

A Cosmic Ray driven Instability

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1 Introduction

The interaction between energetic charged particles and thermal plasma which forms the basis of diffusive shock acceleration leads also to interesting dynamical phenomena. For a compressional mode propagating in a system with homogeneous energetic particle pressure it is well known that 'friction' with the energetic particles leads to damping. The linear theory of this effect has been analyzed in detail by Ptuskin*. Not so obvious is that a non-uniform energetic particle pressure can in addition amplify compressional disturbances. If the pressure gradient is sufficiently steep this growth can dominate the frictional damping and lead to an instability. It is important to note that this effect results from the collective nature of the interaction between the energetic particles and the gas and is not connected with the Parker instability, nor with the resonant amplification of Alfvén waves.

2 Physical description

In diffusing through the thermal plasma the energetic particles produce a reaction force, $-\nabla P_c$, where P_c is the energetic particle pressure. This interaction is mediated by irregularities in the magnetic field, but if their mean motion relative to the thermal plasma is small enough to be ignored the energetic particles can be thought of as exchanging momentum directly with the gas. Consider a small compressional disturbance (which in the absence of energetic particle effects would be simply a sound wave or a magneto-acoustic wave) propagating parallel to the pressure gradient of the energetic particles. For short wavelength disturbances the diffusion timescale of the energetic particles (which decreases quadratically with wavelength) will be much smaller than the dynamical timescale of the disturbance (which decreases linearly) so that the solution of the diffusion equation will approximate a steady solution with constant flux, $\kappa \nabla P_c = \text{const.}$, where κ is the diffusion coefficient. Thus if ρ is the gas density the acceleration resulting from the reaction of the energetic particles is $-\nabla P_c / \rho \propto 1 / \rho \kappa$. Clearly small scale density fluctuations will induce acceleration fluctuations (and hence velocity fluctuations which can amplify the density fluctuations) unless $\kappa \propto 1 / \rho$.

If the energetic particles were scattered directly by the molecules and ions of the thermal plasma one would indeed expect the mean free path, and hence the diffusion coefficient, to be inversely proportional to the gas density. However there is no reason why the effective diffusion coefficient resulting from collective processes should be inversely proportional to the gas density and in general it will not be. The

* V. S. Ptuskin 1981, *Astrophys. Space Sci.* 76 265-78.

instability then results from the fact that the energetic particles couple more strongly to regions of higher (or lower) density and push these through the undisturbed gas. This causes the density to increase in the front of the disturbance and decrease at the back so that the original density fluctuation is amplified. The fluctuations in the gas pressure exert restoring forces which oppose this effect; thus we expect the instability to appear most strongly when the gas is cold.

It is not easy to determine the density dependence of the diffusion coefficient, however if the scattering is produced mainly by Alfvén waves of moderate amplitude, quasilinear theory gives

$$\kappa \approx \frac{1}{3} r_g v \left(\frac{B}{\delta B} \right)^2 \cos^2 \theta$$

where r_g is the particle gyroradius, v the particle speed, B the magnetic field, δB the amplitude of the field fluctuations and θ the angle between field and gradient. If we compress a small region (but one which is still large compared to the gyroradii of the energetic particles and the wavelengths of the scattering waves) parallel to the gradient θ increases (so that $\cos \theta$ decreases), $B \cos \theta$ is constant, $r_g \propto 1/B$ and $\delta B^2 \propto \rho^{3/2}$ so that for this case $\kappa \propto \rho^{-3/2} \cos \theta$ and decreases at least as fast as $\rho^{-1.5}$. We have ignored the problems associated with wave saturation and cross-field diffusion, however the Alfvén wave excitation through the resonant streaming instability, $v_A \cos \theta \nabla P_c \propto B \rho$ where v_A is the Alfvén speed, is also greater in compressed regions. Thus we expect that the energetic particles will couple more strongly to the compressed regions because the magnetic field, in particular the transverse component, will be larger there and the wave activity will be enhanced.

