ON METHOD OF MUON SPECTRUM MEASUREMENTS BY THE SCINTILLATION DETECTORS OF A LARGE THICKNESS $T > 4t_o$.

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The various methods are known for the study of muon spectrum. The direct ones include the muon energy measurements by magnetic spectrometers. The indirect ones deal with the reconstruction of the muon spectrum from the spectrum of secondary particles (γ-quanta, knock-on electrons and e^+e^--pairs) obtained by burst or calorimeter technique. The burst technique is based on the measurement of the number of cascade particles, mainly in the cascade maximum, by the detectors of small thickness $T \leq t_o$ (t_o - radiation unit). The calorimeter method consist in determination of the cascade energy with help of the cascade curve shape. For this purpose the multylayer detectors can be used. They are usually comprised of proportional counters, X-ray emulsion chambers or scintillation counters with the target material placed between them.

Using the scintillation detectors of a large thickness one can measure the total cascade energy directly. In this case the detector works as a true calorimeter. But when the total energy is detected, the cascade spectrum differs from the muon one.

Let us consider the spectrum of cascades generated by muons in the target of infinite thickness with Z=12. The spectrum is measured by the scintillation detector of thickness $T > 4t_o$. The cascades with energy ν > 200 GeV are mainly generated by bremsstrahlung γ-quanta. The contribution of knock-on electrons decreases with energy. The value of $R_{\delta_b} = \frac{F_\delta(\nu)\,d\nu}{F_b(\nu)\,d\nu}$ (where $F_\delta(\nu)d\nu$, $F_b(\nu)d\nu$ are the spectra of knock-on electrons and γ-quanta, is the particle energy) depends on the muon spectrum index very weakly. The values of R_{δ_b} for Z=12 and γ_μ = 1.0, 1.5 and 2.7 are presented in Table 1. One can calculate R_{δ_b} with the accuracy better than 5% according to the formula:

$$R_{\delta_b} \approx 409 \cdot \frac{1}{(Z+1)\nu} \qquad (1),$$

where ν is in GeV.

Table 1

ν, GeV		100	200	300	400	500	1000
R_{δ_b}	γ_μ=1.0	0.315	0.160	0.106	0.079	0.063	0.0306
	γ_μ=1.5	0.314	0.161	0.107	0.079	0.063	0.0303
	γ_μ=2.7	0.307	0.161	0.107	0.070	0.063	0.0299

The contribution of the cascades from e^+e^--pairs is about (3÷4)% for all energies /1,3,4/. Therefore, the spectra of the cascades generated only by muons through bremsstrahlung and inelastic scattering should be considered.

The bremsstrahlung cross section /1,2/ for Z=12 proves to be approximated with accuracy better than 1% by following functions:

$$d\sigma_b(v,\nu) = C_b \cdot f_b(v,\nu,Z)\cdot d\ln v \qquad (2),$$

where $C_b = 4\alpha\left(r_0\frac{m}{\mu}\right)^2\cdot\frac{N_0}{A}\cdot Z(Z+1)$,

$f_b(v,\nu,12) = C_1 = 11.45$ for $v<0.03$

$f_b(v,\nu,12) = A_1(B_1+\ln\frac{1}{v})$; A_1=0.939, B_1=8.75 for $0.03<v<0.178$

$f_b(v,\nu,12) = D\cdot\ln\nu+(K-B\ln\nu)\cdot\ln\frac{1}{v}$, D=0.824, K=5.7, B=0.48 for $0.178<v<0.9$,

where $v=\frac{\nu}{E_\mu}$, ν, E_μ are in GeV.

For all other values of Z, $f_b(v,\nu,Z)$ is

$$f_b(v,\nu,Z) = f_b(v,\nu,12) - \frac{2}{3}\left(\frac{4}{3}-\frac{4}{3}v+v^2\right)\cdot\ln\frac{Z}{12} \qquad (3).$$

The approximations (2) and (3) are convenient for calculations of bremsstrahlung energy losses and of the γ-quanta spectrum, $F_b(\nu)\,d\nu$. The functions (2) and (3) as well as these functions multiplied by v^{γ_μ} can be integrated analytically. Calculated according to (2) the energy losses differ by less than 0.1% from the precise values. For power-law muon spectrum one gets the following expression for $F_b(\nu)\,d\nu$:

$$F_b(\nu)\,d\nu = \frac{d\nu}{\nu^{\gamma_\mu+1}}\,\frac{C_b}{\gamma_\mu}\left[L'(\gamma_\mu)+\left(D'(\gamma_\mu)-B'(\gamma_\mu)\frac{1}{\gamma_\mu}\right)\ln\nu + \frac{K'(\gamma_\mu)}{\gamma_\mu}\right] = \frac{d\nu}{\nu^{\gamma_\mu+1}}\,\frac{C}{\gamma_\mu}\cdot m(\gamma_\mu,\ln\nu) \qquad (4),$$

where in $L'(\gamma_\mu)$ the integration over v up to $v_1=0.178$ is performed,

$D'(\gamma_\mu) = D(1-v_1^{\gamma_\mu})$, $K'(\gamma_\mu) = K\left[1-v_1^{\gamma_\mu}\cdot(1-\ln v_1^{\gamma_\mu})\right]$,

$v_1=0.178$, $B'(\gamma_\mu) = B\left[1-v_1^{\gamma_\mu}\cdot(1-\ln v_1^{\gamma_\mu})\right]$.

