Atmospheric Muons and Neutrinos, and the Neutrino-Induced Muon Flux Underground.

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ABSTRACT

The diffusion equation for neutrino-induced cosmic ray muons underground has been solved. The neutrino-induced muon flux and charge ratio underground have been calculated. The calculated horizontal neutrino-induced muon flux in the energy range 0.1 - 10000 GeV is in agreement with the measured horizontal flux. The calculated vertical flux above 2 GeV is in agreement with the measured vertical flux. The average charge ratio of neutrino-induced muons underground was found to be $\mu^+/\mu^- = 0.40$.

1. Introduction

From the decay of charged mesons in the atmosphere ($\pi^\pm \rightarrow \mu^\pm + \nu$, $K^\pm \rightarrow \mu^\pm + \nu$) the intensity of cosmic ray muons at sea-level is obtained and compared with measured spectra (1). Then we consider neutrinos in stead of muons from the same decays and obtain the neutrino intensity and these neutrinos give neutrino-induced muons.

2. The neutrino Spectrum

We expect the neutrino spectrum to have the form

$$\nu(E_\nu,\theta) = E_\nu^{-(\gamma+1)} f(E_\nu,\theta)$$

and approximate $f(E_\nu,\Theta)$ by

$$f(E_\nu,\theta) = C(E_\nu)\left\{ \frac{G_{N\pi} R_{\nu\pi}^{\gamma}}{1+\frac{E_\nu \cos\theta}{R_{\nu\pi} B_\pi(\theta)}} + C_K \frac{G_{NK} R_{\nu K}^{\gamma}}{1+\frac{E_\nu \cos\theta}{R_{\nu K} B_K(\theta)}} \right\}$$

$R_{\nu\pi} = 0.25$ ($E_\nu = R_{\nu\pi} E_\pi$), $R_{\nu K} = 0.56$ ($E_\nu = R_{\nu K} E_K$)
$C_K = 0.635$ (Branching ratio for $K \rightarrow \mu + \nu$ decay) $G_{N\pi} \approx 0.07$
$G_{NK} \approx 0.009$ (1). $B_\pi(\Theta)$ is the average decay constant for π's at zenith angle θ. $B_K(\Theta)$ is the average decay constant for K's at zenith angle θ. $C(E_\nu)$ is only a little energy dependent and is obtained by comparing $\nu(E_\nu,\theta)$ as given above with the calculated neutrino intensities. $C(E_\nu) \approx 3.57$.

3. Neutrino-Induced Muons

The neutrino nucleon total cross section is given by

$\sigma_\nu = S_\nu \cdot 10^{-38} E_\nu \ cm^2$, E in GeV

The neutrino intensity at depth h (g/cm^2) is then

$$\nu(E_\nu, h, \theta) = \nu(E_\nu, \theta)\, e^{-\frac{h E_\nu}{\Lambda_\nu}}, \qquad \frac{1}{\lambda_\nu} = \frac{E_\nu}{\Lambda_\nu}$$

λ_ν is the interaction length for neutrinos. We introduce a maximum energy, E_{max}, for the validity of the cross sections given above and take $\nu(E_\nu, h, \theta) = 0$ for $E_\nu > E_{max}$ in calculation of muons induced by neutrinos. The diffusion equation for muons is

$$\frac{\partial \mu_\nu(E_\mu, h, \theta)}{\partial h} = \int_{E(E_\mu, h)}^{E_{max}} \nu(E', h, \theta) \frac{E'}{\Lambda_\nu} g_{\nu\mu}(E(E_\mu, h)/E') \frac{dE'}{E} e^{-bh}$$

$E = E(E_\mu, h)$ is muon energy at depth h. E_μ is muon energy at depth h = 0. We further have (4)

$$g_{\nu\mu^-}(u) = \frac{3}{16.75}(5.25 + u^2) \text{ from } \frac{d\sigma^\nu}{dy} = \frac{G^2 M E_\nu}{\pi}\{Q + (1-y)^2 \bar{Q}\}$$

$$g_{\bar{\nu}\mu^+}(u) = \frac{3}{8.25}(1 + 5.25\, u^2) \text{ from } \frac{d\sigma^{\bar{\nu}}}{dy} = \frac{G^2 M E_\nu}{\pi}\{\bar{Q} + (1-y)^2 Q\}$$

$$u = \frac{E(E_\mu, h)}{E'}. \qquad \text{with } \alpha \equiv \bar{Q}/(Q + \bar{Q});\ \alpha = 0.16$$