3 Instability criterion

The ideas outlined above can be put on a firmer footing if we take the equations of ideal energetic particle hydrodynamics,

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial(\rho U)}{\partial x} &= 0, \\ \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} &= -\frac{1}{\rho} \frac{\partial(P_g + P_c)}{\partial x}, \\ \frac{\partial P_g}{\partial t} + U \frac{\partial P_g}{\partial x} + \gamma_g P_g \frac{\partial U}{\partial x} &= 0, \\ \frac{\partial P_c}{\partial t} + U \frac{\partial P_c}{\partial x} + \gamma_c P_c \frac{\partial U}{\partial x} &= \frac{\partial}{\partial x} \left(\kappa \frac{\partial P_c}{\partial x} \right) \end{aligned}$$

(U is the velocity, P_g the gas pressure, γ_g and γ_c the adiabatic exponents of the gas and energetic particles), linearize them and perform a two-scale expansion, *i.e.* look at perturbations of short wavelength and small amplitude on a smoothly varying background. To lowest order the perturbations, denoted by a prefix δ , are simply sound waves with frequency ω and wave number k ,

$$\delta u = \pm a_g \frac{\delta \rho}{\rho}, \quad \delta P_c = 0, \quad (\omega - kU)^2 = a_g^2 k^2,$$

where $a_g = \sqrt{\gamma_g P_g / \rho}$ is the gas sound speed. To this order and in the short wavelength limit the energetic particles are decoupled from the perturbations in the gas.

The next order is more interesting and gives an equation for the wave amplitude in terms of the wave action density, the wave energy density divided by the Doppler shifted frequency, $A = \rho \delta u^2 / (\omega - k u)$;

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x} [A(U \pm a_g)] = A \left[-\frac{\gamma_c P_c}{\rho \kappa} \pm \frac{1}{\rho a_g} \frac{\partial P_c}{\partial x} \left(1 + \frac{\rho \delta \kappa}{\kappa \delta \rho} \right) \right]$$

In the absence of energetic particle effects the RHS of this equation would be zero and we would have conservation of wave action. The first term on the right is proportional to the energetic particle pressure and represents pure Ptuskin damping. The second term shows the destabilizing effect of an energetic particle pressure gradient; unless $\delta(\rho \kappa) = 0$ waves going in one direction will be amplified.

The condition for the instability to occur is clearly that the second term dominates the first,

$$\left| \frac{1}{\rho a_g} \frac{\partial P_c}{\partial x} \left(1 + \frac{\rho \delta \kappa}{\kappa \delta \rho} \right) \right| > \frac{\gamma_c P_c}{\rho \kappa}$$

or, if $\kappa \propto \rho^\beta$, that the length scale of the energetic particle pressure be less than a critical length scale,

$$\frac{P_c}{|\nabla P_c|} < \frac{|1 + \beta| \kappa}{\gamma_c a_g}.$$

As indicated by the physical discussion the instability occurs when a steep energetic particle gradient is established in a cold gas.

4 Implications

This instability has important consequences for the structure of shocks modified by particle acceleration. The length scale associated with the increase of particle pressure in the shock precursor is κ/U where U is the shock speed whereas the critical length scale is of order κ/a_g ; thus we expect the precursor to be unstable for all strong modified shocks. The ratio of the advection time through the precursor to the instability growth time is of order the Mach number of the shock so that any slight density fluctuations in the upstream medium should be strongly amplified in the precursor region.

This probably means that the small-scale structure of the shock is stochastic and irregular. The strongly amplified disturbances we expect to form shocks which contribute to the gas heating and may assist in injecting particles into the diffusive acceleration process. An interesting side effect is that the effective diffusion coefficient in the shock precursor region will be reduced; this will shorten the acceleration time scales and may allow higher particle energies to be reached.

The instability may also be important in determining the structure of the galactic halo and of cooling flows.