A NEW METHOD OF DIFFERENTIAL STRUCTURAL ANALYSIS
OF γ -FAMILY BASIC PARAMETERS

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ABSTRACT

The maximum likelihood method is used for the
first time to restore parameters of electron-
photon cascades registered on X-ray films.
The method allows to carry out a structural
analysis of the γ -quanta family darkening
spots independent of the γ -quanta overlapping
degree and to obtain maximum admissible accu-
racies in estimating the energies of the γ -
quanta composing a family. The parameter esti-
mation accuracy weakly depends on the value of
the parameters themselves and exceeds by an
order the values obtained by integral methods.

In our previous papers /1,2/ we have proposed for the
first time to use the maximum likelihood method for the
solution of the traditional inverse problem of X-ray emul-
sion experiments. The essence of the method consisted in
the reconstruction of source parameters (the energy E,
zenith and azimuth angles Θ and φ , the spot center co-
ordinates X, Y) of the darkening spot on the X-ray emul-
sion film according to the matrix of darkening densities
$\{\rho_{ij}\}$ measured by a densitometer in the cells $(i,j) i =$
= 1, ..., N_i , j = 1,..... N_j . The scanning cell dimen-
sions ΔX , Δy and the number of cells $n_i \cdot n_j = N$ are
fixed. It has been shown in /1,2/ that the maximum likeli-
hood method gives consistent, efficient and nonshifted
estimates for the parameters E, Θ, φ, X, Y in the wide range
of scanning cell dimensions $\Delta X = \Delta y = 10,20...50 \mu$ km. The
accuracy of parameter estimation slightly depends on the
value of the parameters themselves and exceeds by an order
the values obtained by the momenta method and by integral
methods.
　　The results of /1,2/ are obtained for the darkening
spots caused by a single γ -quantum (electron) with the
energy E in the range 2 ÷ 100 TeV and zenith angle
in the interval 1 ÷ 0.5.
　　In /3/ the results of solution of the inverse problem
for the general case - the number of the darkening spot

sources (γ-quanta) is greater than 1 with the strongly marked structure of the spot overlapping, are given.

Remaining within the framework of notations accepted in /1,2/, one may present the likelihood function for the realized density matrix $\{\rho_{ij}\}$ at the unknown but fixed vector of parameters $\vec{\eta} = \eta(E_1, \dots E_n, X_1, \dots X_n, Y_1, \dots Y_n, \Theta, \Psi)$ in the form

$$\phi(\{\rho_{ij}\} | \vec{\eta}) = \prod_{i=1}^{Ni} \prod_{j=1}^{Nj} f_\ell(n_{ij}/\vec{\eta}), \qquad (1)$$

where $n_{ij} = \rho_{ij} \cdot S$ is the number of electrons in the cell (i, j); $S = \Delta x \cdot \Delta y$ is the cell area; N_i, N_j is the corresponding number of scanning cells along the X and Y axes; $f_e(n_{ij}/\vec{\eta})$ is the probability of n_{ij} electrons having been registered in the cell (i, j) at the given $\vec{\eta}$ vector of parameters; $\ell = 1$, if $n_{ij} < m_{min}$; $\ell = 0$, if $n_{min} \leq n_{ij} \leq n_{max}$; $\ell = 1$, if $n_{ij} > n_{max}$. The boundary values of the electron number are defined by the operation properties of the film and are chosen by us as in /1-3/ $\rho_{max} = n_{max} / S = 0.5 \mu km^{-2}$, $\rho_{min} = n_{min} / S = 0.002 \mu km^{-2}$. The distribution function $f_o(n/\vec{\eta})$ is assumed to be Poisson with the mean value $\bar{n} = \sum_{m=1} \bar{n}_m + \bar{n}_\phi$, where n_γ is the number of γ-quanta (electrons), $\bar{n}_m(E_m, X_m, Y_m, \Theta, \Psi)$ is the average number of electrons in the cell (i, j) generated by the source with the energy E_m, whose trajectory crosses the film at the point X_m, Y_m at the zenith and azimuth angles Θ and Ψ respectively; $\bar{n}_\phi$ is the average number of "background" electrons ($\rho_\phi = \bar{n}_\phi / S = 0.04 \mu km$).

