ANALYTICO-NUMERICAL METHODS OF CALCULATIONS OF ENERGY AND THREE-DIMENSIONAL PARTICLE DISTRIBUTIONS IN ELECTROMAGNETIC CASCADES

Ivanenko I.P., Kanevsky B.L., Roganova T.M., Sizcv V.V., Triphonova S.V.

Institute of Nuclear Physics, Moscow State University, Moscow 119899, USSR

The paper reports on analytical and numerical methods of calculation of the energy and three-dimensional EPS characteristics. The angular and lateral functions of electrons in EPS have been obtained to the Landau and small angle approximations A and B and compared with the earlier data /1,2/. A numerical method of solution of cascade equations for the EPS distribution function moments has been constructed. Considering the equilibrium rms angle as an example, we analyse errors appearing when approximating the elementary process cross sections by their asymptotic expressions.

1. Analytical method of solution of a lateral problem for EPS; In /3/, an analytical method to solve ajoint equations for the electron LDF in EPS has been proposed. The method allowed obtaining the accurate moments of LDF and the function itself. The energy-integral LDF being represented as the Mellin inverse transformation

$$N(E_0,>E,t,\rho)=\frac{1}{2\pi i}\int ds (E_0/E)^s \frac{1}{s} H_1(s) e^{\lambda_1(s)t} P_{n_2}(s,\tilde{\rho}), \qquad (1)$$

where $\tilde{\rho}=\rho E/E_k$, the structure function can be presented as the infinite power series

$$P_{n_2}(s,\tilde{\rho})=P^{(0)}+(P^{(1)}-P^{(0)})+(P^{(2)}-P^{(1)})+\dots \qquad (2)$$

Δ is written in the form of a linear combination of the Meier G-functions

$$\Delta = P^{(n)}-P^{(n-1)}=\sum_{k=0}^{2n} C_k(s)\, G^{60}_{26}\left(f(s)\tilde{\rho}^2 \,\middle|\, \begin{matrix} a_p(s) \\ b_q(s) \end{matrix}\right), \qquad (3)$$

where s is the shower age parameter, the first 2n-2 lateral moments of the function transform, $N^{(n)}$, identically coincide with exact function moments.

Proceeding from the properties of Meier G-functions/4/, one can determine the asymptotic behaviour of P_{n_2} at small and large core distances

$$\begin{aligned} &\tilde{\rho}\to 0 \quad P_{n_2} \sim \tilde{\rho}^{\,2-s} \\ &\tilde{\rho}\to\infty \quad P_{n_2} \sim \exp\left[-4f^{1/4}\sqrt{\tilde{\rho}}\right]\tilde{\rho}^{\,2-s+\frac{1-s}{4}}\sum_{k=0}^{\infty} M_k(s)\,\tilde{\rho}^{-k/2} \end{aligned} \qquad (4)$$

2. Asymptotic method of calculation of the EPS lateral-angular functions. The method proposed consists in reducing the solution for angular and lateral functions to a (3)-type contour integral based on the known asymptotics of the solution and calculation of the latter by a saddle-point method. The angular and lateral parts of a cascade equation solution in approximation A can be presented in

the form /2/

$$P(s,\theta)=\frac{1}{4\pi^2 i}\int dp\,\Gamma(p+1)\,m_\theta(p,s)\left(\frac{\theta E}{E_K}\right)^{-2p}\frac{1}{\theta^2} \qquad (5)$$

$$p(s,z)=\frac{1}{4\pi^2 i}\int dp\,\Gamma(p+1)\,m_z(p,s)\left(\frac{zE}{E_K}\right)^{-2p}\frac{1}{z^2}$$

The asymptotics of angular and lateral functions(5) at $p\to\infty$ $(z\to\infty)$ and $t\gg 1$ reads, respectively

$$m_\theta(p,s)\sim\Gamma(p+1)\,a_\theta(s)^{-2p}\;;\quad m_z(p,s)\sim\Gamma(3p+1)\,a_z(s)^{-2p} \qquad (6)$$

In the region $\mathrm{Re}\,p<0\,(z\to 0)$ the energy-integral angular and lateral distribution functions have the closed singularities

$$1/p+1,\quad 1/p+s/2 \qquad (7)$$

Patching of the two asymptotics for the angular and lateral functions yields, respectively:

$$m_\theta(p,s)\sim\frac{\Gamma(p+1)\Gamma(p+s/2)}{\Gamma(s/2)}\,a_\theta(s)^{-2p}\;;\quad m_z(p,s)\simeq\frac{\Gamma(2p+2)\Gamma(2p+s)}{\Gamma(s)}\cdot a_z(s)^{-2p} \qquad (8)$$

The integral in (5) is then represented as a product of singularities by some function $G(p,s)$ without singularities in the region $-s/2<\mathrm{Re}\,p<\infty$.

