EFFICIENCY OF REGULAR ACCELERATION OF PARTICLES BY A SHOCK WAVE AT DIFFERENT INJECTION REGIMES

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1. Introduction. A significant fraction of the inflowing plasma energy in the collisionless shock vicinity might be transferred to the particles accelerated by a regular acceleration mechanism [1,2], [3-6]. The accelerated particles back pressure modifies an infinite planar shock structure so that a profile of the plasma flow velocity $u(x)$ in the shock frame has two characteristic length scales as it is seen in Fig.1.

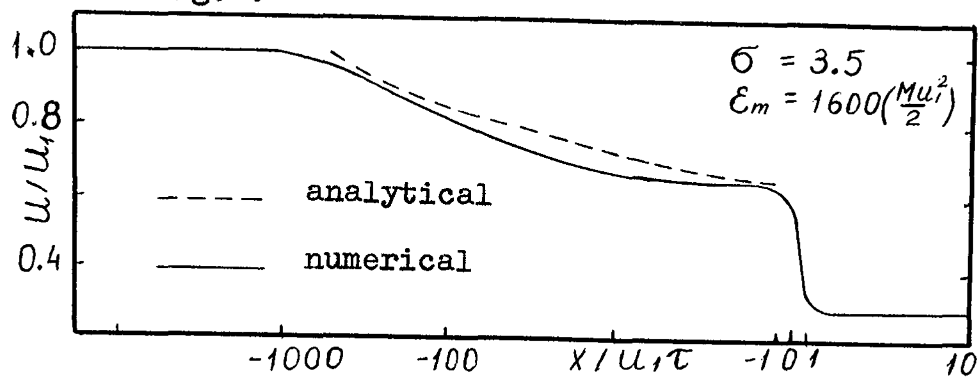


Fig.1.

The smooth velocity transition $u_1 \leqslant u \leqslant u_s$ within the scale $L \simeq \varkappa(\mathcal{E}_m)/u_1$ (the precursor) is due to the accelerated particles pressure; the abrupt decrease from u_s to u_2 within the scale $\ell \simeq \lambda_T$ (the subshock) is caused by the thermal particles. Here λ_T is the mean free path of the thermal particles, $\varkappa$ - the energetic particles diffusion coefficient, $\mathcal{E}_m$ - the maximum (or cutoff) particle energy.

The parameter $\beta = (u_1 - u_s)/u_1$ is introduced to express quantitatively the modification level. If the magnetic field dynamics within the precursor is neglected, β might be related to the fractional pressure P_c of the accelerated particles at the subshock. For nonrelativistic particles $P_c = \beta\rho_1 u_1^2 = P_2 \beta\sigma/(\sigma-1)$, where ρ_1 is the upstream $(x=-\infty)$ plasma mass density, σ -the total compression ratio, P_2 -the post-shock plasma pressure. The interrelation between $P_c, \beta, \mathcal{E}_m$ and the rate of the thermal particles injection to the acceleration process is an important but poorly investigated question.

2. Method and Results. It follows from the transport equation for the accelerated particles under certain approximations [3,4] that within the precursor

$$\ln\frac{\mathcal{E}_m}{\mathcal{E}_s} = \int_{u_s}^{u_1}\frac{du}{u_1}\left(\frac{\rho_1 u_1^2}{P}\right), \tag{1}$$

where the partial pressure $p = dP/d\ln\mathcal{E}$ is related to the plasma flow velocity by

$$\frac{p}{\rho_1 u_1^2} = \frac{u-u_2}{u_1} + \frac{3}{2\sigma}\ln\frac{u-u_2}{u_1} + C\ . \qquad (2)$$

The constant C should be determined by matching $p(\mathcal{E})$ to the thermal spectrum at some energy $\mathcal{E}_s$, which separates thermal particles from the accelerated ones. Assume the injection energy $\mathcal{E}_s$ be equal to $Mu_1^2/2$ (M – the particle mass). Assume also that the accelerated particles with the mean free path larger than ℓ do not interact with the subshock. In such a case p_{inj} should be a function of the subshock compression ratio $\sigma_s = u_s/u_2$ only. Since determination of the injection function $p_{inj}(u_s/u_2)$ is a complicated problem itself, we assumed it to be constant.

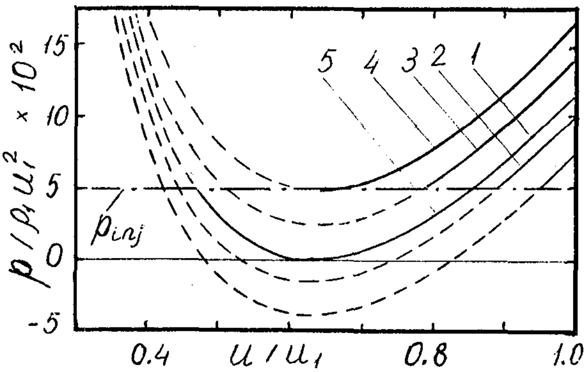


Fig.2.

