NASA
Reference
Publication
1150

September 1985

Joint Research Effort on Vibrations of Twisted Plates

Phase I: Final Results

Robert E. Kielb,
Arthur W. Leissa,
James C. MacBain,
and Kelly S. Carney

NASA

NASA Reference Publication 1150

1985

Joint Research Effort on Vibrations of Twisted Plates

Phase I: Final Results

Robert E. Kielb
Lewis Research Center
Cleveland, Ohio

Arthur W. Leissa
Ohio State University
Columbus, Ohio

James C. MacBain
U.S. Air Force Wright Aeronautical Laboratories
Wright-Patterson Air Force Base, Ohio

Kelly S. Carney
Lewis Research Center
Cleveland, Ohio

NASA
National Aeronautics and Space Administration

Scientific and Technical Information Branch

Summary

This publication gives the complete theoretical and experimental results of the first phase of a joint government/industry/university research study on the vibration characteristics of twisted cantilever plates. The study was conducted to generate an experimental data base and to compare many different theoretical methods with each other and with the experimental results. Plates with aspect ratios, thickness ratios, and twist angles representative of current gas turbine engine blading were investigated. The theoretical results were generated by numerous finite element, shell, and beam analysis methods. The experimental results were obtained by precision machining a set of twisted plates and testing them at two laboratories. The second and final phase of the study will concern the effects of rotation.

Introduction

The vibration analysis of gas turbine engine blading has evolved from simple beam analysis to sophisticated finite element techniques. Although the finite element analysis of this blading has become commonplace, significant differences in the published results for various methods of analysis have raised some doubt concerning the adequacy of these methods to accurately predict the vibratory characteristics of highly twisted blades.

An example of the types of differences existing in the literature is the predicted first bending frequency for pretwisted plates. The predicted dependency of frequency on twist angle for twisted cantilever plates spans the spectrum from increasing significantly to decreasing significantly. Likewise, major differences in predicted frequencies can be found for the other modes of a twisted cantilever plate.

To aid in resolving such discrepancies, to validate analysis methods for twisted blades, and to generate an extensive data base on the vibrations of twisted blades, a joint government/industry/university research effort was conducted. A series of specimens were precision machined and carefully tested at two laboratories. Concurrently, analysts and researchers with known interests in this problem were invited to calculate the free vibration frequencies and mode shapes of these specimens by using their best available methods. Sixteen groups responded to the request and submitted 19 sets of results.

Because of the extent of this project and the volume of data collected, the documentation of the research for phase I has been divided into four publications. Reference 1 compares the theoretical methods with each other and with the experimental results. Reference 2 describes the experimental portion of this project and includes a discussion of the support compliance effect. Reference 3 relates the history of the problem to the results of this research project. Since it was felt that other researchers would want access to a complete set of experimental and theoretical results (which would be inappropriate in a journal article because of the quantity), the results are provided herein.

The authors wish to acknowledge the following organizations and individuals who made substantial contributions to the joint research effort summarized in this work: University of Dayton Research Institute (F. Bogner); The Technical University of Denmark (K. Ravn-Jensen); Indian Institute of Technology-Madras (V. Ramamurti); Imperial College of Science and Technology (D.J. Ewins and B. Hillary); Institute of Sound and Vibration Research; University of Southampton (D.P. Gao and M. Petyt); General Electric Company/AEG (B.G. McClintick, P. Niskode, and M.J. Stallone); General Motors Corporation, Allison Gas Turbine Division (W.N. Dalton and R. Jay); United Technologies Corporation, Pratt & Whitney Aircraft Group (R.D. Aunger and J. Hurchalla); Garrett Turbine Engine Company (R.J. Stockton); McGraw Edison Company, Turbodyne Division (M. Singh); Ohio State University (M. Ewing, J.K. Lee, and A.J. Wang); and U.S. Air Force Wright Aeronautical Laboratories (B. Tavner).

Symbols

A-U method identification

a plate length

b plate width

D plate flexural rigidity, $D = Eh^3/12(1-\nu^2)$

E modulus of elasticity

h plate thickness
l number of finite element nodes spanwise
M number of spanwise nodal lines
m number of finite element nodes chordwise
N number of chordwise nodal lines
n number of finite element nodes through thickness
λ frequency parameter, $\lambda = \omega a^2 \sqrt{\rho h/D}$
φ twist angle (root to tip)
ν Poisson's ratio
ρ mass density
ω frequency

Discussion

The geometry of a twisted plate is shown in figure 1. Twenty geometric configurations were considered. These consisted of all combinations of two aspect ratios ($a/b = 1$ and 3), two thickness ratios ($b/h = 5$ and 20), and five twist angles ($\varphi = 0°$, 15°, 30°, 45°, and 60°). Reference 2 also includes experimental results for $a/b = 2$. These configurations were chosen to represent the range of values existing in gas turbine engine blading. To limit the study to a manageable size and to focus on the important effect of pretwist, only rectangular cross sections were considered (hence, camber and other airfoil cross-section effects were ignored), and taper and rotational effects were not considered. The effects of rotation are being considered and will be discussed in a future publication.

Theoretical Methods

The 19 sets of theoretical results were derived from beam theory (2), shell theory (2), and finite element methods (15). The participants, listed in the acknowledgments, were asked to use their "best" analysis methods to obtain the first 10 frequencies and corresponding mode shapes for the 20 configurations considered, assuming complete fixity at the clamped end. To determine the degree of convergence, the participants were also asked to conduct a convergence study for one of the blades ($a/b = 1$, $\varphi = 30°$, $b/h = 20$). Descriptions of their methods and mathematical models were requested.

A basic description of each theoretical method is given in table I. These methods are identified by the letters A to S (the experimental results are identified by T and U). Included are the type of element, model data, and method of eigensolution. The models are defined by the notation $l \times m \times n$, where l, m, and n represent the number of nodes in the spanwise, chordwise, and thickness directions, respectively. For plate and shell elements, n is always 1 and is omitted. The highlights of the participants' descriptions are given here with the methods grouped into five categories. More detailed information is available from the authors of this paper.

Finite element methods.—Finite element methods based on plate, shell, and solid elements were used.

Plate elements: Methods A to G used finite elements based on plate theory. Methods A and B used the well-known finite element program NASTRAN (COSMIC version, April 1982 release, CTRIA2 elements). The only difference was that the mass matrix formulations were lumped in method A and coupled in method B. Method C used the MARC program (revision J.2-1) and the triangular plate element with membrane and bending stiffness, known as element 49. Methods D to F used triangular elements based on thin-plate theory. Method G used the quadrilateral elements of the program known as SAP.

Shell elements: Methods H to K used finite elements based on shell theory. Method H used NASTRAN (MSC version, level 61, CQUAD4 elements) and included shear deformation effects. Method I used the quadrilateral thin-shell element of the MARC program. Method J used an eight-node quadrilateral shell element. Method K included thick-shell effects.

Solid elements: Methods L to O used three-dimensional elements with 12, 18, 8, and 16 nodes per element, respectively. Except for the convergence study of method N, only one element was used through the thickness in all cases.

Other methods.—Both shell and beam theory were also used.

Shell theory: Methods P and Q used direct shell theory analysis. Method P used the Rayleigh-Ritz method to solve the shallow shell theory equations. Method Q used the finite difference energy method to solve the nonshallow shell theory equations.

Beam theory: Methods R and S used advanced beam theories including warping corrections. However, only method R included shear deformation and rotary inertia effects.

Two slightly different types of "constant thickness" definitions were used by the participants. Those using the plate and shell methods defined the thickness in a direction normal to the local middle surface. For this definition, the intersection of a plane at a constant spanwise coordinate with the twisted blade resulted in a slightly nonrectangular shape (thicker at the ends). Those using three-dimensional models with more than one node through the thickness constructed their meshes such that this intersection resulted in a rectangle. The specimens were manufactured with this latter definition. To estimate the differences introduced into the comparisons as a result of this difference in thickness definition, a numerical study was performed on the short, thick configuration by using method A. Both thickness definitions were used. The results showed that the predicted frequencies were only significantly different for the $a/b=1$ and $\varphi=60°$ case. Even in this case the maximum difference was less than 3 percent. The differences were largest for the bending modes.

Experiment

The specimens (fig. 2) were precision machined at the U.S Air Force Wright Aeronautical Laboratories from 7075-T6 cold-drawn aluminum bar stock. The width b of all of the specimens was 5.08 cm (2 in). The modulus of elasticity was carefully measured by using the method described in reference 4. The specimens were then tested at both the U.S. Air Force Wright Aeronautical Laboratories and the NASA Lewis Research Center by holographic interferometry. A special steel holding fixture was designed and used to minimize the compliance effects of the support. Excitation was accomplished either with a siren (low frequency) or a piezoelectric crystal (high frequency). To check on the repeatability, at least two test runs were made by different technicians. The resonant frequencies were found to be repeatable within 0.5 percent. As previously mentioned, the experimental methods are described in detail in reference 2.

Results

The results of the two experimental tests and the 19 theoretical analyses were collected and compared. The frequencies were put in the form of the standard plate nondimensional frequency parameter, $\lambda=\omega a^2 \sqrt{\rho h/D}$. The vibratory modes of twisted plates consist of beam bending and torsion, chordwise bending, and extension, not all uncoupled from each other. However, it is useful to identify modes according to their predominant type of motion by using the nomenclature coined for beams and flat plates. Therefore the modes are classified as follows: (1) flapwise bending (1B, 2B, . . .), (2) torsion (1T, 2T, . . .), (3) chordwise bending (1C, 2C, . . .), (4) edgewise bending (1E, 2E, . . .), and (5) extension (1A, 2A, . . .). Although the nodal lines of normal displacement determined from the eigenfunctions are, in general, curved and do not lie exactly in the spanwise and chordwise directions, classifying modes by the number of nodal lines lying approximately in these directions is useful. The convention used in this study is to classify modes as M/N, where M and N are the number of normal displacement nodal lines approximately parallel to the spanwise and chordwise directions, respectively. The nodal line at the root is not counted. For the flapwise bending modes, $M=0$; for the torsional modes, $M=1$; and for the chordwise modes, $M>1$. For the edgewise bending and extensional modes, this classification is not appropriate. The discussion to follow uses the terms thin ($b/h=20$), thick ($b/h=5$), long ($a/b=3$), and short ($a/b=1$).

The participants were asked to conduct convergence studies for one configuration. The results of these studies are shown in table II. Typically, three or four mesh densities were used and the results compared. For most of the methods, the difference in frequency between the mesh density used for the vibration study and a slightly coarser mesh was less than 0.2 percent for the first mode and less than 3 percent for the higher frequency modes. Examples of the finite element meshes used for the vibration study are shown in figure 3.

The frequency results have been put in both tabular (table III) and graphical (fig. 4) form. The tables compare the frequencies of all methods as a function of twist angle for each mode of each configuration. The last three rows give the percent difference between the average of the theoretical results and the experimental result T. A positive number indicates that the theoretical average was larger than the experimental value. A dashed line was used when either the experimental or theoretical value was not available. The figures display the dependence of frequency on twist angle for each mode of each configuration. The theoretical data points are connected with solid lines and the experimental points are connected with dashed lines. The symbols represent the methods A to U.

The mode shapes are also important. Five of the participants sent in at least partial sets of mode shapes. In addition, the experimental mode shapes were determined by holographic interferometry. The available analytical and experimental mode shapes total 1200 (20 specimens by 10 modes per specimen by 6 sets of mode shapes). Since it is impractical and of little value to present all 1200 mode shapes in this publication, only those for the limiting twist angles (i.e., 0° and 60°) are shown in figure 5.

Summary of Results

The complete set of theoretical and experimental results of a joint government/industry/university research study of the vibration characteristics of twisted cantilever plates are given. The study included 19 sets of theoretical results obtained by using state-of-the-art techniques and two sets of carefully conducted experimental results.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, May 17, 1985

References

1. Kielb, R.E.; Leissa, A.W.; and MacBain, J.C.: Vibrations of Twisted Cantilever Plates—A Comparison of Theoretical Results. Int. J. for Numerical Methods in Engineering, vol. 21, issue 8, Aug. 1985, pp. 1365–1380.
2. MacBain, J.C.; Kielb, R.E.; and Leissa, A.W.: Vibrations of Twisted Cantilever Plates—Experimental Investigation. ASME Paper 84-GT-96, 1984.
3. Leissa, A.W.; MacBain, J.C.; and Kielb, R.E.: Vibrations of Twisted Cantilever Plates—Summary of Previous and Current Studies. J. Sound Vibr., vol. 96, no. 2, Sept. 22, 1984, pp. 159-173.
4. Mohr, D.G.: Experimental Measurements of Material Damping of Aluminum and Graphite/Epoxy in Free-Fall with Tuneable Excitation. MIT Space Systems Laboratory, Report SSL-11-82, May 1982.

TABLE I.—DESCRIPTION OF METHODS

Method	Element data						Model data						Method of eigensolution	Dynamic reduction
	Type of element		Number of nodes	Number of degrees of freedom	Mass form	Comments	a/b = 1			a/b = 3				
							$l \times m \times n$[f]	Number of degrees of freedom	Number of elements	$l \times m \times n$[f]	Number of degrees of freedom	Number of elements		
A	Plate elements	Triangular plate[a]	3	15	Lumped	Cubic for normal displacements; linear for in-plane displacements	9 × 9	360	128	25 × 9	1080	384	Inverse power	No
B		Triangular plate[a]			Coupled		9 × 9	360	128	25 × 9	1080	384	Inverse power	
C		Triangular plate[b]			Lumped		9 × 9	360	128	25 × 9	1080	384	Inverse power	
D		Triangular plate			Coupled		11 × 9	450	144	31 × 9	1350	480	Subspace iteration	
E		Triangular plate			Coupled		7 × 7	252	72	13 × 5	360	96	Sturm sequence	
F		Triangular plate			Coupled	Cubic in area coordinates for normal displacements; linear for in-plane displacements	7 × 7	252	72	13 × 5	360	96	Sturm sequence	
G		Quadrilateral thin plate[c]	4	20	Lumped	Four triangles; cubic for normal displacements; linear for in-plane displacements	11 × 11	660	100	11 × 11	660	100	Sturm sequence	
H	Shell elements	Quadrilateral shell[d]	4	20	Coupled	Transverse shear included	11 × 11	550	100	11 × 11	550	100	Modified Givens	Yes
I		Curved quadrilateral shell[e]	4	48	Lumped	Double-curved, quadrilateral, thin-shell element	9 × 9	864	64	25 × 9	2592	192	Inverse power	No
J		Quadrilateral shell	8	40	Coupled	Assumed stress hybrid; modified Hellinger-Reissner principle; 2 × 2 integration in plane; analytical through thickness	11 × 91	450	20	--------	----	---	--------	No
K		Superparametric thick shell	8	40	-------	Based on Mindlin's theory including shear deformation	7 × 7	165	9	13 × 5	240	12	----------	Yes
L	Three-dimensional elements	Parabolic conoid shell	12	36	Lumped	Quadratic displacement spanwise; linear in other two directions	7 × 7 × 2	252	18	15 × 7 × 2	588	42	Matrix iteration	No
M		Quadrilateral thick plate	18	54	-------	Parabolic shape functions—in-plane linear through thickness	7 × 7 × 2	294	9	19 × 7 × 2	798	27	----------	----
N		Isoparametric	8	24	Lumped	Corner node brick	11 × 11 × 2	660	100	11 × 11 × 2	660	100	Subspace iteration	----
O		Isoparametric	16	48	Coupled	14-point integration rule	25 × 13 × 2	1872	72	25 × 13 × 2	1872	72	Simulation iteration	No
P	Other methods	Shallow shell theory; Rayleigh-Ritz; 6 × 6 polynomials												
Q		Nonshallow shell theory; finite difference energy method; 12 × 12 rectangles												
R		Beam theory; includes shear deformation, rotational inertia, and torsional warping; Rayleigh-Ritz, 10-term polynomials												
S		Beam theory; includes torsional warping; Galerkin technique; 15 uniform beam modes												
T		Experimental, Air Force Wright Aeronautical Laboratories												
U		Experimental, NASA Lewis Research Center												

[a] NASTRAN COSMIC, CTRIA2 element.
[b] MARC, element 49.
[c] SAP.
[d] MSC NASTRAN, CQUAD4 element.
[e] MARC, element 4.
[f] l, m, and n represent number of nodes in spanwise, chordwise, and thickness directions, respectively.

TABLE II.—CONVERGENCE STUDY

(1) Method G: $a/b=1$; $b/h=20$; $\varphi=15°$

Mode	Meshes studied[a]				
	4×4	4×5	5×4	6×6	[b]10×10
1	3.376	3.376	3.405	3.422	3.444
2	10.38	10.55	10.28	10.46	10.48
3	18.89	19.13	19.25	19.79	20.28
4	23.71	24.81	23.78	25.47	26.50
5	29.06	29.68	29.73	30.83	31.67
6	43.98	44.42	44.36	44.66	44.66
7	46.44	48.56	47.30	51.15	53.72
8	50.55	53.71	51.60	56.87	59.83
9	54.58	55.80	56.00	57.98	61.24
10	59.40	62.64	61.53	66.14	69.38

(2) Method H: $a/b=1$; $b/h=20$; $\varphi=60°$

Mode	Meshes studied[a]		
	5×5	[b]10×10	20×20
1	3.2425	3.2256	3.2209
2	14.593	14.219	14.114
3	22.388	21.927	21.768
4	28.612	27.989	27.829
5	42.576	41.152	40.681
6	48.554	45.943	45.124
7	59.183	55.854	54.912
8	68.408	63.722	62.470
9	68.582	67.213	66.816
10	73.966	69.362	68.145

(3) Method J: a/b = 1; $b/h=20$; $\varphi=30°$

Mode	Meshes studied[a]						
	3×3	4×3	5×3	4×4	[b]5×4	6×4	5×5
1	3.3399	3.3365	3.3350	2.3350	3.3340	3.3336	3.3348
2	14.354	14.332	14.326	14.339	14.331	14.328	14.324
3	18.864	18.702	18.649	18.697	18.647	18.628	18.648
4	26.947	26.882	26.854	26.762	26.749	26.743	26.719
5	34.553	34.274	34.174	34.242	34.170	34.136	34.134
6	46.685	46.381	46.245	46.340	46.201	46.133	46.186
7	55.665	55.017	54.698	55.002	54.645	54.499	54.706
8	63.708	60.873	60.133	60.845	60.051	59.757	60.017
9	66.473	66.240	65.864	64.008	63.817	63.712	62.882
10	73.497	71.613	71.103	70.381	69.504	69.187	69.166

(4) Method K: $a/b=1$; $b/h=20$; $\varphi=30°$

Mode	Meshes studied[a]			
	1×1	2×2	[b]3×3	4×4
1	3.19297	3.07876	3.06964	3.06597
2	8.25274	13.4014	13.3656	13.3367
3	22.1768	18.0637	17.3390	17.2032
4	35.5662	25.6697	25.7292	25.5695
5	38.8702	32.8606	32.4942	32.1968
6	62.0406	45.2402	44.3371	43.9086
7	100.143	55.2076	52.1282	50.3748
8	124.819	63.6912	57.3779	55.2743
9	174.390	70.3630	63.8795	61.2159
10	240.252	81.5775	69.8640	66.4574

(5) Method L: $a/b=1$; $b/h=20$; $\varphi=30°$

Mode	Meshes studied[a]			
	2×4	[b]3×6	4×8	6×12
1	3.398	3.381	3.379	3.374
2	13.98	14.18	14.26	14.31
3	19.21	18.49	18.40	18.37
4	24.33	25.21	25.78	26.27
5	33.96	33.94	34.03	34.09
6	46.36	46.95	46.94	46.88
7	50.30	52.21	52.88	53.51
8	57.80	58.39	59.57	60.87
9	62.60	63.54	61.16	60.67
10	65.10	70.20	68.48	68.51
11	86.22	83.74	86.04	88.86
12	76.94	92.09	91.98	93.09

[a]Number of elements in spanwise, chordwise, and thickness directions, respectively.
[b]Mesh used in vibration study.

TABLE II.—Concluded

(6) Method N: $a/b=1$; $b/h=20$; $\varphi=30°$

Mode	Meshes studied[a]				
	5×5×1	[b]10×10×1	15×15×1	20×20×1	10×10×2
1	3.37	3.35	3.35	3.35	3.38
2	14.14	14.23	14.25	14.26	14.28
3	18.54	18.27	18.24	18.24	18.49
4	25.00	25.91	26.21	26.31	26.12
5	33.29	33.37	33.54	33.58	33.67
6	45.71	45.76	45.80	45.80	45.95
7	53.38	52.52	52.81	52.94	53.22
8	56.72	58.31	58.22	58.22	59.09
9	60.46	58.67	59.60	59.96	59.64
10	70.29	65.96	66.11	66.19	67.10

(7) Method O: $a/b=2$; $b/h=20$; $\varphi=30°$

Mode	Meshes studied[a]				
	4×2	6×3	8×4	10×5	[b]12×6
1	3.812	3.744	3.710	3.710	3.710
2	20.89	20.48	20.36	20.29	20.26
3	22.36	21.07	20.68	20.53	20.45
4	66.20	59.59	58.36	57.78	57.51
5	63.08	60.37	59.61	59.29	59.10
6	78.62	71.13	69.52	68.76	68.66
7	-----	105.04	102.23	101.10	100.56
8	-----	107.94	104.98	103.77	103.16
9	-----	136.77	130.76	127.96	126.58
10	-----	149.02	140.45	137.89	136.82

(8) Method P: $a/b=1$; $b/h=20$; $\varphi=30°$

Mode	Meshes studied[c]			
	4×4	[b]6×6	6×8	5×8
1	3.2812	3.2653	3.2645	3.2659
2	16.444	16.079	16.070	16.077
3	19.888	19.409	19.404	19.410
4	31.467	28.120	27.937	27.942
5	38.357	37.347	37.318	37.352
6	50.347	49.037	49.949	49.098
7	------	------	------	------
8	76.455	67.042	65.144	65.207
9	------	------	------	------
10	------	------	------	------

(9) Method Q: $a/b=1$; $b/h=20$; $\varphi=30°$

Mode	Meshes studied[d]			
	4×4	8×8	[b]12×12	16×16
1	3.37329	3.39250	3.39722	3.39929
2	13.37254	14.18996	14.32722	14.37397
3	18.52758	18.64300	18.68638	18.70206
4	30.18277	27.66221	27.38769	27.30346
5	35.10403	34.89516	34.89738	34.89309
6	48.10618	46.67379	46.51712	46.47342
7	62.90461	57.08569	56.64977	56.51397
8	68.50919	61.70973	61.16853	60.99656
9	81.25857	68.34251	65.81152	64.96341
10	104.05667	72.75561	71.48077	71.15059

[a]Number of elements in spanwise, chordwise, and thickness directions, respectively.
[b]Mesh used in vibration study.
[c]Effective number of terms in polynomial expansions in spanwise and chordwise directions, respectively. Symmetric and antisymmetric analyses were made with half the number of polynomial terms in chordwise direction.
[d]Number of descritized rectangles in the spanwise and chordwise directions.

TABLE III.—FREQUENCY PARAMETERS

(1) First bending mode: 0/0; $a/b = 1$; $b/h = 20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	3.475	3.456	3.424	3.452	3.610
	B	3.451	3.432	3.402	3.430	3.589
	C	3.480	3.467	3.464	3.557	3.814
	D	3.354	3.337	3.290	3.218	3.125
	E	3.404	3.377	3.321	3.264	3.124
	F	3.452	-----	3.236	-----	3.096
	G	3.462	3.444	3.395	3.323	3.234
	H	3.460	3.443	3.395	3.320	3.226
	I	3.472	-----	3.474	-----	3.426
	J	3.460	3.427	3.334	3.202	3.059
	K	3.466	3.352	3.070	2.730	2.408
	L	3.463	3.442	3.381	3.290	3.176
	M	-----	-----	-----	-----	-----
	N	3.451	3.429	3.351	3.243	3.115
	O	3.770	3.760	3.740	3.700	3.600
Shell	P	3.474	3.426	3.265	2.939	2.343
	Q	3.462	3.445	3.397	3.323	3.230
Beam	R	3.347	3.341	3.322	3.294	3.262
	S	3.354	3.359	3.366	3.381	3.401
Experiment	T	3.325	3.055	3.143	2.965	2.842
	U	3.333	3.062	3.149	2.965	2.853
Difference between average of theoretical results and experimental result T, percent	All	3.87	11.03	6.69	9.92	11.55
	Finite element methods (A to O)	4.26	11.38	6.93	10.44	12.75
	Selected methods (D and H to N)	3.53	10.28	5.55	6.38	7.62

TABLE III.—Continued.

(2) First torsion mode: 1/0; $a/b = 1$; $b/h = 20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	8.547	10.840	15.400	19.960	23.750
	B	8.400	10.650	15.120	19.570	23.240
	C	8.589	10.890	15.460	19.990	23.770
	D	8.461	10.770	15.350	19.900	23.650
	E	8.543	11.612	17.830	22.770	27.110
	F	8.504	-----	17.260	-----	26.960
	G	8.455	10.480	14.800	19.330	21.710
	H	8.375	10.370	14.430	18.530	21.930
	I	8.509	-----	14.440	-----	21.560
	J	8.358	10.330	14.330	18.370	21.690
	K	8.377	10.180	13.370	15.940	16.740
	L	8.207	10.150	14.180	18.370	21.980
	M	-----	-----	-----	-----	-----
	N	8.327	10.260	14.230	18.250	21.620
	O	8.610	10.530	14.520	18.640	22.160
Shell	P	8.513	10.660	16.080	24.350	37.320
	Q	8.487	10.400	14.330	18.320	21.640
Beam	R	8.474	11.460	17.230	23.710	30.430
	S	8.473	11.420	17.140	23.820	30.500
Experiment	T	7.809	9.403	13.110	17.120	20.180
	U	7.835	9.438	13.090	17.050	20.220
Difference between average of theoretical results and experimental result T, percent	All	7.65	12.02	14.34	14.35	17.02
	Finite element methods (A to O)	7.56	11.20	12.90	10.53	11.12
	Selected methods (D and H to N)	6.74	9.09	8.53	6.07	5.30

TABLE III.—Continued.

(3) Second bending mode: 0/1; $a/b=1$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	21.420	20.750	18.940	16.730	14.640
	B	20.750	20.150	18.490	16.390	14.360
	C	21.610	20.920	19.110	16.910	14.810
	D	20.660	19.970	18.180	15.980	13.880
	E	20.370	19.620	17.230	15.580	13.630
	F	21.090	-----	17.190	-----	13.600
	G	20.970	20.280	18.270	15.380	12.110
	H	21.270	20.560	18.700	16.410	14.220
	I	21.290	-----	19.180	-----	15.130
	J	21.200	20.380	18.650	16.470	14.400
	K	21.200	20.080	17.340	14.100	11.570
	L	20.810	20.210	18.490	16.270	14.110
	M	-----	-----	-----	-----	-----
	N	20.730	20.080	18.270	15.970	13.740
	O	22.240	21.710	20.150	18.070	16.020
Shell	P	21.310	20.850	19.410	17.180	14.460
	Q	21.290	20.570	18.690	16.390	14.210
Beam	R	20.730	19.980	18.210	16.190	14.310
	S	21.020	20.430	18.900	16.980	15.230
Experiment	T	19.870	18.000	16.920	14.540	12.390
	U	19.950	18.060	16.930	14.530	12.450
Difference between average of theoretical results and experimental result T, percent	All	5.87	11.80	8.65	10.87	12.35
	Finite element methods (A to O)	5.90	11.73	8.25	10.18	11.60
	Selected methods (D and H to N)	5.48	10.95	8.05	8.36	10.63

TABLE III.—Continued.

(4) Chordwise bending mode: 2/0; $a/b=1$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	27.270	27.390	27.990	28.960	29.890
	B	26.130	26.200	26.710	27.620	28.510
	C	27.460	27.560	28.100	28.980	29.810
	D	26.960	27.100	27.680	28.560	29.360
	E	26.960	27.560	28.910	30.550	32.050
	F	27.340	-----	28.300	-----	31.900
	G	26.580	26.500	26.910	28.390	30.610
	H	27.050	26.970	27.110	27.550	27.990
	I	27.200	-----	27.240	-----	28.090
	J	26.770	26.670	26.750	27.120	27.500
	K	26.980	26.530	25.730	24.950	24.470
	L	25.160	25.040	25.210	25.840	26.570
	M	-----	-----	-----	-----	-----
	N	25.950	25.840	25.910	26.290	26.710
	O	29.110	28.770	28.500	28.730	29.130
Shell	P	27.460	27.450	28.120	30.640	37.180
	Q	27.470	27.350	27.390	27.710	28.050
Beam	R	-----	-----	-----	-----	-----
	S	-----	-----	-----	-----	-----
Experiment	T	26.110	24.330	25.310	25.520	25.520
	U	26.200	24.440	25.310	25.470	25.610
Difference between average of theoretical results and experimental result T, percent	All	3.26	9.63	7.24	8.83	12.72
	Finite element methods (A to O)	3.02	9.37	7.01	8.18	11.25
	Selected methods (D and H to N)	1.77	7.70	4.56	4.49	6.32

TABLE III.—Continued.

(5) Second torsion mode: 1/1; $a/b = 1$; $b/h = 20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	31.240	32.670	36.260	40.590	44.640
	B	30.180	31.540	34.920	38.940	42.600
	C	31.590	33.030	36.680	41.090	45.310
	D	30.440	31.940	35.670	40.130	44.170
	E	30.080	32.350	37.070	42.240	42.680
	F	30.070	-----	36.610	41.780	43.800
	G	30.550	31.670	34.520	37.900	41.650
	H	30.500	31.690	34.640	38.120	41.150
	I	30.970	-----	35.150	-----	41.450
	J	31.180	31.330	34.170	37.480	40.320
	K	30.480	31.110	32.490	33.230	32.350
	L	29.840	30.990	33.940	37.570	40.940
	M	-----	-----	-----	-----	-----
	N	29.750	30.790	33.360	36.470	39.200
	O	31.580	32.700	35.520	38.980	42.200
Shell	P	30.980	32.570	37.350	46.450	62.180
	Q	30.980	32.110	34.900	38.170	41.000
Beam	R	30.910	38.870	55.110	73.770	93.420
	S	30.870	38.710	54.760	73.960	93.430
Experiment	T	28.410	27.990	31.330	34.560	37.030
	U	28.510	28.090	31.290	34.510	37.100
Difference between average of theoretical results and experimental result T, percent	All	7.39	14.55	16.22	20.27	23.60
	Finite element methods (A to O)	7.17	12.03	10.67	10.95	10.99
	Selected methods (D and H to N)	6.70	10.60	8.40	7.01	7.29

TABLE III.—Continued.

