PROTON-AIR AND PROTON-PROTON CROSS SECTIONS FROM AIR SHOWER DATA

John Linsley
Department of Physics and Astronomy
University of New Mexico, Albuquerque, NM 87131
USA

ABSTRACT

Data on the fluctuations in depth of maximum development of cosmic ray air showers, corrected for the effects of mixed primary composition and shower development fluctuations, yield values of the inelastic proton-air cross section for laboratory energies in the range 10^8-10^{10} GeV. From these values of σ_{pa}, corresponding values of the proton-proton total cross section are derived by means of Glauber theory and geometrical scaling. The resulting values of σ_{pp} are inconsistent with a well known $\ln^2 s$ extrapolation of ISR data which is consistent with SPS data; they indicate a less rapid rate of increase in the interval $540 < \sqrt{s} < 10^5$ GeV.

1. Introduction. The total inelastic proton-air cross section σ_{pa}, which is interesting in relation to the theory of hadron-hadron and hadron-nucleus interactions, can be derived from data on cosmic ray air showers. Cosmic ray evidence of an increase with increasing energy in σ_{pp}, the total proton-proton cross section, has been confirmed, first at ISR energies and more recently with the CERN SPS collider.[1-4] In the interval $10 < \sqrt{s} < 540$ GeV the increase, amounting to some 75%, agrees well with a $\ln^2 s$ dependence but is compatible with a slower rate.[3] A recent cosmic ray result for $2 \cdot 10^3 < \sqrt{s} < 2 \cdot 10^4$ GeV favors $\ln^2 s$,[5] while another result of this kind, for $\sqrt{s} = 3 \cdot 10^4$ GeV, favors a slower increase.[6] In case of the cosmic ray results, σ_{pp} is derived from σ_{pa} using Glauber theory.

My purpose here is to draw attention to another body of air shower evidence extending to even higher energies ($10^4 < \sqrt{s} < 1.6 \cdot 10^5$ GeV) which also bears on this question.[19] The evidence consists in part of data on the fluctuation of shower elongation x_m, and in part of data on the primary composition. These are combined using an expression for $\{x_m\}$ as a function of $\langle \ln A \rangle$, $\{\ln A\}$ and $\{x_{m,p}\}$, where A is the primary mass number, $\langle \ln A \rangle$, $\{\ln A\}$ are the mean value and dispersion about the mean of lnA for fixed primary energy, $\{x_{m,p}\}$ is the dispersion of x_m for proton-initiated showers of a fixed energy (allowing for fluctuations in shower origin and development), and $\{x_m\}$ is the observed dispersion of x_m, corrected for reception fluctuations (instrumental dispersion).[7] Using experimental evidence on $\langle \ln A \rangle$ and $\{\ln A\}$ from other air shower observations, the correction for contamination of the primary beam with heavy nuclei is shown to be small: $\{x_{m,p}\} \cong \{x_m\}$. Cascade simulations have shown that $\{x_{m,p}\}$ is proportional to λ_{pa}, the proton-air mean free path, with a proportionality constant equal to 1.4. From λ_{pa} one obtains σ_{pa}; σ_{pp} follows by application of Glauber theory.

2. Primary Composition Effect. I have shown elsewhere that if y is an observable such as x_m, whose mean value for proton showers can be graphed vs lnE as a straight line (over reasonably wide intervals, say 2 decades or more), and if the line width (fluctuation of y) for showers initiated

by various nuclei is approximated by $\{y(A)\} = \{y(1)\}(1 - k\ln A)$, where k is constant, then

$$\{y\}^2 = \{y(1)\}^2[1 - 2k\langle\ln A\rangle + k^2(\langle\ln A\rangle^2 + \{\ln A\}^2)] + b^2\{\ln A\}^2 \;, \quad (1)$$

where b is the slope of the straight-line graph.[7] This follows from elementary statistical theory and the superposition principle of Peters. For $y = x_m$, b is just $D_{e,p}$, the proton shower elongation rate. With these substitutions I solve (1) for $\{x_{m,p}\}$, obtaining

$$\{x_{m,p}\} = \{x_m\}\left(\frac{1 - (D_{e,p}\{\ln A\}/\{x_m\})^2}{1 - 2k\langle\ln A\rangle + k^2(\langle\ln A\rangle^2 + \{\ln A\}^2)}\right)^{\frac{1}{2}} . \quad (2)$$

