NASA Contractor Report 3928

A Root-Mean-Square Pressure Fluctuations Model for Internal Flow Applications

Y. S. Chen
Universities Space Research Association
Columbia, Maryland

Prepared for
George C. Marshall Space Flight Center
under Contract NAS8-35918

NASA
National Aeronautics
and Space Administration

Scientific and Technical
Information Branch

1985

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

INTRODUCTION

Pressure fluctuations of turbulent flow are known to be the source of driving force for structural vibrations and radiation of acoustic energy. Theoretically, exact solution of the pressure fluctuations can be obtained by directly integrating the Poisson's equation for pressure. The spectral properties of the pressure fluctuations can then be computed by Fourier transforming the two point correlation of the pressure fluctuations. The transformed equation involves terms with up to fourth-order velocity correlations. Approximate assumptions are employed to solve the higher-order correlation terms.

Application of this approach to a constant-mean-shear homogeneous turbulent flow is described in detail by William K. George, et al. [1]. For wall pressure fluctuations on the surface of a flat plate, R. H. Kraichnan [2] estimated that the ratio of the root-mean-square (rms) wall pressure fluctuations to the wall shear stress is 6. Experimental measurements, reviewed by W. W. Willmarth [3], and computations done by Panton and Linebarger [4] show that this ratio is lower than Kraichnan's estimation. Experimental data of wall pressure fluctuations including compressibility effect have been collected and correlated by Langanelli, Martellucci, and Shaw [5]. They conclude that a value between 1.7 and 3 for the ratio of the rms pressure fluctuations to the wall shear stress may be appropriate for the subsonic turbulent boundary layer. Results of a direct numerical simulation of R. A. Handler, et al. [6] for a 2-D channel flow show a value around 3.32 for this ratio.

The main drawback of the approach using the direct solution to the Poisson's equation for pressure is that the geometry of the boundary has to be simple so that the Green's function for integration can be handled easily. Generally, the Green's function for integrating the Poisson's equation must satisfy the following two conditions [7,8]:

$$\nabla^2 G = \delta(\Delta r) \text{ in the flow field} \quad .$$

$$\frac{\partial G}{\partial n} = 0 \text{ on the solid boundary} \quad .$$

where $\delta(\Delta r)$ denotes the Dirac delta function based on a displacement in space Δr, and n is in the direction normal to the solid boundary. To find such a function for a complicated boundary geometry, such as that of the interior of the piping installations of the space shuttle main engine, would be practically impossible.

In this paper, however, a completely different approach is taken. This new approach stems from the momentum equation for incompressible flow. The Reynolds decomposition concept is employed to derive a transport equation for the root-mean-square pressure fluctuations. This transport equation is then closed by approximately modeling the higher order terms which results in three empirical constants to be determined from experimental measurements.

A sensitivity study on the empirical constants is included for the case of an incompressible high Reynolds number 2-D channel flow from which one of the predictions is compared with experimental measurements and other predictions. Results of an annulus flow prediction are also included.

Major advantages of the present approach are that the concern on the geometric complexity is eliminated and the model can be solved using any CFD code along with the equations governing the flow field.

DERIVATION OF THE TRANSPORT EQUATION FOR THE ROOT-MEAN-SQUARE PRESSURE FLUCTUATIONS

Basic equations involved in deriving the transport equation for the root-mean-square fluctuations are the continuity and momentum equations of incompressible flow. They are

$$\text{Continuity:} \quad \frac{\partial U_i}{\partial x_i} = 0 \tag{1}$$

$$\text{Momentum:} \quad \frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_i \partial x_j} \tag{2}$$

where U_i represents velocity components, ρ is the fluid density, and P denotes the fluid pressure force.

