MUON GROUPS AND PRIMARY COMPOSITION AT 10^{13}-10^{15} EV

E.V.Budko,A.E.Chudakov,V.A.Dogujaev,A.R.Mihelev,
V.A.Padey,V.A.Petkov,P.S.Striganov,O.V.Suvorova,
A.V.Voevodsky

Institute for Nuclear Research,the USSR Academy
of Sciences,Moscow

ABSTRACT

The data on muon groups observed at Baksan underground scintillation telescope is analysed.In this analysis we compare the experimental data with calculations, based on a superposition model in order to obtain the effective atomic number of primary cosmic rays in the energy range 10^{13}-10^{15}ev.

1.Introduction. Our general approach to the problem was presented in Kyoto (1),and the first experimental data from Baksan in (2).The first attempt to analyse Baksan multiplicity spectrum in terms of primary composition was made in Bangalore (3).Now we have to correct one of conclusions made in (3) concerning effective atomic number.The change came for two reasons:1)now a smaller zenith angle interval was accepted $0° \leqslant \theta < 30°$,corrections for individual events inside this interval were made.This affected the obtained decoherence curve,making it 20% narrower,2)the second reason is the choice of more reliable function $N_\mu=F(E_0)$ which represents the mean number of muons from primary nucleon of energy E_0.

2.Experiment. The observed multiplicity or muon number spectrum is shown in fig. 1.It is practically the same as in (3) the difference $m^{-3.4}$ or $m^{-3.3}$ being not significant.All the difference with (3) is in calculated curves.The experimental decoherence curve is shown in fig.2.It deviates slightly from pure exponential one with parameter (12 ± 1) m.The decoherence curve is used to choose the parameters of assumed lateral distribution functions (LDF) in such a way as to fit the experimental curve, $D(R)$. $F_1(r) \div F_4(r)$:examples of tried LDF.
   Then the important function $\triangle(R)$ (see below) does not depend much on the choice of the type of LDF.

3.Calculations of expected muon number spectra for a given primary atomic number A. To do this following assumption have been made:
1.the muon group from A is a superposition of A independent groups from constituent nucleons,
2.LDF does not depend on A and energy per nucleon E_0,
3.fluctuations of the muon numbers are pure Poissonian,
4.the power law energy spectrum of primaries of a given A,
5.the mean number of muons of energy $E_\mu \geqslant E_{th}$ from primary nucleon of energy E_0 was taken as $f(E_0)=0.355(E_0^{0.47}-0.55)^{1.55}$

for $E_{th}=0.22$ Tev, E_0 in Tev on the basis of numerous Monte-Carlo simulations of cascades in atmosphere (5,6).

The problem is presented by a system of equations:

$$N = A * f(E_0) \qquad /1/$$

$$N' = N * \triangle(R) \qquad /2/$$

$$P(E_0)dE_0 \sim E_0^{-\gamma-1}dE_0 \qquad /3/$$

$$J_A(m) = \int_0^\infty e^{-N'} \frac{(N')^m}{m!} \varphi_A(N')dN' /4/$$

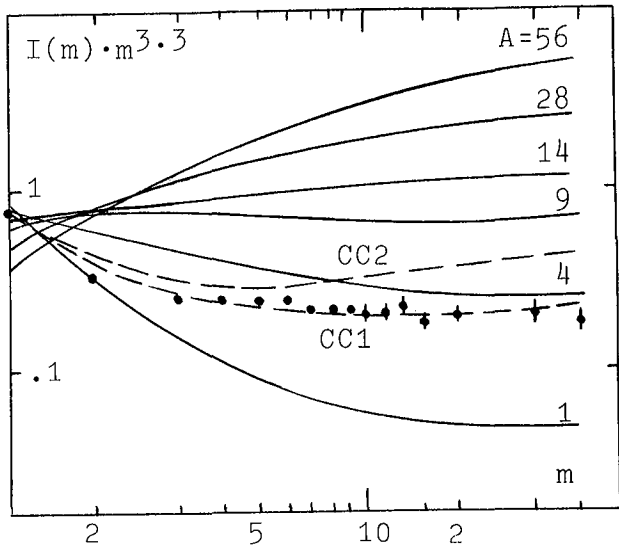$$I_A(m) = J_A(m)/\sum_{m=1}^\infty m * J_A(m) /5/$$

Fig. 1

Here: N is the mean number of muons $E_\mu > E_{th}$ from nucleus A with total energy $A \cdot E_0$; $\triangle(R)$ is the fraction of N which is expected to be inside detecting area when the core of group is at the distance R from telescope centre ($\triangle(R)$ is calculated using assumed LDF); $\varphi_A(N')$ is distribution of N' (expected numbers inside area), which is obtained by solving eqs./1-3/; $J_A(m)$ is the intensity of multiplicities m for a given A; $I_A(m)$ the same, but normalized to the total muon flux through detecting area. The resulting from these calculations $I_A(m)$ are shown in fig. 1 for several A and mixed composition. Obviously, these results depend on $\gamma=1.7$ (which is in agreement with experiment) and on functions $f(E_0)$ and $\triangle(R)$.