For $\gamma_\mu=1$ the ratio $\frac{L'(\gamma_\mu)}{m(\gamma_\mu,\ \ln\nu)}$ is 27% and 23% for

$\nu = 200$ and 2000 GeV respectively. For $\gamma_\mu > 2.5$ the ratio $\frac{L'(\gamma_\mu)}{m(\gamma_\mu, \ln\nu)}$ is less than 2%. According to (4) the bremsstrahlung spectrum can't be approximated by power law: for $\gamma_\mu = 2.5$ when ν is increased by the factor of 10, the value of $m(\gamma_\mu, \ln\nu)$ increases by 29% demonstrating thus the deviation from the power-law spectrum. The photon spectrum is flatter than the muon one. In the more rough approach when the bremsstrahlung spectrum is taken as power-law one, the difference between γ_μ and γ_b is equal to $\Delta\gamma = 0.11$ ($\gamma_\mu = 2.5$). Thus, when the total cascade energy is measured, the spectrum $F_b(\nu)d\nu$ is flatter than the spectrum of the parent muons.

If energy $\varepsilon = \nu \cdot q(\nu)$, where $q(\nu) < 1$, is deposited in the detector, the variation of q with energy can lead to the deformation of the energy release spectrum as compared with the photon one. The value of q is a function of the detector thickness, T, the distance between the point of the cascade generation and the position of the detector, X_b, and of energy, ν. With the detector thickness being constant and with the generator thickness being infinite, the energy release spectrum (in $cm^{-2} \cdot sec^{-1}$) can be calculated as:

$$F_b(\varepsilon)d\varepsilon = d\varepsilon \int_{\varepsilon/q_{max}}^{\infty} \frac{d\nu}{\nu^{\gamma_\mu+1}} \frac{dX_b(\nu)}{d\varepsilon} \frac{C_b}{\gamma_\mu} \left[L'(\gamma_\mu) + (D'(\gamma_\mu) - \frac{B'(\gamma_\mu)}{\gamma_\mu})\ln\nu + \frac{K'(\gamma_\mu)}{\gamma_\mu}\right] \quad (5),$$

where $\frac{dX_b(\nu)}{d\varepsilon}$ is the variation of the thickness of the generation layer at which the cascade with energy ν releases energy in the range $[\varepsilon, \varepsilon + d\varepsilon]$ in the detector. For the steep spectrum it is convenient to use $d\ln\varepsilon$ instead of $d\varepsilon$. Taking into account the cascade curves /5/, which give the dependence q(x) for the detector of thickness T, one can obtain the dependence $\frac{dX_b}{d\ln\varepsilon} = X_b(T)\left[C_2 + \left(\frac{\nu_0}{\nu}\right)^{\gamma_1}\right]$, where $\nu_0 = \varepsilon / q_{max}$ is the minimal energy of the cascade producing the energy release ε, q_{max} is the maximal fraction of released energy, $X_b(T)$ is the function weakly dependent on T, $C_2 \approx 0.23$, $\gamma_1 = 4.5$. The expression (5) can be integrated analytically. For $\varepsilon / q_{max} \geqslant 100$ GeV it is equal to

$$F_b(\varepsilon)\, d\ln\varepsilon = \frac{C_b X_b(T)}{\gamma_\mu} q_{max}^{\gamma_\mu}(T, \varepsilon) \frac{d\ln\varepsilon}{\varepsilon^{\gamma_\mu}} \left(\frac{C_2}{\gamma_\mu} + \frac{1}{\gamma_\mu + \gamma_1}\right) * \left[L'(\gamma_\mu) + (D'(\gamma_\mu) - \frac{B'(\gamma_\mu)}{\gamma_\mu}) \ln\frac{\varepsilon}{q_{max}(\varepsilon)} + \frac{K'(\gamma_\mu)}{\gamma_\mu}\right] \quad (6).$$

The accuracy of (6) is better than 4%. The energy release spectrum is seen from (6) to be of the same shape as the bremsstrahlung spectrum for the very thick detectors and for q_{max}=const. Generally for $\varepsilon > 100$ GeV and $T > 4t_0$ one has

$$q_{max}(T, \varepsilon) = q_{max}(T_0)\left(\frac{T}{T_0}\right)^{\alpha_b} + H_b - M_b \ln\varepsilon \quad (7),$$