Using Renolds' decomposition technique, the velocity components and pressure can be decomposed into a mean quantity and a time-dependent component. They are written in the following form:

$$U_i = \bar{u}_i + u_i$$

$$P = \bar{p} + p$$

Where the variables with over bar denote time-averaged mean values. Substituting these relations into equation (2), a time-dependent momentum equation can be obtained. The result is

$$\frac{\partial u_i}{\partial t} + \bar{u}_j \frac{\partial u_i}{\partial x_j} + u_j \frac{\partial \bar{u}_i}{\partial x_j} + u_j \frac{\partial u_i}{\partial x_j} - \overline{u_j \frac{\partial u_i}{\partial x_j}} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_i \partial x_j} \quad . \tag{3}$$

Multiplying equation (3) by p, taking time-averaging, and taking the sum for i = 1,2,3, the following equation is obtained:

$$\overline{p\frac{\partial u_i}{\partial t}} + \frac{1}{2\rho}\frac{\partial \bar{p}^2}{\partial x_i} + \overline{u_j\, p\frac{\partial u_i}{\partial x_j}} + \overline{u_j\, p}\,\frac{\partial \bar{u}_i}{\partial x_j} + \bar{u}_j\, p\frac{\partial \bar{u}_i}{\partial x_j} - \nu\, \overline{p\frac{\partial^2 u_i}{\partial x_i \partial x_j}} = 0 \quad . \tag{4}$$

Using the time-dependent continuity equation, the following transport equation for the rms pressure fluctuations is obtained:

$$\underbrace{\overline{p\frac{\partial u_i}{\partial t}} + \bar{u}_j\, \overline{p\frac{\partial u_i}{\partial x_j}}}_{\text{(I)}} = -\frac{1}{2\rho}\frac{\partial \bar{p}^2}{\partial x_i} \underbrace{- \overline{p\frac{\partial u_i u_j}{\partial x_j}} + \nu\frac{\partial}{\partial x_i}\left(\overline{p\frac{\partial u_i}{\partial x_j}}\right)}_{\text{(II)}} \underbrace{- \overline{u_j\, p}\,\frac{\partial u_i}{\partial x_j}}_{\text{(III)}} \underbrace{- \nu\, \overline{\left(\frac{\partial p}{\partial x_i}\right)\left(\frac{\partial u_i}{\partial x_j}\right)}}_{\text{(IV)}} \quad . \tag{5}$$

The terms in equation (5) have been labeled with Roman numerals which permit us to state the relation in words: The rate of change of pressure fluctuations through pressure strain redistribution (I) is equal to (II) the convective diffusion and viscous diffusion of mean-square pressure fluctuations, plus (III) the production of pressure fluctuation, plus (IV) the turbulent viscous dissipation.

Closure modeling is required to solve this equation. This is described in the next section where three empirical constants are introduced.

MODELING THE TRANSPORT EQUATION

The terms in equation (5) cannot be solved analytically. Approximate closure modeling to these terms is necessary to make equation (5) solvable.

First, the terms on the left hand side of equation (5) can be written as:

$$\overline{p\frac{\partial u_i}{\partial t}} + \bar{u}_j\, \overline{p\frac{\partial u_i}{\partial x_j}} = \frac{\partial \overline{p u_i}}{\partial t} + \bar{u}_j\frac{\partial \overline{p u_i}}{\partial x_j} - \overline{u_i\frac{\partial p}{\partial t}} - \bar{u}_j\, \overline{u_i\frac{\partial p}{\partial x_j}} \quad . \tag{6}$$

Assuming that the first two terms on the right hand side of equation (6) are small and negligible, equation (6) is approximately modeled as

$$\overline{p\frac{\partial u_i}{\partial t}} + u_j\, \overline{p\frac{\partial u_i}{\partial x_j}} \approx C_1\sqrt{K}\left(\frac{\partial\sqrt{p^2}}{\partial t} + u_j\frac{\partial\sqrt{p^2}}{\partial x_j}\right) \quad , \tag{7}$$

where C_1 is called the coherence function in statistics and K is the turbulent kinetic energy.