1 - $\mathcal{E}_m = 2.5\,\mathcal{E}_s$

2 - $\mathcal{E}_m = 4.8\,\mathcal{E}_s$

3 - $\mathcal{E}_m = 15.0\,\mathcal{E}_s$

4 - $\mathcal{E}_m = 178.0\,\mathcal{E}_s$

5 - $\mathcal{E}_m = \infty$

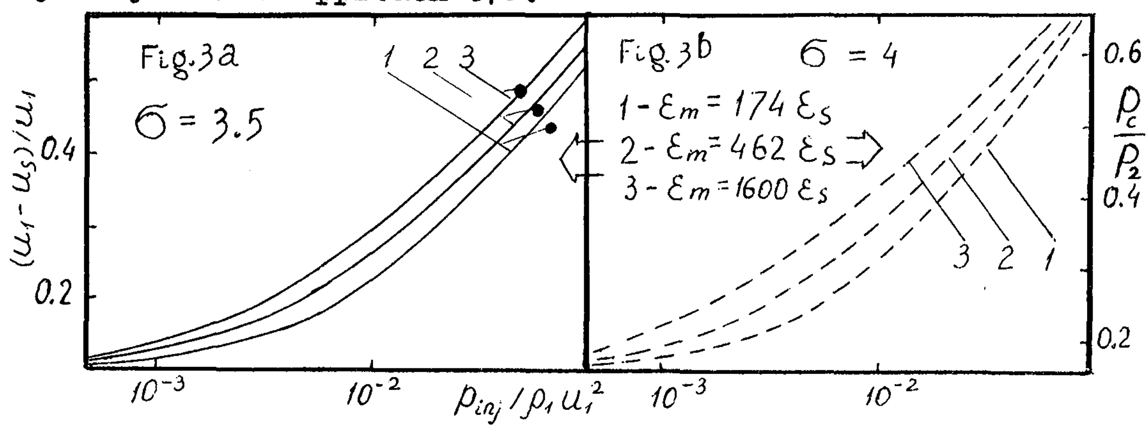
Fig.2 represents a family of curves $p(u)$ vs. u for a set of parameters C corresponding to different u_s and $\mathcal{E}_m$. Solid lines indicate a physically reasonable range $u_s \leq u \leq u_1$ at $p_{inj} = 5\cdot10^{-2}\rho_1 u_1^2$ for $\sigma = 4$. It follows from (1) and (2) that if $u_s < \frac{5}{2\sigma}u_1$

$$u_s \simeq \left[\frac{5}{2\sigma} - \sqrt{\frac{3p_{inj}}{\sigma\rho_1 u_1^2} - \left(\frac{3\pi}{\sigma\ln(\mathcal{E}_m/\mathcal{E}_s)}\right)^2}\,\right]u_1\ . \qquad (3)$$

If $u_s > \frac{5}{2\sigma}u_1$ (curves 1-3) this relationship is :

$$u_s \simeq \left[1 - \frac{2(\sigma-1)}{2\sigma-5}\,\frac{p_{inj}}{\rho_1 u_1^2}\left(\left(\frac{\mathcal{E}_m}{\mathcal{E}_s}\right)^{\frac{2\sigma-5}{2(\sigma-1)}} - 1\right)\right]u_1\ . \qquad (4)$$

It is seen from this expression that the modification level is limited by $\beta_{max} = \frac{2\sigma-5}{2\sigma} + \sqrt{\frac{3p_{inj}}{\sigma\rho_1 u_1^2}}$ corresponding to the highest attainable pressure of the accelerated particles $P_c/P_2 = \frac{2\sigma-5}{2(\sigma-1)} + \frac{\sigma}{\sigma-1}\sqrt{\frac{3p_{inj}}{\sigma\rho_1 u_1^2}}$ which is reached at $\mathcal{E}_m = \infty$ (curve 5). The above conclusion differs from that one derived in the hydrodynamical approach [7].



The existence of the significant dependence of the modification level and the acceleration efficiency on the injection rate is illustrated by Fig.3 representing β and P_c vs. ρ_{inj} at different $\mathcal{E}_m$ for $\sigma=3.5$ (solid lines) and $\sigma=4$ (dashed lines). Circles are results of the numerical shock structure simulation via the Boltzmann equation[6] with $\sigma=3.5$ and $\mathcal{E}_m/\mathcal{E}_s$ = 174, 462, 1600.

After the constant C in (1) has been determined for $\rho_{inj} = 0.05\rho_1 u_1^2$ that corresponds to a numerical simulation, the equation (1), with $\mathcal{E}_s$ and u_s replaced with $\mathcal{E}$ and u respectively, yields u as a function of $\mathcal{E}$ which, in turn, is related to x by $x = \varkappa(\mathcal{E})/u(\mathcal{E})$ and $\varkappa(\mathcal{E}) = 2\mathcal{E}\tau/3M$ (τ =const). The analytical and numerical solutions for velocity profiles at $\sigma=3.5$ and $\mathcal{E}_m$ = 1600 $\mathcal{E}_s$ are compared in Fig.1.

The results of the analytical and numerical solutions represented in Figs.1,3 are in a reasonable agreement confirming the adequacy of the approximations used while deriving (1) and (2). The analogous conclusion was drawn in [8] published when the present paper was in preparation.

3. Summary. Thus, the above analysis leads to the following conclusions:

The acceleration efficiency, the modification level and the precursor structure are well fitted by the analytical approximation [3,4], provided that the injection function $\rho_{inj}(u_s/u_2)$ is known. The modification level and the acceleration efficiency are in the considerable dependence on ρ_{inj}. The fractional energy of the accelerated particles (i.e. the acceleration efficiency) and the shock modification level increase as the cutoff energy $\mathcal{E}_m$ increases. However, for an arbitrarily large Mach numbers there exist the finite limiting values of the acceleration efficiency and modification level corresponding to $\mathcal{E}_m \to \infty$. When taken into account the energy flux, carried away by particles escaping through q might slightly modify the above results. But conclusion on the injection rate role should remain valid. Thus, the investigation of the thermalization process of the particles at the shock seems to be the important problem for the shock acceleration theory.

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