4. Discussion. To demonstrate the sensitivity of A_{eff} to assumed functions $f(E_0)$ and $\triangle(R)$ let us use "N_0-approximation", introduced in (1), valid for $m \gg 1$; which consists of following:
1. at large N $f(E_0) \sim E_0^\delta$, then $\varphi_1(N') \approx C * (N')^{-\gamma/\delta-1}$ at large N',

Fig. 2

2. suppose this power law distribution is valid for $N' > N_0'$ and is cut to zero for $N' < N_0'$; determine N_0' by

$$\int_0^\infty \varphi_1(N') N'dN' = \int_{N_0'}^\infty C * N' * (N')^{-\gamma/\delta-1}$$

so providing the correct total number of muons.

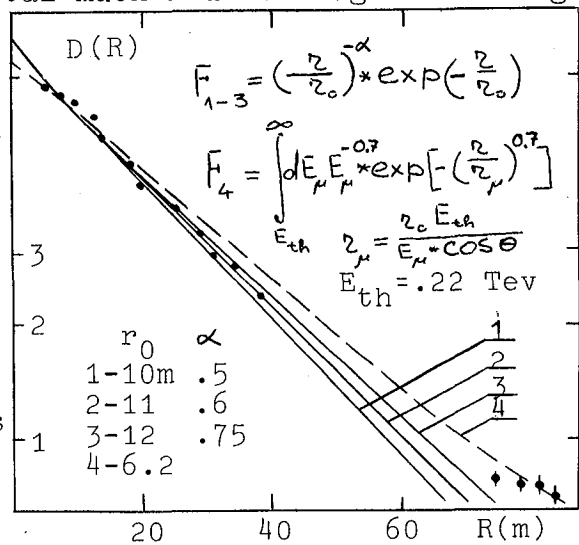In a similar way we determine N_0 which corresponds to the

case $\Delta(R)=1$. Fig.3 shows the original $f(E_0)$ (5) - the depen-
dence of number of muons N,
$E\mu>220$ Gev on the primary
nucleon energy E_0 and its
power law version in "N_0-
approximation".The total
number of muons after integ-
rating over primary spect-
rum should be the same for
both versions.Substituting
the functions $f(E_0)$ and
$\varphi_A(N')$ in /1/ and /4/ by
power law versions using the
mentioned procedure we ob-
tain a simple solution of
eqs. /1-5/:

$$f(E_o)=0.355(E_o^{0.47}-0.55)^{1.55}$$

$$E_{th}=0.22 \ Tev$$

$$E_0 (Tev/n)$$

Fig. 3

$$\mathbb{I}_A(m)=\frac{\gamma-\delta}{\delta}(N_o')^{\frac{\gamma-\delta}{\delta}}\int_{N_o'}^{\infty}e^{-N'}\frac{(N')^{m-\frac{\gamma}{\delta}-1}}{m!}dN' \qquad /6/$$
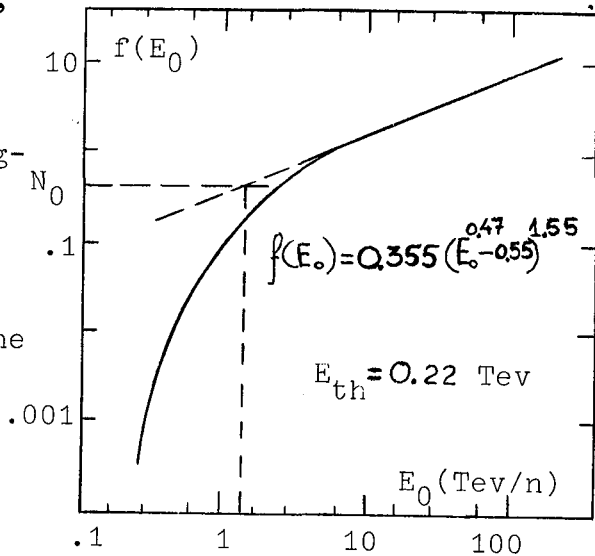
where $N_0'=N_0 \cdot A \cdot \Delta_{eff}, \Delta_{eff}=\left[\frac{1}{S}\int_0^{\infty}2\pi R\,dR\,(\Delta(R))^{\gamma/\delta}\right]^{\frac{\delta}{\gamma-\delta}}$ - is

