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EXACT SOLUTIONS FOR SPORADIC ACCELERATION OF COSMIC RAYS

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ABSTRACT

The steady state spectra of cosmic rays subject to a sporadic acceleration process, wherein the gain in energy in each encounter is a finite fraction of the particle energy, are derived on the basis of a simple mathematical model which includes the possibility of energy dependent leakage of cosmic rays from the galaxy. Comparison with observations allows limits to be placed on the frequency and efficiency of such encounters.

1. Comparison of Fermi-process and Sporadic Acceleration. Among the various processes of cosmic-ray acceleration the possibility suggested by Fermi [1] that the particles, above a minimum threshold energy, gain energy statistically in encounters with moving magnetised interstellar clouds, has been studied most extensively [2-6]. In the mathematical formulation of such a model one assumes that the average change in energy in each encounter is small fraction of the original energy so that in the Taylor-expansion of the Boltzmann collision integral one retains only the first two, convective and diffusive, terms. Exact analytic solutions upto the second order, keeping the leakage from the galaxy constant at all energies, are given in references [5-6]. Learche and Schlickeiser have investigated extensively the problem including energy dependent leakage [7] and one can find an extensive list of references in their paper.

2. A Mathematical Model of Sporadic Acceleration. When in a single encounter the energy of the cosmic ray increases by a large factor we can not treat the transport under the diffusion approximation; the fluctuations in the number of encounters become critically important in determining the spectra of primaries and secondaries. In the mathematical formulation of the problem, to keep it tractable and simple, it is assumed here that the energy of the particle E is multiplied by a factor α after each encounter and the spread in α is taken to be negligible. Let the probability of such an encounter per unit time be A, the leakage probability B(E) and s be the spallation rate. If the injection with the spectral form I(E) started at time t=0, then the spectrum of particles which have suffered exclusively n encounters at time t is given by

$$\mathcal{M}(n,E,t) = \int_0^\infty \int_0^t \mathcal{M}(n-1, E', t')\, A\ (E-\alpha E') \times \exp - [A+B(E)+s]\,(t-t')\, dt'\, dE' \qquad (1)$$

with

$$\mathcal{M}(o,E,t) = \int_0^t I(E,t') \exp - [A+B(E)+s]\,(t-t')\, dt' \qquad (2)$$

Assuming that the cosmic rays are in steady state the spectrum is given by

$$M(E) = \lim_{t \to \infty} \sum_{n=0}^{\infty} \mathcal{M}(n,E,t) \qquad (3)$$

It is straight forward to derive the result

$$M(E) = \sum_{n=0}^{\infty} I\ (E/\alpha^n)\ \frac{\alpha}{A} \prod_{k=0}^{n} A/\alpha\ [A+B(E/\alpha^k) + s] \qquad (4)$$

The spectra of secondaries L(E) is obtained by substituting s M(E) for I(E) in equation (4). Writing $P_n(E)$ for the product one gets

$$L(E) = \sum_{n=0}^{\infty} s\ M(E/\alpha^n)\ \frac{\alpha}{A}\ P_n(E) \qquad (5)$$

Now various cosmic-ray models can be investigated by specifying I, A, B and α. For example for $I = I_o\ E^{-\beta}$, $B = B_o$ independent of energy eqation (4) converges for

$$A\ \alpha^{\beta-1}\ (A + B_o + s)^{-1} < 1 \qquad (6)$$

Or keeping in mind the currently popular models of acceleration by shocks in the interstellar medium [8-11] it is appropriate to choose

$$I = I_o\ [E_o + E]^{\delta-\beta}\ ; \quad B = B_o E^{\delta} + h \qquad (7)$$

with E_o = 1 GeV/nucleon, $h = 10^{-7} yr^{-1}$, $\beta = 2.7$ and $\delta = 0.5$.

3. Comparison with Observations and Results. In figure 1 the ratio L(E)/M(E) is shown for two values of the acceleration parameter α = 1.26, 1.6 with the maximum probability of encounter at low energies $A B_o^{-1}$ = 1, 3 and 6 respectively. These theoretical expectations are compared with the observations [12-14] of the ratio of boron to carbon in cosmic rays at various energies. The theoretical results allow upward or downward scaling to approximately represent change in the spallation rate s due to different choice of the interstellar density.

