

THE COSMIC-RAY SHOCK STRUCTURE PROBLEM  
FOR RELATIVISTIC SHOCKS

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ABSTRACT

The time asymptotic behaviour of a relativistic (parallel) shock wave significantly modified by the diffusive acceleration of cosmic-rays is investigated by means of relativistic hydrodynamical equations for both the cosmic-rays and thermal gas. The form of the shock structure equation and the dispersion relation for both long and short wavelength waves in the system are obtained. The dependence of the shock acceleration efficiency on the upstream fluid speed, long wavelength Mach number and the ratio  $N = P_{CO}/(P_{CO} + P_{GO})$  ( $P_{CO}$  and  $P_{GO}$  are the upstream cosmic-ray and thermal gas pressures respectively) are studied.

1. Introduction. Nonlinear theories of diffusive shock acceleration of cosmic-rays were initially developed by Axford et al. (1977, 1982), Drury and Völk (1981) (see Drury 1983 for a review and further references). These authors discuss how the efficient acceleration of cosmic-rays in shock waves by the first order Fermi mechanism leads to shock broadening or shock structure due to the interaction between the background thermal gas and cosmic-rays. Previous work on this problem has been restricted to non-relativistic shocks. The purpose of the present paper is to extend these models to relativistic shocks in which the background fluid velocity  $V$  is an appreciable fraction of the speed of light.

2. Equations. Consider a one dimensional, steady state inviscid, hydrodynamical model of cosmic-ray acceleration in a (parallel) relativistic shock. The basic conservation laws for mass momentum and energy are:

$$\gamma_n V = J_n \quad (\text{mass conservation}), \quad (1)$$

$$(\gamma^2 - 1)(W_c + W_g) + P_c + P_g + 2\gamma^2 V T_c^{40} = P_n \quad (\text{momentum}), \quad (2)$$

$$\gamma^2 V (W_c + W_g) + (2\gamma^2 - 1) c^2 T_c^{40} = F_n \quad (\text{energy}), \quad (3)$$

where  $J_n$ ,  $P_n$ ,  $F_n$  are the (constant) mass, momentum, and energy fluxes of the system, and

$$W_c = P_c + \rho_c = \gamma_c P_c / (\gamma_c - 1), \quad (4)$$

$$W_g = P_g + \rho_g = \gamma_g P_g / (\gamma_g - 1) + n c^2, \quad (5)$$

are the cosmic-ray and gas enthalpy respectively.  $P_c$ ,  $\rho_c$ ,  $\gamma_c$  are the cosmic-ray pressure, energy density and adiabatic index; whereas  $P_g$ ,  $\rho_g$ ,  $\gamma_g$  are the corresponding gas quantities;  $n$  denotes the gas density,  $T_c^{40}$  represents the cosmic-ray energy flux in the scattering frame (or fluid frame) and  $\gamma = (1 - V^2/c^2)^{-1/2}$  is the Lorentz factor of the fluid speed  $V$ .

Assuming there are no dissipative mechanisms for the gas in the fluid frame, it follows that the thermal gas is compressed adiabatically following the flow (except at gas subshocks) so that  $P_g = P_{g0} (n/n_0)^{\gamma_g}$ , where the subscript 'o' denotes the (constant) upstream state. To complete the system we use the cosmic-ray momentum equation (in the scattering frame):

$$-\nu T_c'^{\prime 0} = \gamma dP_c/dx + (W_c + \alpha c^2 T_c'^{\prime 0}/V) dy/dx + \gamma V dT_c'^{\prime 0}/dx, \quad (6)$$

governing the interaction between the cosmic-rays and gas via the scattering frequency  $\nu$ . (Note the hydrodynamical diffusion coefficient in this development is  $K = c^2(\gamma_c - 1)/\nu$ .)

