

EXPLANATION OF THE SECONDARY TO PRIMARY RATIO WITHIN THE
CONTINUOUS FERMI ACCELERATION MODEL

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ABSTRACT

The secondary to primary ratio in galactic cosmic radiation at relativistic momenta is calculated in a model, where the primaries are continuously accelerated from the thermal galactic background medium by 1st and 2nd order Fermi acceleration. It is shown that the measured decrease with momentum does not exclude that cosmic rays are accelerated in the interstellar medium as a whole, as claimed by Cowsik (1980) and Eichler (1980). Once a momentum dependence of the mean lifetime and the different spatial source distributions are adequately taken into account, the measured decreasing ratio can be explained.

1. Introduction. Hayakawa (1969), Eichler (1980) and Cowsik (1980) claimed that the measured ratio of secondary to primary cosmic rays rules out continuous acceleration of cosmic rays in the interstellar medium as a whole. Here we show that their predicted logarithmically increasing ratio is a sole consequence of the assumed equality of the secondaries and primaries reciprocal sum of fragmentation and mean lifetime. If the momentum dependence of the mean lifetimes and the different source distributions of primary and secondary nuclei are adequately taken into account, the measured decreasing secondary/primary ratio can be successfully explained in the framework of continuous acceleration in the whole interstellar medium.

2. Basic Equations. The steady-state momentum (hereinafter momentum stands for momentum per nucleon) equilibrium spectrum of primary (F) and secondary (S) cosmic rays at the solar system is controlled by two leaky-box type equations

$$p^{-2} \frac{d}{dp} [D(p) p^2 \frac{dF}{dp}] - p^{-2} \frac{d}{dp} [p^2 (\dot{p}_{\text{Gain}} + \dot{p}_{\text{Loss}}) F(p)] - n v \sigma_0 A_F^{2/3} F - \frac{F}{T_F} \cdot \left(\frac{p}{p_1}\right)^b + q_0 \delta(p - p_1) = 0 \quad (1),$$

$$p^{-2} \frac{d}{dp} [D(p) p^2 \frac{dS}{dp}] - p^{-2} \frac{d}{dp} [p^2 (\dot{p}_{\text{Gain}} + \dot{p}_{\text{Loss}}) S(p)] - n v \sigma_0 A_S^{2/3} S - \frac{S}{T_S} \cdot \left(\frac{p}{p_1}\right)^b + \xi F(p, p_1) = 0 \quad (2),$$

where the mean lifetimes T_F and T_S have to be calculated from the respec-

tive age distributions at the solar system (compare Schlickeiser and Lerche 1985, OG 7.2-8, equations (14), (2)).

Equations (1) and (2) are based on the physical model, that primary cosmic rays are continuously accelerated from the hot interstellar medium ($p_i \ll m_p c$) by resonant wave scattering ($D(p)$) and shock wave acceleration (\dot{p}_{Gain}), whereas secondaries result from fragmentation of primaries. Fully ionized particles heavier than protons at the same momentum per nucleon have the same stochastic ($D(p) = a_2 p^{2-\eta}$) and first-order Fermi ($\dot{p}_{\text{Gain}} = a_1 p^{1-\eta}$) acceleration rates as well as the same continuous momentum loss term ($\dot{p}_{\text{Loss}} = -\theta p$, $\theta = 1/3 \text{ div } \vec{v}_w$, due to adiabatic cooling in the galactic wind).

