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SECONDARY TO PRIMARY RATIO AND THE CONTINUOUS ACCELERATION

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1. Introduction. It is well known that the ratio of secondary to primary nuclei in the cosmic radiation is a decreasing function of energy for $E \gtrsim 2$ GeV/n. This fact has to be interpreted in terms of c.r. propagation and acceleration model. An important problem is whether these two processes are separated in time (and in space) or can occur simultaneously. Assuming the leaky box model, Cowsik[1] showed that the decreasing sec/prim ratio is in a strong disagreement with an effective acceleration taking place in the ISM, predicting an increasing sec/prim ratio with energy. However it seems that there is still some confusion whether this conclusion is applicable to other models of c.r. propagation or it is true for the leaky box model only.[2]

In this paper we give a general formula for the sec/prim ratio, independently of any details of the propagation and acceleration model. In the limit of equal fragmentation paths for primaries and secondaries, this ratio at a given momentum nucleon is proportional only to the mean path of the observed primaries at that momentum. We shall show (basing partly on this formula) that it is unlikely to get a decreasing sec/prim ratio with energy if an acceleration process takes place during particle propagation in the ISM.

2. General formula. Let us denote by $f(p,t)$ the vacuum time distribution of primaries observed at the Earth with the momentum/nucleon p. The number of the observed primaries is of course $n_1(p)=\int_0^\infty f(p,t)e^{-t/T_1}dt$, where T is their mean life time against fragmentation. (We shall keep in mind that "time" means "path length" in g/cm^2). Particles arriving with age t have produced secondaries, which must propagate and be accelerated in the same way as their parent particles, if we adopt a reasonable assumption that these processes depend on p only (which is not changed by fragmentation). So they come to the observation point with the same momentum/n and their number is

$$n_{2,t}(p)=\int_0^t f(p,t)e^{-t'/T_1}k\,dt'\,e^{-(t-t')/T_2}=k\left(\frac{1}{T_1}-\frac{1}{T_2}\right)^{-1}\left(e^{-t/T_2}-e^{-t/T_1}\right)f(p,t)\rightarrow \tag{1}$$

$$\rightarrow kte^{-t/T_1}f(p,t) \text{ for } T_2\rightarrow T_1$$

The total number of secondaries is $n_2(p)=\int_0^\infty n_{2,t}(p)\,dt$ and for the sec/prim ratio we have

$$r(p)\equiv\frac{n_2(p)}{n_1(p)}=k\Delta\left(\frac{\int_0^\infty f(p,t)e^{-t/T_2}dt}{\int_0^\infty f(p,t)e^{-t/T_1}dt}-1\right) \quad \text{where } \Delta\equiv\left(\frac{1}{T_1}-\frac{1}{T_2}\right)^{-1} \tag{2}$$

Of course it has been well known for a long time that the sec/prim ratio depends on the path length distribution but we would like to stress here that eq. (2) holds for any model of the Galaxy and for any assumptions about the acceleration or deceleration processes (provided they depend on p only). For $0 < (T_2 - T_1)/T_1 \ll 1$ eq. (2) gives

$$r(p) \simeq k[\bar{t}(p) + 1/2 \cdot \overline{t^2}(p) \cdot (T_2 - T_1)/T_1^2 + \ldots] \tag{3}$$

where $\bar{t}(p) = \int_0^\infty t f(p,t) e^{-t/T_1} dt \Big/ \int_0^\infty f(p,t) e^{-t/T_1} dt$ (and similarly for $\overline{t^2}(p)$) is the mean time (path length) of the observed primary particles at a given p. Thus the sec/prim energy dependence is practically equivalent to the energy dependence of the mean propagation time of the observed particles (for $T_2 - T_1 \ll \bar{t}$ and $\overline{t^2} \sim \bar{t}^2$), but not of the mean vacuum time.

3. Examples. First we shall consider a situation when p is a unique function of t. This could occur if, for example, c.r. were produced with a constant p and then accelerated according to $dp/dt = h(p) > 0$. Then f(p,t) is reduced to F(t) where $n_1(p)dp = F[t(p)]dt(p)e^{-t(p)/T_1}$. For the sec/prim ratio we have from (1) $r(p) = k\Delta(e^{t(p)/\Delta} - 1)$ with $t(p) = \int_{p_0}^{p} du/h(u)$ (4)
so n_2/n_1 is a growing function of p. For the first order Fermi process, when $dp/dt = \beta p$ ($\beta > 0$), we get

$$r(p) = k\Delta\left[(p/p_0)^{1/\beta\Delta} - 1\right] \xrightarrow[T_2 \to T_1]{} \frac{k}{\beta} \ln p/p_0 \tag{5}$$

If particles are produced with a distribution of primary momentum p then $f(p,t)dt = f(p',t')dt'$ where $t' = t + \tau$ with $\tau = \int_p^{p'} du/h(u)$ and

$$r(p') + 1 = \int_0^\infty f(p,t) e^{-(t+\tau)/T_2} dt \Big/ \int_0^\infty f(p,t) e^{-(t+\tau)/T_1} dt \tag{6}$$

so $r(p') + 1 = [r(p) + 1] e^{\tau/\Delta}$; hence $r(p') > r(p)$. (7)

Here τ is independent of t but it is not a necessary condition for r(p) to grow. r(p) will also grow if f(p,t) for higher p' is effectively shifted to longer times so that, for example, $f(p,t)dt = f(p',t')dt'$ with $t' = t + \tau(p,p',t)$ and $\tau > 0$. One would expect that to be rather natural when the acceleration takes place. Growing of r(p) is seen from (6) since $e^{\tau/T_2} > e^{\tau/T_1}$ for any t for $T_2 > T_1$, which is the case for secondaries being lighter.