(6) First edgewise bending mode: -/-; $a/b = 1$; $b/h = 20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	44.420	45.420	47.320	48.020	46.560
	B	-----	45.260	46.730	46.900	45.050
	C	44.670	45.700	47.630	48.690	47.300
	D	44.530	45.410	46.930	46.960	44.860
	E	46.430	47.030	47.320	46.130	47.320
	F	46.590	-----	47.590	-----	47.170
	G	43.710	44.660	46.400	45.880	41.660
	H	43.720	44.740	46.630	47.430	45.940
	I	44.640	-----	47.250	-----	47.690
	J	43.630	44.540	46.200	46.870	45.470
	K	43.800	44.200	44.340	42.140	35.960
	L	44.700	45.470	46.950	48.110	47.300
	M	-----	-----	-----	-----	-----
	N	43.760	44.500	45.750	45.920	43.760
	O	43.680	44.810	47.080	48.830	49.060
Shell	P	43.550	45.040	49.040	52.460	50.180
	Q	43.520	44.580	46.520	47.300	45.810
Beam	R	42.920	43.920	45.840	46.560	45.210
	S	67.080	70.130	77.460	87.920	99.920
Experiment	T	36.680	39.010	39.220	38.310	36.990
	U	-----	-----	-----	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	19.58	16.27	19.13	23.01	24.01
	Finite element methods (A to O)	17.54	13.59	16.06	18.18	18.46
	Selected methods (D and H to N)	16.85	12.94	15.28	17.15	16.74

TABLE III.—Continued.

(7) Chordwise bending mode: 2/1; $a/b=1$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	-----	55.720	57.710	58.220	58.770
	B	51.890	52.770	54.500	55.450	56.640
	C	55.590	56.550	58.600	59.030	59.490
	D	53.380	54.330	55.940	63.190	67.630
	E	54.810	55.850	57.150	57.500	58.410
	F	53.520	53.310	55.760	57.900	58.400
	G	53.050	53.720	54.920	55.710	57.000
	H	52.920	53.810	55.660	55.960	55.350
	I	54.190	-----	57.060	-----	56.350
	J	51.960	52.890	54.650	54.840	54.840
	K	53.640	53.650	52.130	47.090	45.350
	L	49.140	50.140	52.210	52.610	52.070
	M	-----	-----	-----	-----	-----
	N	50.370	51.880	52.510	52.040	51.770
	O	55.030	55.880	57.850	58.660	58.110
Shell	P	55.000	56.120	59.050	63.460	77.370
	Q	54.620	55.380	56.650	56.230	55.910
Beam	R	-----	-----	-----	-----	-----
	S	-----	-----	-----	-----	-----
Experiment	T	49.780	47.540	50.030	55.010	57.030
	U	-----	-----	-----	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	6.56	12.18	10.30	2.68	1.19
	Finite element methods (A to O)	6.14	11.77	9.82	1.79	−1.04
	Selected methods (D and H to N)	4.69	9.93	7.88	−1.33	−4.13

TABLE III.—Continued.

(8) Chordwise bending mode: 3/0; $a/b=1$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	64.660	64.640	64.560	64.470	64.450
	B	59.380	59.390	59.410	59.550	59.760
	C	65.550	65.470	65.260	64.970	64.600
	D	63.800	63.730	63.520	63.230	62.870
	E	56.160	54.810	55.220	60.650	60.650
	F	59.010	-----	58.850	-----	59.750
	G	61.310	61.240	61.280	62.140	63.660
	H	64.770	64.700	64.500	64.180	63.720
	I	64.710	-----	64.130	-----	63.760
	J	60.290	59.970	60.050	62.890	63.360
	K	66.380	65.600	63.880	61.710	59.540
	L	58.420	58.400	58.390	58.440	58.340
	M	-----	-----	-----	-----	-----
	N	59.020	58.910	58.660	58.260	57.710
	O	64.970	64.580	64.000	65.370	65.960
Shell	P	65.740	66.040	65.210	66.960	75.160
	Q	66.080	66.010	65.810	65.490	65.000
Beam	R	-----	-----	-----	-----	-----
	S	-----	-----	-----	-----	-----
Experiment	T	60.090	61.600	-----	-----	-----
	U	-----	-----	-----	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	3.88	1.27	-----	-----	-----
	Finite element methods (A to O)	3.13	0.30	-----	-----	-----
	Selected methods (D and H to N)	3.83	0.46	-----	-----	-----

TABLE III.—Continued.

(9) Third bending mode: 0/2; $a/b=1$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	61.790	61.410	61.240	64.260	-----
	B	-----	58.980	59.440	62.390	66.420
	C	63.100	62.680	62.350	64.980	-----
	D	59.040	58.800	59.480	56.430	57.290
	E	-----	-----	-----	-----	-----
	F	-----	-----	-----	-----	-----
	G	60.170	59.830	60.480	64.900	66.250
	H	61.860	61.410	60.980	63.330	67.210
	I	61.260	-----	61.170	-----	-----
	J	63.930	63.900	63.820	63.660	66.850
	K	64.320	62.200	57.380	55.640	54.080
	L	64.660	64.130	63.540	66.190	68.900
	M	-----	-----	-----	-----	-----
	N	58.910	58.450	58.300	-----	-----
	O	66.900	66.860	66.730	66.440	68.610
Shell	P	-----	-----	-----	-----	-----
	Q	61.580	61.180	61.170	64.060	67.970
Beam	R	57.010	56.790	56.670	57.980	60.830
	S	58.860	58.020	56.780	55.820	55.330
Experiment	T	56.930	53.110	54.230	48.480	46.810
	U	-----	-----	-----	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	7.69	13.00	10.56	21.72	26.41
	Finite element methods (A to O)	8.70	13.92	11.45	22.71	27.37
	Selected methods (D and H to N)	8.17	13.62	10.61	20.33	25.54

TABLE III.—Continued.

(10) Third torsion mode: 1/2; $a/b=1$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	-----	-----	-----	71.630	-----
	B	-----	-----	68.110	67.900	67.500
	C	73.510	73.470	73.220	73.080	72.590
	D	69.050	69.140	69.350	69.500	69.340
	E	-----	-----	-----	-----	-----
	F	-----	-----	-----	-----	-----
	G	69.540	69.380	69.130	68.340	72.310
	H	70.970	70.920	70.730	70.250	69.360
	I	71.040	-----	-----	-----	71.690
	J	69.460	69.490	69.500	69.330	68.850
	K	-----	-----	-----	-----	-----
	L	71.780	71.330	70.200	68.760	67.100
	M	-----	-----	-----	-----	-----
	N	66.790	66.580	65.450	64.970	63.810
	O	74.830	74.870	75.040	75.150	75.000
Shell	P	-----	-----	-----	-----	-----
	Q	71.720	71.680	71.480	70.990	70.110
Beam	R	68.870	78.960	102.400	131.300	162.500
	S	68.530	78.440	101.500	131.100	161.900
Experiment	T	65.580	-----	-----	-----	-----
	U	-----	-----	-----	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	6.99	-----	-----	-----	-----
	Finite element methods (A to O)	7.34	-----	-----	-----	-----
	Selected methods (D and H to N)	6.11	-----	-----	-----	-----

TABLE III.—Continued.

(11) First bending mode: 0/0; $a/b=1$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	3.400	3.369	3.285	3.179	3.087
	B	3.377	3.346	3.264	3.159	3.069
	C	3.480	3.466	3.461	3.546	3.783
	D	3.332	3.314	3.265	3.190	3.096
	E	3.399	3.371	3.315	3.240	3.146
	F	3.455	3.287	3.230	-----	3.090
	G	3.462	3.443	3.391	3.312	3.213
	H	3.365	3.347	3.297	3.220	3.123
	I	3.472	-----	3.397	-----	3.203
	J	-----	-----	-----	-----	-----
	K	3.345	3.230	2.954	2.631	2.329
	L	3.572	3.557	3.512	3.444	3.361
	M	3.371	3.346	3.277	3.177	3.068
	N	3.302	3.277	3.209	3.105	2.981
	O	3.617	3.608	3.584	3.548	3.504
Shell	P	-----	-----	-----	-----	-----
	Q	-----	-----	3.391	-----	-----
Beam	R	3.253	3.247	3.231	3.207	3.180
	S	3.354	3.357	3.364	3.378	3.396
Experiment	T	2.936	2.859	2.816	2.765	2.595
	U	2.950	2.884	2.839	2.778	2.618
Difference between average of theoretical results and experimental result T, percent	All	13.89	15.19	15.16	14.62	17.99
	Finite element methods (A to O)	14.28	15.45	15.11	14.38	17.53
	Selected methods (D and H to N)	13.50	14.53	13.96	11.60	14.16

TABLE III.—Continued.

(12) First bending mode: 1/0; $a/b=1$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	7.947	7.859	8.269	8.414	8.863
	B	7.807	7.722	8.128	8.274	8.722
	C	8.590	8.667	8.872	9.143	9.411
	D	8.272	8.321	8.481	8.698	8.911
	E	8.537	8.612	8.912	9.286	9.810
	F	8.481	-----	8.760	-----	9.380
	G	8.455	8.488	8.609	8.823	9.073
	H	7.541	7.621	7.829	8.100	8.361
	I	8.510	-----	8.630	-----	8.906
	J	-----	-----	-----	-----	-----
	K	7.354	7.326	7.196	6.898	6.398
	L	7.211	7.283	7.484	7.781	8.124
	M	7.556	7.734	8.154	8.635	9.093
	N	7.127	7.191	7.370	7.624	7.920
	O	7.597	7.683	7.917	8.247	8.613
Shell	P	-----	-----	-----	-----	-----
	Q	-----	-----	8.593	-----	-----
Beam	R	7.734	7.948	8.532	9.404	10.480
	S	8.306	8.524	9.138	10.010	11.230
Experiment	T	6.763	6.712	6.903	7.236	7.407
	U	6.783	6.736	6.939	7.257	7.450
Difference between average of theoretical results and experimental result T, percent	All	14.81	15.33	16.70	15.11	17.30
	Finite element methods (A to O)	14.69	14.77	15.68	13.10	14.71
	Selected methods (D and H to N)	11.63	11.44	12.37	9.05	10.16

TABLE III.—Continued.

(13) First edgewise bending mode: -/-; $a/b = 1$; $b/h = 5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	11.110	10.960	10.560	9.980	9.318
	B	11.110	10.950	10.540	9.943	9.269
	C	11.170	11.070	10.800	10.380	9.855
	D	11.130	10.990	10.600	10.020	9.340
	E	11.570	11.460	11.010	10.330	9.800
	F	11.570	-----	11.070	-----	9.820
	G	10.930	10.800	10.420	9.721	8.604
	H	10.930	10.750	10.250	9.577	8.825
	I	11.120	-----	10.670	-----	9.509
	J	-----	-----	-----	-----	-----
	K	10.950	10.620	9.802	8.803	7.894
	L	11.340	11.220	10.880	10.370	9.778
	M	10.930	10.670	10.060	9.364	8.715
	N	10.940	10.790	10.370	9.753	9.019
	O	10.950	10.860	10.580	10.170	9.679
Shell	P	-----	-----	-----	-----	-----
	Q	-----	-----	10.410	-----	-----
Beam	R	10.730	10.630	10.350	9.939	9.458
	S	16.770	16.070	14.760	13.410	12.250
Experiment	T	4.069	5.487	6.044	6.934	8.823
	U	-----	-----	-----	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	64.47	51.33	43.89	31.52	6.59
	Finite element methods (A to O)	63.42	49.79	42.68	29.73	4.56
	Selected methods (D and H to N)	63.17	49.38	41.75	28.13	2.09

TABLE III.—Continued.

(14) Second bending mode: 0/1; $a/b=1$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	19.100	19.110	19.090	18.980	18.720
	B	18.540	18.550	18.560	18.480	18.250
	C	21.600	21.540	21.350	21.040	20.620
	D	19.630	19.590	19.470	19.230	18.820
	E	20.330	20.220	20.220	20.070	19.920
	F	21.000	20.440	20.440	-----	19.990
	G	-----	20.870	20.660	20.410	20.030
	H	18.480	18.530	18.630	18.640	18.460
	I	21.290	-----	20.910	-----	19.970
	J	-----	-----	-----	-----	-----
	K	17.710	17.380	16.460	15.130	13.670
	L	17.180	17.170	17.140	17.040	16.880
	M	18.400	18.600	18.980	19.280	19.370
	N	16.500	16.470	16.360	16.180	15.930
	O	18.670	18.670	18.680	18.640	18.530
Shell	P	-----	-----	-----	-----	-----
	Q	-----	-----	20.930	-----	-----
Beam	R	17.520	17.590	17.770	18.000	18.230
	S	21.020	21.960	23.960	26.460	29.080
Experiment	T	13.900	13.040	13.150	13.880	12.430
	U	-----	-----	-----	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	27.34	31.77	32.18	27.38	35.11
	Finite element methods (A to O)	27.26	31.41	31.04	25.35	32.85
	Selected methods (D and H to N)	24.68	27.38	28.06	21.06	29.32

TABLE III.—Continued.

(15) Chordwise bending mode: 2/0; $a/b = 1$; $b/h = 5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	27.010	26.430	26.540	25.570	25.300
	B	26.050	25.470	25.540	24.620	24.340
	C	31.590	31.380	30.790	29.940	28.940
	D	28.650	28.360	27.770	26.910	25.870
	E	30.780	30.250	29.500	28.600	27.550
	F	30.550	29.420	29.650	29.200	27.180
	G	-----	30.250	29.630	28.540	27.760
	H	25.230	25.110	24.770	24.230	23.530
	I	30.970	-----	29.880	-----	27.330
	J	-----	-----	-----	-----	-----
	K	24.060	23.800	23.090	22.120	21.220
	L	23.210	23.130	22.880	22.530	22.110
	M	25.120	24.930	24.480	24.000	23.690
	N	22.200	22.080	21.690	21.120	20.470
	O	25.100	25.030	24.810	24.440	23.940
Shell	P	-----	-----	-----	-----	-----
	Q	-----	-----	29.910	-----	-----
Beam	R	28.130	28.670	30.190	32.530	35.460
	S	29.650	30.190	31.740	34.220	37.220
Experiment	T	21.520	-----	-----	-----	-----
	U	-----	-----	-----	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	20.94	-----	-----	-----	-----
	Finite element methods (A to O)	20.19	-----	-----	-----	-----
	Selected methods (D and H to N)	16.05	-----	-----	-----	-----

TABLE III.—Continued.

(16) Second torsion mode: 1/1; $a/b=1$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	24.940	24.800	24.450	23.980	23.470
	B	23.830	23.730	23.460	23.100	22.680
	C	26.190	27.140	26.420	25.550	24.640
	D	25.090	24.750	24.330	23.590	22.860
	E	26.950	26.650	26.060	25.310	24.530
	F	27.060	-----	25.830	-----	24.480
	G	26.110	26.010	25.670	24.970	23.420
	H	24.090	23.920	23.470	22.870	22.200
	I	27.200	-----	26.190	-----	24.370
	J	-----	-----	-----	-----	-----
	K	22.680	22.220	21.110	19.740	18.340
	L	21.760	21.760	21.760	21.710	21.550
	M	23.270	23.790	24.330	24.200	24.030
	N	20.200	20.180	20.110	19.480	19.770
	O	24.390	24.290	24.000	23.600	23.130
Shell	P	-----	-----	-----	-----	-----
	Q	-----	-----	26.360	-----	-----
Beam	R	-----	-----	-----	-----	-----
	S	-----	-----	-----	-----	-----
Experiment	T	22.260	21.870	21.200	20.960	-----
	U	-----	-----	-----	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	9.34	9.27	12.53	9.56	-----
	Finite element methods (A to O)	9.34	9.27	11.98	9.56	-----
	Selected methods (D and H to N)	5.16	3.95	8.00	4.43	-----

TABLE III.—Continued.

(17) Extensional mode: -/-; $a/b = 1$; $b/h = 5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	-----	26.010	25.690	25.140	24.440
	B	26.120	26.020	25.730	25.160	24.480
	C	26.190	26.110	25.820	25.330	24.590
	D	26.220	26.110	25.780	25.240	24.510
	E	26.250	26.050	25.750	25.080	24.100
	F	26.250	-----	25.380	-----	23.580
	G	-----	26.310	25.700	25.070	24.770
	H	26.150	26.050	25.730	25.210	24.500
	I	26.500	-----	26.060	-----	24.830
	J	-----	-----	-----	-----	-----
	K	26.140	25.700	24.570	23.010	20.950
	L	27.480	27.390	27.130	26.690	26.090
	M	26.130	26.120	25.950	25.570	25.120
	N	26.180	26.080	25.780	25.300	24.660
	O	26.240	26.180	26.000	25.720	25.350
Shell	P	-----	-----	-----	-----	-----
	Q	-----	-----	25.710	-----	-----
Beam	R	-----	-----	-----	-----	-----
	S	-----	-----	-----	-----	-----
Experiment	T	-----	-----	-----	-----	-----
	U	-----	-----	-----	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	-----	-----	-----	-----	-----
	Finite element methods (A to O)	-----	-----	-----	-----	-----
	Selected methods (D and H to N)	-----	-----	-----	-----	-----

TABLE III.—Continued.

(18) Second edgewise bending mode: -/-; $a/b=1$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	29.310	29.390	29.540	29.590	29.400
	B	29.310	29.350	29.410	29.360	29.070
	C	29.870	30.220	30.670	31.090	31.360
	D	29.790	29.860	30.000	30.100	30.050
	E	31.220	31.450	31.750	32.050	32.040
	F	31.220	-----	31.670	-----	31.890
	G	29.180	29.330	29.520	29.970	30.290
	H	29.530	29.580	29.660	29.560	29.140
	I	29.700	-----	30.210	-----	30.470
	J	-----	-----	-----	-----	-----
	K	29.430	29.200	28.530	27.460	26.030
	L	-----	-----	-----	-----	-----
	M	29.430	29.450	29.690	30.080	30.570
	N	29.070	29.010	28.810	28.390	27.680
	O	29.320	29.390	29.560	29.700	29.720
Shell	P	-----	-----	-----	-----	-----
	Q	-----	-----	30.090	-----	-----
Beam	R	28.640	28.620	28.550	28.440	28.260
	S	105.100	106.100	109.000	113.900	112.000
Experiment	T	-----	-----	-----	-----	-----
	U	-----	-----	-----	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	-----	-----	-----	-----	-----
	Finite element methods (A to O)	-----	-----	-----	-----	-----
	Selected methods (D and H to N)	-----	-----	-----	-----	-----

TABLE III.—Continued.

(19) Chordwise bending mode: 2/1; $a/b = 1$; $b/h = 5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	45.450	45.280	44.680	43.470	41.880
	B	-----	42.530	42.020	41.070	39.730
	C	48.730	48.610	48.290	47.690	-----
	D	48.240	47.740	47.800	45.330	43.320
	E	-----	-----	-----	-----	-----
	F	-----	-----	-----	-----	-----
	G	45.670	45.520	45.090	44.610	44.250
	H	41.400	41.340	40.940	40.210	39.030
	I	46.590	-----	45.960	-----	44.210
	J	-----	-----	-----	-----	-----
	K	38.850	38.150	36.060	32.630	28.740
	L	35.640	35.520	35.190	34.680	34.010
	M	40.500	40.430	39.950	39.310	39.130
	N	33.790	33.660	33.250	32.560	31.570
	O	40.750	40.690	40.590	40.380	40.010
Shell	P	-----	-----	-----	-----	-----
	Q	-----	-----	50.250	-----	-----
Beam	R	-----	-----	-----	-----	-----
	S	-----	-----	-----	-----	-----
Experiment	T	-----	-----	-----	-----	-----
	U	-----	-----	-----	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	-----	-----	-----	-----	-----
	Finite element methods (A to O)	-----	-----	-----	-----	-----
	Selected methods (D and H to N)	-----	-----	-----	-----	-----

TABLE III.—Continued.

(20) Chordwise bending mode: 3/0; $a/b=1$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	-----	46.120	45.590	44.790	43.850
	B	-----	45.970	45.170	44.020	42.870
	C	51.820	51.740	51.390	50.590	47.190
	D	-----	-----	-----	45.730	44.330
	E	-----	-----	-----	-----	-----
	F	-----	-----	-----	-----	-----
	G	52.910	50,010	49.410	47.760	44.590
	H	-----	-----	-----	-----	-----
	I	50.690	-----	50.040	-----	46.820
	J	-----	-----	-----	-----	-----
	K	-----	-----	-----	-----	-----
	L	42.550	42.310	41.570	40.410	38.970
	M	-----	-----	-----	-----	-----
	N	38.840	38.590	37.850	36.590	34.870
	O	46.460	46.140	45.210	44.100	42.920
Shell	P	-----	-----	-----	-----	-----
	Q	-----	-----	46.340	-----	-----
Beam	R	-----	-----	-----	-----	-----
	S	-----	-----	-----	-----	-----
Experiment	T	-----	-----	-----	-----	-----
	U	-----	-----	-----	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	-----	-----	-----	-----	-----
	Finite element methods (A to O)	-----	-----	-----	-----	-----
	Selected methods (D and H to N)	-----	-----	-----	-----	-----

TABLE III.—Continued.

(21) Third bending mode: 0/2; $a/b=1$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	48.620	47.830	-----	45.810	45.360
	B	46.310	45.640	-----	43.690	43.480
	C	-----	-----	-----	-----	-----
	D	-----	-----	-----	-----	-----
	E	-----	-----	-----	-----	-----
	F	-----	-----	-----	-----	-----
	G	50.120	-----	-----	-----	-----
	H	46.300	45.750	44.600	43.690	42.170
	I	-----	-----	-----	-----	-----
	J	-----	-----	-----	-----	-----
	K	44.820	43.720	40.670	37.120	34.220
	L	46.360	45.740	44.200	42.290	40.400
	M	46.810	45.080	43.990	44.200	44.250
	N	-----	-----	-----	-----	-----
	O	-----	-----	-----	-----	-----
Shell	P	-----	-----	-----	-----	-----
	Q	-----	-----	-----	-----	-----
Beam	R	41.690	41.640	41.480	41.240	40.950
	S	58.860	58.280	56.730	54.490	52.130
Experiment	T	-----	-----	-----	-----	-----
	U	-----	-----	-----	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	-----	-----	-----	-----	-----
	Finite element methods (A to O)	-----	-----	-----	-----	-----
	Selected methods (D and H to N)	-----	-----	-----	-----	-----

TABLE III.—Continued.

(22) Third torsion mode: 1/2; $a/b=1$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	-----	52.780	51.720	50.960	49.340
	B	-----	49.120	49.230	48.080	46.680
	C	-----	-----	-----	-----	-----
	D	-----	-----	-----	-----	-----
	E	-----	-----	-----	-----	-----
	F	-----	-----	-----	-----	-----
	G	-----	-----	-----	-----	-----
	H	-----	-----	-----	-----	-----
	I	-----	-----	-----	-----	-----
	J	-----	-----	-----	-----	-----
	K	-----	-----	-----	-----	-----
	L	49.850	49.640	49.040	48.170	47.160
	M	-----	-----	-----	-----	-----
	N	-----	-----	-----	-----	-----
	O	-----	-----	-----	-----	-----
Shell	P	-----	-----	-----	-----	-----
	Q	-----	-----	-----	-----	-----
Beam	R	62.420	63.060	64.910	67.870	71.770
	S	62.900	63.510	65.290	68.260	72.020
Experiment	T	-----	-----	-----	-----	-----
	U	-----	-----	-----	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	-----	-----	-----	-----	-----
	Finite element methods (A to O)	-----	-----	-----	-----	-----
	Selected methods (D and H to N)	-----	-----	-----	-----	-----

TABLE III.—Continued.

(23) First bending mode: 0/0; $a/b=3$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	3.419	3.416	3.437	3.551	3.838
	B	3.416	3.415	3.434	3.548	3.836
	C	3.422	-----	3.473	-----	4.052
	D	3.289	3.293	3.293	3.297	3.301
	E	3.430	3.320	3.300	3.280	3.270
	F	3.416	-----	3.370	-----	3.270
	G	3.417	3.417	3.420	3.429	3.452
	H	3.414	-----	-----	-----	3.428
	I	3.419	-----	3.634	-----	4.019
	J	-----	-----	-----	-----	-----
	K	3.416	3.391	3.318	3.209	3.076
	L	3.424	-----	-----	3.424	-----
	M	-----	-----	-----	-----	-----
	N	3.415	3.415	3.415	3.415	3.415
	O	3.700	3.700	3.700	3.700	3.700
Shell	P	3.426	3.379	3.222	2.902	2.315
	Q	3.414	3.415	3.417	3.420	3.425
Beam	R	3.353	3.355	3.360	3.365	3.381
	S	3.354	3.357	3.366	3.381	3.401
Experiment	T	3.320	3.310	3.279	3.318	3.224
	U	-----	3.335	3.303	3.358	3.225
Difference between average of theoretical results and experimental result T, percent	All	2.93	2.82	3.86	1.79	6.51
	Finite element methods (A to O)	3.22	3.24	4.56	3.21	9.30
	Selected methods (D and H to N)	2.24	1.67	3.98	0.55	6.49

TABLE III.—Continued.

(24) First torsion mode: 1/0; $a/b=3$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	21.120	22.470	26.040	30.950	36.500
	B	20.800	22.130	25.640	30.480	35.950
	C	21.190	-----	26.140	-----	36.650
	D	21.080	22.450	26.030	30.990	36.590
	E	21.240	24.450	32.090	41.120	51.890
	F	21.250	-----	31.520	-----	50.220
	G	21.690	23.850	27.000	30.850	34.370
	H	20.540	-----	-----	-----	34.240
	I	21.140	-----	25.550	-----	33.490
	J	-----	-----	-----	-----	-----
	K	20.980	22.080	24.980	28.910	33.220
	L	20.410	-----	-----	28.780	-----
	M	-----	-----	-----	-----	-----
	N	20.850	22.010	25.100	29.440	34.430
	O	21.200	22.200	25.400	29.700	34.700
Shell	P	21.150	22.380	26.330	34.290	50.770
	Q	21.220	22.280	25.150	29.200	33.900
Beam	R	21.040	22.260	25.550	30.210	35.670
	S	21.040	22.240	25.490	30.300	35.730
Experiment	T	19.580	20.680	23.530	28.270	32.290
	U	-----	20.790	23.610	28.310	32.280
Difference between average of theoretical results and experimental result T, percent	All	7.01	8.36	11.32	9.31	15.07
	Finite element methods (A to O)	6.93	8.92	12.41	9.53	14.32
	Selected methods (D and H to N)	6.02	6.76	7.42	4.27	6.12

TABLE III.—Continued.

(25) Second bending mode: 0/1; $a/b=3$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	21.380	20.720	19.100	17.150	15.320
	B	21.310	20.660	19.040	17.110	15.280
	C	21.410	-----	19.090	------	15.280
	D	20.580	19.930	18.330	16.430	14.560
	E	21.580	21.070	19.390	17.370	15.680
	F	22.140	20.690	19.060	------	15.690
	G	21.120	18.790	16.460	14.110	11.620
	H	21.590	-----	-----	------	15.220
	I	21.360	-----	20.220	------	17.560
	J	-----	-----	-----	------	------
	K	21.380	20.650	18.950	16.900	14.920
	L	21.320	-----	-----	17.000	------
	M	-----	-----	-----	------	------
	N	21.300	20.640	19.010	17.040	15.140
	O	22.900	22.300	20.400	18.300	16.300
Shell	P	21.290	20.840	19.310	17.090	14.360
	Q	21.360	20.670	19.000	16.990	15.070
Beam	R	20.990	20.320	18.680	16.720	14.860
	S	21.020	20.430	18.900	16.980	15.230
Experiment	T	20.760	19.680	18.030	15.990	14.090
	U	-----	19.760	18.120	16.140	14.100
Difference between average of theoretical results and experimental result T, percent	All	3.08	4.43	5.09	5.16	6.88
	Finite element methods (A to O)	3.40	4.49	5.13	4.95	7.39
	Selected methods (D and H to N)	2.33	3.56	5.74	5.06	8.98

TABLE III.—Continued.

(26) Third bending mode: 0/2; $a/b=3$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	60.150	58.440	56.210	53.840	51.280
	B	59.790	58.130	55.900	53.340	50.990
	C	60.300	-----	56.350	------	51.380
	D	57.860	56.450	54.240	51.840	49.240
	E	62.350	61.000	58.640	55.450	53.250
	F	64.370	-----	57.970	------	53.260
	G	-----	-----	-----	------	------
	H	62.200	-----	-----	------	52.510
	I	60.020	-----	59.150	------	58.520
	J	-----	-----	-----	------	------
	K	60.100	58.070	55.480	52.620	49.430
	L	59.840	-----	-----	53.410	------
	M	-----	-----	-----	------	------
	N	60.240	58.370	56.110	53.710	51.080
	O	62.460	61.470	59.790	57.700	55.130
Shell	P	60.100	58.320	56.610	54.380	50.340
	Q	60.120	58.150	55.950	53.580	50.950
Beam	R	58.640	57.040	54.910	52.620	50.120
	S	58.860	58.020	56.780	55.820	55.330
Experiment	T	58.650	51.610	46.290	47.310	47.680
	U	-----	55.840	40.160	49.900	47.540
Difference between average of theoretical results and experimental result T, percent	All	3.00	11.77	18.39	12.43	8.64
	Finite element methods (A to O)	3.55	12.30	18.77	12.37	8.96
	Selected methods (D and H to N)	2.32	10.45	17.70	10.56	8.58

TABLE III.—Continued.

(27) First edgewise bending mode: -/-; $a/b=3$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	63.470	66.720	73.000	80.610	87.910
	B	63.470	66.660	72.920	80.490	87.700
	C	63.540	-----	73.100	------	88.080
	D	63.690	66.620	72.800	80.270	86.980
	E	67.400	73.460	83.570	91.660	99.070
	F	67.460	-----	82.900	------	98.400
	G	-----	-----	-----	------	------
	H	62.470	-----	-----	------	87.930
	I	62.770	-----	71.680	------	83.140
	J	-----	-----	-----	------	------
	K	62.440	65.680	71.340	77.340	83.710
	L	62.900	-----	-----	80.280	------
	M	-----	-----	-----	------	------
	N	62.710	66.000	72.090	79.440	86.800
	O	64.820	67.340	72.980	80.340	88.150
Shell	P	62.290	66.160	74.600	89.410	115.600
	Q	61.820	65.290	71.400	78.820	86.020
Beam	R	61.920	65.100	71.380	79.060	86.380
	S	67.080	70.130	77.460	87.920	99.890
Experiment	T	-----	57.820	57.980	64.870	73.470
	U	-----	60.480	63.180	70.810	76.740
Difference between average of theoretical results and experimental result T, percent	All	-----	13.95	22.04	21.02	19.31
	Finite element methods (A to O)	-----	14.34	22.32	20.21	17.35
	Selected methods (D and H to N)	-----	12.53	19.45	18.23	14.28

TABLE III.—Continued.

