## EMPIRICAL DESCRIPTION OF THE HADRON - HADRON AND HADRON - <br> NUCLEUS INTERACTION AT THE ACCELERATOR ENERGY RANGE

G:Kubiak,J.Szabelski,J.Wdowczyk
Institute of Nuclear Science and Technology, 90-950 Lodz,ul Uniwersytecka 5,Box 447, Poland
J.Kempa and A.Piotrowska

Department of Physics, University of Lodz, 90-236 Lodz, ul Nowotki 149-153, Poland

## 1. INTRODUCTION

It has been demonstrated by Wdowczyk and Wolfendale (1984) that the energy spectrum of the secondary pions produced in $p-p$ interactions with energies up to a few times $10^{4} \mathrm{eV}$ can be described well by a scaling violation formula of the form

$$
\begin{equation*}
P\left(E_{\pi}\right)=\left(E_{0} / 205\right)^{\alpha} q\left(\left(E_{\pi} / E_{0}\right)\left(E_{0} / 205\right)^{\alpha}\right) \frac{k\left(E_{0}, 205\right)}{E_{\pi}}, \tag{1}
\end{equation*}
$$

where the parameter $\alpha$, which is a measure of the scaling violation,is gradually increasing with energy. The coefficient $k(E, 205)=K_{\pi}\left(E_{0}\right) / K(205)$ describes the variation of inelasticity coefficient for charged pion production.

The energy spectra of the others secondary particles can be obtained by similar transformations.

For purpose of numerical calculations the function $q(x)$ is usually taken from Fermilab data at 205 GeV and approximated by the following formula (for $\pi^{+}+\pi^{-}$)
$q(x)=.692(1-x)^{7.4} \exp (2.185 x)+.807(1-x)^{6.2} \exp (2.968 x)$,
which is obtained by integration over $p_{t}$ of the more general expression fitting simultaneously the $x$ and $p_{t}$ variation for the same Fermilab data at 205 GeV (Kafka et al 1977). The expression has the form

$$
\begin{align*}
q\left(x, p_{t}\right) & =9.624(1-x)^{6.2} \exp \left(-5.963 p_{t}^{2} \exp (-2.968 x)\right) \\
& +8.744(1-x)^{7.4} \exp \left(-6.318 p_{t}^{2} \exp (-2.185 x)\right) \tag{3}
\end{align*}
$$

As a reasonable choice for $\alpha$ and $k_{\pi}$ variation it is taken $\alpha=.815\left(\left(E_{0} / 205\right)^{0.135}-1\right) \operatorname{In}\left(E_{0} / 205\right)$, for $E_{0} \leqslant 7.710^{4} \mathrm{GeV}$ $\alpha=.072+.0163 \ln \left(\mathrm{E}_{0} / 205\right), \quad$ for $\mathrm{E}_{0}>7.710^{4} \mathrm{GeV}$.

$$
\begin{equation*}
k_{\pi t o t}\left(E_{0}, 205\right)=\left(E_{0} / 205\right)^{-.042}, k_{\pi \pm}\left(E_{0}, 205\right)=\left(E_{0} / 205\right)^{-.061} \tag{5}
\end{equation*}
$$

It is also assumed that formula (1) and (2) can be used for describing the energy spectra of kaons and baryons (antibaryons ). All those are taken together but relative role of baryons and antibaryons increases with primary energy acoording to the expression

$$
\begin{equation*}
F=1-.77\left(E_{0} / 205\right)^{-.043} \tag{6}
\end{equation*}
$$

It is assumed that the average energy of kaons and baryons ( antibaryons) increases with energy faster than that of pions and the relative inelasticity coefficient for production of those also increases with energy. The expression giving the ratio of the particles average energy and their inelasticity coefficient to those of pions are respectively

$$
\begin{equation*}
b=.45+.253 \ln \sqrt{E_{0}}, \quad R=1.81-1.5\left(E_{0} / 205\right)^{-.042} \tag{7}
\end{equation*}
$$

The interactions of mesons with protons are also assumed to be described by the same expressions except that the total inelasticity coefficient is taken to be 0.6 instead of 0.5 with total multiplicity the same and average energy increaesd by factor of 1.2 .

Taking into account all these assumptions formula (1) is transformed into the following expressions for kaon and baryon plus antibaryon production in proton interaction and for pion production in pion interactions

$$
\begin{align*}
& P(E)=\left(E_{0} / 205\right)^{\alpha} q\left(\left(E / b / E_{0}\right)\left(E_{0} / 205\right)^{\alpha}\right) \frac{R}{E},  \tag{8}\\
& P(E)=\left(E_{0} / 205\right)^{\alpha} q\left(\left(m / 1.2 / E_{0}\right)\left(E_{0} / 205\right)^{\alpha}\right) \frac{k\left(E_{0}, 205\right)}{E} \tag{9}
\end{align*}
$$

Combining both formulae we can obtain expression for the spectrum of kaons and baryons plus antibaryons produced in the meson interactions.
2. APROXIMATE FONMULAE FOR INTERACTIONS WITH AIR NUCLEI

For analysis of the cosmic ray propagation in the atmosphere in actual fact, instead of the formulae for interactions of protons and mesons with protons, we have to use formulae appropriate for interactions with air nuclei. Using the method outlined among others by Elias et al (1980) we have introduced simple corrections to the above given expressions to account for the fact that target is an air nucleus.