3. The Shock Structure Equation. The cosmic-ray momentum equation (6) used in conjunction with equations (1) - (5), reduces to the shock structure equation

$$dy/dx = c \nu y^2 (T_c'^{\prime 0} / n_0 c) / \{ \gamma [ (a_g/c)^2 / (\gamma_g - 1) ] V^2 - a_g^2 \} \quad (7)$$

governing the inverse compression ratio  $y = n_0/n = \gamma V/\gamma_0 V_0$  of the flow. In (7),  $T_c'^{\prime 0}$  can be expressed solely in terms of  $y$  (or  $V$ ) using (2) and (3), and  $a_g = (\gamma_g P_g/n)^{1/2}$  denotes the nonrelativistic thermal gas sound speed.

Note in particular the singularity in the denominator in (7) occurs when the fluid speed matches the short wavelength thermal gas sound speed

$$V_s = a_g / [ 1 + (a_g/c)^2 / (\gamma_g - 1) ]^{1/2}. \quad (8)$$

For non-relativistic shocks  $V_s \rightarrow a_g$ ; also note that there is a strict upper limit to the thermal gas sound speed  $V_s$  in the relativistic case with  $V_s < (\gamma_g - 1)^{1/2} c$ . On the other hand, for a near uniform shock transition we have  $T_c'^{\prime 0} \approx 0$  throughout the flow; the solution of  $T_c'^{\prime 0} = 0$  for the upstream fluid speed  $V_0$  then yields the long wavelength sound speed in the system

$$V_{L0} = \{ (a_{c0}^2 + a_{g0}^2) / [ 1 + (a_{g0}/c)^2 / (\gamma_g - 1) + (a_{c0}/c)^2 / (\gamma_c - 1) ] \}^{1/2} \quad (9)$$

where  $a_{c0}^2 = \gamma_c P_{c0}/n_0$ .

A further complication that arises in the relativistic theory (as opposed to the non-relativistic theory) is that  $T_c'^{\prime 0}$  in (7) develops a singularity when the fluid speed  $V \rightarrow \sqrt{\gamma_c - 1} c$  which is the sound speed of the 'effectively massless' cosmic-ray gas (cf. Weinberg, 1972). The occurrence of this singularity is presumably related to the fact that the adiabatic deceleration rate for a fully relativistic cosmic-ray gas is given by

$$\langle \dot{E} \rangle_{ad} / c = \langle \dot{\eta} \rangle_{ad} = -\eta \frac{\gamma^2}{3} \frac{d}{dx} \left( \frac{V}{\gamma^2} \right) = -\eta \frac{\gamma^2}{3} \frac{dV}{dx} \left( 1 - 3 \frac{V^2}{c^2} \right), \quad (10)$$

where  $\eta = \gamma(x) m' c$  and  $p'$  and  $m'$  represent the individual particle momentum and relativistic mass in the scattering frame. The basic implication of (10) is that cosmic-rays are accelerated in compressive flows with  $V < c/\sqrt{3} = \sqrt{\gamma_c - 1} c$  but are decelerated in compressive flows with  $V > c/\sqrt{3}$  (cf. Webb, 1985).