Let us next consider a second order Fermi acceleration - when p is not a unique function of time. In particular we shall assume that its behaviour with time corresponds to a uniform diffusion along the log p axis. Moreover we adopt a 1-dim. model of the Galaxy, the dimension x being perpendicular to the Galactic plane. C.r. nuclei are produced in the region $0 < x < l$ at a constant rate q (per unit length) with a single momentum p_0. They diffuse, are accelerated and fragment at the same time, leaking out of the Galaxy at x = 0 and x = l. First we shall consider a case of a constant spatial diffusion coefficient D. For that case it is easy to find the function f(p,t):

$$f(p,t)dp = (4\pi K t)^{-1/2} \exp[-\ln^2(p/p_0)/4Kt] \frac{dp}{p} \cdot \frac{4q}{\pi} \sum_{1,3,\ldots} \frac{1}{m} \sin\frac{m\pi x}{l} \cdot \exp\left[-\left(\frac{m\pi}{l}\right)^2 Dt\right] \tag{8}$$

since particles with age t have a gaussian distribution of $y = \ln(p/p_0)$. Denoting $\int_0^\infty f(p,t) e^{-t/T_i} \equiv I_i$ we find

$$I_i = \frac{2q}{\pi\sqrt{K}} \frac{1}{p} \sum_{1,3,\ldots} \frac{1}{m} \sin\left(\frac{m\pi x}{l}\right) \left[\frac{1}{T_i} + \left(\frac{m\pi}{l}\right)^2 D\right]^{-1/2} \cdot (p/p_0)^{-\frac{1}{4K}\left[1/T_i + \left(\frac{m\pi}{l}\right)^2 D\right]} \tag{9}$$

It is evident that $r(p)$ increases with momentum for $T_2 > T_1$. In the limit $p \to \infty$, when the first term in the series dominates, we get

$$r(p) \to (p/p_0)^{\frac{1}{4K\Delta}} - 1 \tag{10}$$

4. Other solutions. It is sometimes difficult to find analytically $f(p,t)$. In particular one would be interested in finding $f(p,t)$ in the above described model if the spatial diffusion depends on p but this does not seem to be an easy task. So we shall treat this problem considering the equilibrium equations. We assume a second order acceleration occuring, for example, as a result of particle collisions with Alfven waves in the ISM. This corresponds to a particle diffusion in the 3-dimensional momentum space with a momentum dependent coefficient $K(p)$. The equation for primary particle density in the phase space ($F_1(x,p) \cdot 4\pi p^2 dp dx = dn_1(x,p)$) is

$$\frac{1}{p^2}\frac{\partial}{\partial p}\left(K(p)p^2\frac{\partial F_1}{\partial p}\right) + D(p)\frac{\partial^2 F_1}{\partial x^2} - \frac{F_1}{T_1(x)} + q\,\delta(p-p_0) = 0 \tag{11}$$

Let us first neglect the term with fragmentation $F_1/T_1(x)$. As it is usually done we look for solutions in the form $F_1(x,p) = F^{I}(p)\cdot F^{II}(x)$. Assuming further $K(p) = B\cdot p^{\eta}$ and $D(p) = A\cdot p^{\alpha}$ we get for $p > p_0$

$$p^{\eta}\frac{\partial^2 F'}{\partial p^2} + (\eta+2)p^{\eta-1}\frac{\partial F'}{\partial p} - \frac{\lambda A}{B}p^{\alpha}F' = 0 \tag{12}$$

We shall look for power law spectra and this implies that $\eta = 2 + \alpha$. (However, for a consistent picture of acceleration and spatial diffusion due to Alfven waves both diffusion coefficients are related by $K(p)\cdot D(p) \sim p^2$ for relativistic particles, but this does not lead to experimentally observed power law spectra). With $F'(p) \sim p^{-\gamma}$ we have from (12)

$$\gamma_{\pm}(\lambda) = \frac{1}{2}\left(3+\alpha \pm \sqrt{(3+\alpha)^2 + 4\lambda A/B}\right) \tag{13}$$

where $F^{I}(p) \sim p^{-\gamma_+}$ for $p > p_0$ and $F'(p) \sim p^{-\gamma_-}$ for $p < p_0$. $F^{II}(x) \sim \sin(\sqrt{\lambda}\,x)$, with $\lambda = (n\pi/l)^2$ and $n = 1,3,5,\ldots$; hence

$$F_1(x,p) = \sum_{1,3,} G_n \sin\left(\frac{n\pi x}{l}\right)\left[(p/p_0)^{-\gamma_n^-}\Theta(p_0-p) + (p/p_0)^{-\gamma_n^+}\Theta(p-p_0)\right] \tag{14}$$

where $\Theta(p-p_0)$ is a step function. Substituting (14) to (11) (with no fragmentation) we get

$$G_n = 2q\cdot\left[n\pi B p_0^{1+\alpha}(\gamma_n^+ - \gamma_n^-)\right]^{-1} \tag{15}$$