The right hand side of (2) contains, besides the observed quantity requiring correction, 2 parameters which describe phenomenologically the relevant features of air showers ($D_{e,p}$ and k), and 2 parameters which describe the composition of the incoming cosmic ray beam ($\langle\ln A\rangle$ and $\{\ln A\}$).

The fluctuation of x_m has been studied experimentally for more than 10 years using the giant air shower array at Haverah Park,[8,9] and almost as long using a similar size array at Yakutsk.[10] It has been measured independently at Dugway using an array of atmospheric Cerenkov light detectors,[11] and at Akeno using data on the electron lateral distribution.[12] Results are given in Table 1.

I have shown elsewhere that under a wide range of assumptions about hadron-nucleus interactions $D_{e,p}$ cannot be appreciably greater than t_o, the radiation length. This insures that the numerator in (2) is approximately 1, so the exact value of $D_{e,p}$ is unimportant. For energy independent composition, $D_{e,p} = D_e$ (observed elongation rate). I adopt $D_{e,p} = 28\ g/cm^2$, practically the experimental value of D_e at these energies.[14] As for the parameter k, Monte Carlo cascade simulations show that its value lies in the range $0.15 \pm .05$, depending on the choice of model.[11,15] The exact value is unimportant if $\langle\ln A\rangle$ is less than 1.

Data on $\langle\ln A\rangle$ and $\{\ln A\}$ at very high energies are summarized in Ref. 7 (see also conference paper OG5.4-4). They can be combined, as in Fig. 1, by plotting $\{\ln A\}$ vs $\langle\ln A\rangle$ in rectangular coordinates. Note that for any pure composition $\{\ln A\} = 0$. It is assumed that the possibilities range from $\langle\ln A\rangle = 0$ (pure protons) to $\langle\ln A\rangle = 4$ (pure Fe). The maximum dispersion for a given mean occurs for a mixture of protons and Fe nuclei. When the scales are chosen as I have done the locus of binary proton-Fe mixtures is a semicircle. The lower boundary of the diagram consists of smaller semicircles corresponding to other binary mixtures. The region inside the boundary represents all possible compositions. The observed equal-energy composition in the low energy region (10^2-10^5 GeV) is indicated by the heavy dashed line. The compositions experimentally allowed at very high energies ($E > 10^8$ GeV) are those within the shaded region. The heavy dotted line

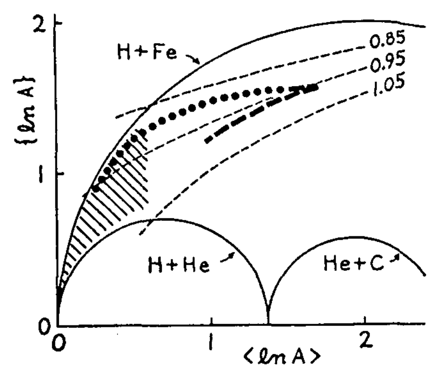

Fig. 1. Cosmic ray composition diagram: 2nd vs 1st moment of the lnA distribution, explained in the accompanying text.