The function $G(p,s)$ is developed into Taylour series in the vicinity of a point p_o

$$G(p,s)=G(p_o,s)+(p-p_o)\,G'(p_o)+(p-p_o)^2/2\;G''(p_o)+\ldots \qquad (9)$$

and a term with the first derivative $G(p_o)$ is required to be zero. From this condition we find a point "p_o" and calculate integrals (5):

$$p(s,\theta)=K_{(2-s)/2}(2a_\theta z_\theta)(a_\theta z_\theta)^{\frac{2+s}{2}}G_\theta(p_o,s)+\ldots;\quad P(s,z)=K_{2-s}(2\sqrt{a_z z_z})(a_z z_z)^{\frac{2+s}{2}}G_z(p_o,s) \qquad (10)$$

where

$$p_0^{(\theta)}=-1+K_{(3-s)/2}(2a_\theta z_\theta)\,a_\theta z_\theta/K_{(2-s)/2}(2a_\theta z_\theta);\quad p_o^{(z)}=-1+K_{3-s}(2\sqrt{a_z z_z})\sqrt{a_z z_z}/K_{2-s}(2\sqrt{a_z z_z}) \qquad (11)$$

The expression for the angular part of the distribution function at $G=1$ coincides with the Belenky's function/1/. However, in the expression for $a_\theta(s)$ in /1/ $a=\sqrt{f(\lambda)}/\lambda$ at $\lambda=\lambda_1(s)$. In the present case, $a_\theta(s)$ is determined from the condition of coincidence of first moments of angular distribution function (9) with a point. If the functions are normalized to the first moment, an accuracy of the functions obtained from (9) is considerably better, and in the core approximation region $(z\to 0)$ these functions practically coincide. At $-s/2<p<3$ $G_\theta(p)\sim 1$, and the term with second derivative is negligible. A parameter $a_z(s)$ for the lateral distribution function is determined from a condition of coincidence of the second moments with a point and approximation function. In the integer points$(p=n)$ the function $G(n,s,t)$ is thought to be known, since the lateral function moments in approximation A are known. In a wide range of the variable $-s/2<p_o<3$, $G_z(p_o)$ varies slightly and a term with $G_z''(p_o)$ can be neglected. Table 2 compares lateral distribution function (9) with Kamata-Nishimura's calculations /2/. Within 5-10%, even zero approximation yields good results. The lateral-angular distributions can be similarly constructed in cascade theory approximation /5/.

3. <u>Numerical method of solution of cascade theory problems for EPC</u>. To solve analytically the cascade theory problems, simplified expressions for elementary process cross secti-

ons are used. The influence of these simplifications on the results of calculations is considered for an "equilibrium" rms angle. A numerical method of cascade equation solution has been reported in /6/. For showers induced in lead and air by a primary electron, Fig.1 shows the rms angles of electrons, (see the dashed curves) devided by the Landau analytical expressions /7/ for these magnitudes(see their table below the figure). The top of Fig.1 presents the values of $\overline{\theta^2_{\rho r}}$ obtained for scattering described in the Landau approximation, in the standard theory approximation A and in approximation C allowing for ionization losses, realistic expressions for bremsstrahlung and pair production cross sections and the Compton effect. The values of $\overline{\theta^2_{\rho r}}$ obtained to Approximation C for air and lead considerably differ from those in Approximation A and from each other. At the bottom of Fig.1, the curves calculated to Approximation A for scattering described realistically are shown - the Thomas-Fermi equation was taken to present atomic potential, a nucleus being considered to be a uniformly charged sphere. By comparing these curves with curves (A) at the figure top a physical accuracy of the Landau approximation can be estimated. On the whole, it can be stated that the differential angular characteristics are sensitive to the details of a description of elementary interactions in the energy range, where their cross section differs from the asymptotic values.

