR transport is evidently realized with a diffusion tensor close to an isotropic one.

REFERENCES

I. Jokipii J.R. Rev.Geophys.Space Phys. I97I, 9, 27.
2. Ginzburg V.L., Ptuskin V.S. Astronomy v. 24 (VINITI ANSSSR), I983, p. 94.
3. Kennel C.F., Engelmann F. Phys.Fl., 1966, 9, 2377.
4. Skilling J. MNRAS, I975, I72, 557.
5. Getmanzev G.G. AZ, I962, 39, 607.
6. Krommes J.A. Suppl. Prog. Theor. Phys. I978, 64, I37.
7. Horton W. in "Principles of Plasma Physics", Energoatomizdat, I984, p. 362.
8. Barge P. et al., Ap.J. I984, 284, 8I7.
9. Ptuskin V.S. Ap.Sp.Sci I979, 6I, 259.
IO.Rechester A.B., Rosenbluth M.N. Phys.Rev.Lett. I978, 40, 38.
II.Skilling J, et al. MNRAS, I974, I67, 87.