Table 1. Summary of $\{x_m\}$ data

$\log E_{lab}$ (GeV)	$\{x_m\}$ g/cm^2	No. of events	Ref.
7.58	89 ± 15	?	12
7.82	90 ± 18	?	12
8.06	86 ± 19	?	12
8.24	80 ± 11	147	11
8.30	70 ± 13	?	12
8.30	69 ± 14	426	8
8.56	62 ± 9	1014	8
8.68	71 ± 6	519	9
8.78	73 ± 10	a	10
8.83	51 ± 7	652	8
8.88	79 ± 4	300	10
9.11	68 ± 10	334	8
9.15	69 ± 6	a	10
9.43	65 ± 11	a	10
9.45	74 ± 11	178	8
9.60	63 ± 5	201	10
10.00	54 ± 11	87	8
10.15	55 ± 13	a	10

[a] total of 464 events

represents a detailed model given in Ref. 7, which describes in a plausible manner the high energy region and the transition from lower energies. The correction factors $\{x_{m,p}\}/\{x_m\}$ for this model to be used with Table 1 according to (2) range in value from 0.91 to 0.95; thus they differ only slightly from 1 (no correction). These corrections have been applied in obtaining the present results, shown in Figures 2 and 3.

Also shown in Fig. 1 are contour lines corresponding to a typical value of $\{x_m\}$ (= 66 g/cm^2) and the indicated values of the correction factor. By examining these contours one sees that correction factors for any other allowed model will differ only slightly from those used here.

3. Development Fluctuations. The next step is to convert from $\{x_{m,p}\}$ to σ_{pa} allowing for shower development fluctuations. These arise primarily because of the leading particle effect. Thus one expects them to scale as λ_{pa}, so there will be a proportionality: $\{x_{m,p}\} = K\lambda_{pa}$ where K is a constant. On the basis of Monte Carlo cascade simulations by Walker and Watson[8] I adopt the value 1.40 for this constant. The resulting values for σ_{pa} are given in Figure 2.

There is of course some uncertainty in the value of K, due mainly to uncertainty about the elasticity of very high energy hadron-air collisions, denoted here by η. The result K = 1.4 rests on a conventional assumption that η is distributed uniformly between 0 and 1 with a mean value of 0.5. It has been pointed out by L.W. Jones that according to accelerator data the value of $\langle\eta\rangle$ for proton collisions with light nuclei is appreciably less, about 0.3.[16] The dependence of K on $\langle\eta\rangle$ can be estimated by assuming 1) that x_{av} of the partial electronic cascade originating from each collision of the leading nucleus is perfectly correlated with the depth of that collision, and 2) that $\{x_{m,p}\} = \{x_{av,p}\}$ (neglecting fluctuations in η). It is then easily shown that

$$K \cong [\Sigma_{j=1}^{\infty} j(1-\eta)\eta^{j-1}]^{\frac{1}{2}} = (1-\eta)^{-\frac{1}{2}}. \qquad (3)$$

For η = 0.5 one obtains K ≅ 1.41; for η = 0.3 the result is K ≅ 1.20. Thus the effect of assuming a smaller value of $\langle\eta\rangle$ is to reduce the apparent cross sections σ_{pa} and σ_{pp} at air shower energies.

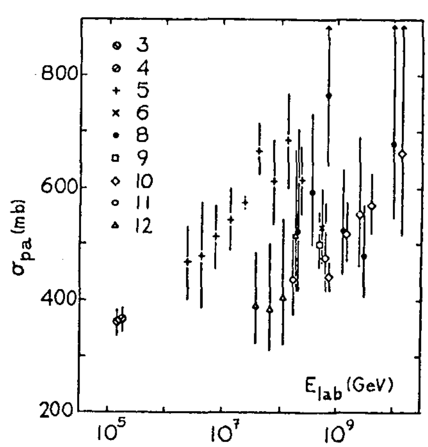

Fig. 2. Results on σ_{pa}. The numbers listed with the symbols are the corresponding references.

4. Results. Examining Fig. 2 one notes good agreement between the present results and one published recently by the Fly's Eye group.[6] The Fly's Eye result is derived from the decrement of the x_m distribution, rather than the dispersion, using a proportionality between Λ_{xm} and λ_{pa}. The value of the constant is taken as 1.6 ± 10% from simulations[15] similar to those of Walker and Watson. The present results disagree with those of Ref. 5, based on the zenith angle distribution of generally smaller air showers for fixed N_e and N_μ. Here also the relation between a measured attenuation length and λ_{pa} is found by means of simulations. There is an approximate proportionality, the value of the constant being 1.45-1.55. The value of $\langle\eta\rangle$ for the Akeno model is 0.44. A smaller value would result in a smaller value of the proportionality constant and a smaller apparent rise in cross sections. However it would not remove the discrepancy since a change in $\langle\eta\rangle$ will have a similar effect on results from Λ_{xm} and $\{x_m\}$.