At high p the momentum spectrum behaves as $p^{-\gamma_1^+}\cdot 4\pi p^2$, so independently of the spatial distribution of the sources which influences only G_n.

We shall find now the secondary particle spectrum. Let $F_2(x_0,x,p)$ denote the phase space density of secondaries observed at x with p, produced at x_0. Then we have

$$F_2(x_0,x,p) = k\,\varrho(x_0)\int_{p_{thr}}^{\infty} F_1(x_0,p')\cdot F_{1p'}(x_0,x,p)\,dp' \tag{16}$$

where $F_{1p'}(x_0,x,p)$ is the solution to the eq. (11) ($T_1 \to \infty$) with the last term $q\delta(p-p_0)$ substituted by $q\delta(x-x_0)\cdot\delta(p-p_0)$. This solution differs from (14) only by different coefficients $G_n(x_0)$, the p dependence being the same. Solving (16) we get for $p > p_0$ $F_2(x_0,x,p) = I_1 + I_2$ where

$$I_1 = \frac{k\varrho(x_0)}{p_0^{1+2\alpha}} \sum_n G_n \sin\left(\frac{n\pi x_0}{L}\right)\left\{\sum_m G_m(x_0)\sin\left(\frac{m\pi x}{L}\right)(\gamma_n^+ - \gamma_m^+ + \alpha)^{-1}\left[(p/p_0)^{-\gamma_m^+} - (p/p_0)^{-(\gamma_n^+ + \alpha)}\right]\right\}$$

and (17)

$$I_2 = \frac{k\varrho(x_0)}{p_0^{1+2\alpha}} \sum_n G_n \sin\left(\frac{n\pi x_0}{L}\right)\left[\sum_m G_m(x_0)\sin\left(\frac{m\pi x}{L}\right)(\gamma_n^+ - \gamma_m^- + \alpha)^{-1}(p/p_0)^{-(\gamma_n^+ + \alpha)}\right]$$

For simplicity we have put $p_{thr} = p_0$. I_1 (I_2) corresponds to the secondaries that have been produced with momenta smaller (larger) than p. To find $F_2(x,p)$ we have to integrate $\int_0^L F_2(x_0,x,p)\,dx_0$ but its momentum dependence is already seen from (17). The terms with $p^{-\gamma_m^+}$ and $p^{-(\gamma_n^+ + \alpha)}$ dominate for $p \gg p_0$, so the sec/prim ratio increases with p as $1-(p/p_0)^{-\alpha}$, practically independently of the gas density distribution $\varrho(x)$. For $\varrho(x)$ = const. and $\alpha = 0.6$ F_2/F_1 (for $p/p_0 = 10$) reaches $\sim 80\%$ of its maximum value.

Taking now into account the fragmentation term in (11) we look, as before, for solutions in the form $F_1(x,p) = F(p)F'(x)$, if $T_1(x)$ = const. For $F'(p) \sim p^{-\gamma}$ we get

$$[\gamma(\gamma+1) - \gamma(\eta+2)]\,p^{\eta-2} - (\lambda A/B)\,p^{\alpha} - 1/BT_1 = 0 \tag{18}$$

This can only be fulfilled at $p \to \infty$ and it can be seen that then the fragmentation term does not play any rôle. In particular, if $\eta - 2 = \alpha$, γ has the same form as in (13). If we assume that the secondary spectrum has a form $\sim p^{-\Gamma}$ for $p \to \infty$ then we get

$$[\Gamma(\Gamma+1) - \Gamma(\eta+2)]\,p^{\eta-2} - (\lambda A/B)\,p^{\alpha} - 1/BT_2 + k\,p^{\Gamma-\gamma} = 0 \tag{19}$$

This can only be fulfilled at high momenta if $\Gamma = \gamma$. So at $p \to \infty$ the sec/prim ratio $\to$ const. even if we take into account fragmentation.

5. Conclusions. We conclude that, contrary to some suggestions, a simultaneous acceleration and propagation in the ISM would lead to the sec/prim ratio increasing with momentum (tending in some cases to a constant for $p \to \infty$). That is in a strong discrepancy with observation. The logarithmic rise, stressed by Cowsik[1], is obtained for some particular cases only. Moreover the shape of the particle spectra at $p \to \infty$ does not depend on the spatial distribution of their sources.

References

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