$H_b = M_b \ln 100$, $\alpha_b = \alpha_b(T)$, $M_b = 2.93\cdot10^{-2}, 3.26\cdot10^{-2}, 3.48\cdot10^{-2}, 3.48\cdot10^{-2}, 3.46\cdot10^{-2}$ for $T=12t_o$, $10t_o$, $8t_o$, $6t_o$, $4t_o$ respectively. For $4t_o \leq T < 6t_o$ one must take $T_o=4t_o$, $\alpha_b^o=0.83$, while for $6t_o < T \leq 13t_o$ – – $T_o=6t_o$, $\alpha_b^o \simeq 0.56$. For $T \geq 13t_o$ and for the large range of $\ln\varepsilon$ $q_{max}(T,\varepsilon)=1$. The value of $q_{max}^{\gamma_\mu}$ can be written as:

$$q_{max}^{\gamma_\mu}(T,\varepsilon) = q_{max}^{\gamma_\mu}(T_o)\left(\frac{T}{T_o}\right)^{\alpha_b\cdot\gamma_\mu}\left[1 - \frac{\gamma_\mu\cdot(M_b\ln\varepsilon - H_b)}{q_{max}(T_o)\left(\frac{T}{T_o}\right)^{\alpha_b}}\right] \quad (8).$$

Then the energy release spectrum $F_b(\varepsilon)$ can be given as:

$$F_b(\varepsilon)d\ln\varepsilon = \frac{C_b X_b(T)}{\gamma_\mu} q_{max}^{\gamma_\mu}(T_o)\left(\frac{T}{T_o}\right)^{\alpha_b\cdot\gamma_\mu}\left(\frac{C}{\gamma_\mu} + \frac{1}{\gamma_\mu+\gamma_1}\right)\frac{d\ln\varepsilon}{\varepsilon^{\gamma_\mu}}\Big[L'(\gamma_\mu) + (D'(\gamma_\mu) - \frac{B'(\gamma_\mu)}{\gamma_\mu})\ln\frac{\varepsilon}{q_{max}(\varepsilon)} + \frac{K'(\gamma_\mu)}{\gamma_\mu}\Big]\cdot\left[1 - \frac{\gamma_\mu\cdot(M_b\ln\varepsilon - H_b)}{q_{max}(T_o)\left(\frac{T}{T_o}\right)^{\alpha_b}}\right] \quad (9).$$

The deviation of the spectrum $F_b(\varepsilon)d\ln\varepsilon$ from the power-law function $d\ln\varepsilon/\varepsilon^{\gamma_\mu}$ is connected with two terms depending logarithmically on ε:

$$m' = L'(\gamma_\mu) + (D'(\gamma_\mu) - \frac{B'(\gamma_\mu)}{\gamma_\mu})\ln\frac{\varepsilon}{q_{max}(\varepsilon)} + \frac{K'(\gamma_\mu)}{\gamma_\mu} \quad \text{and} \quad \lambda = 1 - \frac{\gamma_\mu(M_b\ln\varepsilon - H_b)}{q_{max}(T_o)\left(\frac{T}{T_o}\right)^{\alpha_b}}$$

If the spectrum (9) is approximated by the power law, the spectral index for $T < 6t_o$ is somewhat greater than γ_μ and for $T \geq 8t_o$ is hardly less than γ_μ. The values of $\Delta\gamma = \Delta\gamma(T,\gamma_\mu)$ are presented in Table 2:

T		4	6	8	10	12
$\Delta\gamma$	$\gamma_\mu=1$	0.038	0.013	0.004	-0.0013	-0.0013
	$\gamma_\mu=2.5$	0.12	0.036	0.004	-0.009	-0.021

Measuring the energy release spectrum by the detector with $T \approx 8t_o$, the energy dependence of the cascade energy deficit compencates for the increasing of the bremsstrahlung cross section. q_{max} varies from 82% to 66% for $\varepsilon=100$ and 10000 GeV respectively. In real measurements cascades come from different directions where the detector has the various thickness. The total energy release spectrum in this case must be found as a sum over various thicknesses T_i involved in the measurements, i.e. $F_b(\varepsilon)d\varepsilon = \sum_i F_b(\varepsilon,T_i)d\varepsilon$. If the detector thickness varies from $6t_o$ to $12t_o$ and $\langle T\rangle = 8t_o$, the spectral index of $F_b(\varepsilon)d\varepsilon$ is almost equal to the muon spectrum index for $1 \leq \gamma_\mu \leq 3$. It is the Artyomovsk 100-ton scintillation counter that operates in this way /6/.

References

1. E.V.Bugaev, Yu.D.Kotov, I.L.Rozental. Kosmicheskie muony i neutrino, Atomizdat, Moscow (1970)
2. L.B.Bezrukov and E.V.Bugaev, 17th ICRC, 7, 102 (1981)
3. R.P.Kokoulin and A.A.Petrukhin, 12th ICRC, 6, 2436 (1971)
4. O.G.Ryazhskaya. Thesis, FIAN (1970)
5. T.A.Chuykova, 18th ICRC, 5, 316 (1983)
6. R.I.Enikeev et al. 18th ICRC, 7, 36 (1983)