The diffusion terms (II) in equation (5) are modeled by dimensional analysis which takes the following form:

$$(\mathrm{II}) \approx \frac{\partial}{\partial x_j}\left(\frac{\ell}{\rho C_2}\frac{\partial \overline{p^2}}{\partial x_j}\right) = \frac{\partial}{\partial x_j}\left(\frac{2\ell}{\rho C_2}\sqrt{\overline{p^2}}\;\frac{\partial\sqrt{\overline{p^2}}}{\partial x_j}\right) \quad , \tag{8}$$

where C_2 is an empirical constant used to adjust the coefficient of diffusivity and ℓ represents a length scale which is related to the turbulent kinetic energy, K, and its dissipation rate, ϵ, through the following equation:

$$\ell = C_\mu^{\;3/4}\,\frac{K^{3/2}}{\epsilon}$$

where the empirical constant $C_\mu = 0.09$.

The production term (III) in equation (5) is expressed in terms of the mean quantities of turbulent flow using dimensional analysis. This gives

$$(\mathrm{III}) \approx C_3\,\mu_t\sqrt{K}\left(\frac{\partial \bar{u}_i}{\partial x_j}\right)^2 \quad , \tag{9}$$

where C_3 is an empirical constant and μ_t represents the turbulent viscosity. μ_t can be calculated from the following equation:

$$\mu_t = \rho\, C_\mu\,\frac{K^2}{\epsilon} \quad .$$

The dissipation term (IV) in equation (5) is also approximated using dimensional analysis. The result is

$$(\mathrm{IV}) \approx -\,C_4\left(\frac{K}{\ell}\right)\sqrt{\overline{p^2}} \tag{10}$$

where C_4 is another empirical constant.

Finally, by letting $P' \equiv \sqrt{\overline{p^2}}$, the model of equation (5) can be written as:

$$\frac{\partial P'}{\partial t} + \bar{u}_j\frac{\partial P'}{\partial x_j} = \frac{1}{\sqrt{K}}\frac{\partial}{\partial x_j}\left(\frac{\ell P'}{\rho\sigma_p}\frac{\partial P'}{\partial x_j}\right) + C_b\,\mu_t\left(\frac{\partial \bar{u}_i}{\partial x_j}\right)^2 - C_a\frac{\sqrt{K}}{\ell}P' \quad , \tag{11}$$

where $\sigma_p = C_1C_2/2$, $C_b = C_3/C_1$ and $C_a = C_4/C_1$ are the three empirical constants of the present model.

Boundary conditions for solving equation (11) are:

a) Production term $\approx$ dissipation term near the wall.

b) Gradient of P' across the line of symmetry is zero.

Two of the empirical constants, C_a and C_b, can be estimated from data of homogeneous turbulence and wall pressure fluctuations measurements.

For nearly isotropic homogeneous turbulence, equation (11) can be written as:

$$\frac{dP'}{P'} = - C_a\left(\frac{\sqrt{K}}{\ell}\right)dt \quad .$$

Integrating the above equation once,

$$C_a = - \frac{\ln\left(\dfrac{P'}{P'_o}\right)}{\displaystyle\int_o^t \left(\frac{\sqrt{K}}{\ell}\right)dt} \quad . \tag{12}$$

Also, the K-ϵ model [9] can be written as:

$$\frac{dK}{dt} = - \epsilon \tag{13a}$$

$$\frac{d\epsilon}{dt} = - C_{\epsilon 2}\frac{\epsilon^2}{K} \quad . \tag{13b}$$

Integrating equation (13b) once, the following relation is obtained.

$$\int_o^t \left(\frac{\sqrt{K}}{\ell}\right)dt = - \frac{\ln(\epsilon/\epsilon_o)}{C_{\epsilon 2}\, C_\mu^{3/4}} \quad . \tag{14}$$

From numerical and experimental investigations [10,11], the decay of isotropic turbulent kinetic energy can be represented by:

$$K = K_o(t-t_o)^{-n}$$

or

$$\frac{dK}{dt} = -\, nK_o(t-t_o)^{-(n+1)} = -\,\epsilon \quad , \tag{15}$$

where the decay constant $n \approx 1.087$ for very high Reynolds number flow.

