A NUMERICAL STUDY OF DIFFUSIVE SHOCK ACCELERATION OF COSMIC RAYS IN SUPERNOVA SHOCKS

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Abstract. The evolution of the energy spectrum of cosmic rays accelerated by the first order Fermi mechanism, by a supernova remnant shock wave, including adiabatic deceleration effects behind the front, is carried out by means of a time-dependent numerical code. The calculations apply to the adiabatic stage (or Sedov stage) of the supernova explosion, and the energetic particle spectrum is calculated in the test particle limit (i.e., the back reaction of the cosmic rays on the flow is not included). The particles are injected mono-energetically at the shock. We show the radial distribution, and the spectrum of the accelerated and decelerated particles.

I. Introduction. Diffusive acceleration of cosmic rays at collisionless astrophysical shock waves, where the particles pass (diffuse) across a plane shock repeatedly is a promising candidate for the origin of cosmic rays (Axford, Leer and Skadron, 1977; Krymskii, 1977; Bell, 1978; Blandford and Ostriker, 1978).

Supernova explosions have long been supposed to be the origin of cosmic rays, primarily because of the energy they liberate. The diffusive shock acceleration mechanism has been applied to spherically symmetric supernova shocks (Krymskii and Petukhov, 1980, Prishchep and Ptuskin, 1981; Drury, 1983) and analytical solutions can be obtained with the assumption that $K/R\dot{R}$ is a constant (see also Blandford and Ostriker, 1980 Bogdan and Völk, 1983; Moraal and Axford, 1983).

In this paper, we solve the time dependent problem of cosmic rays diffusively accelerated in a spherical shock by numerical integration.

II. The Model. Under the assumption that there is sufficient scattering that the pitch-angle distribution is nearly isotropic (diffusion limit) the cosmic ray transport equation (Parker, 1965) may be written (for spherical symmetry) as:

$$\frac{\partial f}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(Kr^2\frac{\partial f}{\partial r}\right) - U\frac{\partial f}{\partial r} + \frac{1}{3}\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2U)\right)p\frac{\partial f}{\partial p} + S \quad , \qquad (1)$$

where $f(t,\vec{r},\vec{p})$ is the phase space distribution function, U is the flow velocity, and S is the source.

If there is a shock at radius R, we require f to be continuous at the shock (Toptygin, 1980), and the jump in streaming flux can be obtained by integrating (1) across the shock. Subscript 1 corresponds to the region outside the shock (upstream region), while subscript 2 corresponds to the region inside the shock (downstream region), the boundary conditions at the shock are:

$$f_1 = f_2 \quad \text{and} \quad \left[K\frac{\partial f}{\partial r} + \frac{Up}{3}\frac{\partial f}{\partial p}\right]_2^1 = -\lim_{\epsilon\downarrow 0}\int_{R-\epsilon}^{R+\epsilon} S\,dr \quad . \qquad (2)$$

III Supernova Shock If the shock front is moving, i.e., R = R(t), then it is useful to use the following dimensionless quantities:

$$\xi = r/R(t) \quad , \quad \tau = K_0 t/R^2(t) \quad \text{and} \quad q = \ln(P/P_0) \quad , \qquad (3)$$

where K_0 and P_0 are constants to be defined later. This transformation breaks down if $2t\dot{R}/R = 1$ because the Jacobian of the transformation is then zero. Note that if K is constant and so is $R\dot{R}/K$, then $R \propto t^{1/2}$.

Assume a velocity profile $U = \dot{R}V(\xi)\theta(1-\xi)$ (e.g. Sedov's blast wave; Sedov, 1959) and consider particle injection at the shock, i.e.,

$$S = Q\delta(r-R)\delta(p-p_0) = \frac{Q}{Rp_0}\delta(\xi-1)\delta(q) \quad , \tag{4}$$

where the units of Q are #/sec/length^2/momentum^2. If $V(\xi)$ goes to zero faster than ξ^k, $k > 1$ (which is true for Sedov's blast wave approximation), then $V(0)=0$ and $\partial(\xi^2 V)/\partial\xi|_0 = 0$ (i.e., the boundary condition at $\xi=0$ is $\partial f/\partial\xi = 0$). If we assume self-similar solutions to the shock, $R = At^\alpha$, then by equation (3) we have:

$$\tau = \frac{K_0}{A^2} t^{(1-2\alpha)} \quad , \quad R = A\left(\frac{A^2\tau}{K_0}\right)^{\alpha/(1-2\alpha)} \text{ and } R_\tau/R = \frac{1}{\tau}\left(\frac{\alpha}{1-2\alpha}\right) . \tag{5}$$

Substituting (3), (4) and (5), together with $K = K_2 e^{a_2 q} - (K_2 e^{a_2 q} - K_1 e^{a_1 q})\theta(\xi-1)$ into (1) and (2), the equations for the transport of cosmic rays in a moving spherical shock can be obtained. In order to have a forward in time equation, α has to be smaller than 1/2 (e.g. Sedov's blast wave, $\alpha = 2/5$).