(28) Second torsion mode: 1/1; $a/b=3$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	66.420	69.690	78.500	91.090	105.600
	B	65.380	68.590	77.300	89.630	103.900
	C	66.700	-----	78.890	------	106.100
	D	66.050	69.370	78.320	90.980	105.600
	E	73.460	76.830	88.960	109.200	130.900
	F	73.450	-----	87.610	------	129.390
	G	-----	-----	-----	------	------
	H	65.220	-----	-----	------	100.600
	I	66.370	-----	77.870	------	99.330
	J	-----	-----	-----	------	------
	K	65.890	68.520	75.570	85.300	96.090
	L	64.080	-----	-----	85.380	------
	M	-----	-----	-----	------	------
	N	65.490	68.350	76.270	87.490	100.600
	O	67.000	69.240	77.320	88.220	101.100
Shell	P	66.420	69.430	79.400	100.100	144.100
	Q	66.680	69.320	76.570	87.050	99.410
Beam	R	66.050	69.570	79.130	92.750	108.850
	S	66.040	69.510	78.950	93.000	109.000
Experiment	T	62.430	64.140	71.610	82.700	94.020
	U	-----	64.300	71.700	82.760	93.920
Difference between average of theoretical results and experimental result T, percent	All	6.71	8.18	9.73	9.80	14.04
	Finite element methods (A to O)	7.00	8.48	10.11	9.03	12.30
	Selected methods (D and H to N)	4.71	6.70	7.01	5.26	6.40

TABLE III.—Continued.

(29) Third torsion mode: 1/2; $a/b=3$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	120.400	124.000	134.100	149.100	167.200
	B	118.300	121.900	131.800	146.500	164.100
	C	121.000	------	134.900	------	168.100
	D	118.800	122.600	132.900	148.200	166.500
	E	122.000	130.700	140.200	156.400	173.900
	F	124.000	------	137.500	------	171.200
	G	------	------	------	------	------
	H	120.500	------	------	------	162.700
	I	120.000	------	135.100	------	164.500
	J	------	------	------	------	------
	K	119.300	122.400	130.600	142.400	155.700
	L	115.800	------	------	141.400	------
	M	------	------	------	------	------
	N	118.300	121.500	130.500	143.900	159.800
	O	129.000	128.600	134.400	147.200	162.800
Shell	P	120.100	123.600	135.700	161.900	222.000
	Q	120.800	123.800	132.400	145.200	160.700
Beam	R	119.200	124.700	139.700	161.400	187.300
	S	119.200	124.600	139.400	161.700	187.500
Experiment	T	113.500	114.700	123.400	136.600	151.200
	U	------	115.000	123.800	136.900	151.000
Difference between average of theoretical results and experimental result T, percent	All	5.75	7.80	8.55	9.20	11.89
	Finite element methods (A to O)	5.90	7.89	8.05	7.00	8.44
	Selected methods (D and H to N)	4.45	6.11	6.71	5.12	6.57

TABLE III.—Continued.

(30) Fourth bending mode: 0/3; $a/b=3$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	118.500	118.100	117.300	116.300	116.200
	B	117.400	117.100	116.200	115.300	115.200
	C	119.000	------	117.800	------	116.700
	D	113.900	113.600	112.800	112.100	112.600
	E	128.700	124.000	124.000	122.700	------
	F	128.700	------	120.630	------	122.650
	G	------	------	------	------	------
	H	127.100	------	------	------	123.100
	I	118.000	------	123.300	------	129.500
	J	------	------	------	------	------
	K	119.500	118.900	117.000	114.500	112.100
	L	117.700	------	------	115.700	------
	M	------	------	------	------	------
	N	120.400	120.000	119.000	117.500	116.500
	O	122.800	125.800	127.600	126.300	125.200
Shell	P	125.600	126.200	128.600	133.900	146.300
	Q	118.600	118.300	117.400	116.200	115.800
Beam	R	114.600	114.300	113.600	112.600	113.300
	S	-------	------	------	------	------
Experiment	T	115.200	112.700	112.300	109.100	108.100
	U	------	113.300	112.800	110.200	108.600
Difference between average of theoretical results and experimental result T, percent	All	4.56	5.79	6.13	7.90	10.21
	Finite element methods (A to O)	4.77	5.80	6.07	7.19	9.14
	Selected methods (D and H to N)	3.54	4.09	4.85	5.09	8.98

TABLE III.—Continued.

(31) Fourth torsion mode: 1/3; $a/b=3$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	187.300	190.300	198.800	212.000	228.500
	B	183.800	186.600	195.000	207.700	223.800
	C	188.700	------	193.600	------	205.700
	D	183.600	186.700	195.500	------	------
	E	------	------	------	------	------
	F	------	------	------	------	------
	G	------	------	------	------	------
	H	193.300	------	------	------	213.800
	I	186.400	------	203.500	------	------
	J	------	------	------	------	------
	K	187.000	189.300	195.500	206.200	------
	L	180.000	------	------	202.500	------
	M	------	------	------	------	------
	N	183.900	186.500	193.800	204.500	------
	O	193.900	196.300	203.500	207.300	204.900
Shell	P	193.300	196.300	206.600	------	------
	Q	188.100	190.700	198.100	209.600	------
Beam	R	184.500	191.600	211.000	239.600	274.200
	S	------	------	------	------	------
Experiment	T	177.000	176.200	184.400	195.000	194.300
	U	------	------	184.900	195.500	------
Difference between average of theoretical results and experimental result T, percent	All	5.46	7.50	7.59	7.66	13.70
	Finite element methods (A to O)	5.24	6.91	6.59	5.66	9.77
	Selected methods (D and H to N)	4.68	6.03	6.43	4.60	9.12

TABLE III.—Continued.

(32) Chordwise bending mode: 2/0; $a/b=3$; $b/h=20$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	270.500	207.300	206.800	206.200	205.600
	B	200.500	200.100	199.300	198.200	197.300
	C	202.100	------	200.300	------	------
	D	------	------	------	204.200	203.600
	E	------	------	------	------	------
	F	------	------	------	------	------
	G	------	------	------	------	------
	H	203.400	------	------	------	202.000
	I	------	------	------	------	212.000
	J	------	------	------	------	------
	K	------	------	------	------	209.000
	L	185.700	------	------	185.300	------
	M	------	------	------	------	------
	N	193.300	193.200	193.000	192.500	191.500
	O	------	------	------	------	------
Shell	P	203.800	204.000	204.800	206.700	211.600
	Q	------	------	------	------	211.600
Beam	R	------	------	------	------	------
	S	------	------	------	------	------
Experiment	T	207.000	201.000	203.400	200.900	202.200
	U	------	184.700	202.500	200.000	202.700
Difference between average of theoretical results and experimental result T, percent	All	−3.77	0.07	−1.27	−1.03	1.32
	Finite element methods (A to O)	−4.15	−0.40	−1.78	−1.83	0.39
	Selected methods (D and H to N)	−6.63	−4.04	−5.39	−3.56	0.70

TABLE III.—Continued.

(33) First bending mode: 0/0; $a/b=3$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	3.399	3.385	3.351	3.318	3.313
	B	3.396	3.383	3.349	3.315	3.311
	C	3.422	-----	3.472	-----	4.024
	D	3.287	3.287	3.287	3.287	3.293
	E	3.312	3.375	3.310	3.320	3.320
	F	3.354	-----	3.312	-----	3.330
	G	3.417	3.416	3.413	3.412	3.411
	H	3.394	-----	-----	-----	3.403
	I	3.419	-----	3.406	-----	3.286
	J	-----	-----	-----	-----	-----
	K	3.394	3.366	3.287	3.167	3.020
	L	3.609	-----	-----	3.609	-----
	M	3.397	3.397	3.396	3.394	3.389
	N	3.384	3.382	3.381	3.378	3.375
	O	3.610	3.620	3.620	3.630	3.640
Shell	P	-----	-----	-----	-----	-----
	Q	-----	-----	-----	-----	-----
Beam	R	3.342	3.344	3.349	3.357	3.369
	S	3.354	3.357	3.364	3.378	3.396
Experiment	T	3.213	3.192	3.206	3.190	3.222
	U	3.213	3.200	3.206	3.198	3.226
Difference between average of theoretical results and experimental result T, percent	All	5.66	5.90	5.10	5.63	5.01
	Finite element methods (A to O)	5.88	6.15	5.20	5.70	5.05
	Selected methods (D and H to N)	5.83	4.94	4.34	5.26	2.20

TABLE III.—Continued.

(34) First edgewise bending mode: -/-; $a/b=3$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	15.870	15.440	14.510	13.470	12.470
	B	15.870	15.430	14.500	13.450	12.460
	C	15.890	-----	14.460	-----	12.350
	D	15.920	15.340	14.180	12.970	11.830
	E	18.370	17.350	15.750	14.240	12.890
	F	18.370	-----	15.760	-----	12.890
	G	-----	-----	-----	-----	-----
	H	15.620	-----	-----	-----	11.860
	I	15.690	-----	14.260	-----	11.720
	J	-----	-----	-----	-----	-----
	K	15.610	15.170	14.150	12.950	11.760
	L	15.820	-----	-----	13.340	-----
	M	15.570	15.050	13.950	12.750	11.620
	N	15.680	15.180	14.140	12.990	11.900
	O	15.650	15.260	14.380	13.360	12.340
Shell	P	------	------	------	-----	-----
	Q	------	------	------	-----	-----
Beam	R	15.480	14.990	13.970	12.850	11.800
	S	16.770	16.070	14.760	13.410	12.250
Experiment	T	------	------	------	-----	-----
	U	------	------	------	-----	-----
Difference between average of theoretical results and experimental result T, percent	All	------	------	------	-----	-----
	Finite element methods (A to O)	------	------	------	-----	-----
	Selected methods (D and H to N)	------	------	------	-----	-----

TABLE III.—Continued.

(35) First torsion mode: 1/0; $a/b = 3$; $b/h = 5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	19.800	19.860	20.050	20.340	20.740
	B	19.500	19.560	19.740	20.040	20.430
	C	21.220	-----	21.450	-----	22.170
	D	20.700	20.740	20.900	21.150	21.510
	E	21.230	21.400	21.900	22.740	23.750
	F	21.230	-----	21.230	-----	23.250
	G	21.690	21.510	21.420	21.240	20.990
	H	19.240	-----	-----	-----	20.060
	I	21.360	-----	21.270	-----	21.680
	J	-----	-----	-----	-----	-----
	K	19.240	19.310	19.530	19.920	20.410
	L	18.510	-----	-----	18.960	-----
	M	19.470	19.550	19.810	20.200	20.710
	N	18.560	18.610	18.750	19.000	19.340
	O	19.460	19.510	19.680	19.950	20.310
Shell	P	------	------	------	-----	-----
	Q	------	------	------	-----	-----
Beam	R	19.270	19.340	19.560	19.910	20.390
	S	19.090	19.170	19.390	19.780	20.300
Experiment	T	18.430	18.450	18.670	18.890	19.320
	U	18.460	18.500	18.700	18.910	19.460
Difference between average of theoretical results and experimental result T, percent	All	7.73	7.14	8.18	6.80	8.30
	Finite element methods (A to O)	8.25	7.78	8.83	7.19	8.79
	Selected methods (D and H to N)	5.89	5.64	6.89	4.82	6.30

TABLE III.—Continued.

(36) Second bending mode: 0/1; $a/b=3$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	21.020	21.600	22.990	24.720	26.580
	B	20.950	21.540	22.930	24.660	26.520
	C	21.360	-----	23.600	-----	27.780
	D	20.470	21.200	22.810	24.760	26.770
	E	21.560	22.580	24.260	27.130	29.480
	F	22.070	-----	24.780	-----	29.310
	G	21.120	21.100	21.020	20.950	20.950
	H	21.160	-----	-----	-----	27.150
	I	21.140	-----	23.250	-----	26.330
	J	-----	-----	-----	-----	-----
	K	20.800	21.290	22.390	23.650	24.850
	L	21.880	-----	-----	25.470	-----
	M	20.910	21.580	23.120	24.950	26.810
	N	20.590	21.210	22.670	24.440	26.340
	O	22.130	22.660	23.960	25.670	27.540
Shell	P	------	------	------	-----	-----
	Q	------	------	------	-----	-----
Beam	R	20.520	21.170	22.660	24.520	26.520
	S	21.020	21.960	23.960	26.460	29.080
Experiment	T	19.910	19.670	21.190	21.580	22.800
	U	20.000	20.180	21.370	22.570	24.170
Difference between average of theoretical results and experimental result T, percent	All	5.95	9.05	8.55	12.92	14.93
	Finite element methods (A to O)	6.20	9.10	8.46	12.42	14.44
	Selected methods (D and H to N)	5.16	7.74	7.26	12.47	13.55

TABLE III.—Continued.

(37) Second torsion mode: 1/1; $a/b=3$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	61.740	61.880	62.300	62.960	63.850
	B	60.760	60.890	61.300	61.960	62.840
	C	66.740	------	67.270	------	68.800
	D	64.590	64.710	65.020	65.540	66.260
	E	67.390	67.050	67.380	68.740	70.750
	F	65.730	------	66.380	------	69.410
	G	------	------	------	------	------
	H	60.400	------	------	------	62.240
	I	66.380	------	66.520	------	66.850
	J	------	------	------	------	------
	K	59.620	59.700	59.830	59.910	59.890
	L	57.440	------	------	58.460	------
	M	60.430	60.580	61.000	61.720	62.880
	N	57.080	57.200	57.540	58.090	58.850
	O	60.640	60.770	61.140	61.750	62.580
Shell	P	------	------	------	------	------
	Q	------	------	------	------	------
Beam	R	60.430	60.630	61.250	62.270	63.660
	S	64.600	64.810	65.460	66.580	68.050
Experiment	T	57.080	57.100	57.660	58.210	59.040
	U	57.290	57.340	57.890	58.520	59.220
Difference between average of theoretical results and experimental result T, percent	All	8.33	7.64	8.85	6.93	8.86
	Finite element methods (A to O)	8.27	7.30	8.83	6.30	8.61
	Selected methods (D and H to N)	6.19	5.69	6.97	4.17	6.03

TABLE III.—Continued.

(38) Third bending mode: 0/2; $a/b=3$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	58.010	57.160	55.160	52.750	50.340
	B	57.650	56.830	54.870	52.500	50.120
	C	60.320	------	56.760	------	51.220
	D	57.070	56.200	54.070	51.480	48.850
	E	62.330	60.310	57.950	55.590	52.900
	F	64.350	------	57.950	------	53.240
	G	------	------	------	------	------
	H	59.420	------	------	------	50.030
	I	60.030	------	56.170	------	48.720
	J	------	------	------	------	------
	K	56.780	55.780	53.390	50.410	47.270
	L	58.980	------	------	53.650	------
	M	57.320	56.280	53.910	51.170	48.540
	N	55.780	54.970	53.620	50.580	48.040
	O	60.420	59.320	56.900	54.150	51.440
Shell	P	------	------	------	------	------
	Q	------	------	------	------	------
Beam	R	55.670	54.850	52.820	50.370	47.890
	S	58.860	58.280	56.730	54.490	52.130
Experiment	T	53.980	52.350	50.570	47.460	45.600
	U	54.020	52.350	50.530	47.900	45.530
Difference between average of theoretical results and experimental result T, percent	All	8.30	8.15	8.73	9.54	8.89
	Finite element methods (A to O)	8.68	8.33	8.92	9.56	8.91
	Selected methods (D and H to N)	6.79	6.20	6.75	7.77	6.12

TABLE III.—Continued.

(39) Second edgewise bending mode: -/-; $a/b = 3$; $b/h = 5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	72.350	73.390	75.940	79.160	82.460
	B	72.350	73.360	75.860	79.020	82.250
	C	72.630	------	77.320	------	84.650
	D	72.740	73.800	76.380	79.660	83.020
	E	83.500	84.230	86.920	89.670	92.990
	F	83.500	------	86.930	------	92.990
	G	------	------	------	------	------
	H	72.590	------	------	------	84.390
	I	71.780	------	76.100	------	82.050
	J	------	------	------	------	------
	K	71.740	72.510	74.130	75.000	74.550
	L	------	------	------	------	------
	M	71.400	72.580	75.320	78.540	81.480
	N	71.300	72.230	74.540	77.490	80.500
	O	71.530	72.810	75.730	78.000	77.750
Shell	P	------	------	------	------	------
	Q	------	------	------	------	------
Beam	R	70.530	71.590	74.220	77.590	81.090
	S	105.100	106.100	109.000	113.900	120.000
Experiment	T	62.470	60.670	61.260	62.990	66.610
	U	------	------	------	------	------
Difference between average of theoretical results and experimental result T, percent	All	17.73	21.47	23.31	23.93	22.30
	Finite element methods (A to O)	15.52	18.41	21.20	20.83	19.99
	Selected methods (D and H to N)	13.15	16.64	18.64	18.90	17.76

TABLE III.—Continued.

(40) Extensional mode: -/-; $a/b = 3$; $b/h = 5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	78.170	78.130	78.020	77.830	77.570
	B	78.170	78.130	78.010	77.820	77.560
	C	78.190	------	77.740	------	77.480
	D	78.320	78.290	78.170	77.980	77.710
	E	78.170	78.170	78.160	78.170	77.490
	F	78.160	------	78.170	------	77.490
	G	------	------	------	------	------
	H	78.280	------	------	------	77.690
	I	78.500	------	78.290	------	76.920
	J	------	------	------	------	------
	K	78.170	78.080	78.010	78.830	80.620
	L	79.220	------	------	78.960	------
	M	78.130	78.080	77.930	77.690	77.390
	N	78.490	78.440	78.320	78.130	77.860
	O	78.320	78.280	78.180	79.240	82.850
Shell	P	------	------	------	------	------
	Q	------	------	------	------	------
Beam	R	------	------	------	------	------
	S	------	------	------	------	------
Experiment	T	------	------	------	------	------
	U	------	------	------	------	------
Difference between average of theoretical results and experimental result T, percent	All	------	------	------	------	------
	Finite element methods (A to O)	------	------	------	------	------
	Selected methods (D and H to N)	------	------	------	------	------

TABLE III.—Continued.

(41) Third torsion mode: 1/2; $a/b=3$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	110.200	110.300	110.700	111.200	112.000
	B	108.200	108.400	108.700	109.200	110.000
	C	121.000	------	121.400	------	122.600
	D	115.500	115.500	115.600	115.900	116.300
	E	------	------	------	------	------
	F	------	------	------	------	------
	G	------	------	------	------	------
	H	109.200	------	------	------	110.500
	I	120.000	------	119.600	------	117.600
	J	------	------	------	------	------
	K	105.400	105.000	103.500	100.800	97.460
	L	101.500	------	------	102.400	------
	M	107.000	106.900	106.700	106.700	107.700
	N	99.130	99.220	99.480	99.920	100.500
	O	107.000	108.100	108.400	109.000	109.700
Shell	P	------	------	------	------	------
	Q	------	------	------	------	------
Beam	R	108.900	109.200	110.200	111.700	113.900
	S	115.900	116.300	117.300	119.000	121.300
Experiment	T	100.600	100.100	101.100	101.500	102.200
	U	100.800	100.500	101.300	101.800	102.400
Difference between average of theoretical results and experimental result T, percent	All	8.48	7.97	8.96	6.52	8.45
	Finite element methods (A to O)	8.10	7.00	8.47	5.04	7.46
	Selected methods (D and H to N)	7.06	6.15	7.23	3.47	5.67

TABLE III.—Continued.

(42) Fourth bending mode: 0/3; $a/b = 3$; $b/h = 5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	111.300	110.800	109.700	108.000	106.100
	B	110.200	109.800	108.600	107.000	105.300
	C	119.100	------	116.900	------	112.000
	D	111.100	110.000	109.300	107.300	105.200
	E	------	------	------	------	------
	F	------	------	------	------	------
	G	------	------	------	------	------
	H	117.100	------	------	------	110.000
	I	118.000	------	115.100	------	105.500
	J	------	------	------	------	------
	K	108.200	107.800	107.100	106.500	105.900
	L	110.000	------	------	107.900	------
	M	109.100	108.600	107.100	105.000	102.900
	N	104.800	104.400	103.300	101.800	100.000
	O	114.300	113.800	112.600	110.700	108.600
Shell	P	------	------	------	------	------
	Q	------	------	------	------	------
Beam	R	104.700	104.400	103.300	101.800	100.100
	S	------	------	------	------	------
Experiment	T	102.000	106.000	99.400	97.710	93.710
	U	------	------	------	------	------
Difference between average of theoretical results and experimental result T, percent	All	8.51	2.48	9.06	8.01	11.26
	Finite element methods (A to O)	9.02	3.03	9.61	8.49	11.72
	Selected methods (D and H to N)	8.26	1.58	8.29	7.56	10.68

TABLE III.—Continued.

(43) Fourth torsion mode: 1/3; $a/b=3$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	168.200	168.200	168.200	168.300	168.400
	B	164.600	164.600	164.700	164.800	164.900
	C	------	------	------	------	------
	D	------	------	------	------	------
	E	------	------	------	------	------
	F	------	------	------	------	------
	G	------	------	------	------	------
	H	------	------	------	------	------
	I	------	------	------	------	------
	J	------	------	------	------	------
	K	------	------	------	------	------
	L	152.700	------	------	153.000	------
	M	------	------	------	------	------
	N	------	------	------	------	------
	O	------	------	------	------	------
Shell	P	------	------	------	------	------
	Q	------	------	------	------	------
Beam	R	------	------	------	------	------
	S	------	------	------	------	------
Experiment	T	148.300	152.300	150.700	150.400	150.800
	U	------	------	------	------	------
Difference between average of theoretical results and experimental result T, percent	All	8.36	8.47	9.46	7.18	9.51
	Finite element methods (A to O)	8.36	8.47	9.46	7.18	9.51
	Selected methods (D and H to N)	2.88	------	------	1.70	------

TABLE III.—Concluded.

(44) Chordwise bending mode: 2/0; $a/b=3$; $b/h=5$.

Type of data	Method	Twist angle, φ, deg				
		0	15	30	45	60
Finite element	A	158.500	158.900	160.100	161.700	163.000
	B	158.500	158.900	159.900	161.300	162.200
	C	------	------	------	------	------
	D	------	------	------	------	------
	E	------	------	------	------	------
	F	------	------	------	------	------
	G	------	------	------	------	------
	H	------	------	------	------	------
	I	------	------	------	------	------
	J	------	------	------	------	------
	K	------	------	------	------	------
	L	163.700	------	------	162.100	------
	M	------	------	------	------	------
	N	------	------	------	------	------
	O	------	------	------	------	------
Shell	P	------	------	------	------	------
	Q	------	------	------	------	------
Beam	R	------	------	------	------	------
	S	------	------	------	------	------
Experiment	T	------	183.500	------	------	------
	U	------	------	------	------	------
Difference between average of theoretical results and experimental result T, percent	All	------	−13.41	------	------	------
	Finite element methods (A to O)	------	−13.41	------	------	------
	Selected methods (D and H to N)	------	------	------	------	------

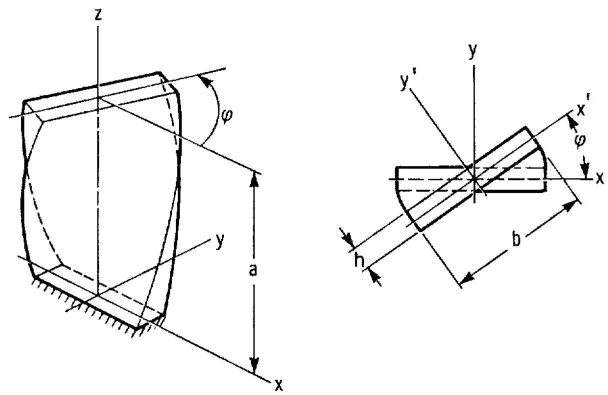


Figure 1.—Twisted cantilever plate.

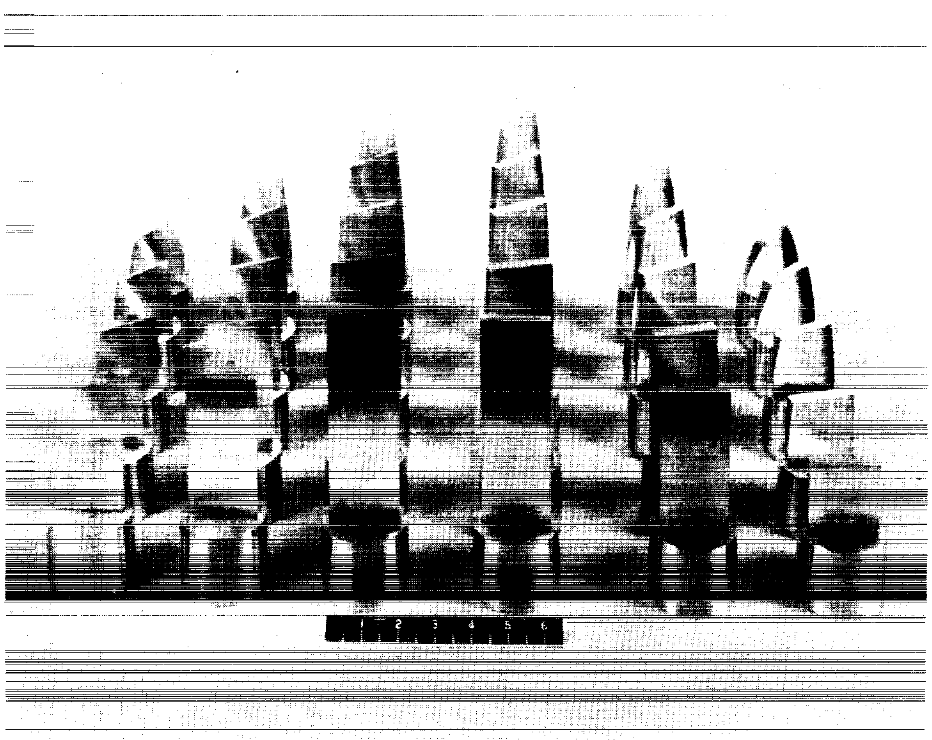
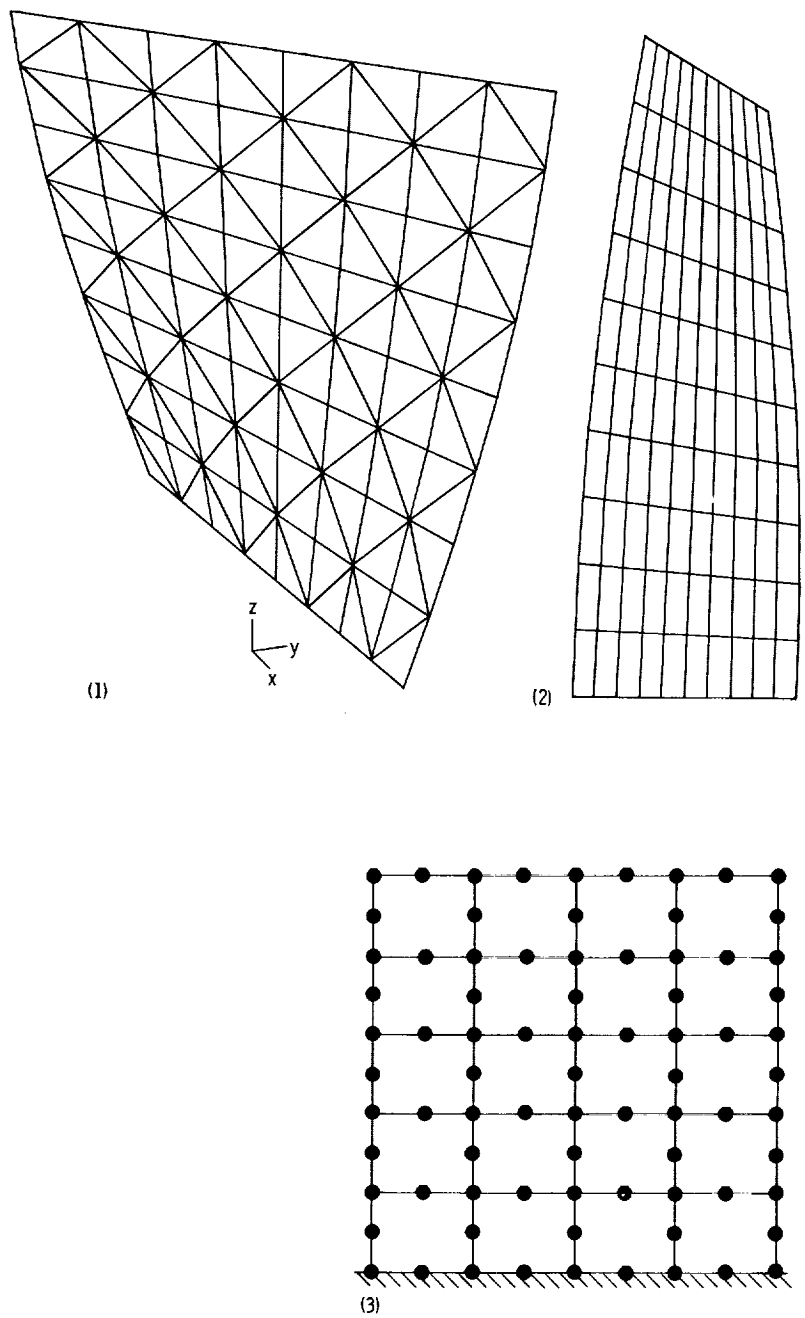
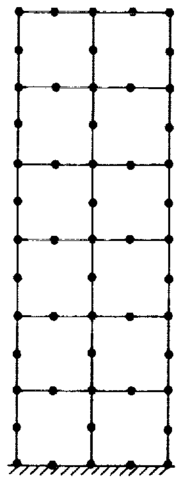


Figure 2.—Precision-machined specimens.

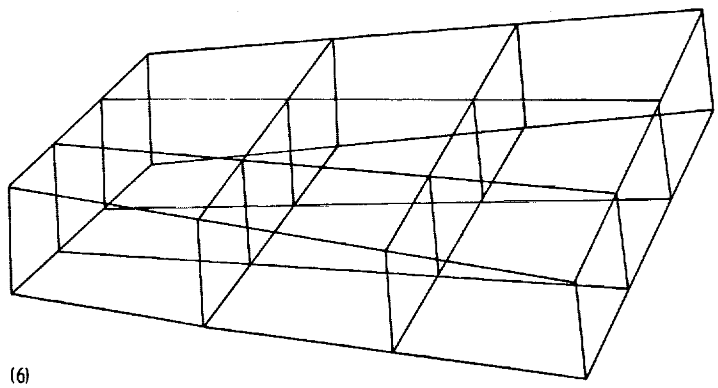


(1) Methods A, B, C, and I: $a/b=1$; $\varphi=45°$.
(2) Method H: $a/b=3$; $\varphi=60°$.
(3) Method J: $a/b=1$; $\varphi=0°$.