The energy loss of muons in standard rock was given by

$$-\frac{dE}{dh} = a + bE \qquad E(E_\mu, h) = e^{-bh}\left\{\frac{a}{b} + E_\mu\right\} - \frac{a}{b} \qquad a, b = \text{const.}$$

$$E_\mu = e^{bh}\left\{\frac{a}{b} + E\right\} - \frac{a}{b} \qquad dE_\mu = e^{bh} dE$$

$$\frac{\partial \mu_\nu(E_\mu, h, \theta)}{\partial h} = \int_{E/E_{max}}^{1} \nu(E/u, h, \theta) \frac{E}{u\Lambda_\nu} g_{\nu\mu}(u) \frac{du}{u} e^{-bh}$$

$$\mu_\nu(E_\nu, h, \theta) = \int_0^h \int_{E/E_{max}}^{1} \nu(E'/u, h', \theta) \underbrace{}_{\frac{E'}{u\Lambda_\nu}} g_{\nu\mu}(u) \frac{du}{u} e^{-bh'} dh' \qquad E' = E'(E_\mu, h')$$

$$\mu_\nu(E, h, \theta) = \mu_\nu(E_\mu, h, \theta)\, e^{bh}$$

$$\mu_\nu(E, h, \theta) = \int_0^h \int_{\frac{E}{E_{max}}}^{1} f(E'/u, \theta)\, E'^{-\gamma} \frac{1}{\Lambda_\nu} u^{\gamma - 1} e^{-\frac{h' E'}{u \Lambda_\nu}} g_{\nu\mu}(u)\, du\, e^{b(h-h')} dh'$$

$$h = \frac{1}{b}\{\ln(a+bE_\mu) - \ln(a+bE)\} \qquad E' = E'(E_\mu, h')$$

$$h' = \frac{1}{b}\{\ln(a+bE_\mu) - \ln(a+bE')\}$$

$$b(h-h') = \ln\frac{a+bE'}{a+bE} \qquad h' = h - \frac{1}{b}\ln\frac{a+bE'}{a+bE}$$

$$\mu_\nu(E,h,\theta) = \int_0^h \int_{E/E_{max}}^1 f(E'/u,\theta)E'^{-\gamma}\frac{1}{\Lambda_\nu}u^{\gamma-1}g_{\nu\mu}(u)\left\{\frac{a+bE'}{a+bE}\right\}^{\frac{E'}{u\Lambda_\nu b}} e^{-\frac{hE'}{u\Lambda_\nu}}\cdot$$

$$\cdot\frac{a+bE'}{a+bE}\,du\,dh'. \qquad -\frac{dE'}{dh'} = a+bE' \qquad -dh' = \frac{dE'}{a+bE'}$$

$$\left.\begin{array}{l} h'=0 \Rightarrow E'=E_\mu \\ h'=h \Rightarrow E'=E \end{array}\right\} \Rightarrow \int_0^h \cdots dh' = \int_E^{E_\mu} \cdots \frac{dE'}{a+bE'}$$

$$\mu_\nu(E,h,\theta) = \int_E^{E_\mu}\int_{\frac{E'}{E_{max}}}^1 f(E'/u)E'^{-\gamma}\frac{1}{\Lambda_\nu}u^{\gamma-1}g_{\nu\mu}(u)\left\{\frac{a+bE'}{a+bE}\right\}^{\frac{E'}{u\Lambda_\nu b}}\cdot$$

$$\cdot e^{-\frac{hE'}{u\Lambda_\nu}}\,du\,\frac{dE'}{a+bE}.$$

The solution of the diffusion equation for neutrino-induced muons under ground is now

$$\mu_\nu(E,h,\theta) = \frac{E^{1-\gamma}}{a+bE}\frac{1}{\Lambda_\nu}\int_{\frac{E}{E_\mu}}^1\int_{\frac{E}{wE_{max}}}^1 f\left(\frac{E}{wu},\theta\right)w^{\gamma-2}u^{\gamma-1}g_{\nu\mu}(u)\cdot$$

$$\cdot\left\{\frac{a+bE/w}{a+bE}\right\}^{\frac{E}{wu\Lambda_\nu}} e^{-\frac{hE}{wu\Lambda_\nu}}\,du\,dw, \quad \frac{E}{w} < E_{max}$$

Deep underground we put $E_\mu = E_{max}$.

4. Discussion and Conclusion

The diffusion equation for neutrino-induced muons underground has been solved analytically, and the muon intensity is given in an integral form which we evaluate with a computer. We then obtain the calculated vertical muon flux for energies above 2 GeV: 2.02 10^{-13} $sec^{-1}cm^{-2}sr^{-1}$ in agreement with the measured vertical flux above 2 GeV (2) which is (1.92±0.44) 10^{-13} $sec^{-1}cm^{-2}sr^{-1}$. For the horizontal muon flux in the energy range 0.1 GeV - 10000 GeV we obtain the calculated flux 4.62 10^{-13} $sec^{-1}cm^{-2}sr^{-1}$ in agreement with the measured horizontal flux which is (4.59±0.42) 10^{-13} $sec^{-1}cm^{-2}sr^{-1}$ (3). We may conclude that there is no strong diffuse extraterrestrial neutrino source. The average muon charge ratio was found to be 0.40. The charge ratio varies between 0.34 at low energies and 0.47 at medium energies.

References

1. Klemke, G. et al.,The primary energy spectrum 0.1 - 50 TeV derived from high precision muon measurements 17'th ICRC, Paris 1981

2. M.M. Boliev et al., 17'th ICRC, Paris 1981. Vol.7, p.106.

3. M. F. Crouch et al., Phys. Rev. D 18, 2239 (1978)

4. B. C. Barish, Phys. Reports 39C (281) 1978