By letting $\epsilon_o = nK_o$ in equation (15), the following expression can be obtained:

$$\ln\left(\frac{\epsilon}{\epsilon_o}\right) = -\,(n+1)\ \ln(t-t_o) \quad . \tag{16}$$

Substituting equations (14) and (16) into equation (12), the result is:

$$C_a = -\,\frac{C_{\epsilon 2}\, C_\mu{}^{3/4}}{(n+1)}\ \frac{\ln\left(\dfrac{P'}{P'_o}\right)}{\ln(t-t_0)} \quad . \tag{17}$$

It is also known [10,11] that the pressure fluctuations in isotropic turbulence has approximately the same decay constant as the turbulent kinetic energy. Therefore,

$$\ln\left(\frac{P'}{P'_o}\right) \approx -n\ \ln(t-t_o)$$

and

$$C_a \approx \frac{C_{\epsilon 2}\, C_\mu{}^{3/4}\, n}{(n+1)} = C_\mu{}^{3/4} = 0.1643$$

where $C_{\epsilon 2}$ = 1.92 in standard K-ϵ model.

The constant C_b can also be estimated by applying the wall boundary condition to the present model. That is,

$$C_b \approx C_a \left[\frac{\sqrt{K}\, P'}{\mu_t \ell \left(\dfrac{du}{dy}\right)^2}\right] \text{ near the wall}$$

where $du/dy = C_\mu^{1/4}\sqrt{K}/\kappa y$ by using the logarithmic law of wall with the von Karman constant $\kappa = 0.4$.

It has been concluded from experimental measurements of wall pressure fluctuations in boundary layer flows [3,5] that the range of rms pressure fluctuations, normalized by wall shear stress, is between 1.7 and 3.0.

This is equivalent to conclude that the ratio of wall pressure fluctuations to ρK just outside the viscous sublayer is roughly between 0.5 and 0.85. If the higher value of this ratio is accepted, then

$$C_b \approx C_a \left[\frac{P'}{C_\mu^{3/4}\, \rho K} \right]_{\text{near the wall}} \approx 0.85 \quad .$$

Therefore, only one empirical constant, σ_p, remained to be determined by matching the prediction to other sources of data in the flow field. To the author's knowledge, no pressure fluctuations measurements have been made away from boundaries in any confined flow region. However, results of higher level numerical simulations, e.g., large eddy simulations for channel and annulus flows done by Schumann [12], can be used to estimate the constant σ_p.

SENSITIVITY STUDY ON σ_p AND DATA COMPARISONS

Effects of σ_p on te predictions of rms pressure fluctuations are investigated numerically for an incompressible fully developed 2-D channel flow. Equations for solving this problem are:

Momentum equation:

$$\frac{1}{y^j}\frac{d}{dy}\left(y^j \mu_t \frac{du}{dy} \right) = \frac{dP}{dx} = -\, 2\rho u_\tau^2 \quad . \tag{18a}$$

K - ϵ model:

$$\frac{1}{y^i}\frac{d}{dy}\left(\frac{y^j \mu_t}{\sigma_K}\frac{dK}{dy} \right) = \epsilon - \mu_t \left(\frac{du}{dy} \right)^2 \quad , \tag{18b}$$

$$\frac{1}{y^i}\frac{d}{dy}\left(\frac{y^j \mu_t}{\sigma_\epsilon}\frac{d\epsilon}{dy} \right) = C_\epsilon\, 2 \left(\frac{\epsilon^2}{K} \right) - C_{\epsilon 1}\, C_\mu\, K \left(\frac{du}{dy} \right)^2 \quad . \tag{18c}$$

Pressure fluctuations model:

$$\frac{1}{y^j}\frac{d}{dy}\left(y^j \frac{\ell p'}{\rho \sigma_p}\frac{dp'}{dy}\right) = C_a \frac{K}{\ell} p' - C_b \mu_t \sqrt{K}\left(\frac{du}{dy}\right)^2 \quad , \qquad (18d)$$

where σ_K = 1.0, σ_ϵ = 1.3, $C_{\epsilon 2}$ = 1.92, $C_{\epsilon 1}$ = 1.43, C_a = 0.1643 and C_b = 0.85. The parameter j = 0 is for 2-D channel flow problems and j = 1 is for axisymmetric pipe flow problems.