IV. Results and Discussion. We employ a finite differencing scheme implicit in the spatial variable but explicit in the momentum variable; it is also second order in spatial variable and first order in the momentum variable. To satisfy causality, we use downstream differencing for momentum except at the shock, where we use upstream differencing. We replace $\delta(q)$ by a Gaussian with a spread of the order of the step size in q during computation.

In the case of a moving spherical shock, we use the supernova shock as an example. We concentrate on the adiabatic stage (or Sedov's stage) of a supernova explosion only.

The self-similar solution (Sedov, 1959) to the blast wave equations is $R = At^\alpha$ where $A = (E/\rho_1)^{1/5}$ and $\alpha = 2/5$. The energy, E, is proportional to the explosion energy, E_0. If $\gamma = 5/3$, then $E = 2.02\,E_0$. ρ_1 is the density of the undisturbed (i.e. upstream) gas. The velocity $V(\xi)$ is given by $V(\xi) = 5/2\,\xi\, v(\xi)$ where $v(\xi)$ is a complicated but monotonically increasing function and $0.24 \leq v(\xi) \leq 0.3$.

The adiabatic stage of a supernova remnant begins roughly when the mass of interstellar gas swept by the shock is equal to the expelled mass of the supernova (e.g. Spitzer, 1978). At the beginning of this stage the shock radius is:

$$R_i = \left(\frac{3\beta M_\odot}{4\pi\rho_1}\right)^{1/3} \tag{6}$$

where β is the expelled mass in solar mass units ($M_\odot$). Substituting (4) and $R_i = At_i^\alpha$ into (6), we get:

$$\tau_i = K_0\left(\frac{\rho_1}{E}\right)^{1/2}\left(\frac{3\beta M_\odot}{4\pi\rho_1}\right)^{1/6} \tag{7}$$

The adiabatic stage will last roughly until the temperature behind the shock drops below the recombination temperature (i.e. the shock becomes radiative) (e.g. Spitzer, 1978). Let subscript f represent this. The relation between upstream and downstream temperature for a strong shock is (e.g. Landau and Lifshitz, 1959):

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2}\mathcal{M}^2 = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2}\frac{\rho_1\dot{R}^2}{\gamma P_1} . \tag{8}$$

Assume an ideal gas law in the undisturbed (upstream) region $P_1/\rho_1 = kT_1/\mu$ where μ is mean molecular weight. This, together with $\dot{R}_f = \alpha\, t_f^{(\alpha-1)}$ and $T_2 = T_r$ (the recombination temperature), gives:

$$\tau_f = K_0\left(\frac{\rho_1}{E}\right)^{1/3}\left(\frac{8}{25}\frac{(\gamma-1)}{(\gamma+1)^2}\frac{\mu}{kT_r}\right)^{1/6} \tag{9}$$

Both τ_i and τ_f of the adiabatic stage are proportional to $K_0\rho_1^{1/3}$. We have studied cases with $\gamma = 5/3$ (which gives $E = 2.02\,E_0$(Sedov, 1959)) and the ratio of the density of helium to that of hydrogen, ρ_{He}/ρ_H, $= 0.4$.

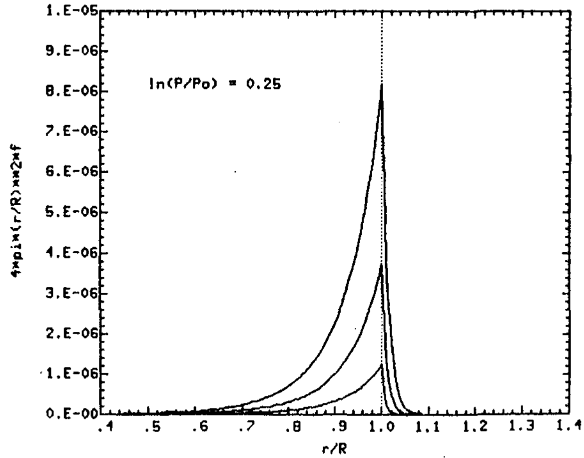

Figure 1. Spatial distribution of particles accelerated by a moving spherical shock. Particles are injected at the shock.