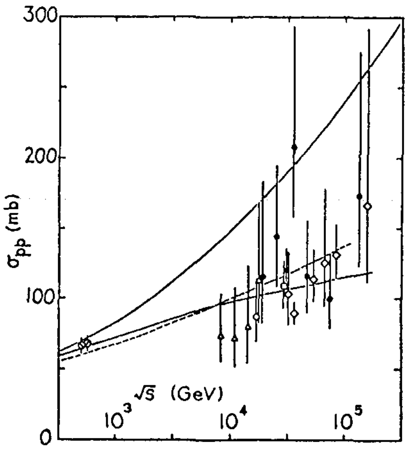

Fig. 3. Results on σ_{pp} (same symbols as Fig. 2). The lower solid curve, from Ref. 17, is a best fit to ISR data allowing deviation from the Froissart bound. The upper solid curve (same Ref.) is a best fit with Froissart bound form. The dashed curve is for the 'impact picture'.[18]

Fig. 3 shows the result of converting the new σ_{pa} values to σ_{pp}. This has been done the same way as in Ref. 6, using Glauber theory with an assumption that the nuclear slope parameter has the same energy dependence as σ_{pp} (geometrical scaling). In parametric form, the relation assumed is $\sigma_{pa} = 24.0\ \sigma_{pp}^{0.648}$ (cross sections in mb). Shown for comparison are the SPS results,[3,4] and well known extrapolations by Block and Cahn, based on ISR data, which successfully predicted those results.[17] Also shown is a recent prediction by Bourrely *et al.*, for the so-called impact picture.[18] The lower Block-Cahn curve is the best fit with $a \neq 0$, where parameter a takes on small positive values to allow for deviations from Froissart bound form. The upper curve is for $a = 0$.

References. [1]YODH *et al.* 1972, Phys. Rev. Lett. 28, 1005; [2]AMALDI *et al.* 1977, Phys. Lett. 66B, 390; [3]BATTISTON *et al.* 1982, Phys. Lett. 117B, 126; [4]ARNISON *et al.* 1983, Phys. Lett. 128B, 336; [5]HARA *et al.* 1983, Phys. Rev. Lett. 50, 2058 and Proc. 18th ICRC 11, 354; [6]BALTRUSAITIS *et al.* 1984, Phys. Rev. Lett. 52, 1380; [7]LINSLEY 1983, Proc. 18th ICRC 12, 135; [8]WALKER and WATSON 1982, J. Phys. G: Nucl. Phys. 8, 1131 (also 1983, Proc. 18th ICRC 6, 114); [9]COY *et al.* 1981, Proc. 17th ICRC 6, 43; [10]DYAKONOV *et al.* 1981, Proc. 17th ICRC 6, 110 and Proc. 18th ICRC 6, 111; [11]CHANTLER *et al.* 1982, J. Phys. G: Nucl. Phys. 8, L51; [12]HARA *et al.* 1983, Proc. 18th ICRC 11, 272; [13]LINSLEY and WATSON 1981, Phys. Rev. Lett. 46, 459 and references therein; [14]WALKER and WATSON 1981, J. Phys. G: Nucl. Phys. 7, 1297 (see also Ref. 13); [15]ELLSWORTH *et al.* 1982, Phys. Rev. D 26, 336; [16]JONES 1983, Proc. 18th ICRC 5, 17; [17]BLOCK and CAHN 1983, Phys. Lett. 120B, 224 and in Proc. 18th *Rencontre de Moriond*, Vol. 3; [18]BOURRELY *et al.* 1985, Phys. Rev. Lett. 54, 757; [19]see also LINSLEY 1985, *Lett. Nuovo Cimento* 42, 403.