HADRON CROSS SECTIONS AT ULTRA HIGH ENERGIES AND UNITARITY BOUNDS ON DIFFRACTION DISSOCIATION

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1. Introduction: The behavior of total cross sections at ultra high energies, above $\sqrt{s} \sim 1$ TeV has been derived from analysis of air shower observation.[1] The proton-air inelastic cross section must be related to the basic proton-proton interaction to determine which of the different models for the asymptotic behavior of the scattering amplitude are consistent with cosmic ray data which may be ruled out. The adjective inelastic for proton-air cross section (σ^{inel}_{p-air}) describes the fact that cosmic ray experiments do not measure all of the absorptive cross section because cascade development is not sensitive to processes that lead to quasi-elastic excitation of the air nucleus (σ_{qe}) or to diffractive excitation of one of the nucleons of the air nucleus (σ_{DD}). The method generally used to calculate p-air inelastic cross section from proton-proton parameters is the Glauber multiple scattering technique.[2] Application of this method leads to the relation

$$\sigma^{inel}_{P-air} = \sigma^{tot}_{P-air} - \sigma^{el}_{P-air} - \sigma_{qe} - \sigma_{D} - \Delta\sigma(\text{inelastic screening}) \qquad (1)$$

The term $\Delta\sigma$(inelastic screening) accounts for screening due to multiple scattering with excited nucleon intermediate states. To calculate terms on the right hand side of this relation it is necessary to know the values of σ^{PP}_{tot}, slope parameter $B^{PP}(t=0)$, single and double diffractive cross sections σ^{PP}_{SD}, σ^{PP}_{DD} the shape of $d^2\sigma/dtdM^2$ at t_{min} for the diffractive process $P+P \to P+X$ and the nuclear density profile.[3]

2. Discussion of Models of Elementary Interaction: Many different models for the high energy behavior of scattering amplitudes have been proposed, all of which agree with PP and $\bar{P}P$ data up to SPS-$\bar{P}P$ collider energies but give different extrapolations at higher energies. They may be classified into three types: (1) Geometric scaling models,[4] (2) Diffraction dominance models,[3] (3) Chou-Yang type models.[5]

The "Geometrical Scaling" models are those in which the interaction radius increases logarithmically with energy. The ratio σ_{el}/σ_{tot} is assumed to be energy independent. The rise in the total cross section comes from all three components, diffractive and inelastic processes.

The diffractive dominance models ascribe all the rise in total cross sections to σ_{el}, σ_{SD} and σ_{DD}, the "inelastic" cross section remaining constant. The ratio $\sigma_{inel}/\sigma_{tot}$ decreases with energy, σ_{el}/σ_t becomes energy independent, σ_{SD}/σ_{tot} slowly decreases with energy while σ_{DD}/σ_{tot} will be asymptotically constant.

In Chou-Yang type models the restriction of geometrical scaling that σ_{el}/σ_t = constant is released. However the detailed behavior of σ_{SD} or σ_{DD} is not generally prescribed.

This it may seem at first glance that with so much freedom it would be possible to fit the cosmic ray data with a large range of models. This is not so, however, because the range of variation of the diffractive component cross sections are limited by unitarity bounds. There is only one proviso for this statement which is that Glauber techniques are valid at these energies.

3. Unitarity Bounds and Limits on Diffraction: The unitarity bound on elastic and diffractive cross section can be stated as[6]

$$\sigma_{el} + \sigma_{Diff} \leq \tfrac{1}{2}\sigma_{tot} \qquad (2)$$

One can write σ_{tot} in terms of its parts

$$\sigma_{tot} = \sigma_{el} + \sigma_{Diff} + \sigma_{ND} \qquad (3)$$

where σ_{ND} is the non-diffractive cross section. It follows from these relations that

$$\sigma_{ND} \geq \tfrac{1}{2}\sigma_{tot} \qquad (4)$$

which means that if $\sigma_{tot}(E)$ increases with energy then σ_{ND} cannot be energy independent.

Assuming the general validity of the bounds we apply it to the specific model of diffractive dominance proposed by Goulianos. The energy variation of σ_{el}, $2\sigma_{SD}$, σ_{DD} and σ_{tot} for this model is shown in figure 1. The value of $\tfrac{1}{2}\sigma_{total}$ is also graphed. The unitarity bound given by equation (2) is violated by this model at $\sqrt{s}$ ~ 200 GeV so is the inequality $\sigma_{ND} \geq \tfrac{1}{2}\sigma_{tot}$.