Differences occur in the catastrophic fragmentation loss term where the total cross section varies proportional to the number of nucleons, $\sigma_F = \sigma_0 A_F^{2/3}$, $\sigma_S = \sigma_0 A_S^{2/3}$ (Silberberg and Tsao 1973). Likewise the sources of primaries and secondaries are different: whereas primaries are accelerated according to our model out of the thermal pool ($p_i \ll m_p c$), secondaries result solely from spallation of primaries, so that their source momentum spectrum is $\xi F(p, p_i)$ where $\xi = n v \sigma_{F \rightarrow S}$ depends on the gas density of the interstellar medium (n) and the partial spallation cross section $\sigma_{F \rightarrow S}$. Also, the spatial distribution of primary and secondary sources may be different: primaries probably are accelerated in the hot coronal phase of the interstellar medium which has a broad distribution with respect to the galactic plane. Secondaries are mainly produced in dense molecular clouds where the interstellar gas is clumped, so that their source scale height is given by the scale height of galactic molecular clouds, $B_S \approx 60 \text{ pc}$ (Burton 1976), which is much smaller than the scale height of the hot coronal gas, $B_F = 500 (T/10^5 \text{ K}) \text{ pc}$, T : temperature of coronal gas (McKee and Ostriker 1977). This difference in the spatial distribution of their respective sources determines the age distributions of particles at the solar system (Lerche and Schlickeiser 1985a, OG 8.3-1). As a result (see Figure 1, OG 8.3-1), secondaries arriving at the solar system on average are younger than primaries, $T_S < T_F$, since their sources on average are closer to the solar system than the primary sources, and, as shown in Schlickeiser and Lerche (1985), these lifetimes enter the leaky-box equations. According to equation (12) of OG 8.3-1, T_S is 13 percent smaller than T_F , if a confinement volume size of $L = 5 \text{ kpc}$, $B_S = 60 \text{ pc}$, and $B_F = 2 \text{ kpc}$ are taken. This difference in the mean ages of particles arriving at the solar system is the physical reason why the secondary to primary ratio decreases with momentum at momenta larger than $10 \text{ GeV}/(c \cdot \text{nucl.})$, as we will demonstrate now.

3. Secondary/Primary-Ratio at Relativistic Momenta. Introducing the secondary to primary ratio $R(p) \equiv S(p)/F(p)$ we find from (1) and (2) at relativistic momenta (Lerche and Schlickeiser 1985b)

$$p^{2-\eta} \frac{d^2 R}{dp^2} + [2 p^{2-\eta} \Delta(p) + (4-a-\eta) p^{1-\eta} + \beta p] \frac{dR}{dp} + [(\lambda_F - \lambda_S) \left(\frac{p}{p_1}\right)^b + \xi_0 (A_F^{2/3} - A_S^{2/3})] R \approx -\frac{\xi}{a_2} \quad (3)$$

with $\beta = \theta/a_2$, $\lambda_F = (a_2 T_F)^{-1}$, $\lambda_S = (a_2 T_S)^{-1}$, $\xi_0 = n v \sigma_0 / a_2$, $\Delta(p) =$

$[F(p)]^{-1}(dF/dp)$. A decreasing ratio $R(p)$ is observed in the momentum range $10 \text{ GeV}/(c.\text{nucl.}) - 10^5 \text{ GeV}/(c.\text{nucl.})$ where the primary momentum spectrum is a straight power law $F(p) \propto p^{-\chi}$, $\chi = \text{const.}$, so that $\Delta(p) = -\chi/p$.

We consider the behaviour of $R(p)$ as $p \rightarrow \infty$ to demonstrate under what conditions the secondary to primary ratio $R(p)$ is decreasing with momentum. Table 1 summarizes the results for various combinations of η (momentum dependence of acceleration rates) and b (momentum dependence of mean lifetime). If the spatial transport is diffusion along the magnetic field, $\eta = b$.

From Table 1 we see that:

- (i) a logarithmically increasing ratio $R(p)$ is a sole consequence of the assumed equality of the secondaries and primaries reciprocal sum of fragmentation and escape lifetime, i.e. $\lambda_S(p/p_1)^b + \xi_0 A_S^{2/3} = \lambda_F(p/p_1)^b + \xi_0 A_F^{2/3}$. This logarithmic dependence is a highly isolated and artificial solution to the problem which even for equal escape lifetimes ($\lambda_S = \lambda_F$) does not hold due to the well-established mass dependence of the total fragmentation cross sections. And it was this isolated solution which Cowsik (1980) used to rule out continuous acceleration of cosmic rays in the Galaxy as a whole.
- (ii) For equal primary and secondary escape lifetimes ($\lambda_F = \lambda_S$) we find a decreasing power law solution in case $\eta > 0$ whose spectral index is determined by the ratio of catastrophic fragmentation losses to continuous momentum losses, which, however, for $p \rightarrow \infty$ approaches a negative constant.
- (iii) Agreement with observations is reached once a momentum dependence of secondaries and primaries escape lifetimes, which have the same momentum shape but different absolute values ($T_{(F)}(p) = T_F(p/p_1)^{-b}$, $T_{(S)}(p) = T_S(p/p_1)^{-b}$, $b > 0$, $p_1 = \text{const.}$, $T_S < T_F$, is allowed. For this case the secondary/primary ratio at large momenta approaches

