THE MODEL OF INDEPENDENT PARTICLES EMISSION IN THE MULTIPARTICLE PRODUCTION THEORY

V.V.Uchaikin, V.A.Litvinov
Altai State University, Barnaul, USSR

ABSTRACT

It is shown, that it is possible to obtain the degree dependence of average multiplicity from energy, KNO-scaling and the effect of leading , taking into account the law of energy preservation.

The following model of multiple process is offered for discussion. Particles are born independently of each other from some distribution $f(E)$, but a chance set of $E_1, \dots, E_n$ turns into realization of the multiple process only under the condition, that the sum

$$S_n = \sum_{i=1}^{n} E_i$$

will fall into the small interval ΔE_0 about E_0 (primary energy). The density of distribution $f(E)$ is assumed to be independent of E_0 .

The distribution of the thus obtained events on multiplicity n will be:

$$P_n = Prob\{S_n \in \Delta E_0\} / \sum_n Prob\{S_n \in \Delta E_0\} = f_n(E_0) / \sum_n f_n(E_0), \qquad (I)$$

where

$$f_n(E) = \underbrace{f(E) * f(E) * \dots * f(E)}_{n}$$

is fold single particle distributions. From the theory of probabilities it is known that if the second distribution $f(E)$ moment exists, then the sum S_n is distributed according to the normal law:

$$f_n(E) \sim \frac{1}{\sqrt{2\pi n}\,\sigma} \exp\left\{-\frac{(E - n\bar{E})^2}{2n\sigma^2}\right\}, \quad n \to \infty, \qquad (2)$$

where

$$\bar{E} = \int_0^\infty E f(E) dE , \quad \sigma^2 = \int_0^\infty E^2 f(E) dE - \bar{E}^2 .$$

It is clear from here that the maximum of distribution falls on

$$\tilde{n} \sim E_0 / \bar{E} \tag{3}$$

and the relative distribution width is

$$\Delta n / \tilde{n} \sim (\sigma / \bar{E}) \tilde{n}^{-1/2} . \tag{4}$$

Thus, in this case we have the linear increase of the average multiplicity and the decrease of relative fluctuation according to the Poisson's law - both are inconsistent with the character of multiple processes.

Another case is discussed in the theory of probability too: when no second and even first moments of distribution $f(E)$ exist [1,2].

If that satisfies the condition

$$\int_E^\infty f(E') dE' \sim a E^{-\alpha} , \quad E \to \infty , \quad \alpha \in (0,2), \tag{5}$$

then instead of (2) there is a tendency to the stable law with the index α and parameter $\beta = 1$ (for positive random values)

$$\mathrm{Prob} \left\{ \frac{S_n}{B_n} - A_n < x \right\} \Rightarrow G(x, \alpha, 1),$$

here

$$B_n = [\alpha a n]^{1/\alpha}$$

and A_n is determined by the index α . For $\alpha > 1$ the first moment exists and can be used for centering consequence $A_n = n\bar{E}/B_n$, for $\alpha = 1$ $A_n \sim \ln n$ and for $\alpha < 1$ $A_n = 0$ [3].
Using this result for each of these cases, we obtain:

$$\alpha > 1 : \quad \tilde{n} \sim E_0 , \quad \Delta n / \tilde{n} \sim \tilde{n}^{\frac{1}{\alpha} - 1} ; \tag{6}$$

$$\alpha = 1 : \quad \tilde{n} \ln \tilde{n} \sim E_0 , \quad \Delta n / \tilde{n} \sim [\ln \tilde{n}]^{-1} ; \tag{7}$$

$$\alpha < 1 : \quad \tilde{n} \sim E_0^{\alpha} , \quad \Delta n / \tilde{n} \to const \neq 0 . \tag{8}$$

From here it is clear, that at $\alpha \in (1,2)$, when only the second moment doesn't exist $\tilde{n}$ is proportional E_0 , although the relative fluctuations decrease at a slower rate; at $\alpha = 1$

when the first moment ceases to exist, the linear dependence $\bar{n}(E_0)$ is violated and the decrease of fluctuations is slow, untill finally at $\alpha<1$ the dependence of multiplicity from E acquires a degree character (with index α), and the distribution width of quantity ($n/\bar{n}$) stops to change with the increase of energy. Thus, at $\alpha<1$ we find the characteristic features of the multiple processes.

At $\alpha=1/2$, the analytical form of density of the stable law is

$$g(x,\alpha,1) = G'(x,\alpha,1) = \frac{1}{2\sqrt{\pi}}\, x^{-3/2} e^{-1/4x}$$

leading to one of the known approximation KNO-distribution

$$\psi(z) = \frac{\pi z}{2}\, e^{-\pi z^2/4}$$

and the average multiplicity $\bar{n} \sim E_0^{1/2}$, which doesn't disagree the experiment (if E_0 means the energy of the particles in the center-of-mass system).