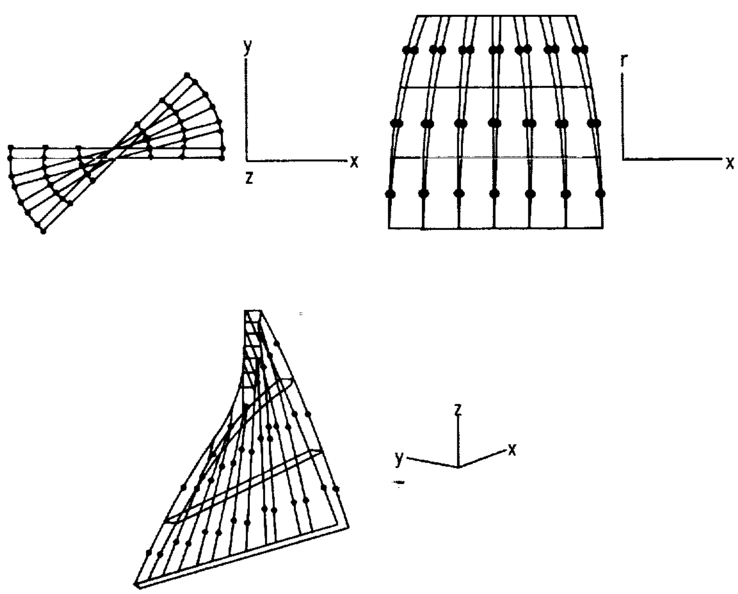
Figure 3.—Meshes used in vibration study.



(4)



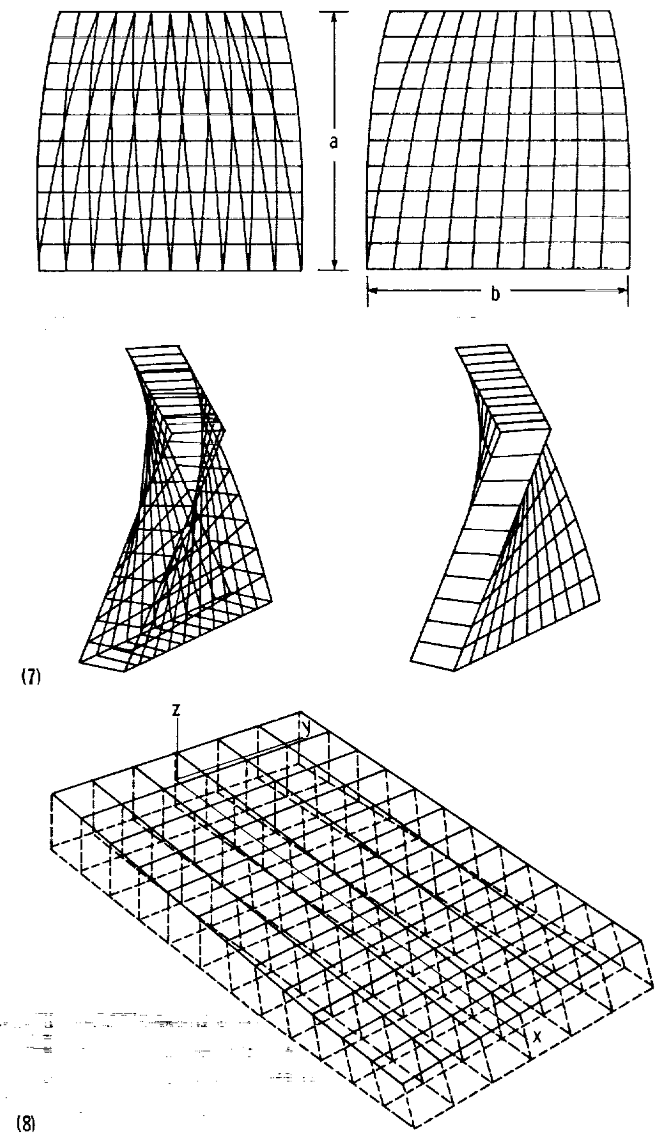
(6)



(5)

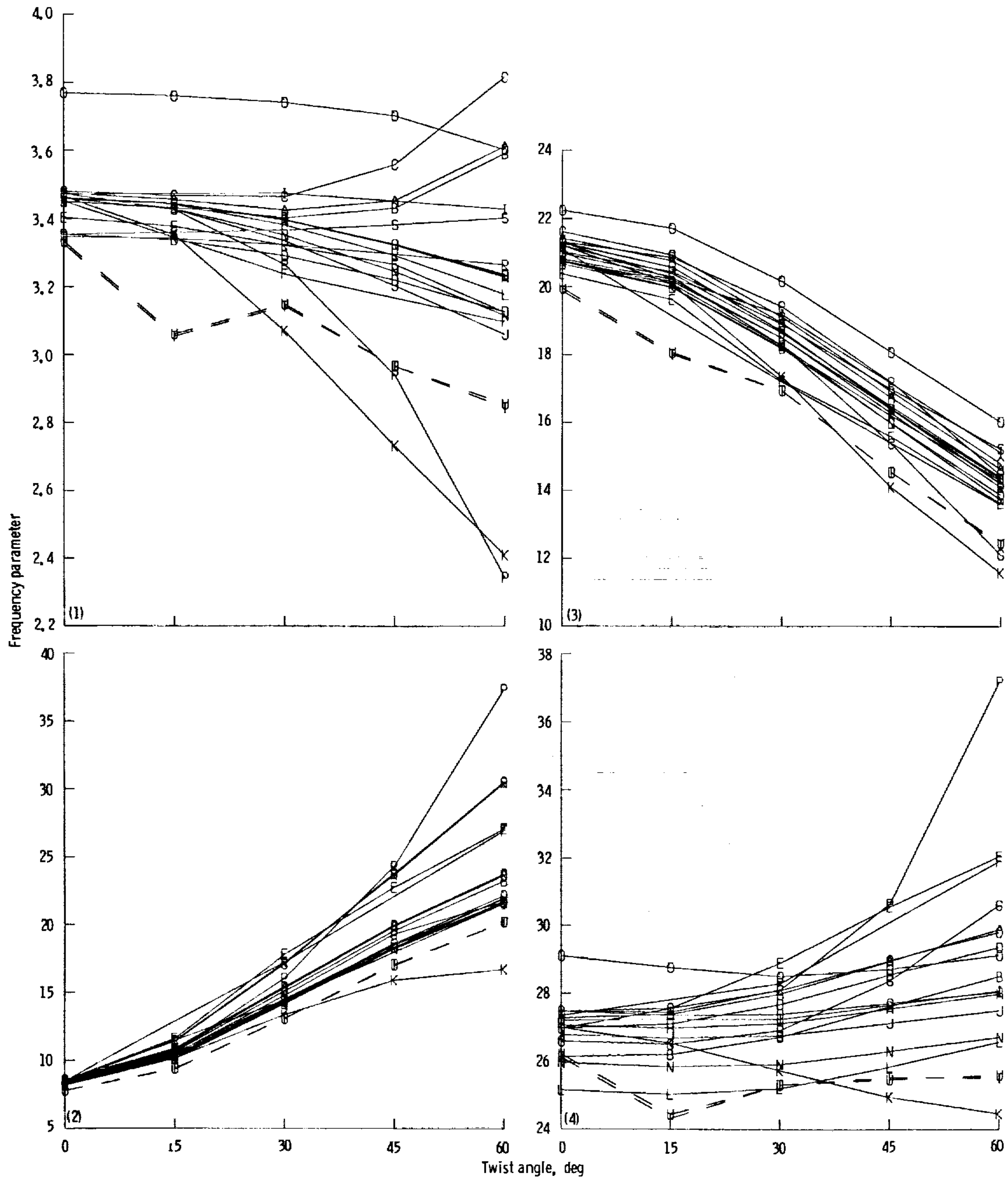
(4) Method K: $a/b=3$; $\varphi=0°$.
(5) Method L: $a/b=1$; $\varphi=45°$; $b/h=20$.
(6) Method M: $a/b=1$; $\varphi=15°$; $b/h=5$.

Figure 3.—Continued.



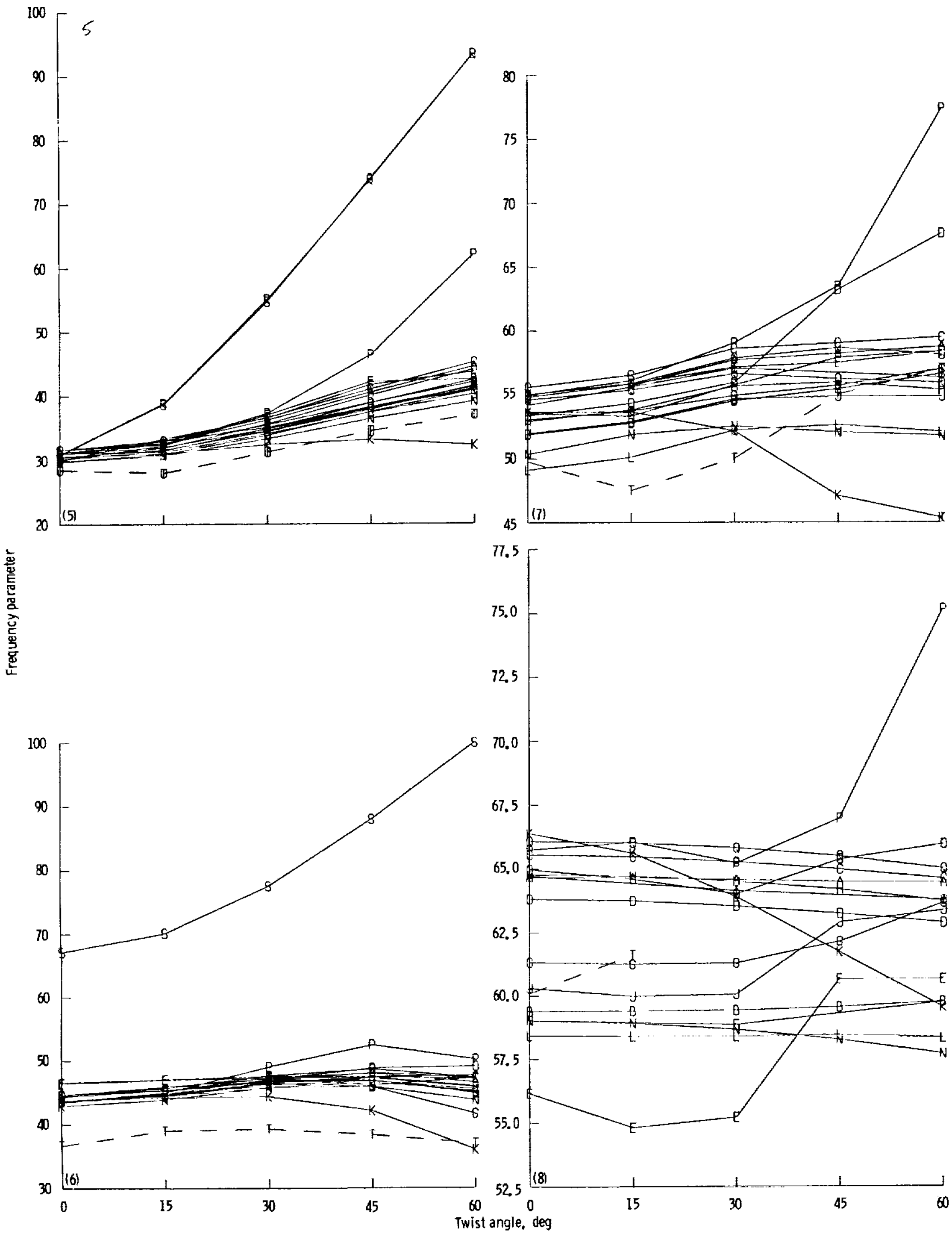
(7) Method N: $a/b = 1$; $\varphi = 45°$; $b/h = 5$.
(8) Method O: $a/b = 2$; $\varphi = 15°$; $b/h = 5$.

Figure 3.—Concluded.



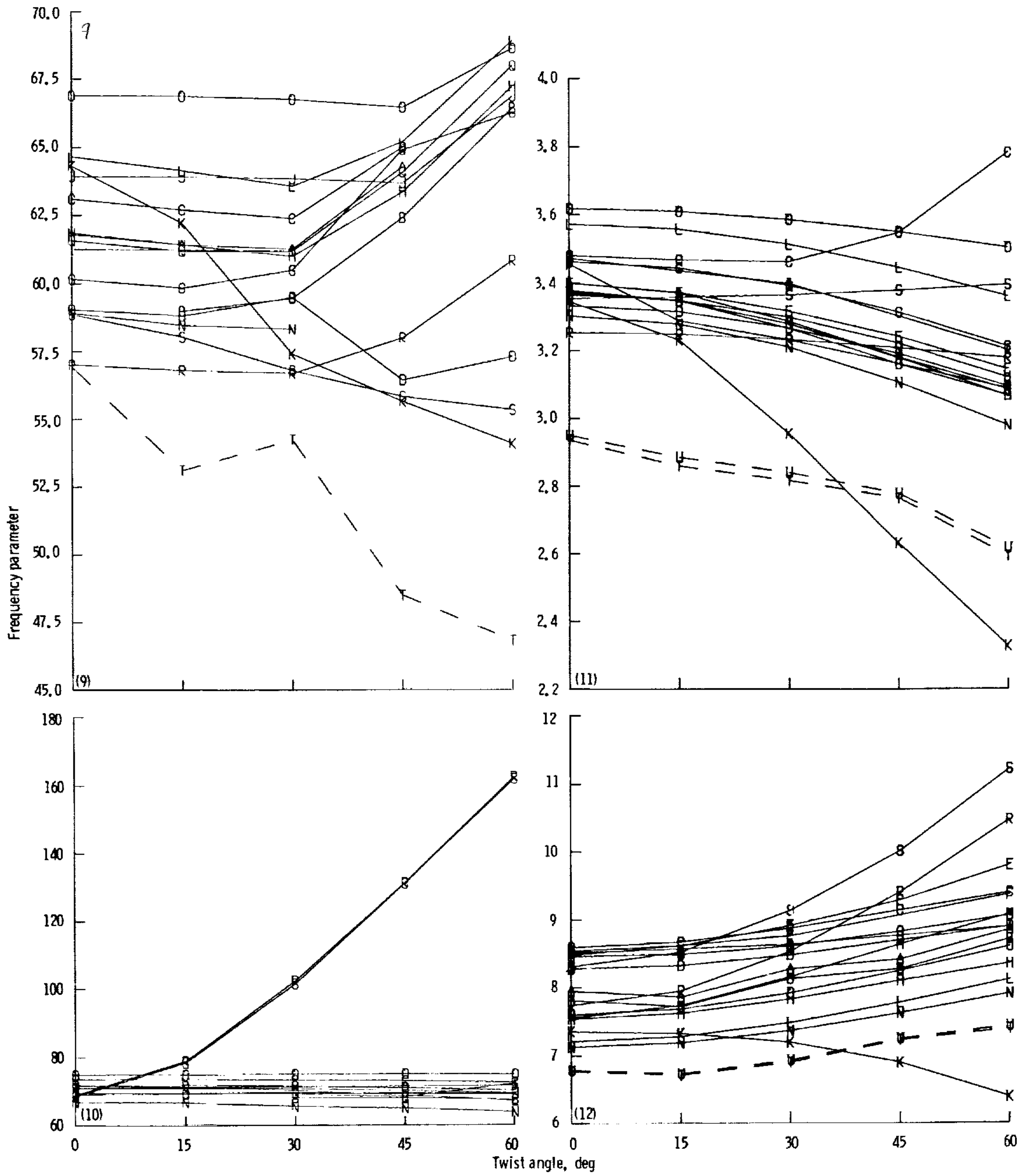
(1) First bending mode: 0/0; $a/b=1$; $b/h=20$.
(2) First torsion mode: 1/0; $a/b=1$; $b/h=20$.
(3) Second bending mode: 0/1; $a/b=1$; $b/h=20$.
(4) Chordwise bending mode: 2/0; $a/b=1$; $b/h=20$.

Figure 4.—Dependence of frequency parameter on twist angle.



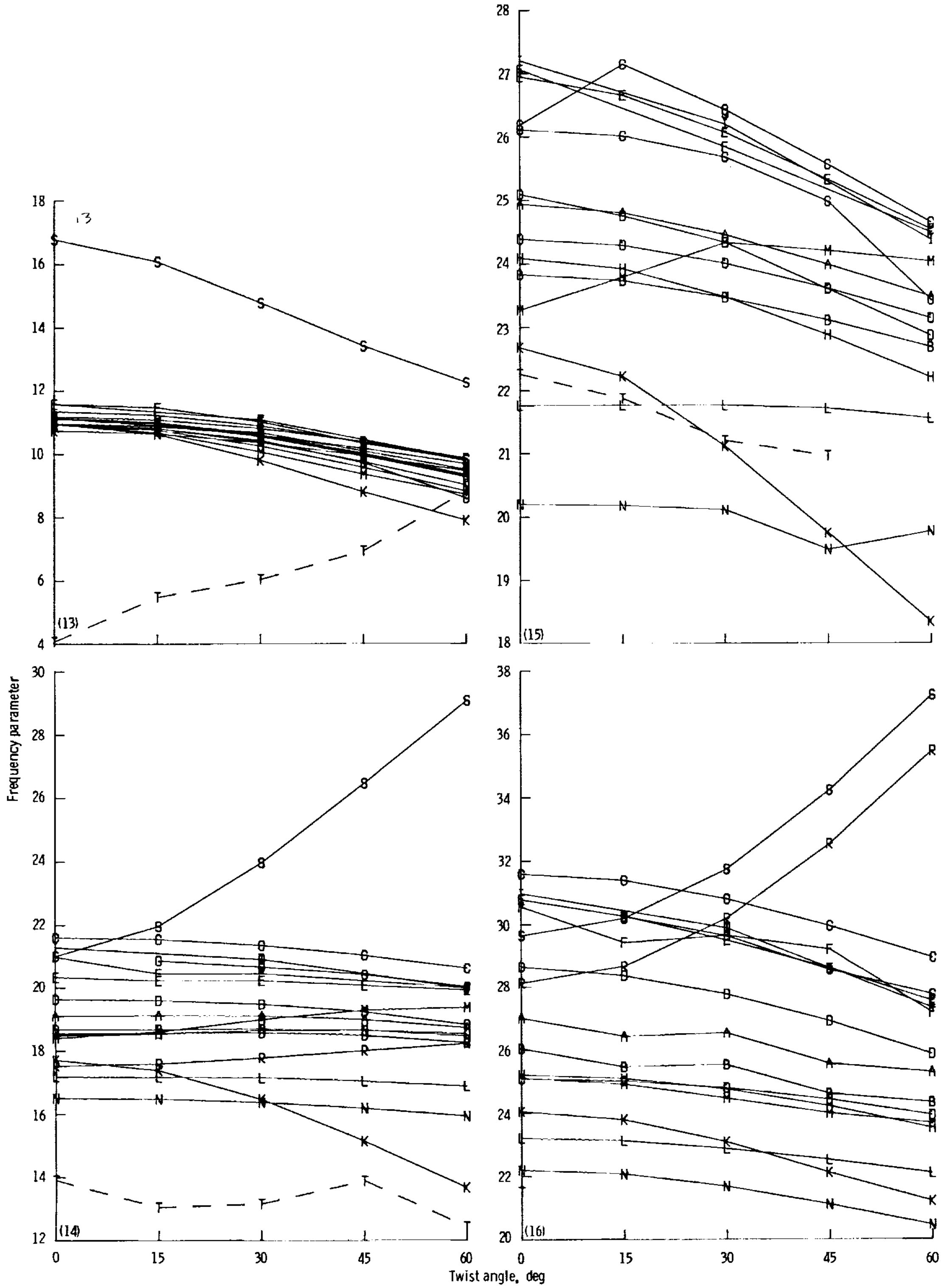
(5) Second torsion mode: 1/1; $a/b = 1$; $b/h = 20$.
(6) First edgewise bending mode: –/–; $a/b = 1$; $b/h = 20$.
(7) Chordwise bending mode: 2/1; $a/b = 1$; $b/h = 20$.
(8) Chordwise bending mode: 3/0; $a/b = 1$; $b/h = 20$.

Figure 4.—Continued.



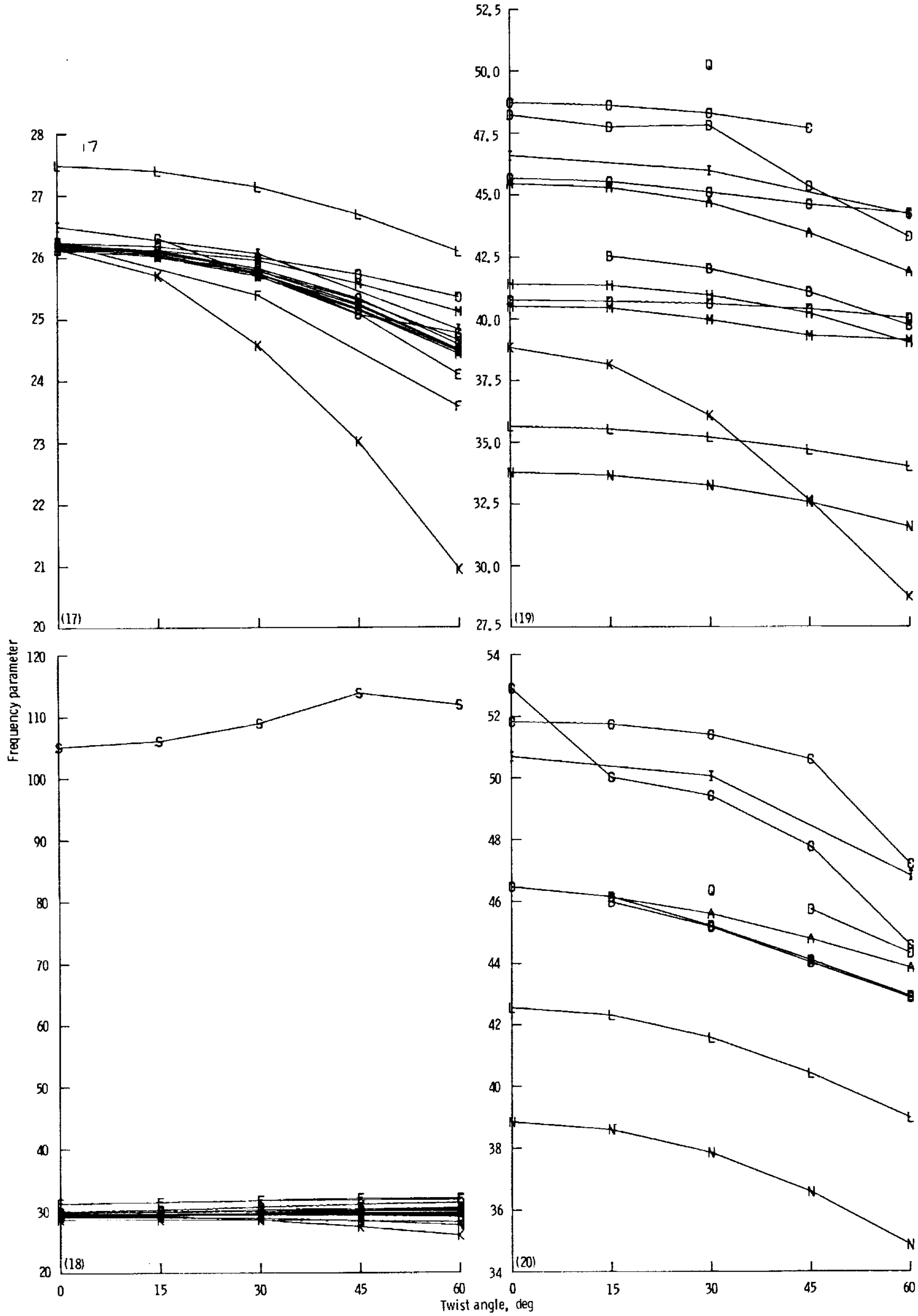
(9) Third bending mode: 0/2; $a/b = 1$; $b/h = 20$.
(10) Third torsion mode: 1/2; $a/b = 1$; $b/h = 20$.
(11) First bending mode: 0/0; $a/b = 1$; $b/h = 5$.
(12) First torsion mode: 1/0; $a/b = 1$; $b/h = 5$.

Figure 4.—Continued.



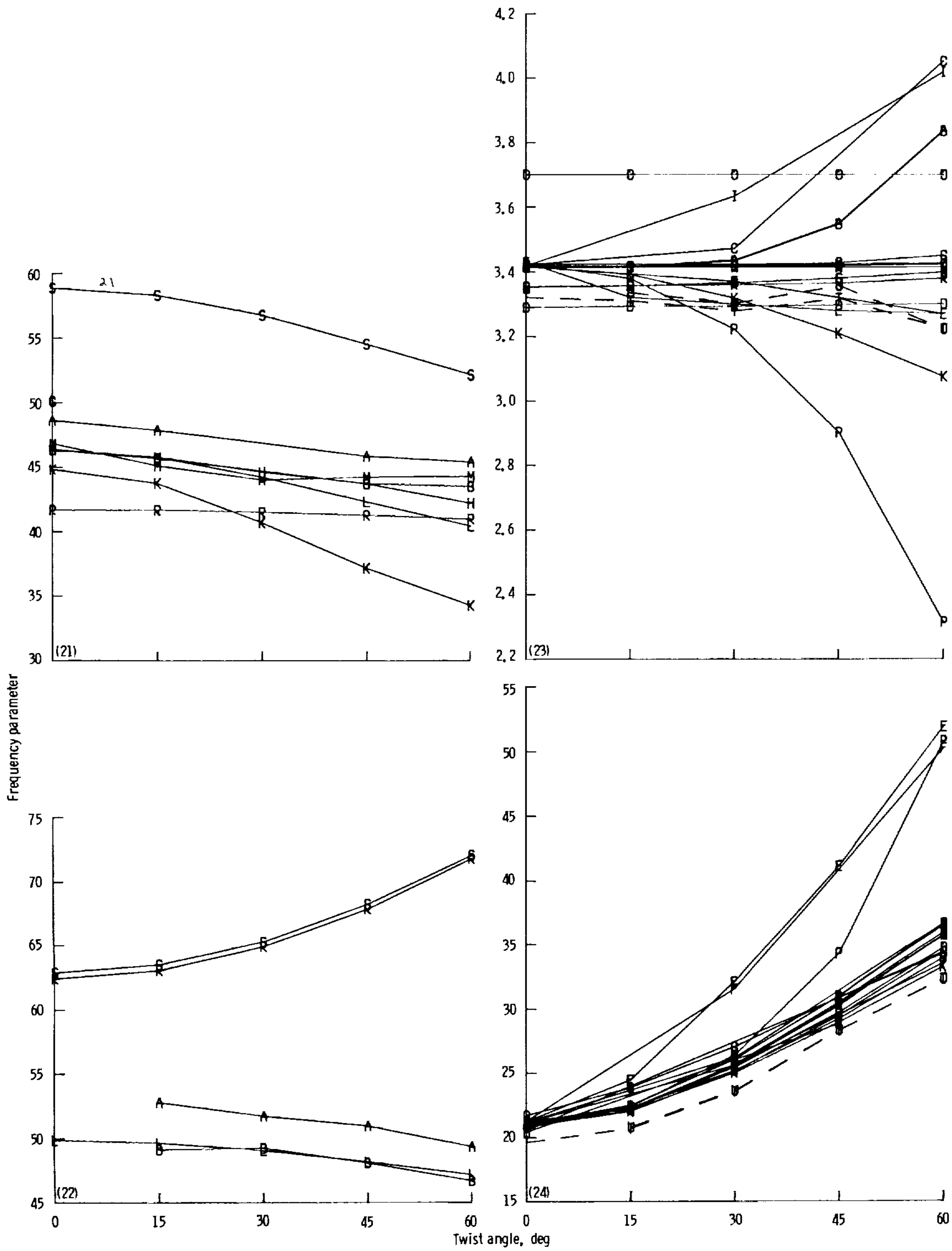
(13) First edgewise bending mode: -/-; $a/b = 1$; $b/h = 5$.
(14) Second bending mode: 0/1; $a/b = 1$; $b/h = 5$.
(15) Chordwise bending mode: 2/0; $a/b = 1$; $b/h = 5$.
(16) Second torsion mode: 1/1; $a/b = 1$; $b/h = 5$.

Figure 4.—Continued.



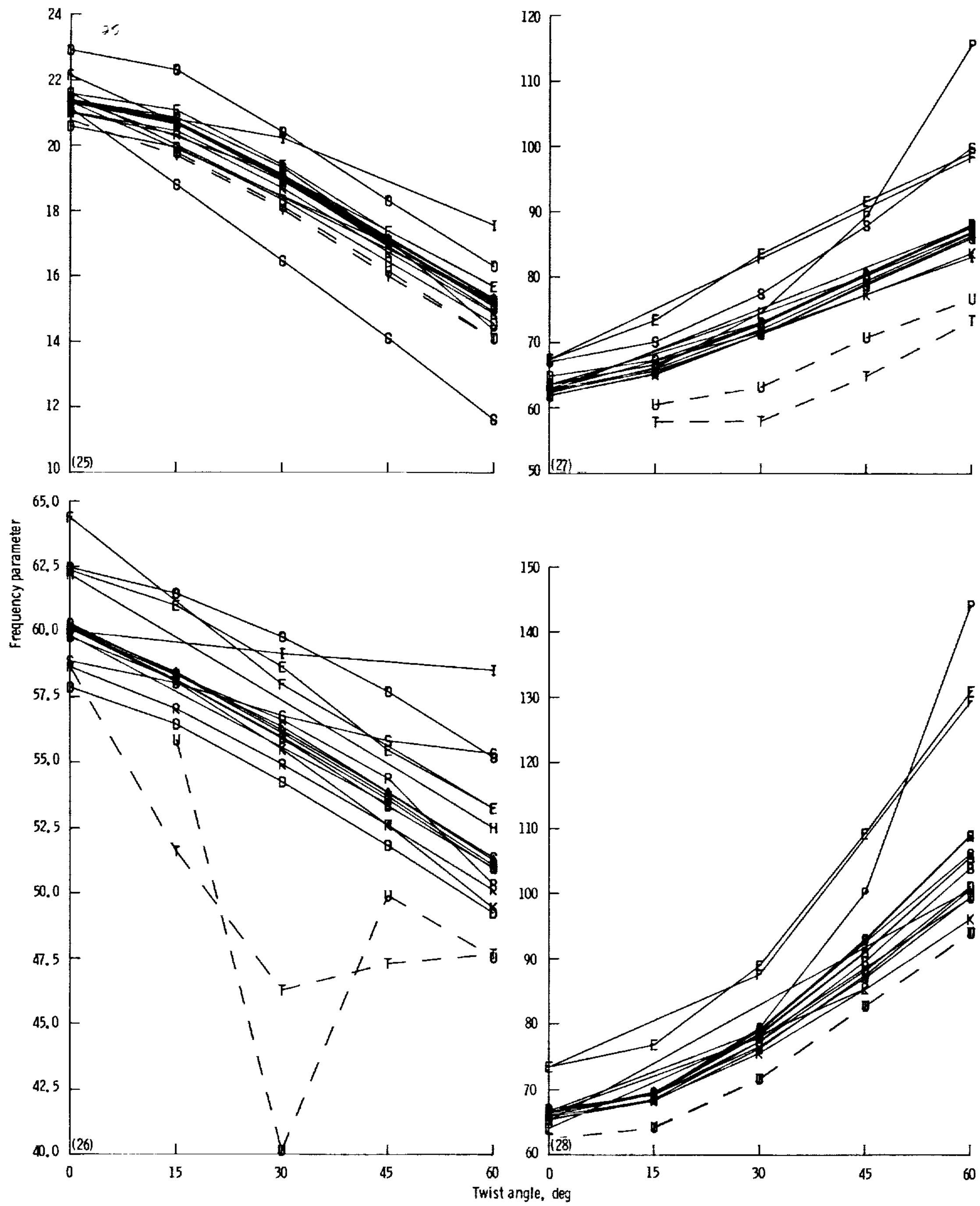
(17) Extensional mode: -/-; $a/b=1$; $b/h=5$
(18) Second edgewise bending mode: -/-; $a/b=1$; $b/h=5$.
(19) Chordwise bending mode: 2/1; $a/b=1$; $b/h=5$.
(20) Chordwise bending mode: 3/0; $a/b=1$; $b/h=5$.