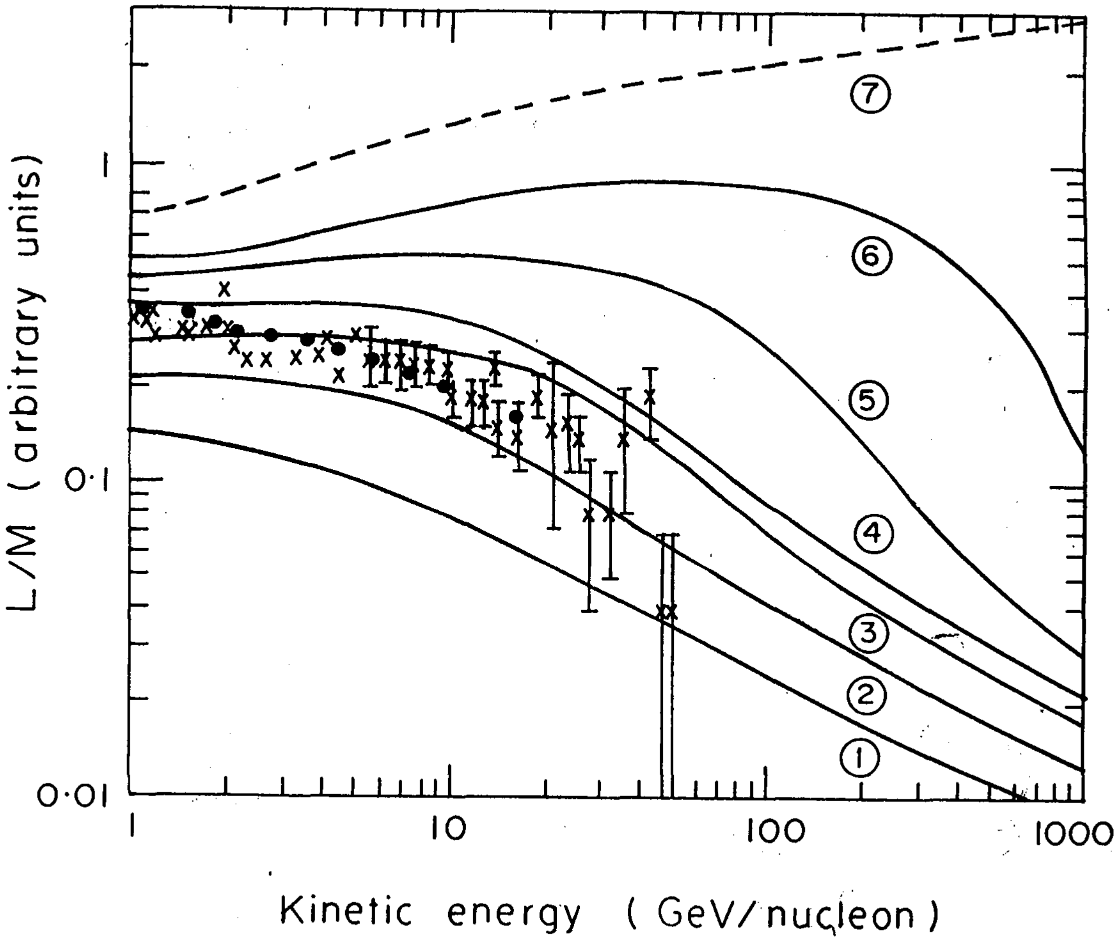

Fig. 1. Theoretical ratios of secondary to primary cosmic rays for (α, AB_o^{-1}) = (1.26, 1), (1.26, 3), (1.26, 6), (1.6, 1), (1.6, 3), are labelled 1-6 and the standard Fermi-process [6] is labelled 7. The observed boron/carbon ratios are from references [12-14].

One can draw several conclusions : (a) All curves tend to $\sim E^{-\delta}$ at high energies as expected, (b) As the rate of acceleration increases L/M ratios increase in magnitude and also become flatter thereby losing the $E^{-\delta}$ behaviour upto progressively higher energies, (c) Thus $\alpha = 1.6$ and $AB_o^{-1} = 3$ is the maximum acceleration rate in the interstellar medium, corresponding to a mean increase of $(1.6)^3 \approx 4$ in the energy of the low energy cosmic rays. As energy increases $P_n(E)$ decreases rapidly so that the expected net acceleration decreases also rapidly to negligible levels even at > 10 GeV/nucleon. On the basis of this analysis and the earlier work on Fermi-process [1-6] it appears that cosmic rays suffer negligible reacceleration in the interstellar medium after they emerge from their sources.

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