Similarly as Elias et al (1980) we have assumed that an interaction of hadron with nucleus is an superposition of subsequent interactions of leading nucleon. The nucleon interacting with nucleus is assumed to undergo with certain probabilities one, two or more interactions with nucleons in the nucl-
eus. The probabilities for the number of interactions used by us are given in the table I. The energy spectrum of the secondaries and the inelasticity coefficient distribution the case of one interaction are the same as for interactions with protons. When the number of interactions is higher those distributions are obtained by summing the contribution from subsequent interactions.

$$
\begin{equation*}
P_{n}\left(E, E_{o}\right)=P_{p p}\left(E, E_{o}\right)+\sum_{i}^{n} P_{p p}\left(E, E_{o}\left(1-K_{1}\right)\left(1-K_{2}\right) \cdots\left(1-K_{i}\right)\right), \tag{10}
\end{equation*}
$$

where $K_{i}$ is the inelasticity coefficient in the $i$ - th subsequent interaction.

In similar way the inelasticity coefficient for p-nucleus interaction can be obtained. Overall spectra of secondary particles can be easily obtained adding the contributions from various number of interactions with probabilities given in table I. The distribution of the inelasticity coeffioient for $p A$ collision can be obtained by Monte Carlo allowing the subsequent inelasticities in p-nucleon interactions to fluctuate according to the assumed fluctuations of the inelasticity coefficient in pp interactions.

Table I

Probability of an incident proton having $n$ collision within a carbon nucleus

| $n$ | 1 | 2 | 3 | 4 | 5 |
| :---: | ---: | ---: | ---: | ---: | ---: |
| $P_{n}$ | .511 | .301 | .131 | .044 | .013 |

Detailed calculations showed that the obtained energy spectra of the secondaries can be well represented by very simple modification of the expression (1) and also the expressions (8) and (9). The modified formula (1) becomes

$$
\begin{gather*}
P\left(E_{\pi}\right)=(F C)^{1.5}\left(\mathrm{E}_{0} / 205\right) \mathrm{q}\left(\left(\mathrm{E}_{\pi} / \mathrm{E}_{0}\right)\left(\mathrm{E}_{0} / 205\right) \mathrm{PC} \cdot 5\right) \frac{\mathrm{k}\left(\mathrm{E}_{0}, 205\right)}{\mathrm{E}_{\pi}}  \tag{11}\\
\mathrm{FC}=1.32
\end{gather*}
$$

The inelasticity coefficient distribution calculated by the outiined above method is consistent with an expression which we have developed (Kempa and Wdowczyk 1985) on the basis of experimental data compiled by Jones (1983).

The expression is
$P(K)=1.9 \exp (-2(1-K))$, for $K \geqslant .1$ and $P(K)=\sqrt{K}$, for $K<.1$.

Kempa and Wdowczyk (1985) gives also additional expression for the quasielastic interactions but those practically can be neglected here. In spite of the fact that they give almost $20 \%$ of the total cross - section their contribution to the flux of secondaries is negligible. Their role practically amounts to an increase of the effective mean free path by a factor of 1.2

Predictions based on the formula (11) are compared with data of Elias et al (1980) in figure 1 where the distributions of pseudorapidities are given for various assumptions about relation of between $p_{\text {tpa }}$ and $p_{t p p}$. It seems that the best val-
ue is obtained if we take

$$
\left\langle\mathrm{p}_{\mathrm{tpA}}\right\rangle=1.15\left\langle\mathrm{p}_{\mathrm{tpp}}\right\rangle
$$

The above described picture of hadron-hadron interaction have been used in our calculations of cosmic ray propagation in the atmosphere presented elsewhere in this proceedings.


Figure 1
REFERENCES
Elias J E et al 1980 Phys.Rev.D, p. 13
Jones L W 1983 18th ICxC,Bangalore,5, p. 17
Kafka $T$ et al 1977 Phys. Rev.D, 16, p. 1261
Kempa $J$ and Wdowczyk $J 1985$ Presented to the 1st Symposium on Underground Physics,Italy, Saint-Vincent,25-28 April
Wdowczyk $J$ and Wolfendale AW 1984 J.Phys.G:Nucl.Phys.10, p. 257