Figure 4.—Continued.



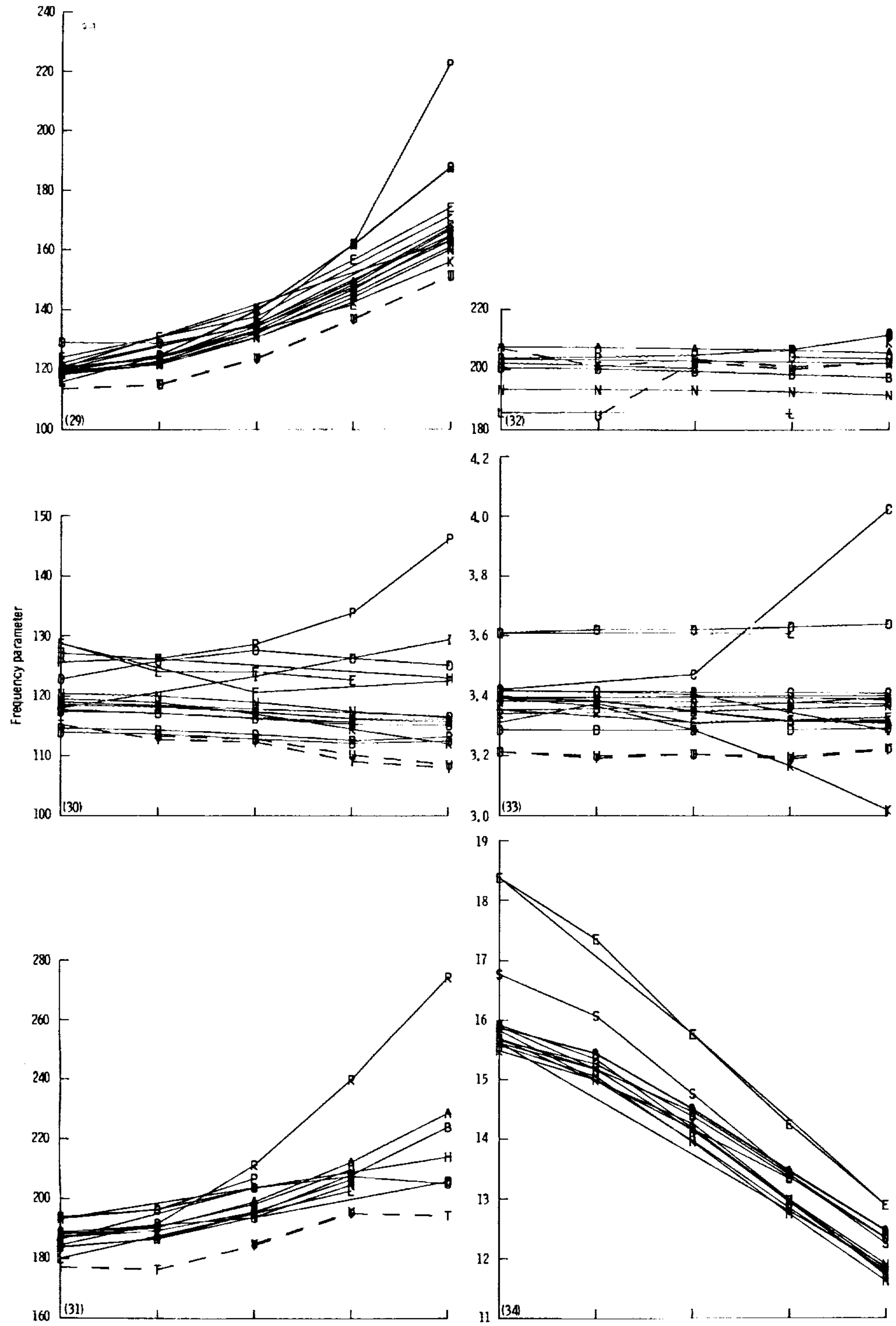
(21) Third bending mode: 0/2; $a/b = 1$; $b/h = 5$.
(22) Third torsion mode: 1/2; $a/b = 1$; $b/h = 5$.
(23) First bending mode: 0/0; $a/b = 3$; $b/h = 20$.
(24) First torsion mode: 1/0; $a/b = 3$; $b/h = 20$.

Figure 4.—Continued.



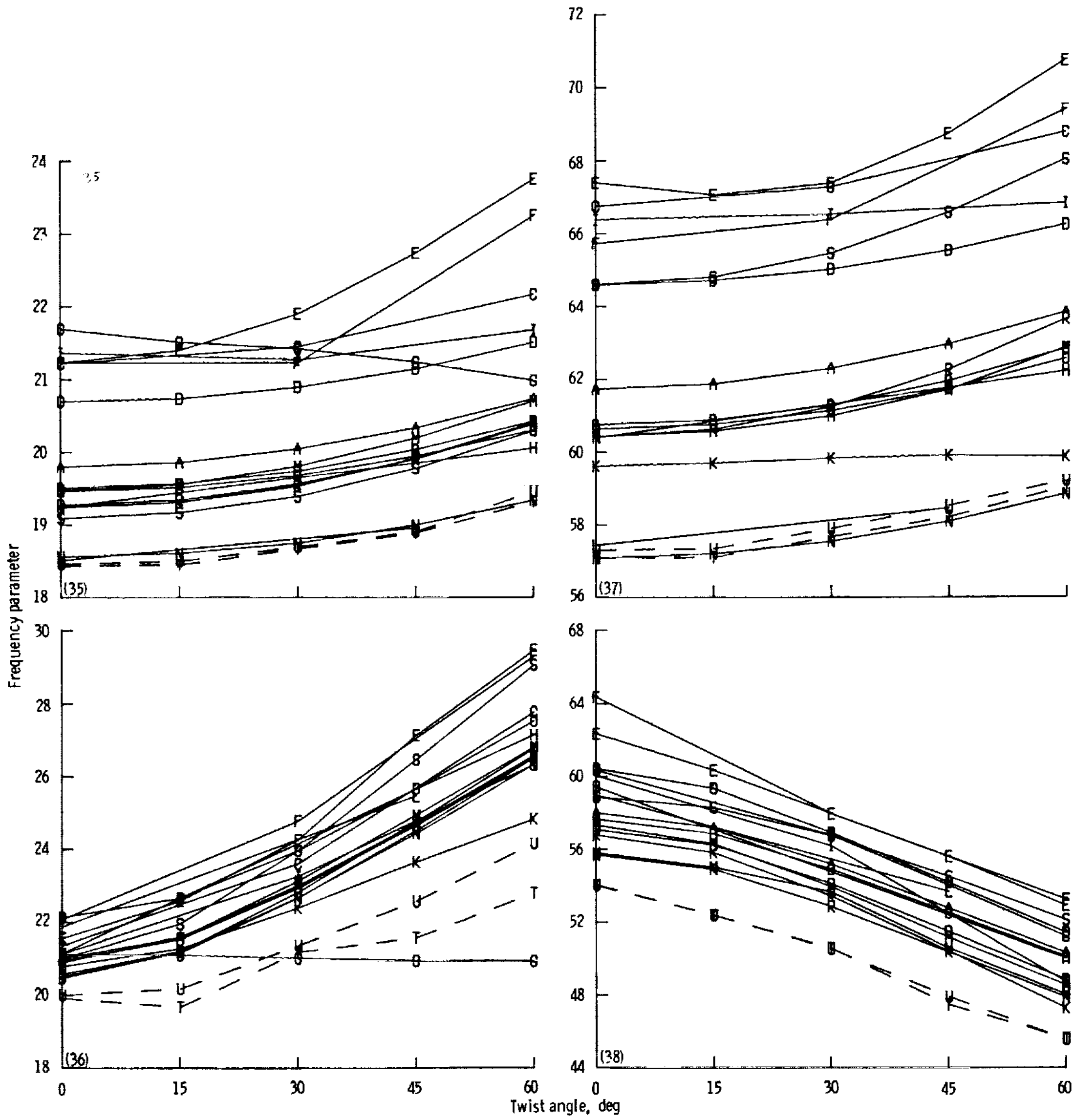
(25) Second bending mode: 0/1; $a/b=3$; $b/h=20$.
(26) Third bending mode: 0/2; $a/b=3$; $b/h=20$.
(27) First edgewise bending mode: -/-; $a/b=3$; $b/h=20$.
(28) Second torsion mode: 1/1; $a/b=3$; $b/h=20$.

Figure 4.—Continued.



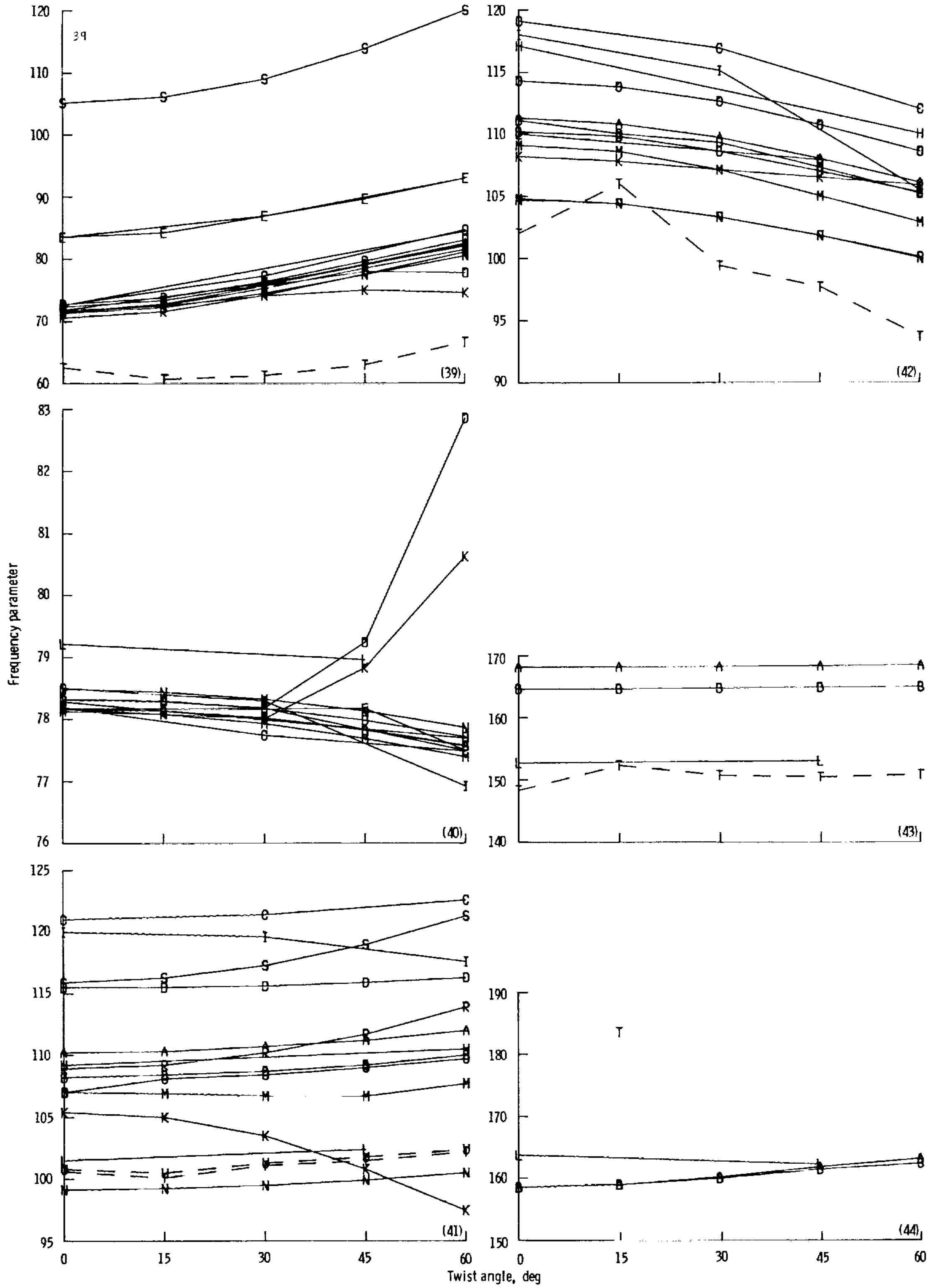
(29) Third torsion mode: 1/2; $a/b=3$; $b/h=20$.
(30) Fourth bending mode: 0/3; $a/b=3$; $b/h=20$.
(31) Fourth torsion mode: 1/3; $a/b=3$; $b/h=20$.
(32) Chordwise bending mode: 2/0; $a/b=3$; $b/h=2$.
(33) First bending mode: 0/0; $a/b=3$; $b/h=5$.
(34) First edgewise bending mode: -/-; $a/b=3$; $b/h=5$.

Figure 4.—Continued.



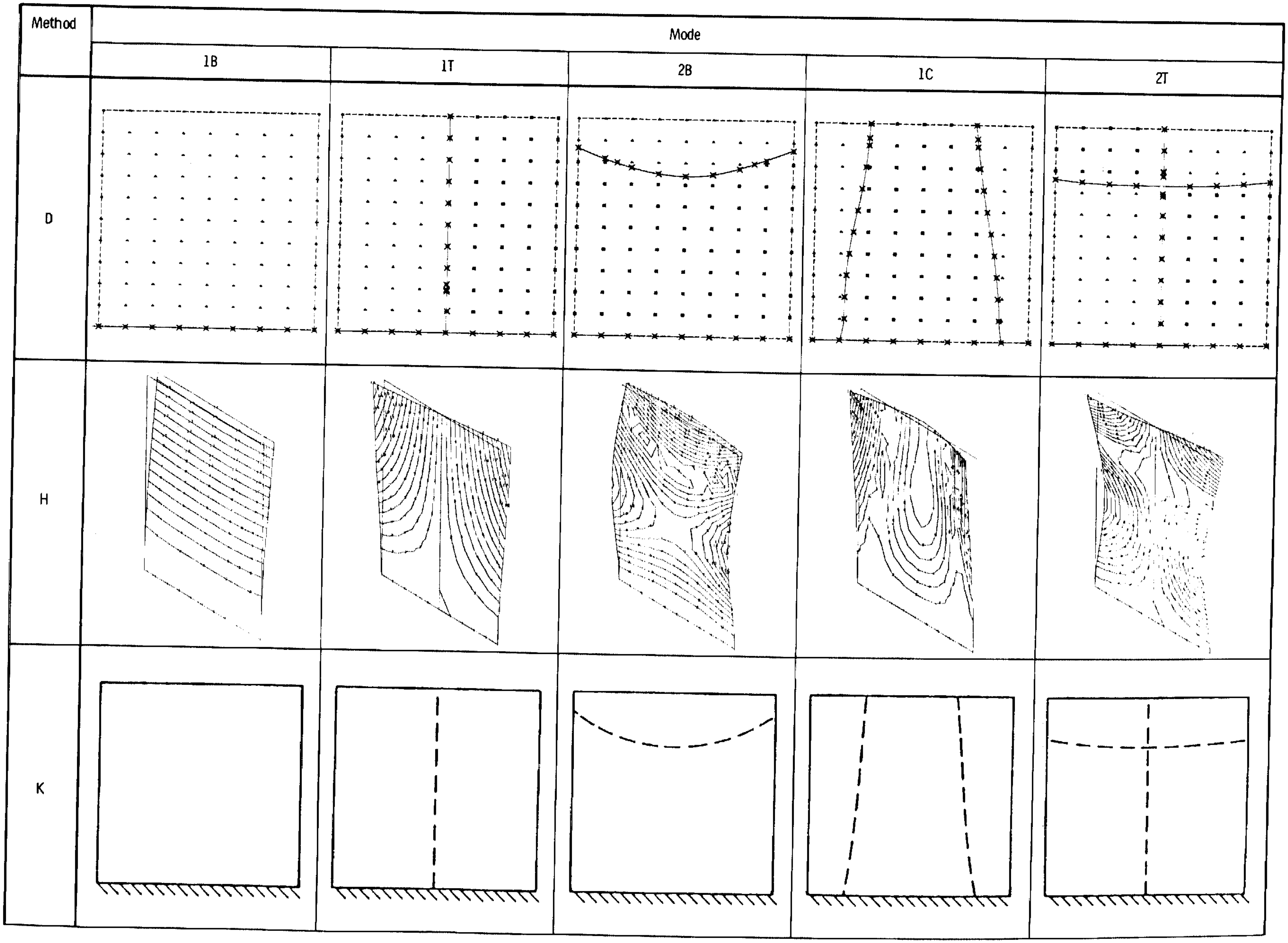
(35) First torsion mode: 1/0; $a/b = 3$; $b/h = 5$.
(36) Second torsion mode: 1/1; $a/b = 3$; $b/h = 5$.
(37) Second torsion mode: 1/1; $a/b = 3$; $b/h = 5$.
(38) Third bending mode: 0/2; $a/b = 3$; $b/h = 5$.

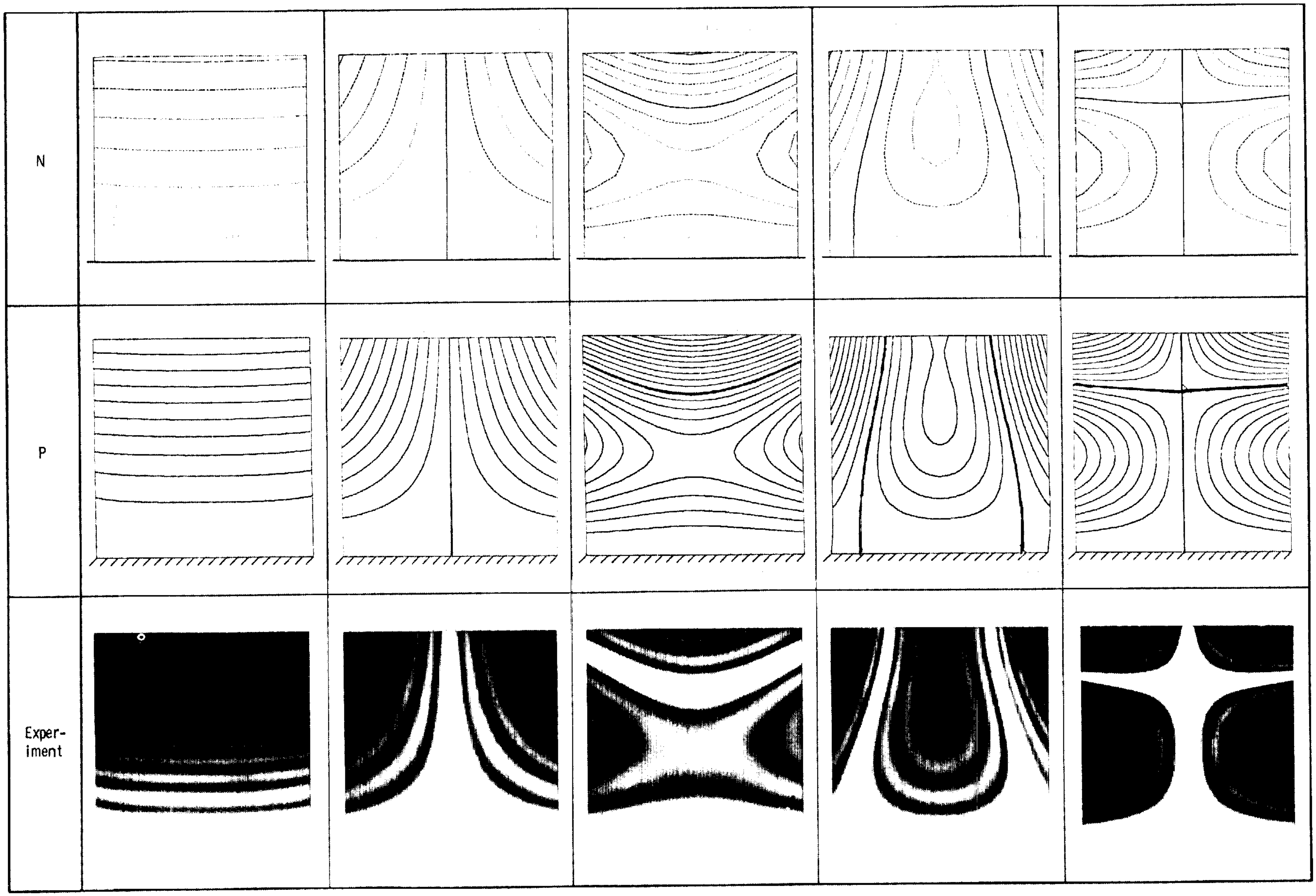
Figure 4.—Continued.



(39) Second edgewise bending mode: -/-; $a/b = 3$; $b/h = 5$.
(40) Extensional mode: -/-; $a/b = 3$; $b/h = 5$.
(41) Third torsion mode: 1/2; $a/b = 3$; $b/h = 5$.
(42) Fourth bending mode: 0/3; $a/b = 3$; $b/h = 5$.
(43) Fourth torsion mode: 1/3; $a/b = 3$; $b/h = 5$.
(44) Chordwise bending mode: 2/0; $a/b = 3$; $b/h = 5$.

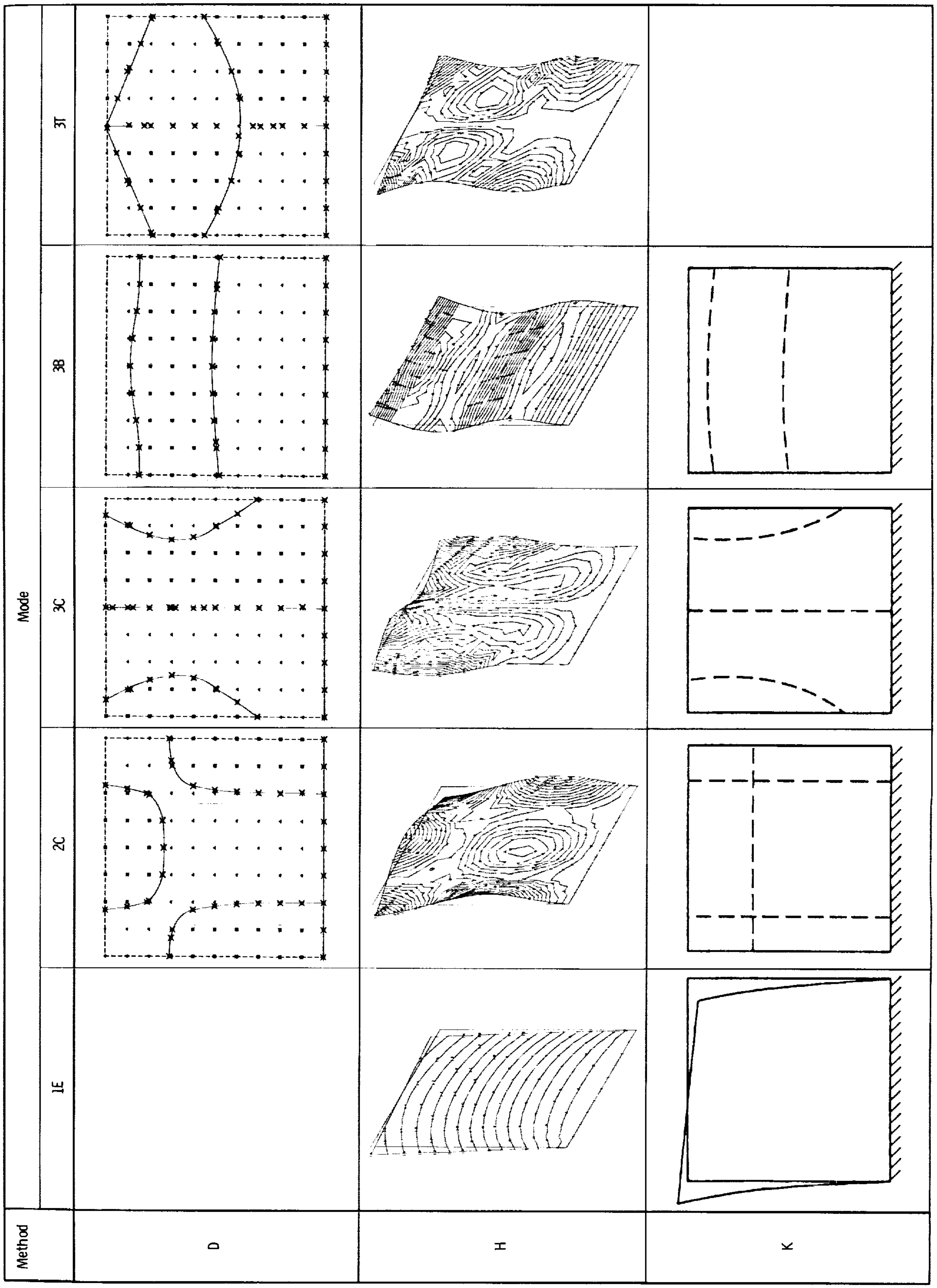
Figure 4.—Concluded.


Method
Mode
1B
1T
2B
1C
2T
D
H
K

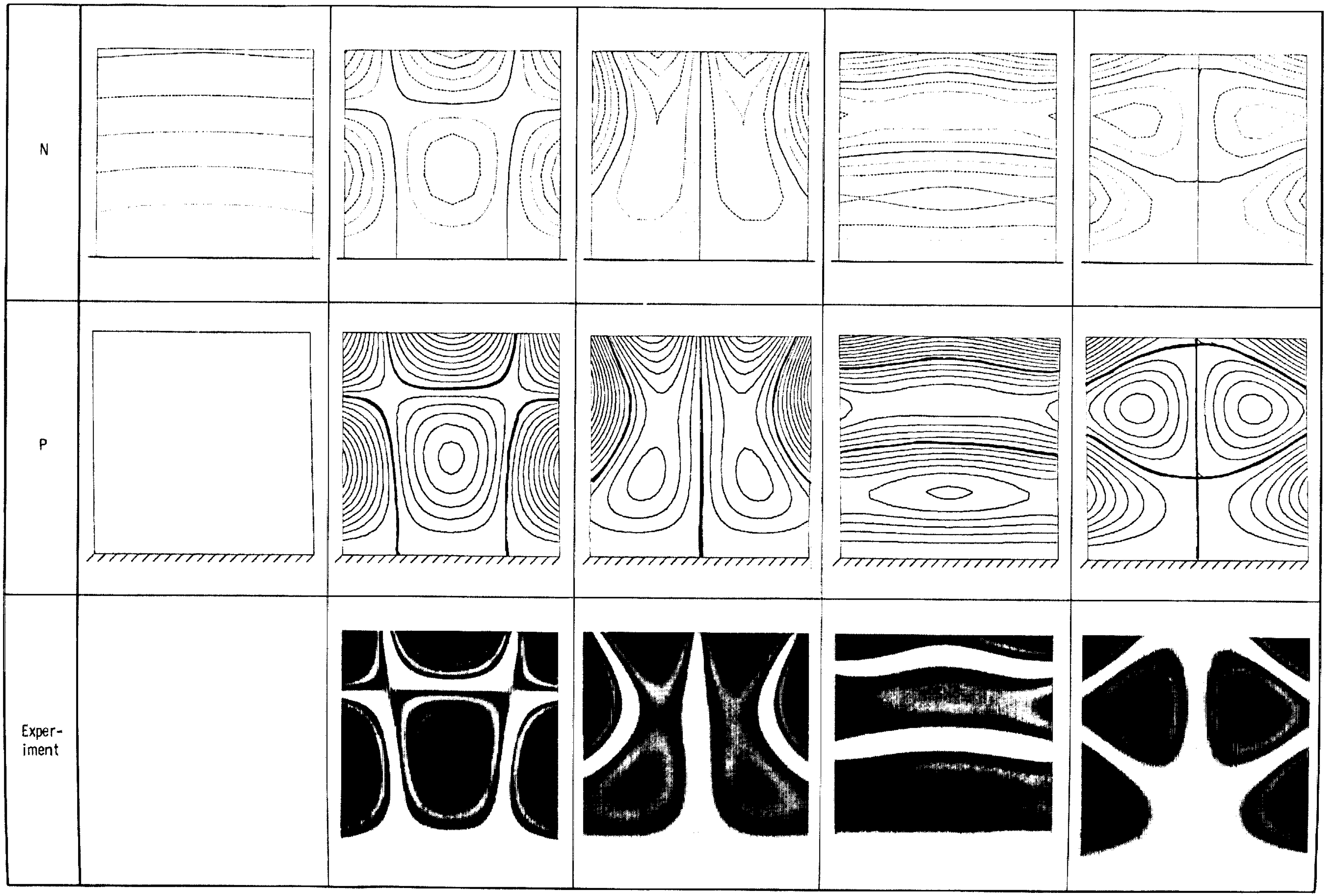


(1) $a/b = 1$; $b/h = 20$; $\varphi = 0°$.

Figure 5.—Comparison of mode shapes.