the "effective fraction" of total number of muons in the group
expected in detection area.For $m\gg N_0'$ and $m\gg 1$

$$\mathbb{I}_A(m)=\frac{\gamma-\delta}{\delta}*(N_o*A*\Delta_{eff})^{\frac{\gamma-\delta}{\delta}}*m^{-\frac{\gamma}{\delta}-1}*\lambda(m) \qquad /7/$$

where $\lambda(m)\to 1$ if $m\to\infty$.When γ and δ are fixed the results de-
pends only on the product $N_0 \cdot A \cdot \Delta_{eff}$.The eq. /7/ describes the
asymptotic behavior of expression /5/ shown as solid curves
on fig.1.By definition the effective atomic number for a mixed
composition is:

$$A_{eff}=\left[\frac{\sum_{i=1}^{6}\rho_i A_i^{\gamma/\delta}}{\sum_{i=1}^{6}\rho_i A_i}\right]^{\frac{\delta}{\gamma-\delta}} \qquad /8/$$

which we can evaluate comparing /7/ with experimental data.
For $\gamma=1.7$, $\delta=0.73$, $N_0=0.43\pm0.12$, $\Delta_{eff}=0.15\pm0.02$ (of them only
N_0 is model dependent (5,6))the new value of A_{eff} is $A_{eff}=3.9\pm1.5$ (using experimental value of $I(m)$ at $m=10$).

Table 1

	A	1	4	9	14	28	56
ρ (Gev/n)	ρ_i	.939	.055	.0009	.0035	.0011	.0003
CC1	γ_i	1.7	1.7	1.7	1.7	1.7	1.7
CC2	γ_i	1.7	1.7	1.7	1.7	1.6	1.6

Table 1 shows two suggested examples of chemical composi-
tion, ρ_i-fraction of nuclei A_i with a given energy per nucle-
on (at 1 Gev/n).Number spectra for these two cases calculated
using eqs. /1-5/ shown in fig.1 by dashed lines.A good fit to

experimental data is provided by CC1 which corresponds to the case of a constant chemical composition (a standard one) in all energy range from 1 Gev/n.The A_{eff} for CC1 according to /8/ is A_{eff}=4.8,which is about 20% bigger than obtained using "N_0-approximation" at m=10;this is because $f(E_0)$ did not reach asymptotic behavior at m=10 for heavy primaries.Different multiplicities m represent different primary energies but also different selection of A.Table 2 shows,assuming CC1 and eqs./1-5/,relative contributions of different A,also their mean energy,to different m,q is relative contribution.

Table 2

		1	2	3	5	10	30	
	m =							
A = 1	E =	6.1	25	86	240	520	2500	
	q =	.83	.64	.47	.32	.22	.16	
A = 4	E =	20	44	94	170	340	1600	E in
	q =	.14	.26	.33	.33	.26	.2	Tev/nucleus
A = 14	E =	53	78	106	160	250	1050	
	q =	.02	.07	.13	.2	.23	.2	
A = 56	E =	170	200	210	220	245	730	
	q =	.005	.03	.07	.15	.3	.44	

4.Conclusions. The Baksan muon number spectrum at $E_\mu > E_{th}$= =220 Gev is well explained suggesting a constant chemical composition (the same as at 1Gev/n) till ~20 Tev/n.

The integral exponent of the heavy primaries power law spectrum should be γ = 1.7 $\pm$ 0.1 till 10^{15} ev/nucleus.

<div align="center">References</div>

1. A.E.Chudakov et al. Proc. of 16 ICRC,v.10,p.188,1979.
2. A.E.Chudakov et al. Proc. of 16 ICRC,v.10,p.175,1979.
3. V.N.Bakatanov et al. Proc. of 18 ICRC,v.7,p.51,1983.
4. A.E.Chudakov et al. Proc. of 16 ICRC,v.10,p.192,1979.
5. A.D.Erlykin Private communication.
6. J.W.Elbert and P.Summers Phys. Dep. University of Utah,
                    UUHEP 83/20,1983.