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1. Introduction. The forward-backward charged multiplicity distribution $P\left(n_{F}, n_{B}\right)$ of events in the $540 \mathrm{GeV} \overline{\mathrm{p}} \mathrm{p}$ collider has been extensively studied by the UA5 Collaboration. It was pointed out that the distribution with respect to $n=n_{F}+n_{B}$ satisfies approximate $K N O$ scaling and that with respect to $Z=n_{F}-n_{B}$ is binomial [l]. The geometrical model of hadron-hadron collision interprets [2] the large multiplicity fluctuation as due to the widely different nature of collisions at different impact parameters b. For a single impact parameter b, the collision in the geometrical model should exhibit stochastic behavior. This separation of the stochastic and non-stochastic (KNO) aspects of multiparticle production processes gives conceptually a lucid and attractive picture of such collisions [3], leading to the concept of partition temperature $T_{p}$ and the single particle momentum spectrum to be discussed in detail below.
2. Description of Model. Assuming the separation of stochastic from non-stochastic aspects of collision to remain valid as $n \rightarrow \infty$, we expect [l] that the distribution in the two-dimensional $\left(n_{F} / \bar{n}\right)-\left(n_{B} / \bar{n}\right)$ plane would become more and more concentrated in a narrow region. For 540 GeV $\bar{p} p$ collisions this region is in the form of an ellipse as shown in Fig. l(a). When $\bar{n}$ becomes large, it becomes thinner and eventually collapses into a line segment (Fig. l(b)). This line segment is a collection of points, at each of which $n_{F} \simeq n_{B}$ and both $n_{F}$ and $n_{B}$ fluctuate only to the extent of $\sqrt{n_{F}}$ (i.e., like a stochastic distribution). For example; if $n=2 \times 10^{6}$, then $n_{F}$ could easily be as small as $0.5 \times 10^{6}$ or as large as $2 \times 10^{6}$. But in either case, one can predict that $n_{B} \simeq n_{F}$ with fractional
errors of the order $(\overline{\mathrm{n}})^{-\frac{1}{2}} \sim 10^{-3}$.
Accepting this picture for very high energies, we see that for fixed $n_{F}$, the distribution of $n_{B}$ is stochastic. How then is the energy partitioned in the backward hemisphere? We shall assume that the energy partition for each hemisphere for a fixed $z=\left(n_{F}+n_{B}\right) / \bar{n}$ is also stochastic but subject to a number of conditions: (a) that the total energy of all outgoing particles on each side is $\mathrm{E}_{0} \mathrm{~h}$, (b) Bloch-Nordsieck factor $d^{3} p / E$ for each particle, and (c) transverse momentum ( $p_{T}$ ) cutoff factor $g\left(p_{T}\right)$. In other words, the probability distribution for central particles on each side will be taken as

$$
\begin{equation*}
\delta\left(\sum_{i} E_{i}-E_{0} h\right) \prod_{i}\left(d^{3} p_{i} / E_{i}\right) g\left(p_{T i}\right) \tag{1}
\end{equation*}
$$

where $E_{0}=\sqrt{s} / 2, E_{0}(1-h)=$ total energy of all leading particles, and i $=1,2, \ldots$ ranges over all the particles (positive, negative, and neutral) on one side minus the leading particles. $h$ is a parameter that describes the fraction of $\mathrm{E}_{0}$ that fragments into all particles in the central region.

Now the mathematical problem (1) is well-known in statistical mechanics as describing a microcanonical ensemble. By the well-known Darwin-Fowler method the single particle distribution of such an ensemble is given by the canonical ensemble:

$$
\begin{equation*}
\mathrm{dn}=\mathrm{K}\left(\mathrm{~d}^{3} \mathrm{p} / E\right) \mathrm{g}\left(\mathrm{p}_{\mathrm{T}}\right) \exp \left(-E / T_{\mathrm{p}}\right) \tag{2}
\end{equation*}
$$

where $T_{p}$ will be called the partition temperature and $K$ is a normalization constant. Notice that all particles, positive, negative, and neutral, kaons, nucleons as well as pions, share the same $T_{p}$.
3. Comparison with Experimental Angular Distribution at 540 GeV . As Fig. l(a) shows, at the $540 \mathrm{p} p$ collider, the distribution is still an ellipse. We shall nevertheless test the validity of (2) at 540 GeV by evaluating the single particle angular distribution from it. We write

$$
\begin{equation*}
d n / d \eta=2 \pi K \sin ^{2} \theta \int_{0}^{E_{0} h} p^{2}(d p / E) g(p \sin \theta) \exp \left(-E / T_{p}\right) \tag{3}
\end{equation*}
$$

where $\eta=$ pseudo-rapidity, $\cosh \eta=\csc \theta$, and

$$
\begin{equation*}
g(p \sin \theta)=\exp (-\alpha p \sin \theta) \tag{4}
\end{equation*}
$$



Fig. I Schematic diagram for for-ward-backward multiplicity distribution at very high energies. (a) The contour lines represent $P\left(n_{F}, n_{B}\right)$ at constant fractions of its ${ }^{\text {F }}$ maximum value at $\sqrt{s}=540 \mathrm{GeV}$ where $\overline{\mathrm{n}} \simeq 29$. (b) The same contour lines degenerate to straight lines for extremely large $\bar{n}$.


Fig. 2 Calculated and experimental $\mathrm{dn} / \mathrm{dn}$ vs. $\eta$ at $\sqrt{\mathrm{s}}=540 \mathrm{Gev}$.



Fig. 3 Calculated $d n / d \eta$ versus $\eta$ at $\sqrt{s}=2 \mathrm{TeV}$ and 40 TeV .

We take $\alpha$ to be equal to $5.25(\mathrm{GeV} / \mathrm{c})^{-1}$. Only pions are included in this calculation. The angular distribution is evaluated from (3) and compared with the results [4] of UA5 in Fig. 2. It is found that the UA5 curve for each multiplicity $n$ is well fitted by (3) for one value of $T_{p}$. We emphasize that there are no adjustable parameters in this computation, the cutoff $\alpha$ having been taken from experiments [5] concerning $p_{T}$ distributions. The parameter $h$ and normalization constant $K$ are both determined from the curves themselves.

We conclude that the angular distribution (3) that results from (2) is in excellent agreement with experiment. Furthermore, we believe (2) would give a complete description of the single particle momentum distribution for central particles.
4. Extrapolation to Higher Energies. We made extrapolations of the angular distribution to $\sqrt{\mathrm{s}}=2 \mathrm{TeV}$ (Tevatron) and 40 TeV (SSC). The assumptions made in these computations are as follows: (i) The values of $\alpha$ for these energies are taken to be 5.0 and $4.4(\mathrm{GeV} / \mathrm{c})^{-1}$, respectively [5]. (ii) The parameter $h$ is taken to be a function of the kNO variable $z=n / \bar{n}$ only. (iii) The values of $\bar{n}_{c h}$ for these energies are taken to be 41 and 78, respectively, by extrapolation. The results are presented in Fig. 3.
5. Acknowledgments. This work is supported in part by the U.S. DOE Grant No. DE-F'G09-84ER40160, and by the U.S. NSF Grant No. PHY 8109110 A01.

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