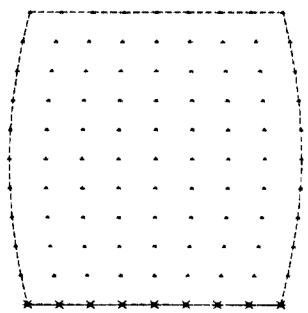
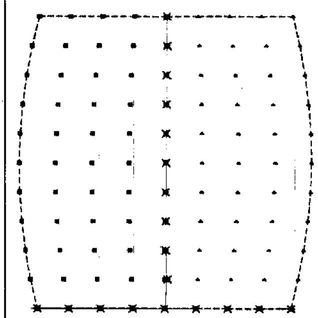
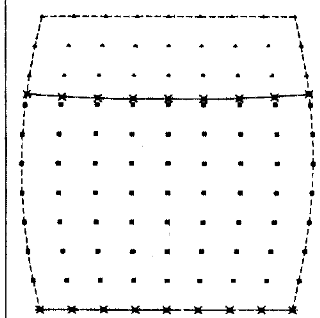
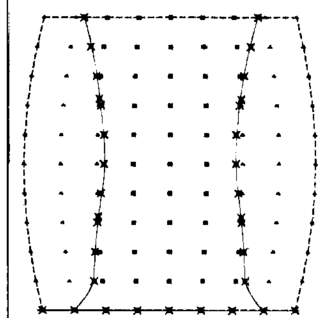
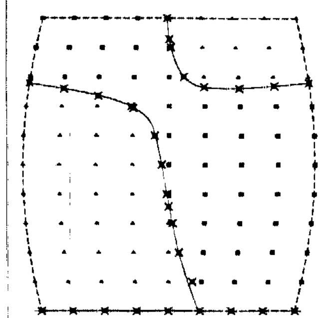
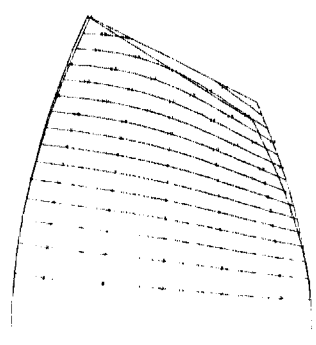
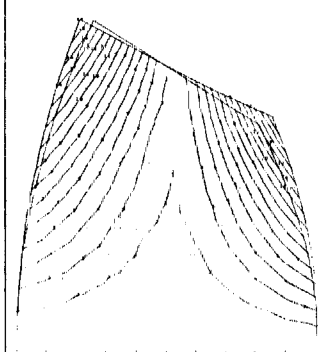
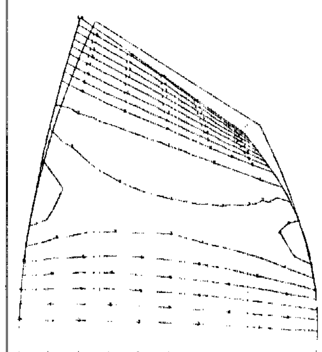
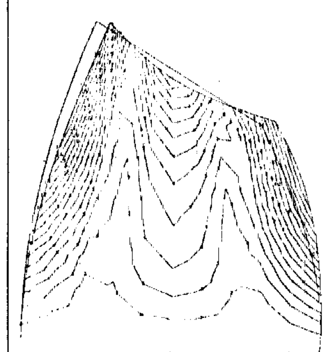
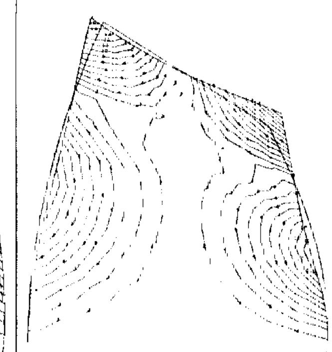


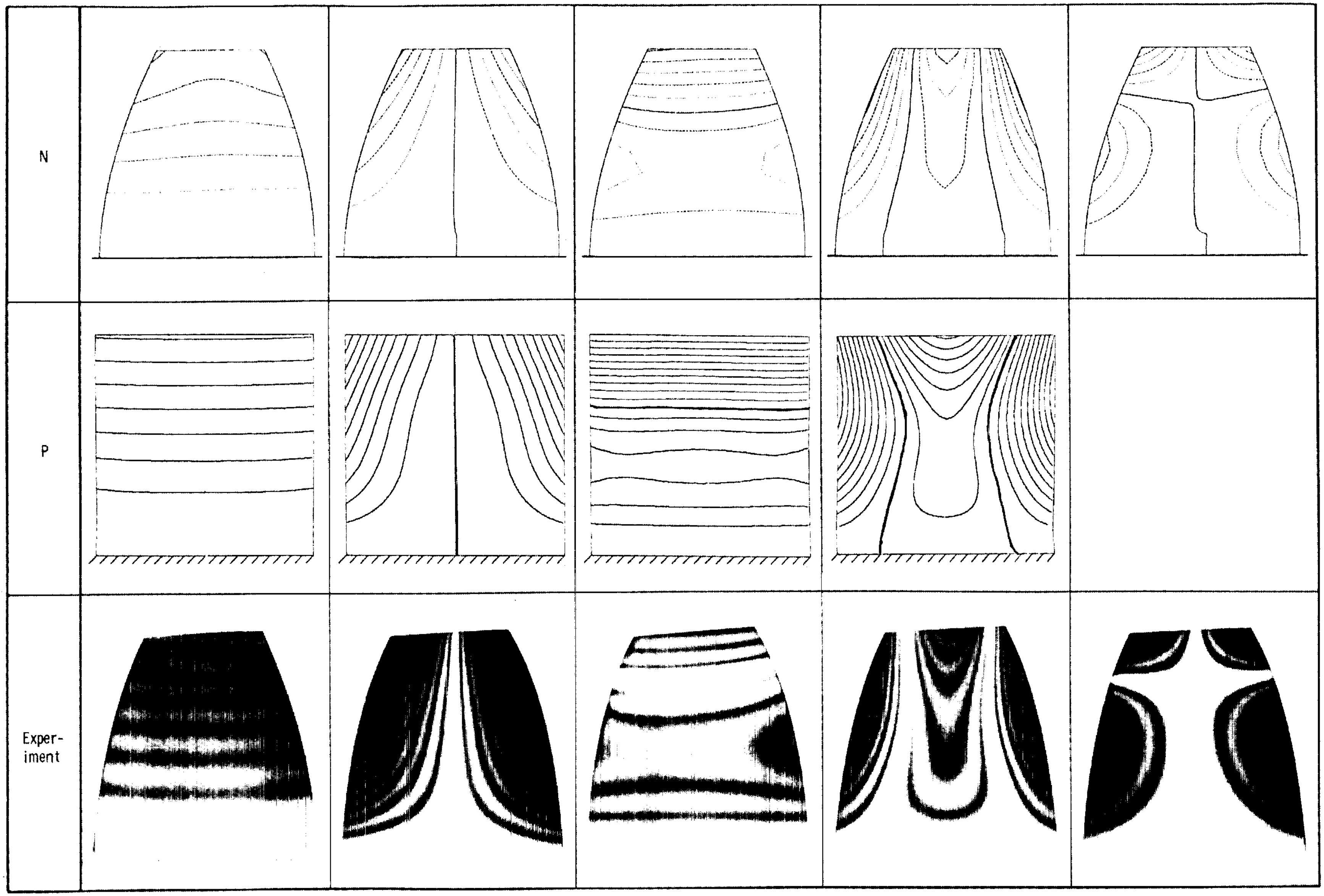
Method
Mode
1E
2C
3C
3B
3T
D
H
K



(l) Concluded.

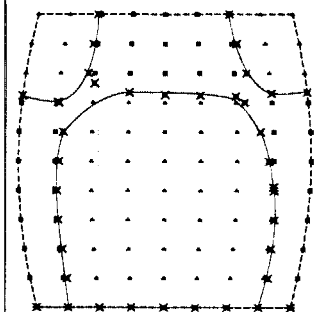
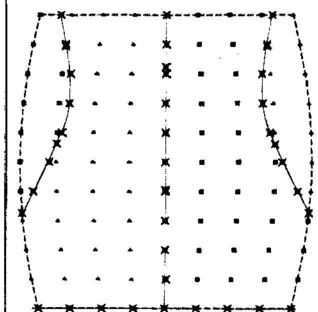
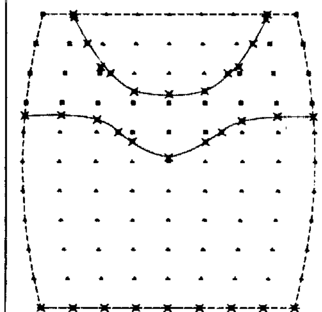
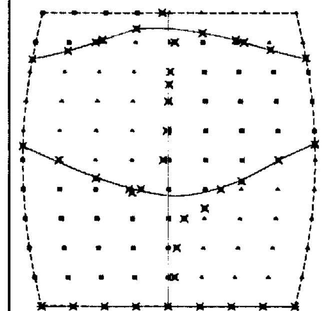
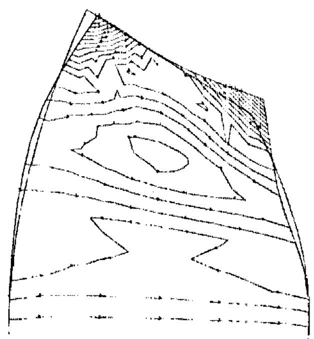
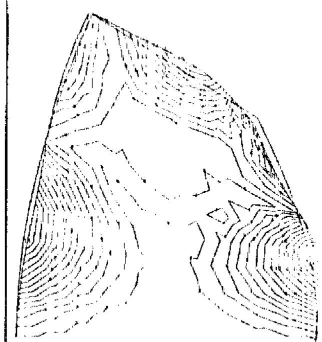
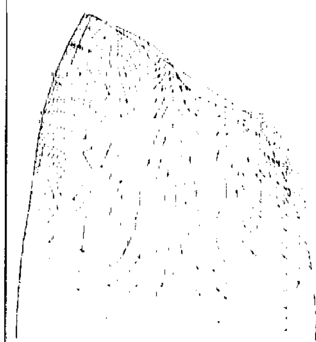
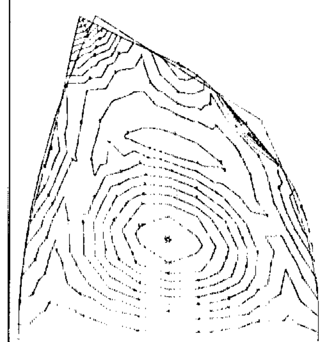
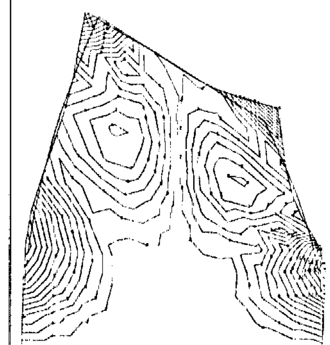
Figure 5—Continued.

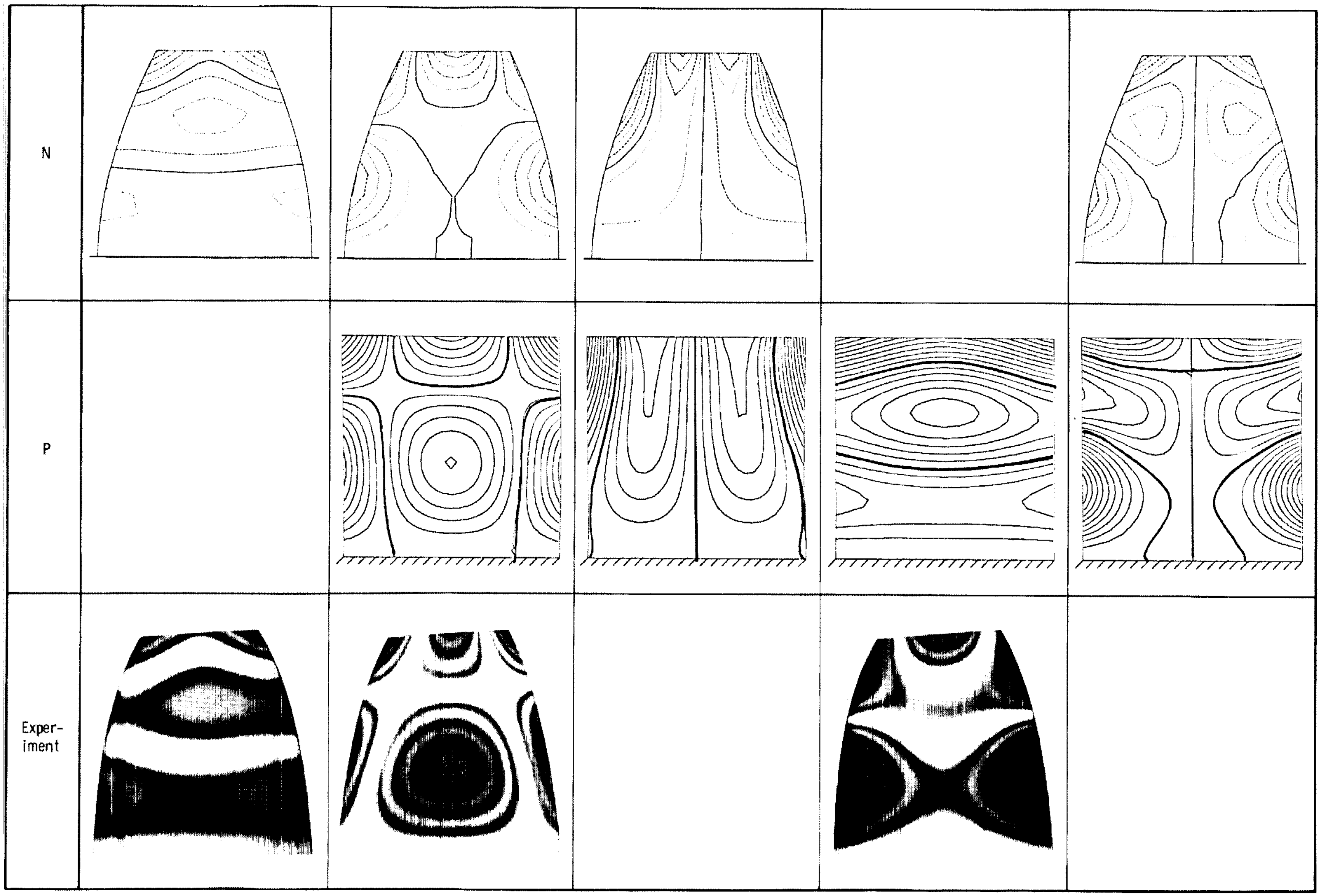
Method	Mode				
	1B	1T	2B	1C	2T
D					
H					
K					



(2) $a/b=1$; $b/h=20$; $\varphi=60°$.

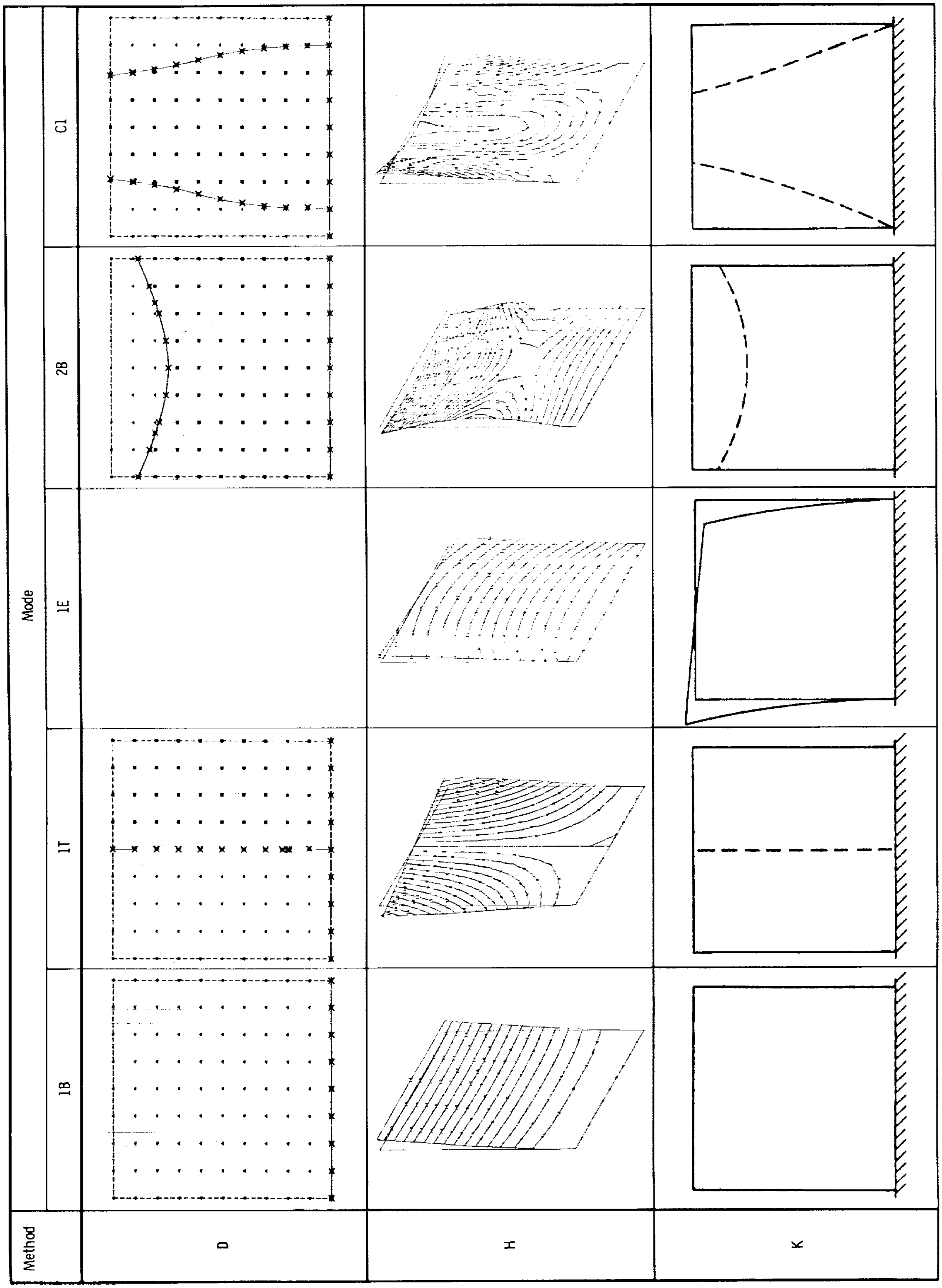
Figure 5—Continued.

Method	Mode				
	1E	2C	3C	3B	3T
D					
H					
K					

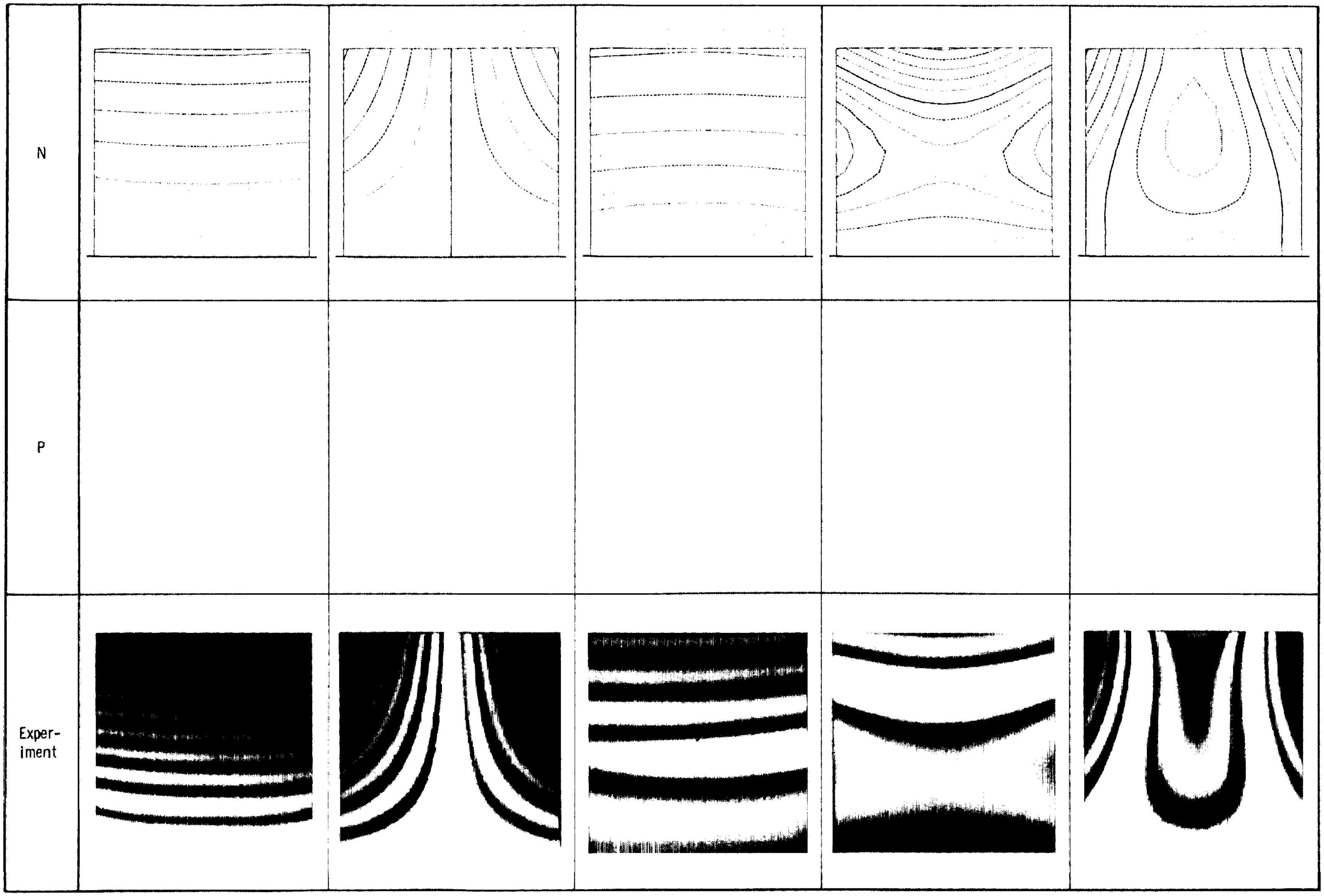


(2) Concluded.

Figure 5—Continued.

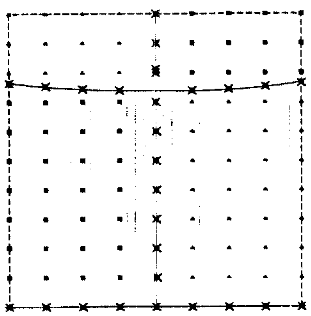
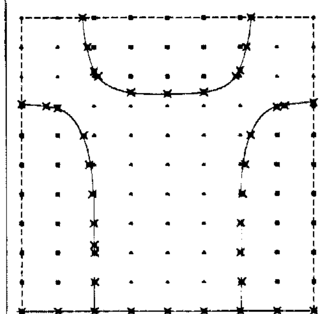
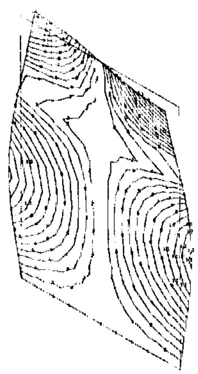
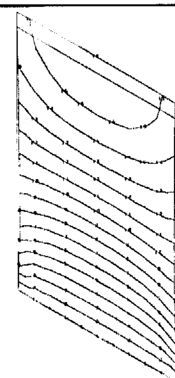
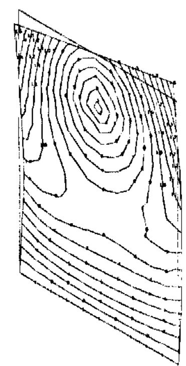
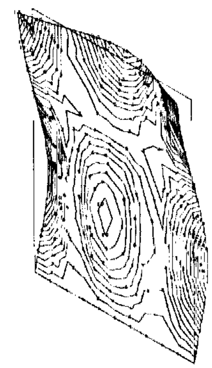
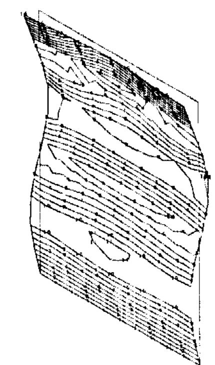
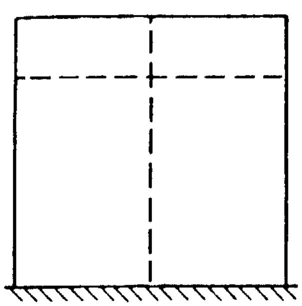
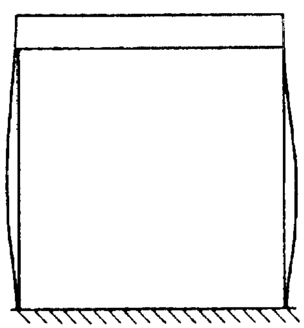
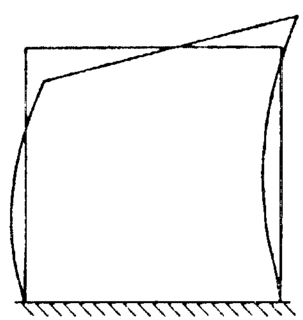
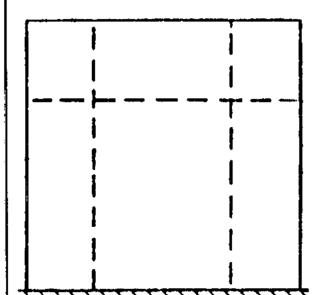
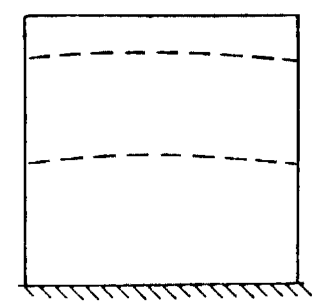


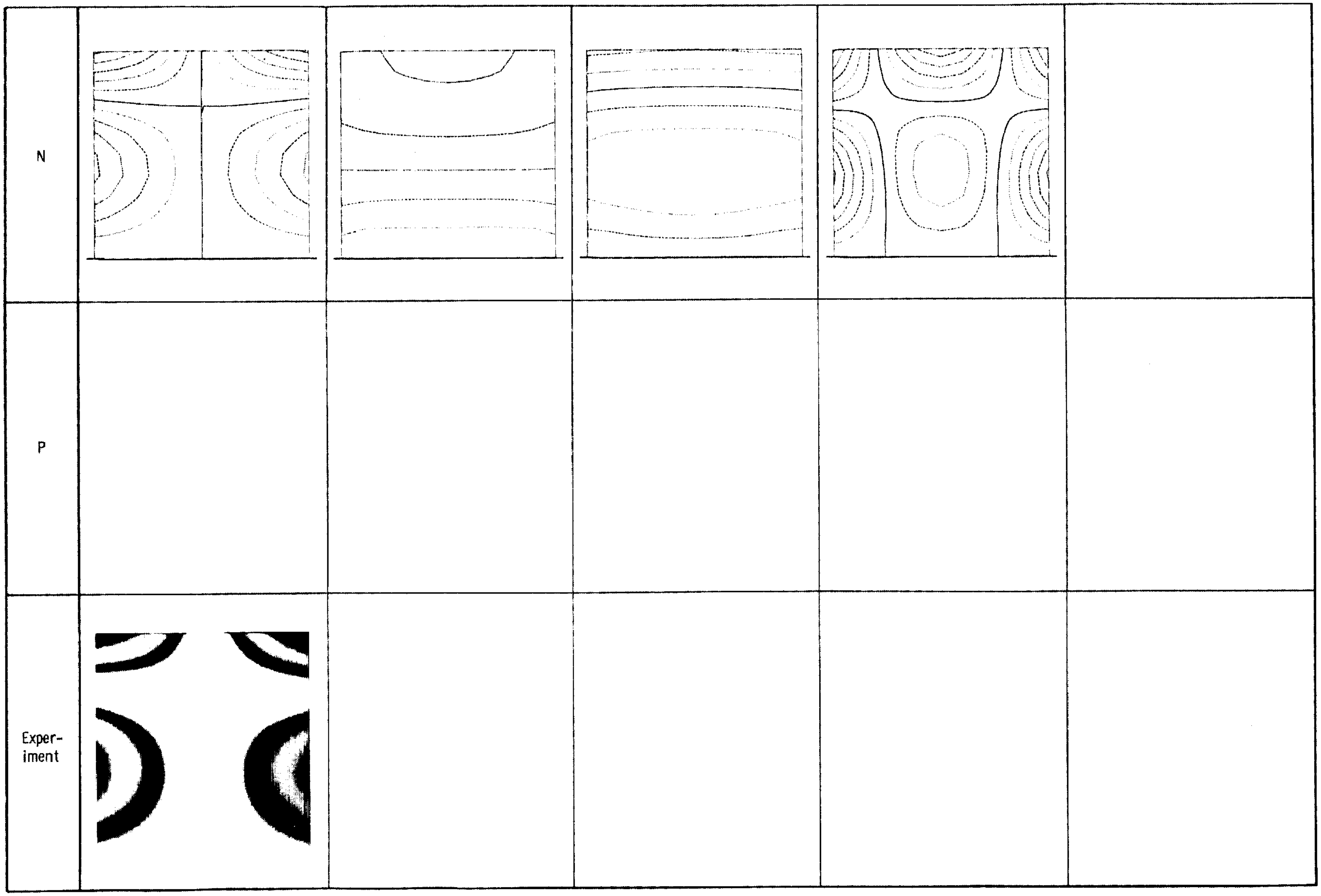
Method
Mode
1B
1T
1E
2B
C1
D
H
K



(3) $a/b = 1$; $b/h = 5$; $\varphi = 0°$.

