GEOPHYSICAL SEARCHES FOR THREE-NEUTRINO OSCILLATIONS

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Introduction. The possibility of using cosmic ray induced neutrinos to detect oscillations in deep underground experiments has been considered by many authors.[1-5,12] Most of the estimates are made neglecting matter effects, or in the two neutrino case. The matter effects, however, are non-negligible:[3,4] in the two neutrino case, they reduce a mixing angle of 45° to 7.5° for 1 GeV neutrinos of squared mass difference 10^{-4} eV^2 going through the earth (average density = 5.52 g/cm^3) making the oscillation totally unobservable. Also, they produce a natural oscillation length of about 6000 km in the case of massless neutrinos, invalidating all estimates based on wavelength arguments. Adding a third neutrino flavor considerably modifies the oscillation pattern and suggests that scales down to 5 x 10^{-5} eV^2 could be observable even when we take into account matter effects and the electron contribution to the incoming flux.

In this paper, after defining the model of the earth used in the calculation, we will show the effect of matter on the probability curves for different cases, varying the masses and the mixing matrix. We will then give the prediction for the ratio upward ν + $\bar{\nu}$/downward ν + $\bar{\nu}$ as a function of the zenith angle at Cleveland, neglecting angular smearing and energy threshold effects.

1. Simple Model for the Earth: The 3 neutrino oscillation formalism has no analytic solution in the case of a continuous medium with varying density.[7] We therefore approximated the real density profile of the earth by a 3 layers model. The earth's density, especially in the core, is rather poorly known.[8] The only firm constraint, i.e. - that the average density is 5.52 g/cm^3 and the diameter 12756 km, has been included in our model: the three layers have respective densities of 3.5 g/cm^3, 5.5 g/cm^3 and 10.17 g/cm^3 and respective radial thicknesses of 928 km, 1950 km and 3500 km. These densities and thicknesses come from a crude estimate made on one of the models quoted in Reference 6.

2. Probabilities. We now study oscillations through a diameter of the earth using the notations of Reference 7.

The main problem is the choice of a mixing matrix: in the case of Dirac neutrinos, the mixing matrix depends on 3 angles and one phase. We do not know of any model predicting a 3 neutrino mixing matrix with mixing angles large enough to be observed.

Accordingly, we consider the following matrices, with $U_{\alpha i}$ the amplitude to go from a charged-current eigenstate $|\nu_\alpha>$ to a mass eigenstate $|\nu_i>$ of mass m_i:

Case 1: 2 neutrino mixing matrix "Cabibbo like":

$$U_2 = \begin{bmatrix} \cos\alpha & -\sin\alpha & . \\ \sin\alpha & \cos\alpha & . \\ . & . & 1 \end{bmatrix}$$

with $\alpha = 45°$ (maximal vacuum mixing)

$22.5°$ (lifting the δm^2 sign degeneracy[7])   (1)

Case 2: 3 neutrino mixing matrix with maximal muon mixing:

$$U_2 = \begin{bmatrix} \tfrac{1}{2} & 1/\sqrt{2} & -\tfrac{1}{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -\tfrac{1}{2} & 1/\sqrt{2} & \tfrac{1}{2} \end{bmatrix}$$

This reduces the outgoing ν_μ flux most.

(2)

Case 3: Big 3-neutrino mixing with a phase. To study the phase-dependence we use the Kobayashi-Maskawa matrix with a phase δ and all angles 45°:

$$U_3 = \begin{bmatrix} 1/\sqrt{2} & . & . \\ . & 1/\sqrt{8} - \tfrac{1}{2}e^{i\delta} & 1/\sqrt{8} + \tfrac{1}{2}e^{i\delta} \\ . & 1/\sqrt{8} + \tfrac{1}{2}e^{i\delta} & 1/\sqrt{8} - \tfrac{1}{2}e^{i\delta} \end{bmatrix}$$

(3)

We study the probability P_α to get flavor α from a beam of νe or ν_μ when we let it evolve through matter and assume one of the previous mixings. The results are shown in Fig. 1 for 1 GeV neutrinos.