For more detailed study of the given model, a Monte-Carlo simulation was carried out with the distribution function:

$$f(E) = \begin{cases} \alpha E^{-\alpha-1}, & E \geqslant 1, \\ 0, & E < 1, \end{cases}$$

$\alpha = 1/4, 1/2, 1, 2, 4.$

Results are shown in fig.1-3. Besides the confirmation of the above theoretical conclusions (3-8), it is clear from these calculation, that at $\alpha<1$ there exists a particle separated by energy, which carries away about a half all the energy of the born particles (effect of leading), and the distribution along the variable $z=n/\bar{n}$ has the typical form of KNO distribution and doesn't depend from primary energy (KNO-scaling). This circumstance, in particular, may be used for the working out of the simple algorithm for simulation the act of multipartical production by Monte-Carlo method in calculations of extensive air showers and perhaps, is of interest to the theory of multiparticle production.

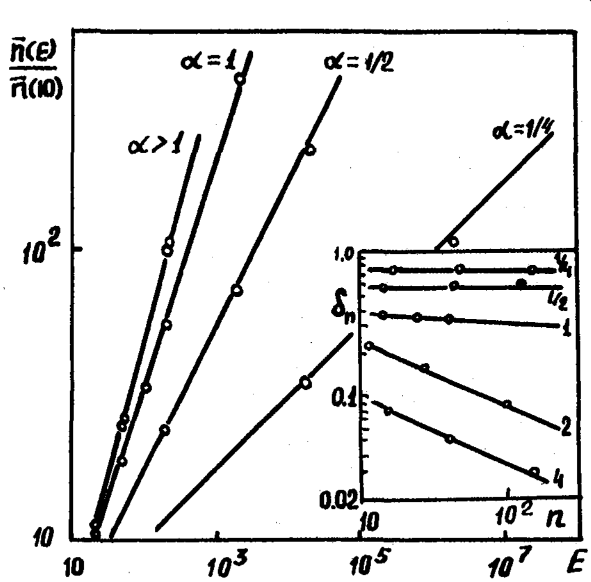

Fig.1.Average multiplicity and relative fluctuations.

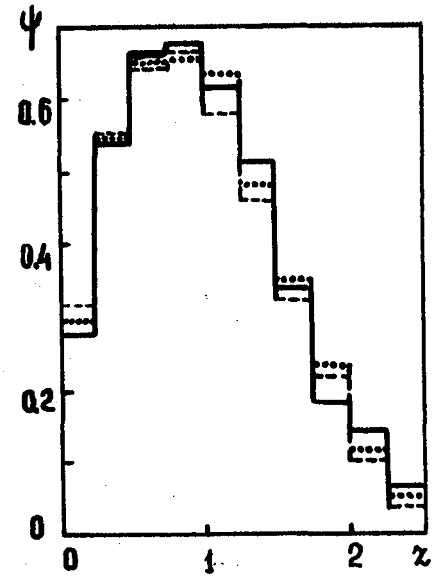

Fig.2.Distributions of multiplicity for E_{01}, $E_{02} = 10E_{01}$, $E_{03} = 100E_{01}$ ($\alpha = 1/2$)

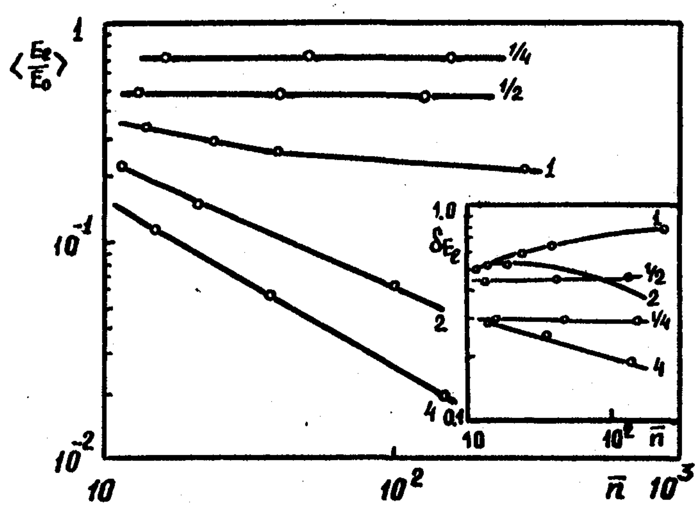

Fig.3.Average value and relative fluctuations of the leading particle energy.

References:

1. Gnedenko B.V., Kolmogorov A.N. Limit distributions for sums independence random values. Moscow, Gostehizdat, 1949, 264 p.
2. Zolotarev V.M. One dimensional stable distributions. Moscow, Nauka, 1983, 304 p.
3. Borovkov A.A. The theory probability lectures. Moskow, Nauka, 1972, 287 p.