4. Numerical Results. The nature of the shock transition can be analyzed by a geometrical method employed by Drury and Völk (1981) in their work on non-relativistic hydrodynamical models. This is illustrated in Figure 1 for a shock in which the long wavelength Mach number  $M = 2$ ,  $N = P_{c0}/(P_{c0} + P_{g0}) = 0.3$ ,  $z_0 = V_0/c = 0.5$ ,  $\gamma_c = 4/3$  and  $\gamma_g = 5/3$ . It shows the Hugoniot, gas adiabat, short wavelength sonic line and  $P_c = 0$  curve in the  $(P_g, V/c)$  plane. The Hugoniot corresponds to the locus  $T'_c = 0$  in the  $(P_g, V/c)$  plane, whereas the short wavelength sonic line corresponds to the zero of the denominator in (7). The upstream state (A) and downstream state (C) lie on the Hugoniot. In the initial part of the transition the gas is compressed adiabatically (AB) followed by a subshock to a uniform downstream state (the straight line segment BC). Across the subshock the momentum and energy fluxes of the cosmic-rays and thermal gas are balanced separately. Figure 2 shows the variation of the cosmic-ray shock acceleration efficiency  $\eta_s$  as a function of the upstream fluid velocity  $V_0$  for a range of upstream Mach numbers ( $M = 2, 4, 5, 10$ ),  $N = 0.3$ ,  $\gamma_c = 4/3$  and  $\gamma_g = 5/3$  ( $\eta_s$  is defined as the fraction of flow kinetic energy that is converted to cosmic-ray energy in the transition, i.e.,  $\eta_s = -\Delta(\gamma^2 V W_c) / \Delta(\gamma \gamma_0 n_0 c^2)$ ). For  $z_0 < \sqrt{\gamma_c - 1} = 0.5773$ , there is a weak dependence of  $\eta_s$  on  $z_0$ , and  $\eta_s$  increases monotonically with  $M$ . At  $0.577 < z_0 < 0.9$ , there are no solutions and the solutions with  $z_0 > 0.9$  are of low efficiency. The negative efficiency solutions at  $z_0 > 0.9$  are obtained in expanding flows ( $V$  increases initially) followed by a subshock to a downstream state  $z_2 = V_2/c < \sqrt{\gamma_c - 1}$ . In compressive flows with  $V_0 > c(\gamma_c - 1)^{1/2} = 0.577c$ , the cosmic-ray pressure decreases as the thermal gas is compressed adiabatically, and the cosmic-ray pressure may eventually become negative; it is then necessary to insert a subshock in the flow while  $P_c > 0$ ; this behaviour is in accord with the result (10). Figure 3 shows the variation of the downstream cosmic-ray pressure  $P_{c2}$  as a function of the downstream cosmic-ray adiabatic index  $\gamma_{c2}$  ( $\gamma_{c0} = 1.55$ ,  $\gamma_g = 5/3$ ,  $M = 5$ ,  $z_0 = 0.5$ ) for a range of  $N$ . The dotted curves correspond to smooth transition solutions; whereas solutions with subshocks are given by the full (solid) curves (cf. Achterberg et al. 1984; Heavens 1984 for the non-relativistic case).

5. Conclusions. The main point to emerge from the present study is that to efficiently accelerate cosmic-rays in relativistic shocks, the upstream fluid speed  $V_0$  needs to be less than  $V_{cr} = \sqrt{\gamma_c - 1} c$ ; compressive flows with  $V > V_{cr}$  lead to a loss of internal energy to the C.R. gas.

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#### 7. References.

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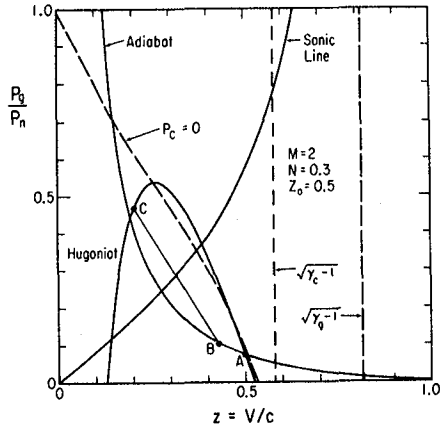


Fig. 1

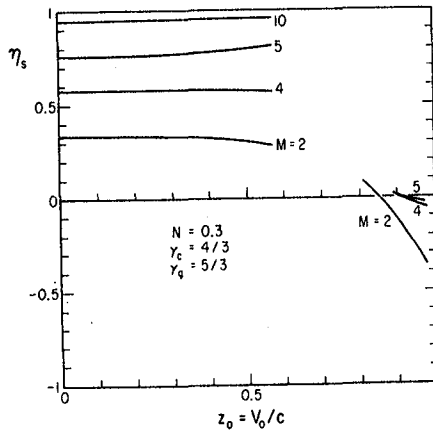


Fig. 2

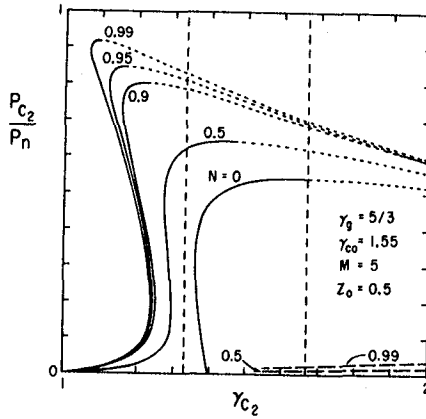


Fig. 3