We see that a squared mass difference of 10^{-4} eV^2 in the 2 neutrino case is unobservable due to matter effects.[3,4] However, this estimate is violated in the 3 neutrino case: for an appropriate mixing matrix, δm^2 scales of order 10^{-4} eV^2 are attainable. Furthermore, graphs (b) and (c) clearly show that the oscillation pattern is totally modified by matter effects. Notice that the probabilities do not go back to their original value in the course of the oscillation. This is a matter effect which one always encounters in the presence of more than two uniform media. Also, we see that 3 neutrino oscillations are very sensitive to the phase of the mixing matrix and that matter effects can, in principle, distinguish between $\delta m^2 > 0$ and $\delta m^2 < 0$ as has already been pointed out.[7]

3. <u>Upward to Downward Ratios at Cleveland.</u> We now present results for the upward to downward ratio of contained events at Cleveland, corresponding to incoming fluxes partitioned into zenith angle bins such that $.1 \times n < \cos\theta \le .1 \times (n+1)$ where θ is the zenith angle and $n \in [0,9]$. The energy is integrated for each bin from .2 GeV to 3 GeV. We neglect the effect of the layer above the detector and define the geometry assuming the detector at the surface of the earth. The zenith angle dependent fluxes that we use have been described elsewhere.[2,9,10] The calculation is made at solar minimum and we assume a ratio $\nu/\bar{\nu}$ of 1.3. We take a fiducial volume of 2×10^{33} nucleons[11] and the inclusive cross sections[2]: $\sigma_\nu = .737 \times E \times 10^{-38}$ cm^2 and $\sigma_{\bar\nu} = E \times .31 \times 10^{-38}$ cm^2 with E the neutrino energy in GeV. We show in Fig. 2 the results of our calculations in the form of ratios upward $\nu + \bar{\nu}$ /downward $\nu + \bar{\nu}$. We first exhibit the effect of matter on 2 neutrino oscillations at the scale $\delta m^2 = 10^{-4}$ eV^2. Although one would expect a big depletion (Fig. 2d) of the ν_μ flux from a naive argument based on vacuum oscillations, we see in Fig. 2c that matter effects completely modify the picture and make the phenomenon unobservable in this case. We also show results in the 3ν case which suggest that it is possible to observe δm^2 scales of order 10^{-4} eV^2.

4. Conclusions: We have shown in this work that matter effects totally modify the expectations for neutrino oscillations through the earth, making the accessible range for the squared mass difference of the order of 10^{-2} eV^2 to 10^{-3} eV^2 in the 2 neutrino case and of the order of 10^{-4} eV^2 to 10^{-5} eV^2 in the 3 neutrino case. In both cases, the mixing needed to reach this precision must be unrealistically big.

In the energy averaging of § 3, we have taken the same electron and muon thresholds. An interesting possibility exists for the case where the electron threshold is lower than the muon threshold, as for the example of contained interactions shown in Fig. 2. One can then imagine a situation in which the upward ν_e interaction rate is increased without a significant decrease in the corresponding rate of ν_μ interactions. (This is possible because, at a given neutrino energy, $\nu_\mu/\nu_e \approx 2$). An oscillation effect maximum around 500 MeV could therefore increase the ν_e interaction rate via $\nu_\mu \rightarrow \nu_e$ more than it would be depleted via $\nu_e \rightarrow \nu_\mu$ without affecting the ν_μ interaction rate significantly since the latter is due primarily to neutrinos of energy $E \gtrsim 1$ GeV.