A finite difference routine is used to solve the above equations. Equations (18a), (18b) and (18c) are solved first until a converged solution is obtained. Equation (18d) is then used to solve for the rms pressure fluctuations.

Experimental measured [13] and numerically computed results of the mean velocity profile, turbulent kinetic energy and its dissipation rate for the fully developed 2-D channel flow are illustrated in Figure 1. Reynolds number based on the maximum mean velocity and the channel half width is 62,000 which is about the same as the experiment.

Effects of σ_p on the predictions of rms pressure fluctuations are shown in Figure 2. Notice that the variation of σ_p does not affect the predictions near the wall where zero diffusion conditions are applied. The near-wall rms pressure fluctuations are fixed by the ratio of C_a and C_b. The levels of predicted p′ increase slightly in the vicinity of the wall with increasing value of σ_p while the levels of p′ decrease strongly with increasing σ_p elsewhere.

One of the cases studied above is selected (σ_p = 6.0) and compared with experimental measurements and other predictions. This is shown in Figure 3 where the wall pressure fluctuations measured by Ball [14] and Elliott [15] and predicted by Handler [6] are used for comparisons. Pressure fluctuations data comparisons away from the wall are provided by Schumann [12] and estimated by a constant-mean-shear isotropic turbulence formula given by George [1]. This formula is

$$P' = \rho \quad 0.33\left(\frac{du}{dy}\right)^2 \ell^2 u^2 + 0.42 u^4 \quad , \qquad (19)$$

where $u^2 \equiv 1/3\ (u^2 + v^2 + w^2) = 2/3$ K, $\ell \equiv u^3/\epsilon$ and du/dy stands for the mean shear (i.e., an average slope of the mean velocity profile). In the present data comparisons, the local mean velocity gradient is used for du/dy to estimate p′ across the channel.

It is shown in Figure 3 that the predictions of the present model with σ_p = 6.0 compare well with experimental measurements and other predictions. Especially, the shape of the p′ profile is very similar to the predictions of Schumann using large eddy simulation for high Reynolds number flow. This suggests that σ_p = 6.0 is a good approximation for the present model.

This set of empirical constants (i.e., σ_p = 6.0, C_a = 0.1643 and C_b = 0.85) of the present model are used to predict pressure fluctuations in a fully-developed annulus flow and compared with Schumann's prediction. This is shown in Figure 4. Ratio of the larger pipe radius to the smaller pipe radius of the annulus is 5.0. It is shown in Figure 4 that agreement between the present predictions and the Schumann's predictions is generally good except the discrepancies near the wall.

SUMMARY

The transport equation for the root-mean-square pressure fluctuations has been derived from the time-dependent momentum equation. Approximate closure modeling for this transport equation has been proposed with three empirical constants.

Two of the empirical constants have been estimated from data of homogeneous turbulence and wall pressure fluctuations measurements. The third constant for the diffusion coefficient of the present model has been estimated by comparing the results of a large eddy simulation for 2-D channel flow.

Data comparisons between the current predictions and other sources of data have been included for a 2-D channel flow and an annulus flow. Results of the comparisons show that the predictions of the present model are at least qualitatively good. More tests of the present model are required to fine-tune the constants of the present model before it can be confidently used in engineering design analysis.

