

ON MAGNETIC PAIR PRODUCTION
ABOVE FAST PULSAR POLAR CAPS

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ABSTRACT

Magnetic pair production is one of high-energy electromagnetic conversion processes important to the development of pair-photon cascades in pulsars. On the basis of current polar cap models, this paper is concerned with the properties of magnetic pair production in fast pulsars. Suppose there is a roughly dipole magnetic field at the stellar surface, the author estimates the effects of non-zero curvature of magnetic field lines upon curvature radiation from primary particles and pair production rate near the surface of pulsars.

INTRODUCTION

In current pulsar theories^{1,2}, primary particles accelerated to extreme relativistic energies by the electric field component along the magnetic field near the polar cap emit curvature radiation, the photons of which, with sufficient energies, will convert into electron-positron pairs, and the processes will further develop into pair-photon cascade. The secondary pairs resulting from the cascade (Lorentz factor $\gamma \sim 10^2 - 10^3$) then produce radio or optical emission via a coherent process, and the surviving curvature photons constitute the observable high energy radiation from pulsars. It is believed that magnetic pair production is a very important attenuation mechanism governing the hard γ -ray emission from pulsars. The main aim of this paper is to estimate the effect of non-zero curvature of magnetic field lines in fast pulsars on the magnetic pair production and the curvature radiation from primary particles.

CURVATURE RADIATION AND MAGNETIC PAIR PRODUCTION

In the calculation concerned with pair-photon cascade above polar

cap, it is usually thought that primary particles move along curved magnetic field lines with vanishing pitch angle, and that most of the curvature radiation from primaries is emitted into a forward cone of half-angle $\sim 1/\gamma$, the power of which, according to the current estimate, is

$$P_{\text{curv}} = 2\gamma^4 e^2 c / 3R_{\text{mc}}^2, \quad (1)$$

where R_{mc} denotes the curvature radius of field lines. Theoretically, a high-energy photon can be absorbed in an external magnetic field to create e^\pm pair only if the following kinematic condition holds, i.e.,³

$$\hbar\omega \sin\theta \geq 2mc^2, \quad (2)$$

where $\hbar\omega$ is the photon energy, and θ the angle at which the photon propagates to the magnetic field. According to this view, at the moment when the photon is just emitted by a primary particle the threshold condition (2) will not be satisfied because $\theta=0$. Thus, the pair can not be produced until θ grows to exceed the threshold condition with the propagating of the photon in the curved and rotating dipole magnetic field.

This point of view, however, would be partially corrected, if the effects of non-zero curvature of magnetic field lines on motion of charged particles are taken into account. Within the framework of the classical theory the associated calculation (see another conference paper by the author, OG 6.2-11) shows that the primary particle with a longitudinal initial velocity parallel to a field line possesses in fact non-zero pitch angle ψ for which one can approximately write

$$\psi \approx \beta c / \omega_b R_{\text{mc}} \approx 1.7 \times 10^3 \gamma / B_p R_{\text{mc}} \ll 1, \quad (3)$$

where $\beta = v/c$, $\omega_b = eB_p/\gamma mc$ is the relativistic cyclotron frequency, and B_p the magnetic field intensity at the pole of pulsar. The curvature radius of the particle orbit R_{oc} is then

$$R_{\text{oc}} \approx R_{\text{mc}} / 2 |\sin(\omega_b t / 2)|, \quad (4)$$

which differs from the curvature radius of the field line R_{mc} . This implies that the radiation from primaries is of the synchro-curvature radiation, whose power (averaged over a cyclotron period) can be easily found with a direct calculation:

$$\bar{P}_{\text{synch-curv}} = \frac{1}{2\pi} \int_0^{2\pi} \frac{2}{3} \gamma^4 (e^2 c / R_{\text{oc}}^2) d(\omega_b t) \approx \frac{4}{3} \gamma^4 (e^2 c / R_{\text{mc}}^2), \quad (5)$$

which is just twice that of the net curvature radiation. Bearing in mind that the power of synchrotron radiation, arising from the helical motion of a relativistic particle with the pitch angle ψ and Lorentz factor γ in a constant uniform magnetic field, may be put into the form⁴

$$P_{\text{synch}} = \frac{2}{3}(e^4/m^2c^3)(\gamma^2-1)B_p^2 \sin^2\psi = \frac{2}{3}\gamma^4(e^2c/R_{\text{mc}}^2)$$

which is formally identical with (1), it is immediately evident that in the case represented by (3) both the synchrotron radiation and the curvature radiation would make roughly the same contributions to the total power. A direct consequence is that one can establish a quantitative criterion for estimating the components of synchro-curvature radiation from a relativistic electron with various possible initial conditions, that is, the curvature radiation becomes significant only when the pitch angle of the electron does not increase beyond the value given by (3), otherwise the synchrotron radiation will be dominant.