$$R(p) \rightarrow \frac{\xi(p/p_1)^{-b}}{a_2(\lambda_S - \lambda_F)} = \frac{nc \sigma_{F \rightarrow S}}{\frac{1}{T_S} - \frac{1}{T_F}} \left(\frac{p}{p_1}\right)^{-b}.$$

These results indicate that the measured secondary to primary ratio of galactic cosmic radiation can be explained in the framework of continuous acceleration models in the general interstellar medium, once the momentum dependence of the mean lifetimes at the highest momenta and the different source distributions are adequately taken into account. So the strongest argument in the past against continuous acceleration of cosmic rays (see e.g. Cowsik 1980) has been invalidated.

References

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Table 1: Secondary/Primary - Ratio R(p) at Large Momenta for $\Delta = -X/p$

Case	$\lambda_S \left(\frac{p}{p_1}\right)^b + \epsilon_0 A_S^{2/3} = \lambda_F \left(\frac{p}{p_1}\right)^b + \epsilon_0 A_F^{2/3}$ Cowsik's case	$\lambda_S = \lambda_F$	$\lambda_S + \epsilon_0 A_S^{2/3} > \lambda_F + \epsilon_0 A_F^{2/3}$	$\lambda_S + \epsilon_0 A_S^{2/3} < \lambda_F + \epsilon_0 A_F^{2/3}$
$\eta = b = 0$	$\frac{\xi \ln p}{a_2 (2\chi + a - \beta - 3)} + \text{const.}$	$R \propto p^\Gamma, \Gamma > 0$ if $(2\chi + a - \beta - 3) > 0$ $R \rightarrow \frac{-\xi}{a_2 \epsilon_0 (A_F^{2/3} - A_S^{2/3})}$ if $(2\chi + a - \beta - 3) < 0$	ξ $\frac{a_2 [\lambda_S - \lambda_F + \epsilon_0 (A_S^{2/3} - A_F^{2/3})]}{\xi}$	$R \propto p^\Gamma, \Gamma > 0$ if $(2\chi + a - \beta - 3) > 0$ $R \rightarrow \frac{-\xi}{a_2 [\lambda_F + \lambda_S + \epsilon_0 (A_F^{2/3} - A_S^{2/3})]}$ if $(2\chi + a - \beta - 3) < 0$
$0 = b < \eta$	$\frac{-\xi}{a_2 \beta}$ const. + $\ln p$	$\frac{\epsilon_0}{-\beta} (A_F^{2/3} - A_S^{2/3})$ $R \propto p$ $R(p = \infty) < 0$	ξ $\frac{a_2 [\lambda_S - \lambda_F + \epsilon_0 (A_S^{2/3} - A_F^{2/3})]}{\xi}$	$R \propto p$ $R(p = \infty) < 0$ $-\frac{1}{\beta} [\lambda_F - \lambda_S + \epsilon_0 (A_F^{2/3} - A_S^{2/3})]$
$b > 0, \eta > 0$	$\frac{-\xi}{a_2 \beta}$ const. + $\ln p$	$\frac{\epsilon_0}{-\beta} (A_F^{2/3} - A_S^{2/3})$ $R \propto p$ $R(p = \infty) < 0$	$\frac{\xi}{a_2 (\lambda_S - \lambda_F)} \left(\frac{p}{p_1}\right)^{-b}$	$\frac{\xi}{a_2 (\lambda_S - \lambda_F)} \left(\frac{p}{p_1}\right)^{-b}$