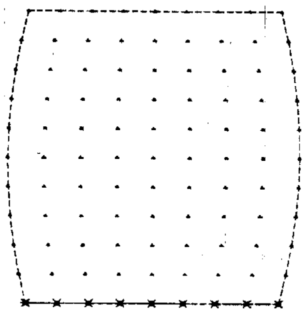
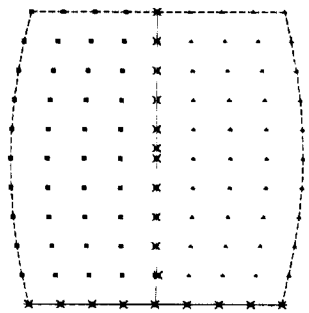
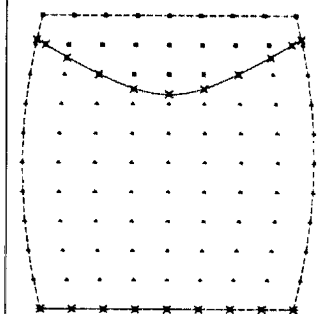
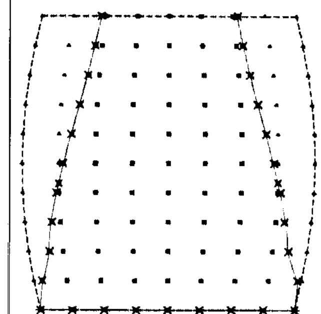
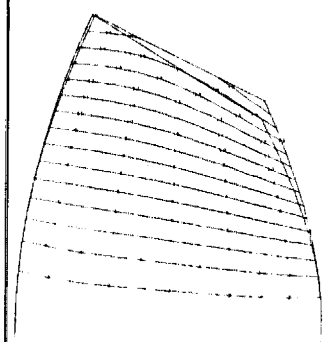
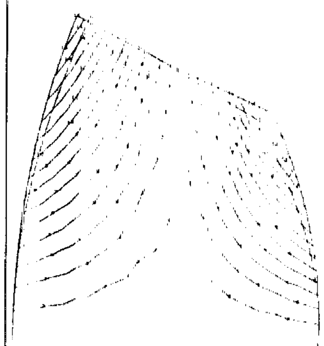
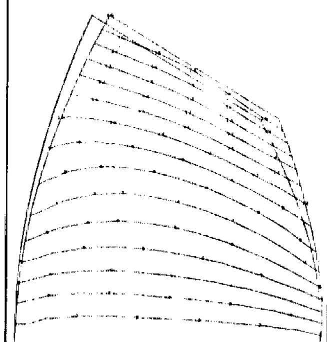
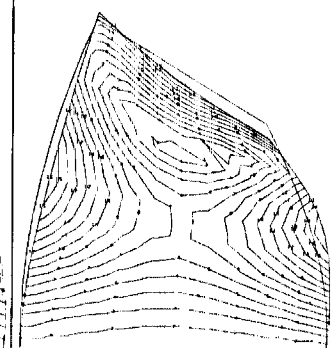
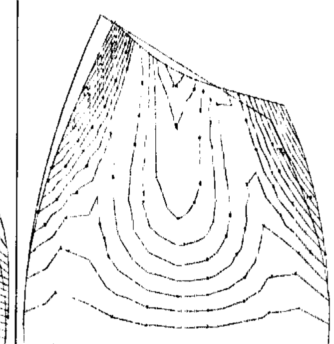
Figure 5—Continued.

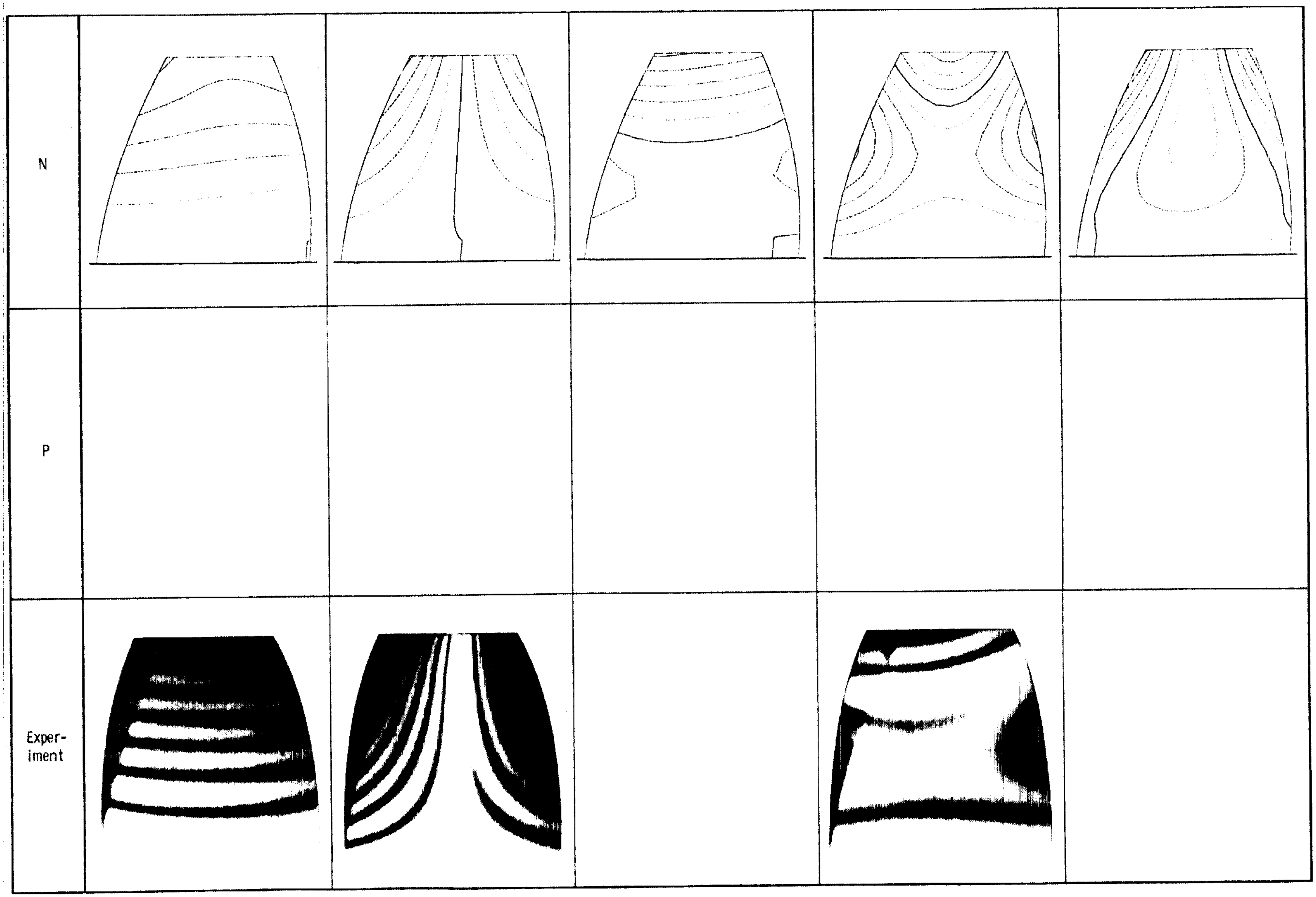
Method	Mode				
	2T	1A	2E	2C	3B
D					
H					
K					



(3) Concluded.

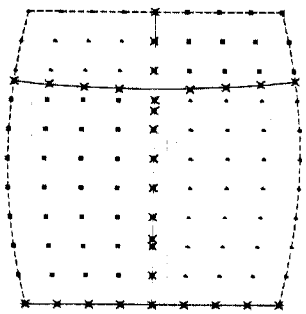
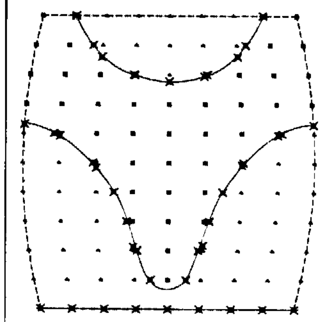
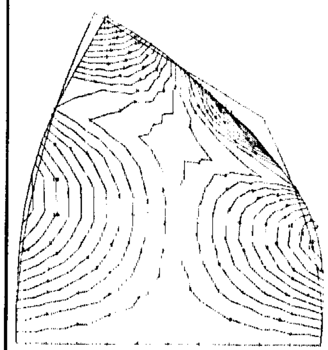
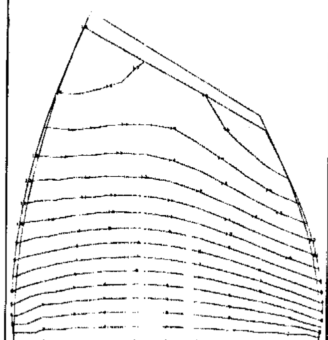
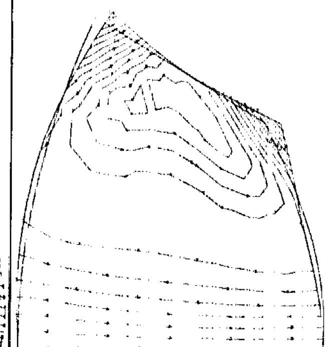
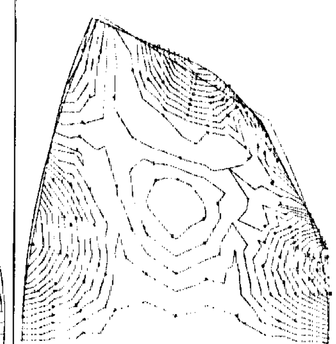
Figure 5—Continued.

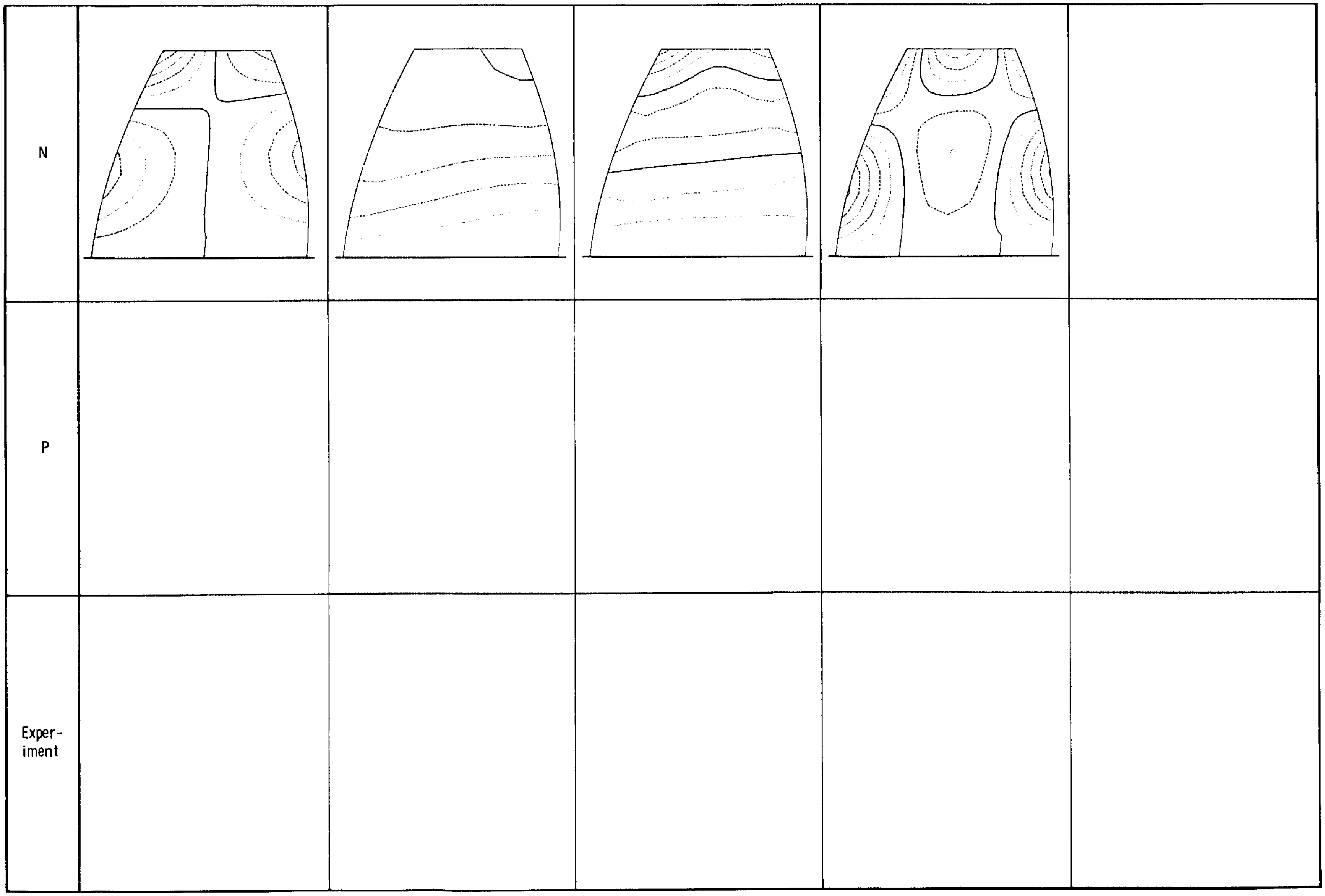
Method	Mode				
	1B	1T	1E	2B	C1
D					
H					
K					



(4) $a/b=1$; $b/h=5$; $\varphi=60°$.

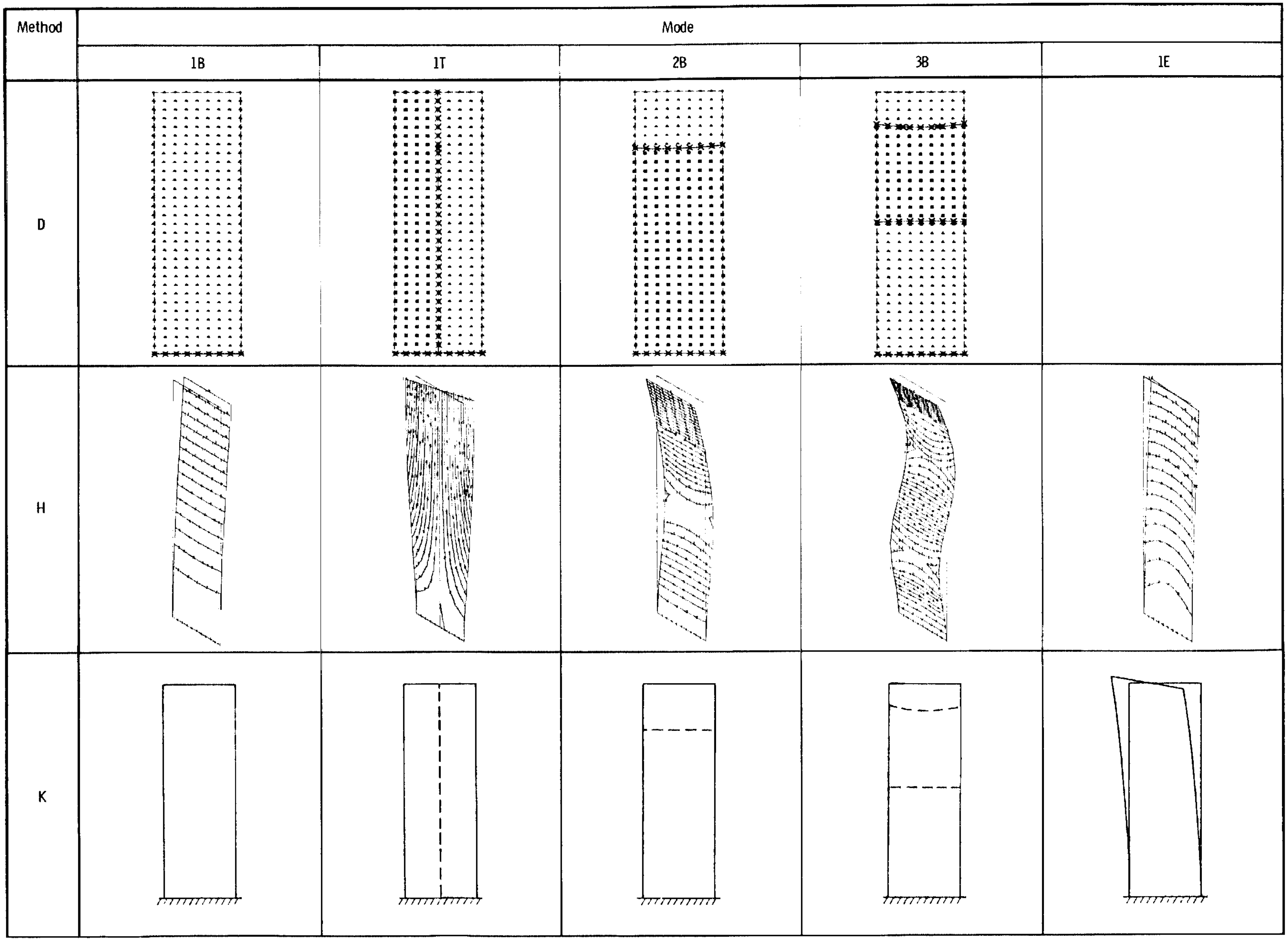
Figure 5—Continued.

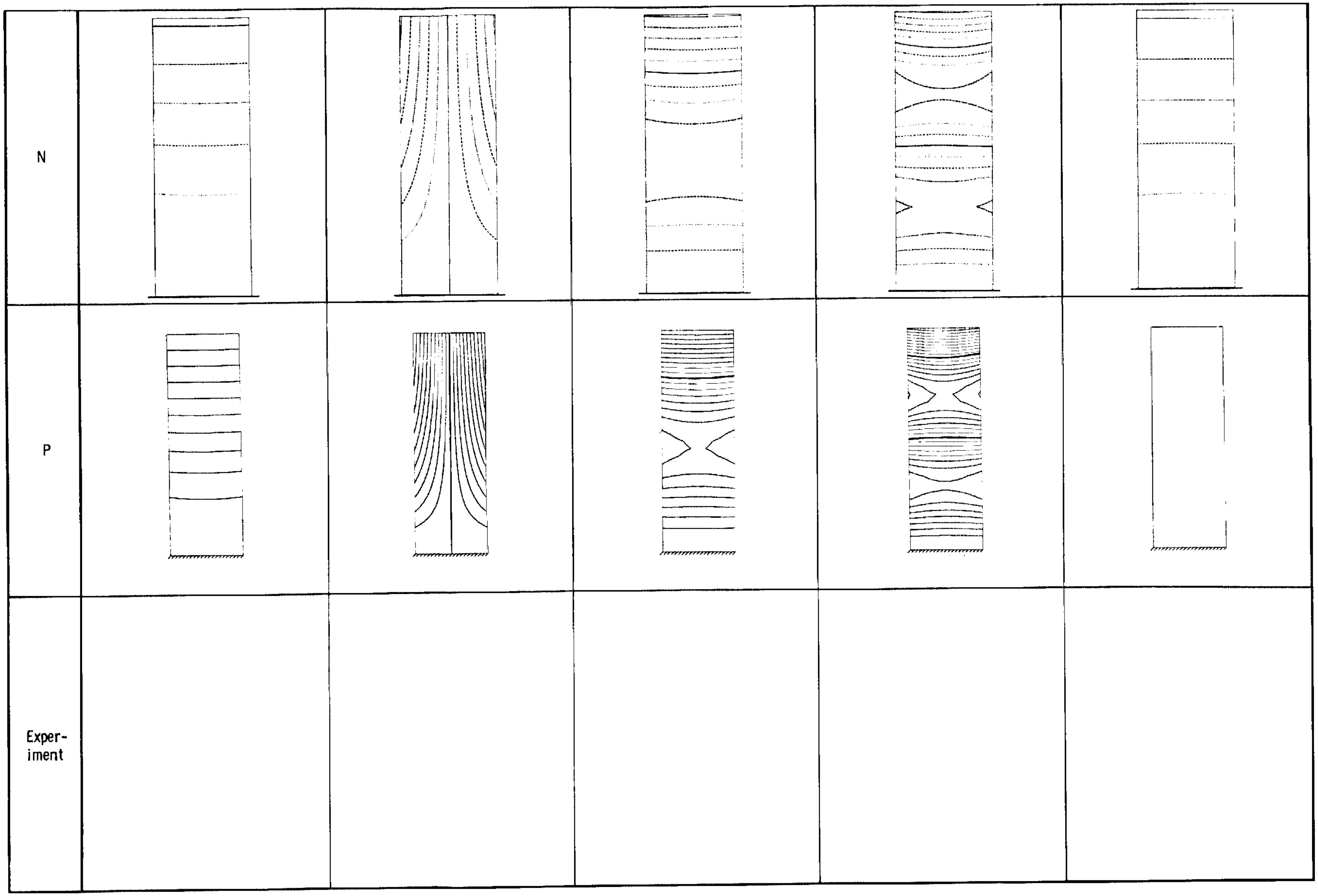
Method	Mode				
	2T	1A	2E	2C	3B
D					
H					
K					



(4) Concluded.

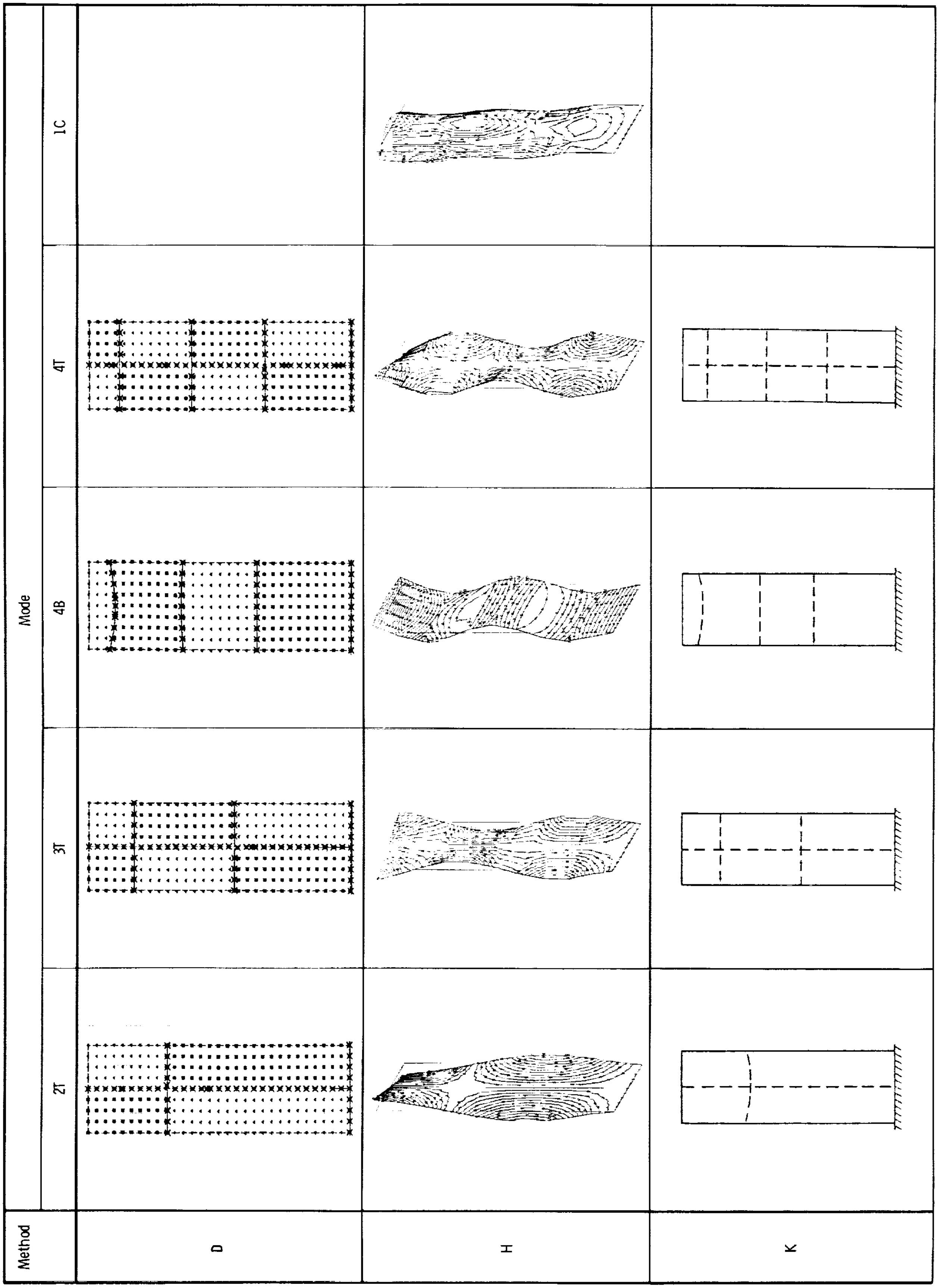
Figure 5—Continued.

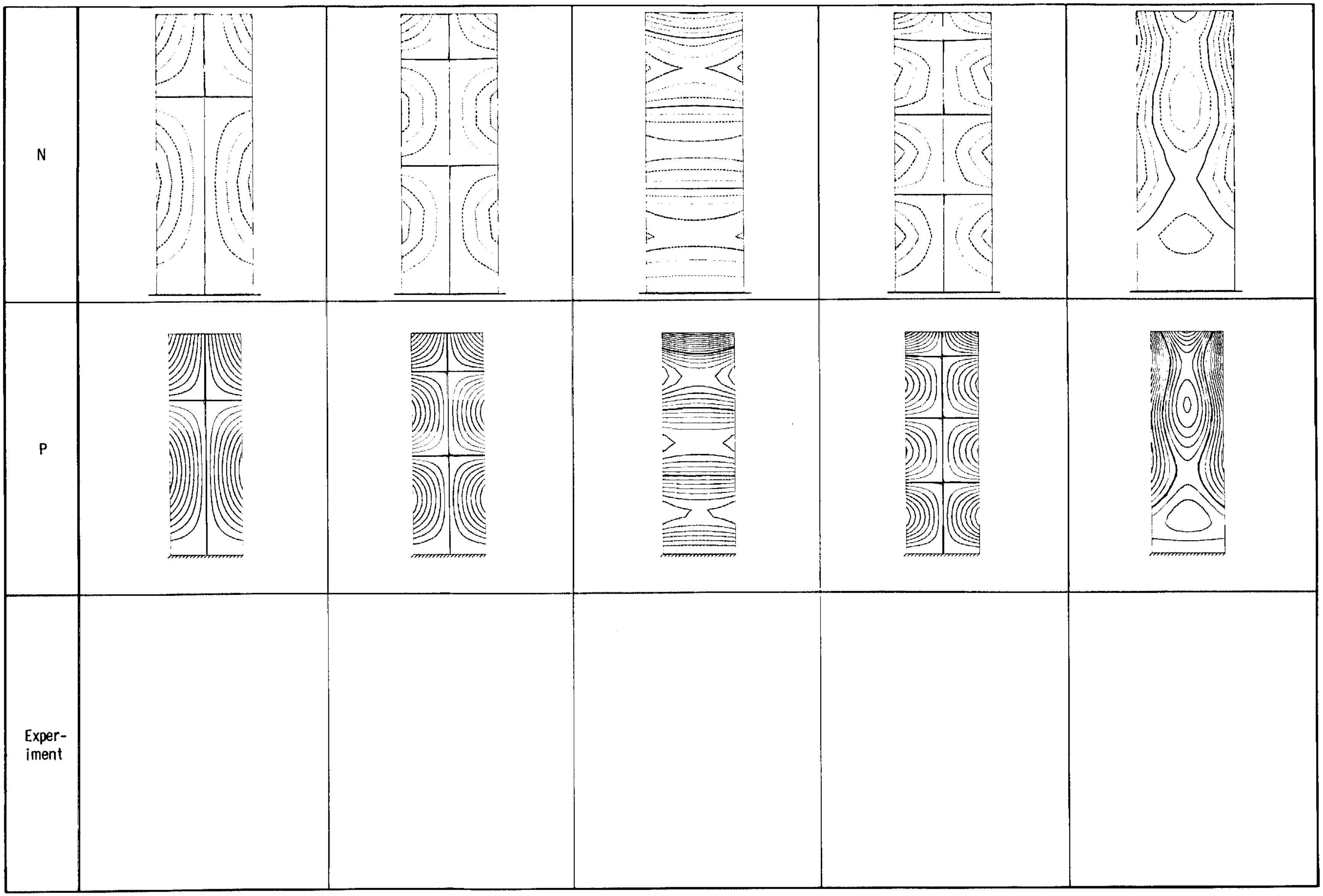

Method
Mode
1B
1T
2B
3B
1E
D
H
K



(5) $a/b = 3$; $b/h = 20$; $\varphi = 0°$.

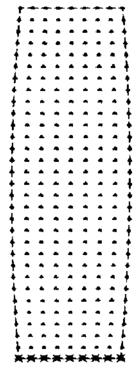
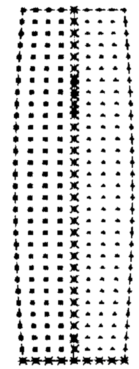
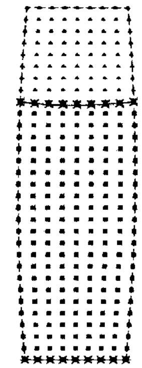
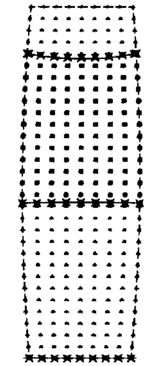
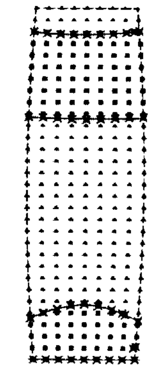
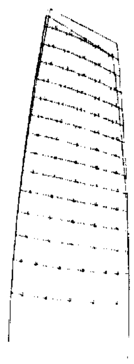
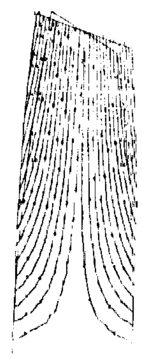
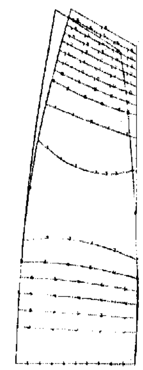
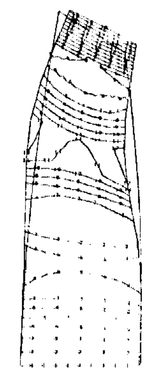
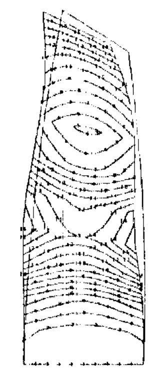
Figure 5—Continued.