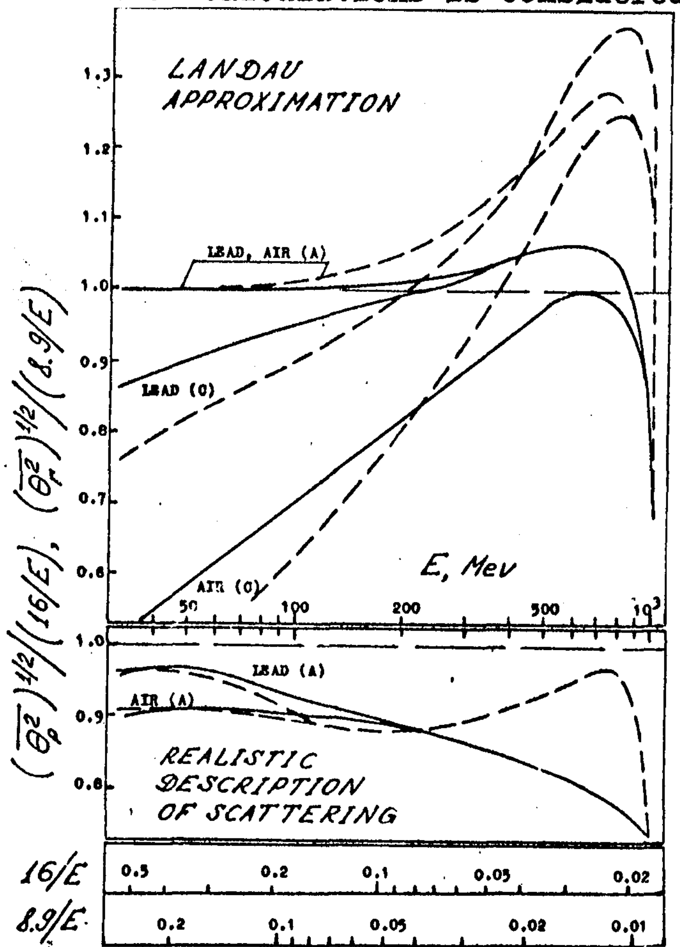

Fig.1

References.

1. Belenky S.Z. Shower processes in cosmic rays, M.: Gastechizdat, 1949, 243p.
2. Nishimura J.N. Handbuch der Physic, Berlin, Springer Verl., 1967, 46/2, 1-113.
3. Roganova T.M., Triphonova S.V., Proc. 18th ICRC, 1983,

5, 32.

4. Luke J.L. The special functions and their approximations, New York, 1969.
5. Ivanenko I.P., Kanevsky B.L. Asymptotic methods of solution of lateral-angular problems in cascade theory (in print).
6. Ivanenko I.P., Ivanova E.V., Maximenko V.M., Sizov V.V. Preprint FIAN, 1985.
7. Landau L.D. JETP, 1940, 10, 1107.

Table 1

The ratio of moments of exact and approximate function for normalization to the first moment and at

	$s = 0.6$		$s = 1$		$s = 2$	
1	1	0.82	1	0.87	1	0.95
2	0.90	0.60	0.92	0.67	0.96	0.83
3	0.82	0.41	0.87	0.48	0.90	0.60

Table 2

The EPS lateral function in cascade theory Approximation A. Comparison with the Nishimura-Kamata's calculation /2/ for the zero approximation, parameter $a_p(s)$ being normalized to the second moment

$s \backslash \tilde{\varrho}$	0	0.01	0.03	0.1	0.3	1.0	3.0
0.6	0.38 0.38	0.33 0.34	0.26 0.27	0.15 0.17	--- 0.05	0.04 0.04	0.00006 0.00009
1	1.12 1.01	0.91 0.91	0.71 0.70	0.4 0.38	--- 0.15	0.017 0.019	0.00050 0.00065
1.4	2.5 2.5	1.8 1.9	1.31 1.35	0.72 0.69	--- 0.27	0.038 0.041	0.0016 0.0019
	0.53 0.54	3.0 2.7	1.9 1.8	0.94 0.99	--- 0.37	0.060 0.060	0.0040 0.0037