On the other hand, it is possible that at the moment when the photon is just emitted by the primary the threshold condition (2) with $\theta=\theta_0=\psi$ might be satisfied due to $\psi \neq 0$, so the pair could be created. In particular, in the cases where $\theta_0=\psi \gg 1/\gamma$, the curvature photon could nearly satisfy even a much more severe condition imposed by dynamics of magnetic pair production,^{3,5} i.e.,

$$\hbar\omega \sin\theta \gtrsim 0.2(B_{\text{cr}}/B_p)mc^2, \quad (6)$$

where

$$B_{\text{cr}} = m^2c^3/e\hbar \doteq 4.414 \times 10^{13} \text{ G.}$$

Under this condition one would anticipate for a considerable pair conversion rate.

IN APPLICATION TO FAST PULSARS

Fast pulsars possess usually shorter period, and sometimes weaker magnetic field. Most spectacular and extreme is the millisecond pulsars such as PSR 1937+214 for which the period $P=1.558\text{ms}$ and field intensity $B_p \doteq 3 \times 10^8 \text{ G.}$ ⁶ Suppose there is a dipole magnetic field at the stellar surface, and take the stellar radius $R \doteq 10^6 \text{ cm}$, one readily finds the curvature radius of field lines in the polar cap

$$R_{\text{mc}} \doteq (CRP/2\pi)^{1/2} = 2.7 \times 10^6 \text{ cm} \quad (7)$$

and the Lorentz factor of primary particles

$$\gamma \approx 3.1 \times 10^6 (R_{mc}/10^6 \text{ cm})^{4/7} (P/1\text{s})^{-1/7} (B_p/10^{12} \text{ G})^{-1/7} \approx 4.4 \times 10^7 \quad (8)$$

for PSR 1937+214. Inserting (7) and (8) into (3) we obtain the pitch angle of primary particles

$$\psi \approx 1.7 \times 10^3 \gamma / B_p R_{mc} \approx 10^{-4} \gg 1/\gamma. \quad (9)$$

It is easy to see from (4) and (5) that considerable radiations occur only at about $R_{oc} = R_{mc}/2$, and so the critical energy of the curvature photon can be written, approximately,

$$\hbar\omega_{cr} = 3\hbar c \gamma^3 / 2R_{oc} \approx 3\hbar c \gamma^3 / R_{mc} \approx 1.8 \times 10^{12} \text{ ev}. \quad (10)$$

By using (9) and (10) we have

$$\hbar\omega_{cr} \sin\theta_0 \approx \hbar\omega_{cr} \psi \approx 370 mc^2 \gg 2mc^2 \quad (11)$$

for $B_p \approx 3 \times 10^8 \text{ G}$. This representation shows clearly that at the moment when the curvature photon is just emitted by the primary the threshold condition (2) is already exceeded greatly.

Datta⁷ has recently placed an upper limit on B_p of 10^7 G for PSR 1937+214 based on rotating neutron star models and realistic equations of state. If so, one would yield

$$\hbar\omega_{cr} \sin\theta_0 \approx 6.7 \times 10^4 mc^2 \quad (12)$$

which is closer to the value imposed by the dynamic condition (6) than (11).

CONCLUSION

Our results point out that, when the effects of non-zero curvature of magnetic field lines on the motion of an electron with initial velocity exactly parallel to field lines (such as the primary particle in pulsars) are taken into account, the radiation power would be stronger than that given by current curvature radiation theories, and the curvature photons would convert into pairs more readily. One would expect that these results would affect to a certain extent the theoretical estimate for high-energy radiations from the millisecond pulsars.

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