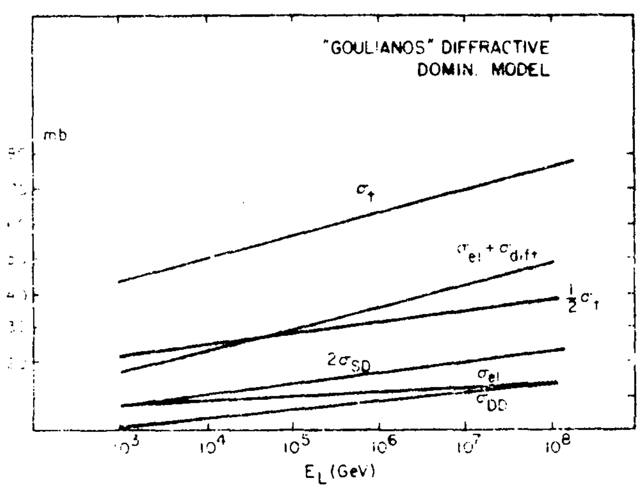

Figure 1: Predictions from a diffractive dominance model (Goulianos 1983) and its relation to unitarity bounds which require $(\sigma_{el} + \sigma_d)/\sigma_t$ to be less than or equal to 0.5

In fact any model which ascribes the increase in cross section entirely to diffractive processes will violate conventional unitarity requirements.

As representative of other models let us consider the model proposed by Block and Cahn. The model gives explicit fits for total and elastic cross sections as well as the slope parameter, (B), as a function of energy. To obtain $\sigma_{p\text{-}air}$ (inelastic) using equation (1) we must calculate the five terms on the right hand side using Glauber methods. The total, elastic and quasi-elastic proton-air cross sections can be calculated in straight forward manner using the model parameters: σ_{pp}^{tot}, σ_{pp}^{el}, B and ρ. To estimate σ_D and $\Delta\sigma$(inel-screening) a knowledge of the single diffractive cross section is needed, which is not given by the model. Of these two terms, $\Delta\sigma$(inel-screening) depends only on the value of $(d\sigma/dtdM^2)_{tmin}$ which varies as $1/M^2$.[2] Most of the contribution to this term comes from small values of M^2 and numerical evaluation gives a value for this correction which varies from about 8 mb at ISR energies to saturation at ~ 14 mb at ultra high energies.

The correction for diffractive dissociation of the target nucleon is given by

$$\sigma_D = (\sigma_{SD}^{PP}/\sigma_{inel}^{PP})\ \sigma_{P\text{-}air}^{1} \qquad (5)$$

Here $\sigma_{p\text{-}air}^{1}$ is the cross section for an absorptive -p-nucleus interaction involving exactly one elementary inelastic scattering encounter. It is easy to show that $\sigma_{p\text{-}air}^{1} \equiv 2/3\ \pi <r^2>$ which corresponds to 142 mb for a root mean square radius of 2.6 fermis. The correction depends on the energy dependence of $\sigma_{SD}^{pp}/\sigma_{inel}^{pp}$ where $\sigma_{inel}^{pp} \equiv \sigma_{tot} - \sigma_{el} = \sigma_{ND} + 2\sigma_{SD} + \sigma_{DD}$.

What does the unitarity bound tell us about the size of this ratio? From the unitarity bound in equation (2) gives

$$2\sigma_{SD} + \sigma_{DD} \leq \tfrac{1}{2}\ \sigma_{tot} - \sigma_{el} \qquad (6)$$

The maximum value of σ_{SD} then is obtained by putting $\sigma_{DD} = 0$. The ratio $\sigma_{SD}/\sigma_{inel}$ is bounded by

$$\frac{\sigma_{SD}}{\sigma_{inel}} \leq \tfrac{1}{4}\ [1 - \frac{\sigma_{el}/\sigma_{tot}}{1 - \sigma_{el}/\sigma_{tot}}] \leq 0.25 \qquad (7)$$