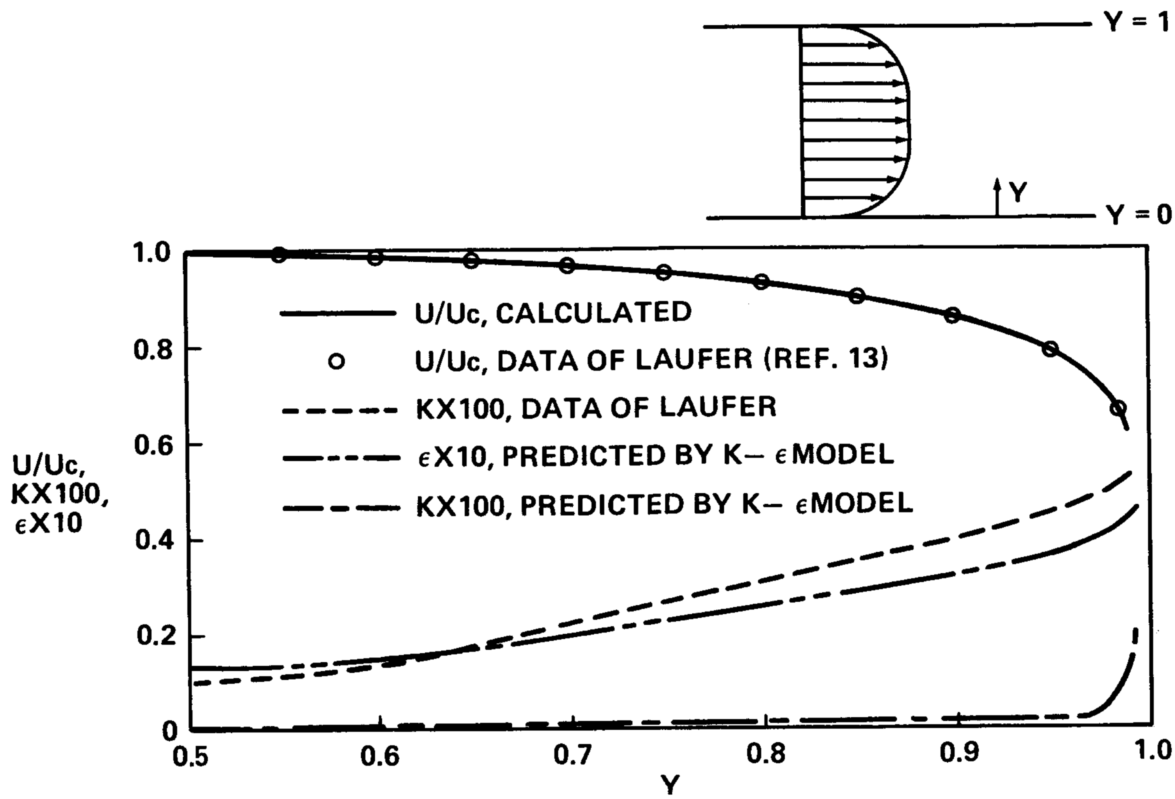

Figure 1. Velocity profile, turbulent kinetic energy and its dissipation rate of a fully-developed 2-D channel flow.