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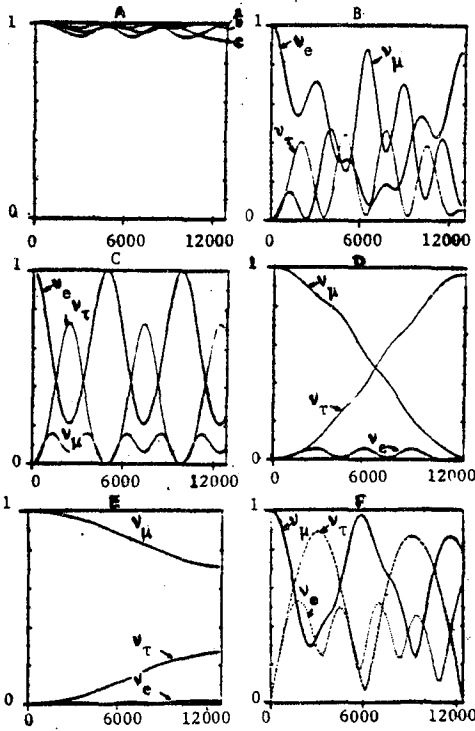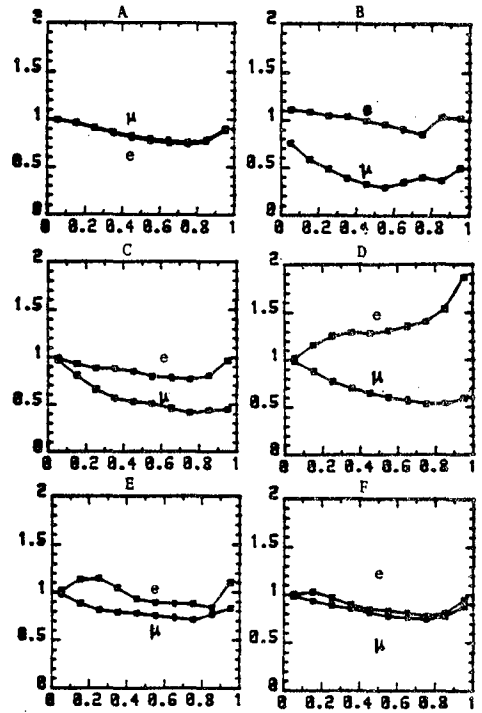
FIGURE 1.

FIGURE 2.

FIGURE 1. Neutrino oscillations through a diameter of the earth. Horizontal axis: distance traveled through the earth; Vertical axis: probability P_α.

(a) Case 1 with $|\delta m^2| = 10^{-4}$ eV^2. The three curves give P_e for a pure ν_μ incoming beam, for $\alpha = 45°$ (curve a), $\alpha = 22.5°$ and $\delta m^2 < 0$ (curve b), $\alpha = 22.5°$ and $\delta m^2 > 0$ (curve c).

(b) Case 3 with $\delta = 0°$, $m_1^2 = m_2^2 = 10^{-8}$ eV^2, $m_3^2 = 10^{-3}$ eV^2.

(c) Same as (b) considering the earth as a vacuum.

(d) Case 3 with $\delta = 270°$, $m_1^2 = 0$, $m_2^2 = 1.5 \times 10^{-4}$ eV^2, $m_3^2 = 5 \times 10^{-5}$ eV^2.

(e) Same as (d) but with $\delta = 0°$.

(f) Case 2 with $m_1^2 = 0$, $m_2^2 = 10^{-4}$ eV^2, $m_3^2 = 5 \times 10^{-4}$ eV^2.

FIGURE 2. Ratios $\dfrac{(\nu+\bar\nu)\,\text{up}}{(\nu+\bar\nu)\,\text{down}}$ for 10 even bins in cos θ.

Horizontal axis: cos θ; vertical axis: ratio $\dfrac{\text{up}}{\text{down}}$.

(a) Without oscillations.

(b) Case 2 with $m_1^2 = 0$, $m_2^2 = 10^{-4}$ eV^2, $m_3^2 = 5 \times 10^{-4}$ eV^2.

(c) Case 3 with $\delta = 270°$, $m_1^2 = 0$, $m_2^2 = 1.5 \times 10^{-4}$ eV^2, $m_3^2 = 5 \times 10^{-5}$ eV^2.

(d) Case 1 with $\alpha = 45°$, $\delta m^2 = 10^{-4}$ eV^2, neglecting matter effects.
(e) Same as (d) but with matter effects.
(f) Same as (e) but with $\alpha = 22.5°$.