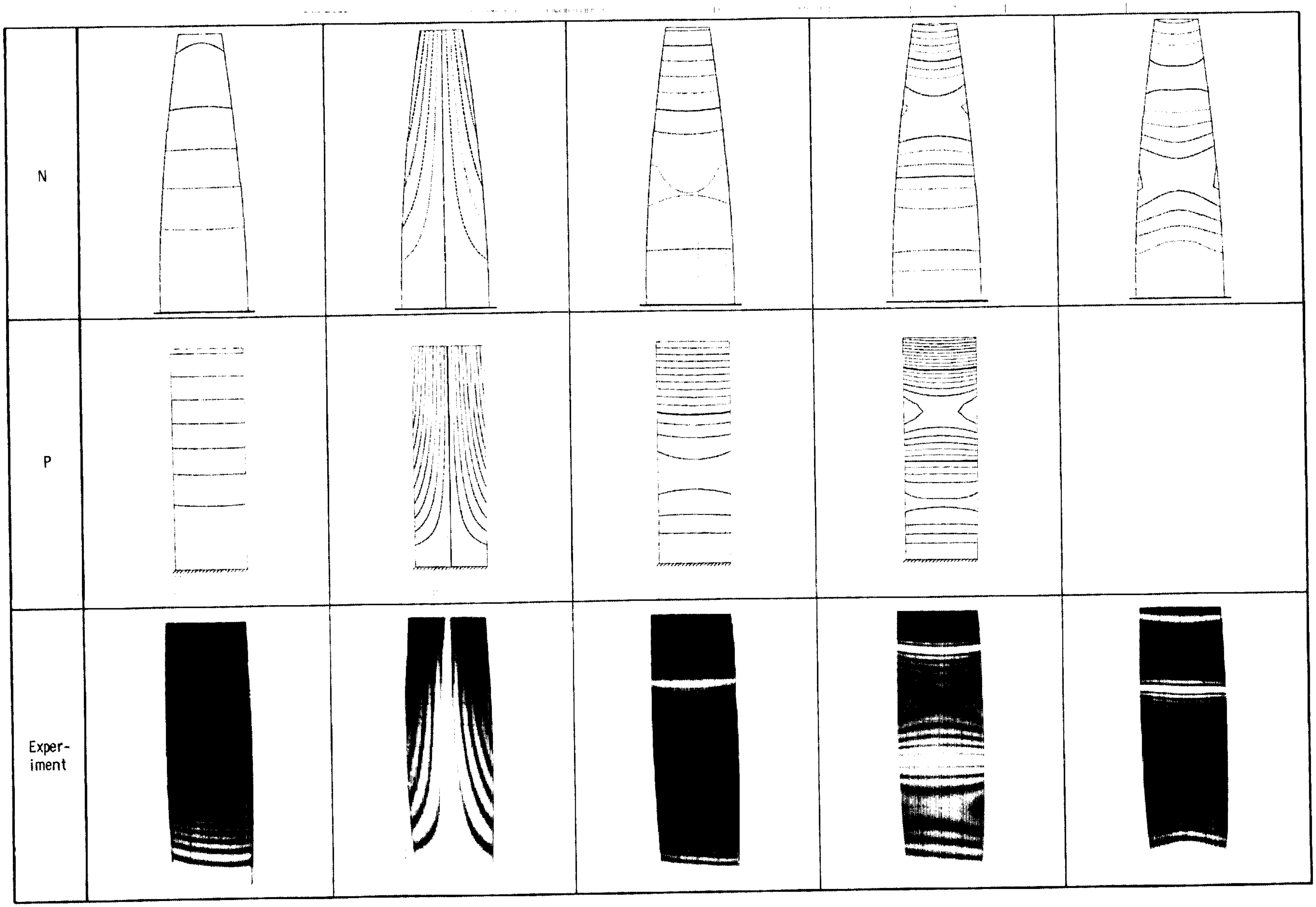

Method
Mode
2T
3T
4B
4T
1C
D
H
K



(5) Concluded.

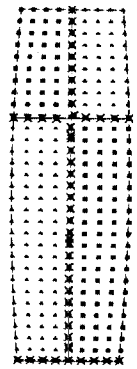
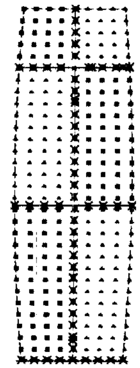
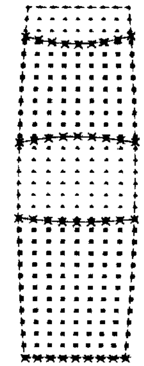
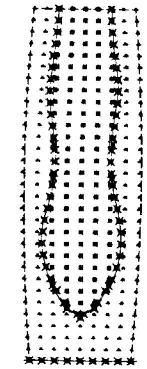
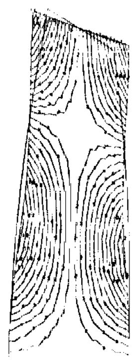
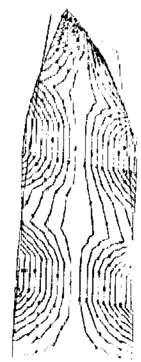
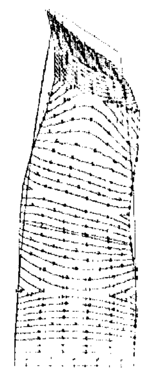
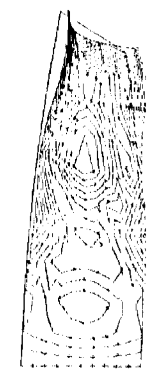
Figure 5—Continued.

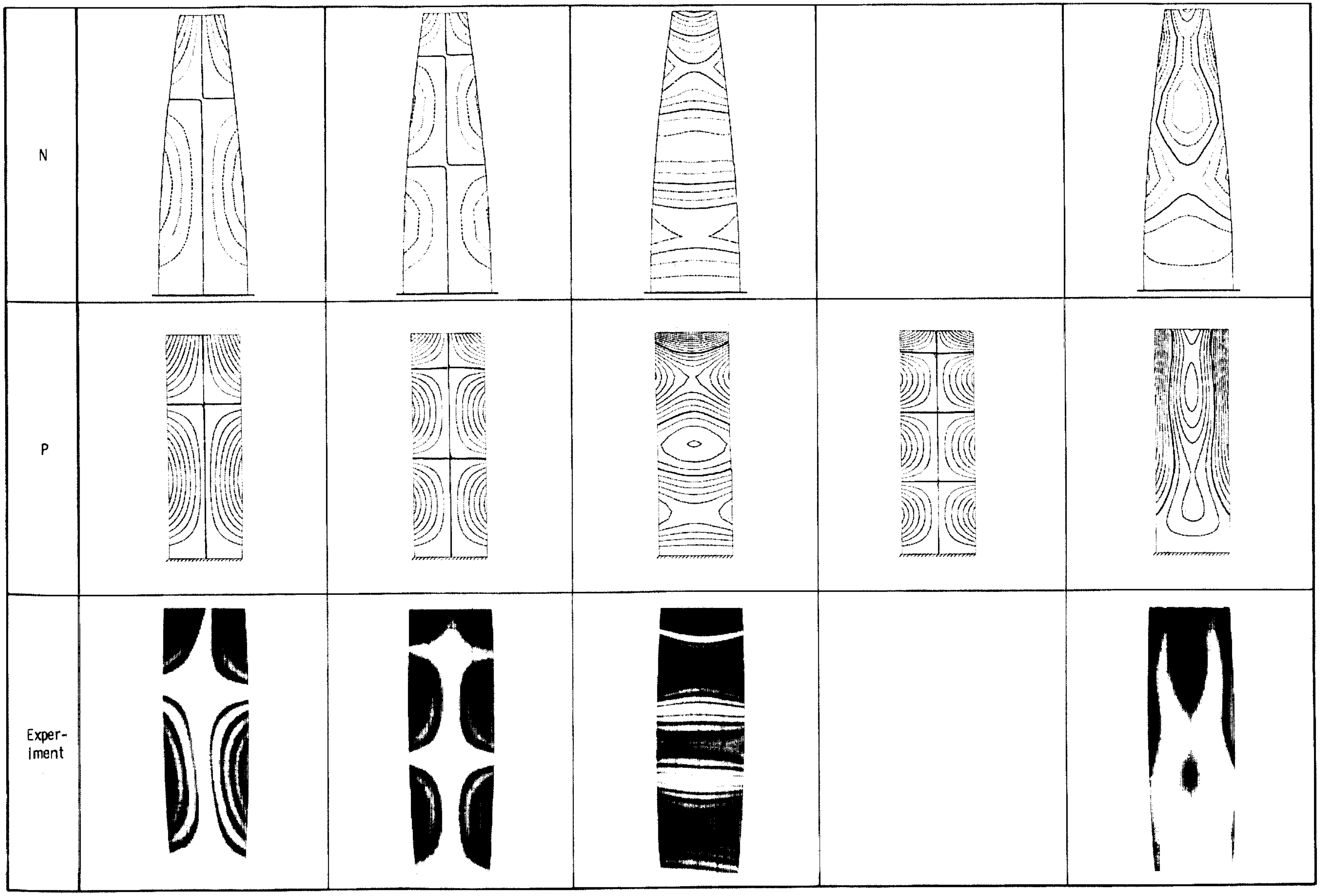
Method	Mode				
	1B	1T	2B	3B	1E
D					
H					
K					



(6) $a/b = 3$; $b/h = 20$; $\varphi = 60^\circ$.

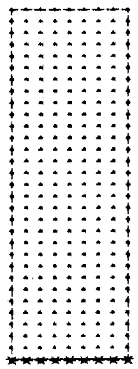
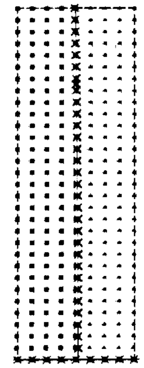
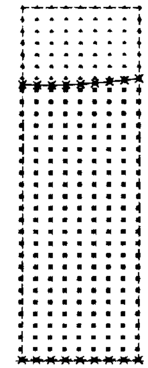
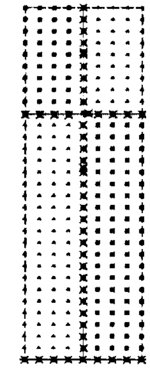
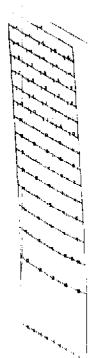
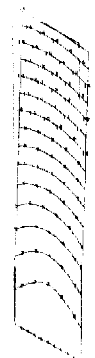
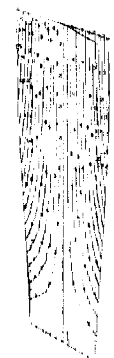
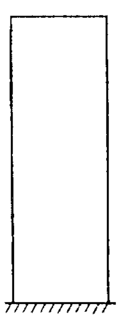
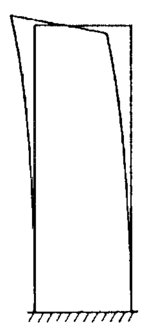
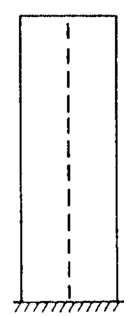
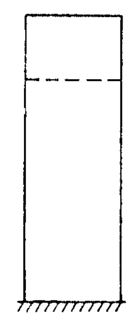
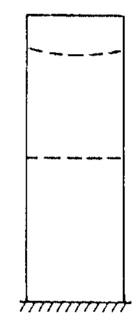
Figure 5—Continued.

Method	Mode				
	2T	3T	4B	4T	1C
D					
H					
K					



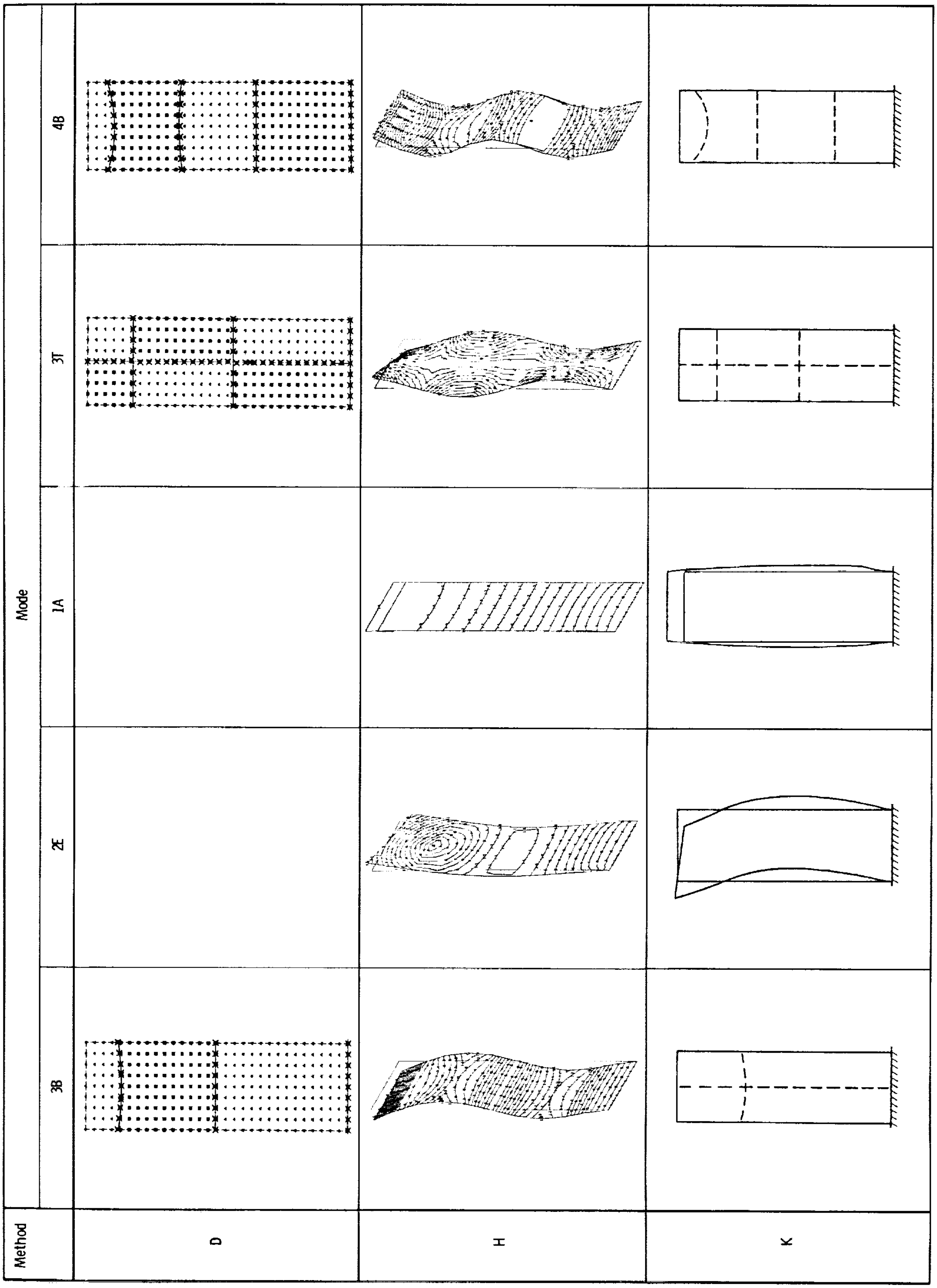
(6) Concluded.

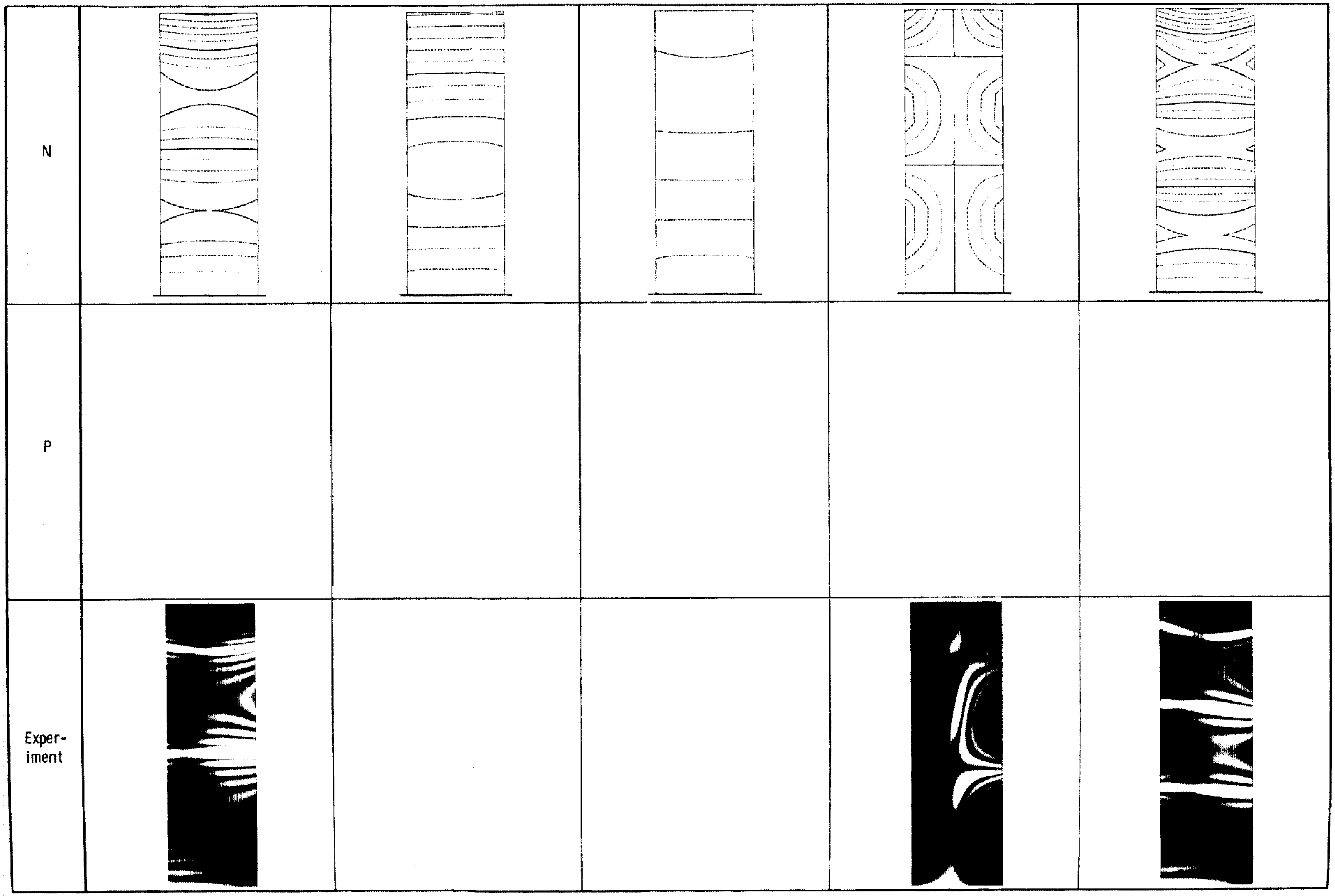
Figure 5—Continued.

Method	Mode				
	1B	1E	1T	2B	2T
D					
H					
K					

(7) $a/b = 3$; $b/h = 5$; $\varphi = 0°$.

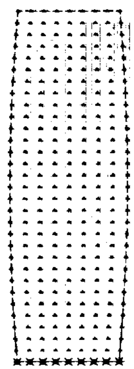
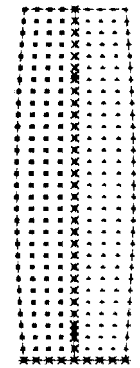
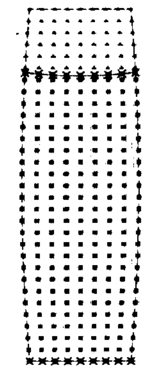
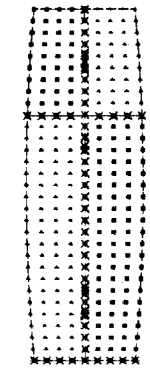
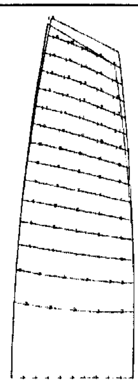
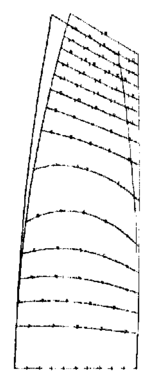
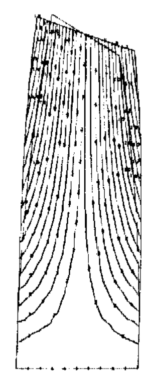
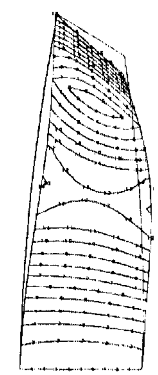
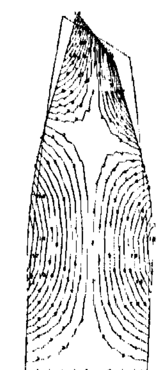
Figure 5—Continued.

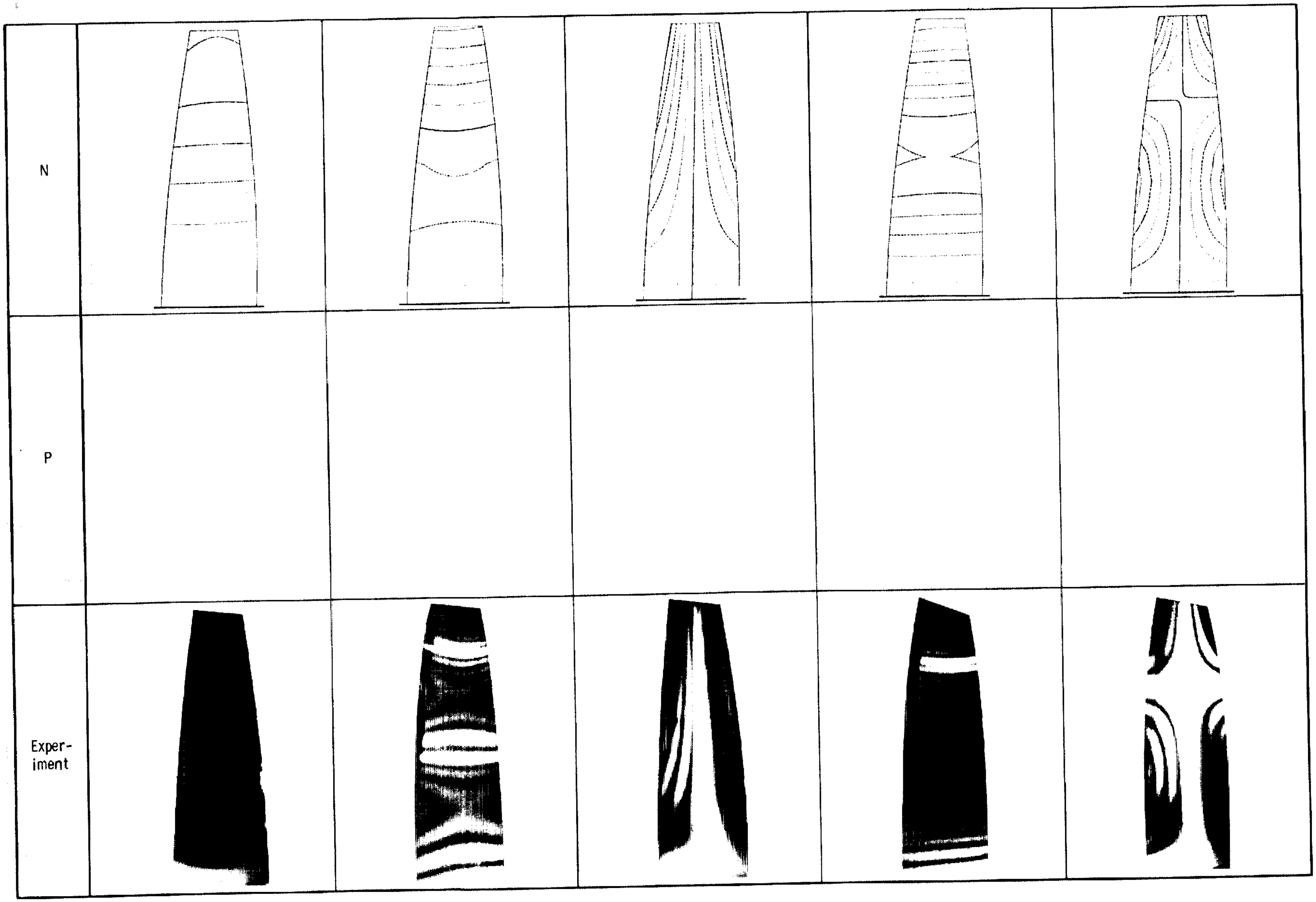

Mode
Method
3B
2E
1A
3T
4B
D
H
K



(7) Concluded.

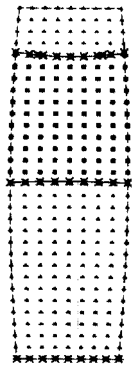
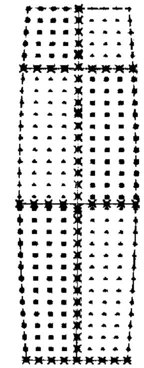
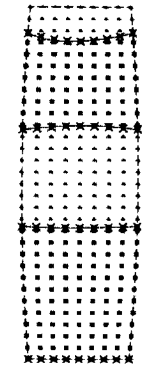
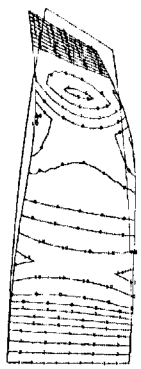
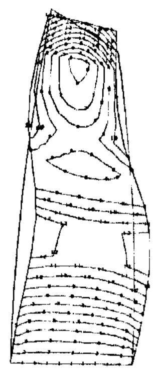
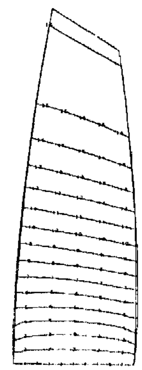
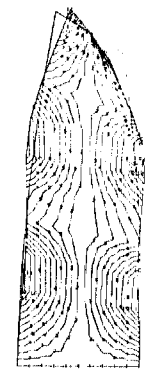
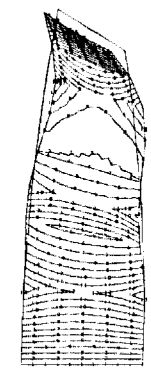
Figure 5—Continued.

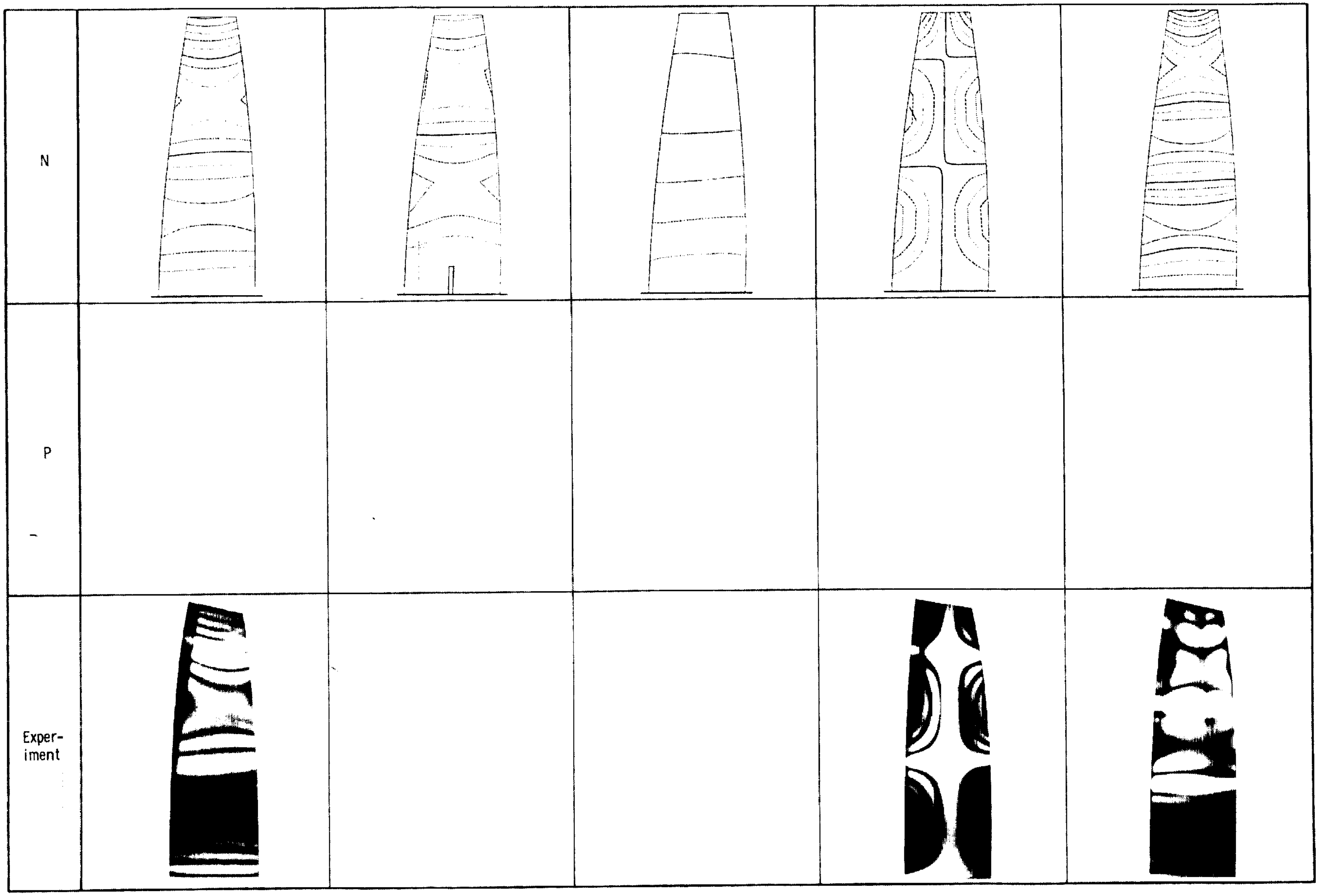
Method	Mode				
	1B	1E	1T	2B	2T
D					
H					
K					



(8) $a/b=3$; $b/h=5$; $\varphi=60°$.

Figure 5—Continued.

Method	Mode				
	3B	2E	1A	3T	4B
D					
H					
K					



(8) Concluded.

Figure 5—Concluded.

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA RP-1150		

4. Title and Subtitle	5. Report Date
Joint Research Effort on Vibrations of Twisted Plates Phase I: Final Results	September 1985
	6. Performing Organization Code 505-33-7B
7. Author(s) Robert E. Kielb, Arthur W. Leissa, James C. MacBain, and Kelly S. Carney	8. Performing Organization Report No. E-2576
	10. Work Unit No.
9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135	11. Contract or Grant No.
	13. Type of Report and Period Covered Reference Publication
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

Robert E. Kielb, Lewis Research Center; Arthur W. Leissa, Ohio State University, Columbus, Ohio (work done under NASA grant NAG3-36); James C. MacBain, U.S. Air Force Wright Aeronautical Laboratories, Wright-Patterson Air Force Base, Ohio; Kelly S. Carney, Lewis Research Center.

16. Abstract

This publication gives the complete theoretical and experimental results of the first phase of a joint government/industry/university research study on the vibration characteristics of twisted cantilever plates. The study was conducted to generate an experimental data base and to compare many different theoretical methods with each other and with the experimental results. Plates with aspect ratios, thickness ratios, and twist angles representative of current gas turbine engine blading were investigated. The theoretical results were generated by numerous finite element, shell, and beam analysis methods. The experimental results were obtained by precision machining a set of twisted plates and testing them at two laboratories. The second and final phase of the study will concern the effects of rotation.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Vibration Twisted plates Finite element Shells	Unclassified - unlimited STAR Category 39

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price*
Unclassified	Unclassified	99	A05

*For sale by the National Technical Information Service, Springfield, Virginia 22161

NASA-Langley, 1985