The maximum value of σ_D is 36 mb if $\sigma_{el} = 0$! In the Block-Cahn model corresponding to a $\ell n^2 s$ energy dependence[5] the ratio of σ_{el} to σ_{tot} varies from 0.175 to 0.37 as energy is varied from $\sqrt{s}$ ~ 20 GeV to $\sqrt{s}$ ~ 10 TeV, corresponding upper limits to σ_D are 28 mb and 15 mb respectively. At ISR, however, there are direct measurements of σ_{SD} and σ_{inel} which give a 14 mb cross section for σ_D. A reasonable measure of the allowed range of σ_D can be obtained by assuming that the lower bound to $\sigma_{SD}/\sigma_{inel}$ is the ISR value and the upper bound is given by equation (8) with σ_{el}/σ_{tot} being taken from Block and Cahn's model. The result is shown in figure (2). Also shown in the figure is the sum of the last three terms in equation (1), i.e. $\sigma_{qe} + \sigma_D + \Delta\sigma$(inel. screening). The

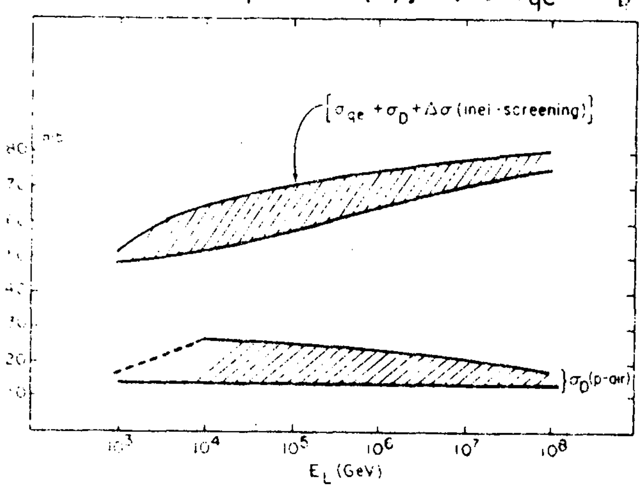

Figure 2: The allowed ranges for the correction for diffractive dissociation of the target nucleon. Also shown is the total correction as a function of energy.

fractional uncertainty in the value of $\sigma_{p\text{-}air}$(inelastic) is less than three percent.

In figure (3) we compare the predictions of the ℓns and $\ell n^2 s$ models of Block and Cahn with air shower cosmic ray cross sections.[7,8,9] It is seen that the data clearly favor the faster energy dependence.

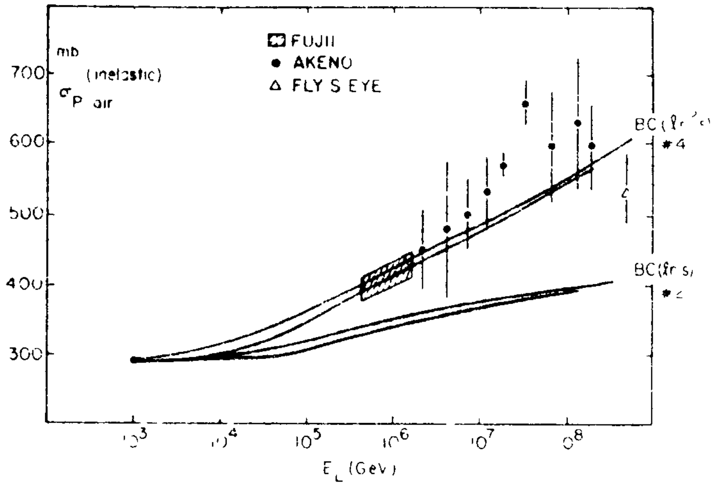

Figure 3: Comparison of predictions for $\sigma_{p\text{-}air}$ (inelastic) from two different models of Block and Cahn: No. 4 for which σ^t_{pp} increases as $\ell n^2 n$ and No. 2 which has a ℓns dependence

4. Conclusion: We have shown that if unitarity bounds on diffractive cross sections are valid at ultra high energies then (a) diffractive dominance models which ascribe the increase in total hadron-hadron cross sections to diffractive processes only are ruled out and (2) that cosmic ray cross sections derived from air shower experiments at ultra high energies clearly rule out models for hadron-hadron cross sections with ℓns energy dependence and favor those with $\ell n^2 s$ variation.

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