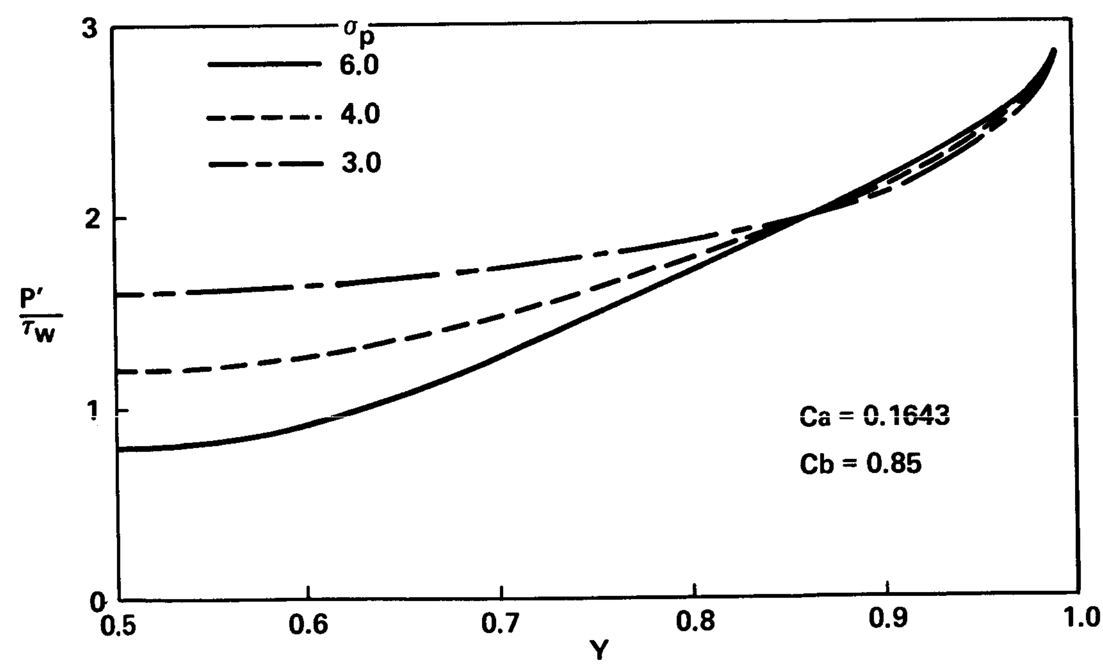

Figure 2. Effect of σ_p on the predictions of rms pressure fluctuations for the 2-D channel flow.

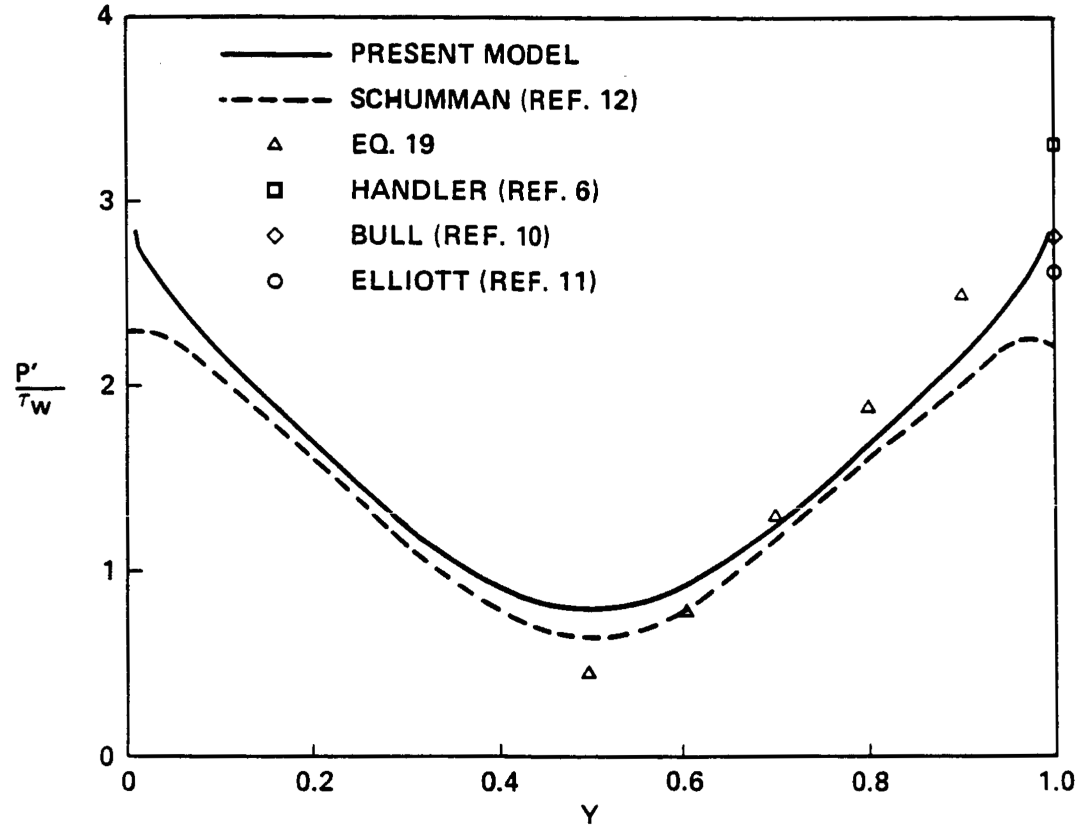

Figure 3. Comparisons of the predicted and measured rms pressure fluctuations for a fully developed 2-D channel flow.