The determination of the $\vec{\eta}$ vector components is performed numerically by minimizing the likelihood function negative logarithm:

$$\Omega_o \equiv \min \Delta(\vec{\eta})_{n_\gamma} \equiv \min \left\{ -\ell n \, \phi(\{\rho_{ij}\} / \vec{\eta}_{n_\gamma}) \right\} \qquad (2)$$

The inverse problem may also be solved in the case when the number of darkening spot sources n_γ is unknown. In this case one should minimize the functional

$$\Omega_1 \equiv \min \left\{ \min \Delta(\vec{\eta}) \right\}_{n_\gamma}, \quad n_\gamma = n_o, n_o + \Delta n, \dots, n', \qquad (3)$$

where n_o and n' determine the boundaries of search for the darkening source number.

The inverse problem was solved by the maximum likelihood method basing on the density matrix $\{\rho_{ij}\}$ generated by the Monte-Carlo method at the given vector of parameters $\{\rho_{ij}\}$. The average number of equivalent electrons in the cell (i, j) was calculated according to the cascade theory axial approximation. Fluctuations were assumed to be Poisson-distributed. The BESM-6 computer main memory allowed one to analyze the rasters with the cell number $N_i \cdot N_j \leq 7000$ and the summary number of unknown parameters $3(n_\gamma + 1) \leq 99$.

Consider the possibilities of the elaborated method at

1) $n_\gamma = 1$, 2) $n_\gamma = 2$, that is of interest for calibrating the emulsion technique on $\pi^\circ \to 2\gamma$ decays; 3) $n_\gamma > 2$ for processing γ-families; 4) $n_\gamma = ?$, i.e. when the number of the darkening spot sources is the component of parameter vector and is unknown.

1. $n_\gamma = 1$

The simulated rasters parameter reconstruction was carried out for discrete series of γ-quanta energy ($E_\circ = 2,5,10,20,50,100$ TeV) $\Delta x = \Delta y = 10,20 \mu$ km cell dimensions and various thicknesses of absorbers (d).

The relative accuracy of estimates of E, Θ, φ made a few per cent. The accuracy of the definition of the spot center coordinates is $2-5 \mu$ km.

2. $n_\gamma = 2$

For the darkening spot pair the calculations were performed at various energies $E_1 = 2 \cdot E_2 (2,4,5,10,20$ TeV) and distances between the spot centers $\mathcal{D} = (0.1 \div 100 \mu$ km) at the scanning step of $10,20,50 \mu$ km.

The results may be formulated as follows:
a) the accuracy of determining the zenith Θ and azimuth φ angles differs insignificantly from the single spot results and makes $\sigma_{\cos\Theta} \leq 5 \div 7 \%$;
b) the accuracy of determining the spot center coordinates is $\sigma_x \approx \sigma_y \leq \Delta x/2$, the distances between centers are $\sigma_{\mathcal{D}} \approx \Delta x$;

c) the accuracy of determining the energy for the overlapping spots is not worse than 10% at the summary energy reconstruction accuracy of 5-7%.

3. $n_\gamma > 2$

For the overlapping families of γ-quanta the inverse problem was solved for the energies distributed uniformly in the 2-20 TeV interval. The number of γ-quanta was varied in the region 2-20 over the area 0.25 mm.

The accuracies of determining the parameter vector components at $3 \leq n_\gamma \leq 10$ were independent of the n_γ number and on the average equalled the corresponding values at $n_\gamma = 2$.

The accuracy of determining the zenith angle cosine made $4 \div 5 \%$ and also was independent of n_γ.

4. $n_\gamma = ?$

The n_γ value varied in the interval $1 \div 5$ due to the complexity of solving the problem at the unknown number of the darkening spot sources.

No noticeable variation of the estimation accuracy was observed. Note also that at the known values of $n_\gamma \leq 10$ the efficiency of solving the inverse problem equalled $90 \div 100 \%$. At the unknown n_γ the efficiency decreased to 80%, and the computer counting time grew 5 - 7 times.

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