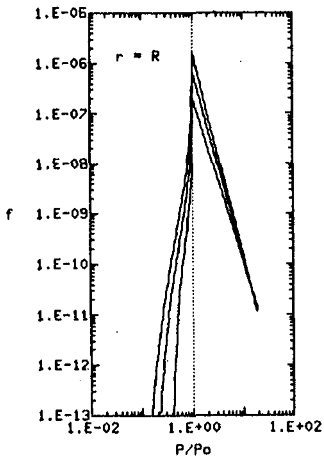

Figure 2. Spectrum of particles accelerated by a moving spherical shock. $\dot{R}R/K$ is large.

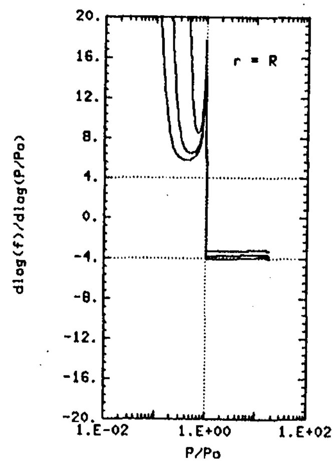

Figure 3. The spectral index with small τ_i and τ_f.

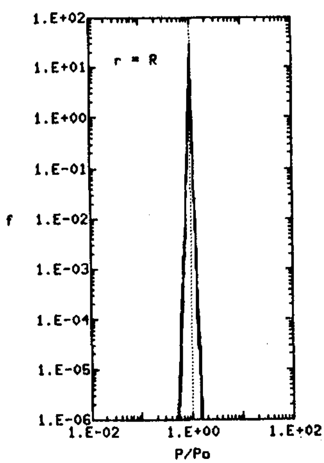

Figure 4. Spectrum of particles accelerated by a moving spherical shock. $\dot{R}R/K$ is small.

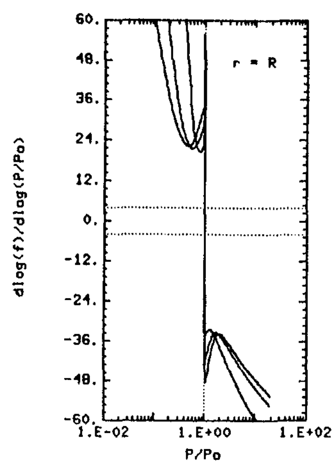

Figure 5. The spectral index with large τ_i and τ_f.

In Figure 1 to 3 we used $E_0 = 3 \times 10^{50}$ erg, $\beta = 0.3$, $K_1 = K_2 = K_0 = 10^{26}$ $cm^2\ sec^{-1}$, $a_1 = a_2 = 0$, $Tr = 10^{26}$ K (the recombination temperature of proton and electron) and the number density of hydrogen $n_H = 3 \times 10^{-3}$ cm^{-3} (hot interstellar medium); i.e., $\tau_i = 1.78 \times 10^{-3}$ and $\tau_f = 5.56 \times 10^{-3}$. An absorbing boundary is set up at a radius 2R.

By studying Figure 1, we see that only a very small portion of particle (high or low energies) stays outside the shock, the absorbing boundary can be considered as being at infinity where the particle distribution, f, is supposed to be zero. This can be understood by the fact that there is no convection outside the shock and the diffusion time scale (R^2/K_0) is much larger than the accelerating time scale (K_0/U^2); and the adiabatic stage ends before the steady state is attained.

The low energy spectrum is very steep and process a cut-off (see Figures 2 and 3), which indicates the adiabatic deceleration is not very effective. The spectral index for accelerated particles ($p > p_0$) starts from values larger than -4 and then relaxes to pass -4 (see Figure 3).

In Figures 4 to 5 we used $E_0 = 3 \times 10^{50}$ erg and $\beta = 0.3$, $K_1 = K_2 = K_0 = 10^{28}$ $cm^2 sec^{-1}$, $a_1 = a_2 = 0$, and $T_r = 10^6$ K, $n_H = 0.1 cm^{-3}$; i.e. $\tau_i = 0.572$ and $\tau_f = 1.79$. The absorbing boundary is at a radius 5R. The spectrum (see Figures 4 to 5) is very steep due to the large curvature of the shock and the escape of particles out of the system.

We have studied the case of momentum dependent diffusion coefficient. The basic features are the same; some detailed changes are consistent with results of a steady state plane shock.

V. Conclusion. The results discussed above may be understood as a consequence of the length scale or time scale involved (e.g. Prishchep and Ptuskin, 1980). There are three time scales involved: diffusion time scale R^2/K, convection time scale $R/\dot{R}$ and acceleration time scale $K/\dot{R}^2$. If the radius of curvature parameter $R\dot{R}/K$ is large, then the acceleration is very efficient and the spectrum will approach that from plane shock and vice versa.

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