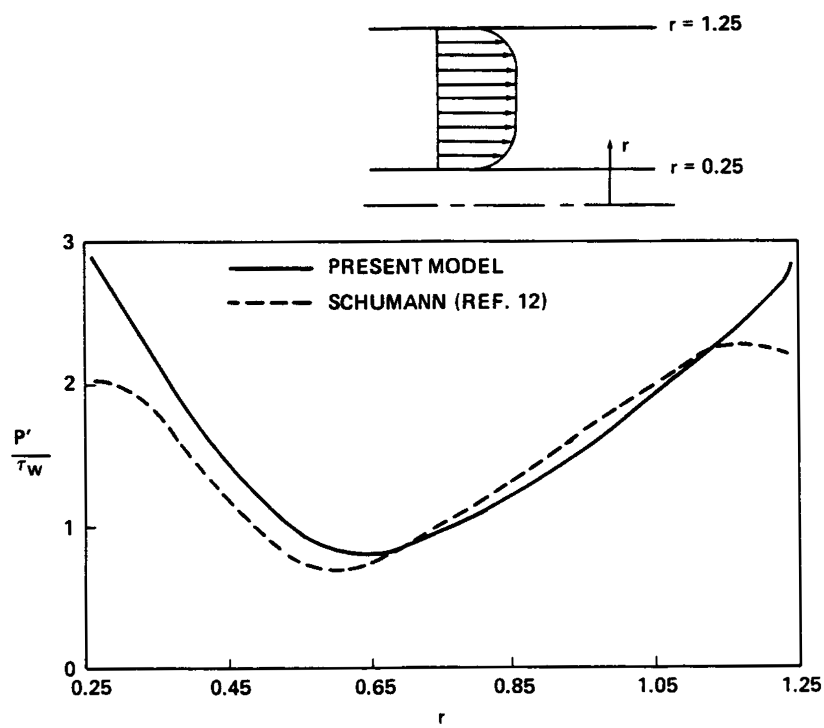

Figure 4. Comparisons of the predicted rms pressure fluctuations for a fully developed annulus flow.

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TECHNICAL REPORT STANDARD TITLE PAGE

1. REPORT NO.	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.
NASA CR-3928		

4. TITLE AND SUBTITLE	
A Root-Mean-Square Pressure Fluctuations Model for Internal Flow Applications	5. REPORT DATE September 1985
	6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S) Y. S. Chen	8. PERFORMING ORGANIZATION REPORT #
9. PERFORMING ORGANIZATION NAME AND ADDRESS Universities Space Research Association, The American City Building, Suite 311, Columbia, MD 21044	10. WORK UNIT NO. M-496
	11. CONTRACT OR GRANT NO. NAS8-35918
12. SPONSORING AGENCY NAME AND ADDRESS National Aeronautics and Space Administration, Washington, D.C. 20546	13. TYPE OF REPORT & PERIOD COVERED Contractor Report
	14. SPONSORING AGENCY CODE

15. SUPPLEMENTARY NOTES

Prepared for the Atmospheric Science Division, Systems Dynamics Laboratory, Science and Engineering Directorate. Contract Monitor: N. C. Costes

16. ABSTRACT

A transport equation for the root-mean-square pressure fluctuations of turbulent flow is derived from the time-dependent momentum equation for incompressible flow. Approximate modeling of this transport equation is included to relate terms with higher order correlations to the mean quantities of turbulent flow. Three empirical constants are introduced in the model. Two of the empirical constants are estimated from homogeneous turbulence data and wall pressure fluctuations measurements. The third constant is determined by comparing the results of large eddy simulations for a plane channel flow and an annulus flow.

17. KEY WORDS	18. DISTRIBUTION STATEMENT
Pressure Fluctuations Internal Flow Turbulence	Unclassified-Unlimited STAR Category: 34

19. SECURITY CLASSIF. (of this report)	20. SECURITY CLASSIF. (of this page)	21. NO. OF PAGES	22. PRICE
Unclassified	Unclassified	16	A02

MSFC - Form 3292 (May 1969)

For sale by National Technical Information Service, Springfield, Virginia 22161

NASA-Langley, 1985