ON THE COMPUTATION OF STRUCTURAL VIBRATIONS

INDUCED BY A LOW-SPEED TURBULENT FLOW

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This paper discusses a method for numerical evaluation of the vibrations of a cylindrical shell structure induced by a low-speed external turbulent flow. The direction of flow is along the axis of revolution of the shell (see Figure 1), and the source of excitation is the pressure fluctuations in the turbulent boundary layer (TBL).

For the investigation of vibration and noise problems it is usually more desirable to utilize the modal expansion approach. The axisymmetric shell structure shown in Figure 1 can be modeled by the assemblage of conical-shell finite-elements. This modeling allows the eigenfunction $\psi_{mn}(x,\theta)$ to be represented in a rectangular product of a longitudinal modal function $f_{mn}(x)$ and a circular harmonic function cos $m\theta$ (or sin $m\theta$), i.e.,

$$\psi_{mn}(x,\theta) = f_{mn}(x) \cos m\theta \qquad m=0,\ 1,\ 2,\ \ldots \qquad n=1,\ 2,\ 3,\ \ldots \tag{1}$$

The forcing function from the TBL is assumed to be spatially homogeneous and temporally stationary. It is commonly expressed in terms of the wavevector-frequency spectrum $\Phi_p(k_1,k_3,\omega)$ where k_1 is the streamwise wavenumber and k_3 is the transverse wavenumber. For the calculation of the structural acceptance with this forcing function, the structural modes must also be expressed in wavenumber space. This can be accomplished by taking a spatial Fourier transform of the modes. The finite-element modeling provides the computed eigenfunction defined at a set of discrete points. If the grid points on the flow surface are equally spaced, a Fast Fourier Transform (FFT) routine may be used. From the FFT spectral coefficients, we may express

$$f_{mn}(x) = a_o + \sum_{\nu=1}^{U} [a_\nu \cos \frac{2\nu\pi}{L} x + b_\nu \sin \frac{2\nu\pi}{L} x] \tag{2}$$

where U is one less than one half of the total number of FFT data points and L is the axial length of the structure.

The effective modal input spectral density $\pi_{mn}(\omega)$ from the TBL can be evaluated as follows,

$$\pi_{mn}(\omega) = A^2\ \phi(\omega)\ J^2_{mn,mn}(\omega) \tag{3}$$

where A is the total area of the flow surface, $J^2_{mn,mn}(\omega)$ is the modal joint-acceptance (or called self-acceptance), which provides a measure of the degree of coupling between the turbulent pressure field and the structure, and $\phi(\omega)$ is the frequency spectrum of the TBL pressure fluctuations. The joint-acceptance can be computed by the following summation, i.e.,

$$J^2_{mn,mn}(\omega) = [(2\pi)^3/A\phi(\omega)]$$

$$\times \left[a_0^2 \, \Phi_p(0,\frac{m}{R},\omega) + \frac{1}{2}\sum_{\nu=1}^{U} (a_\nu^2+b_\nu^2)\, \Phi_p(\frac{2\nu\pi}{L}, \frac{m}{R}, \omega)\right] \qquad (4)$$

where R is the radius of the cylindrical shell.

Often it is required to evaluate the summation up to the convection wavenumber. This requires that the number of FFT data points to be approximately $\omega L/\pi U_c$, where U_c is the convection velocity of the TBL. If the length L of the structure is large and the frequencies of interest are high, the required data points will generally exceed the number of finite-element grid points. This difficulty can be overcome by obtaining additional data points from spline fitting and interpolation of the eigenfunctions.

If the size of structure is larger than the correlation length of the pressure field, the cross-modal acceptance $J^2_{mn,m'n'}(\omega)$, can be neglected. In this case, the structural displacement response spectrum $[s(\omega)]$ evaluated at $\theta = \theta_o$ can be calculated as follows:

$$[s(\omega)] = [\psi][H^*_{mn}(\omega)][\pi_{mn}(\omega)][H_{mn}(\omega)][\psi]^T \qquad (5)$$

where $[\psi]$ is the assembly of eigenvectors, each column represents one eigenvector $\{f_{mn}(x_i) \cos m\theta_o\}$. $H_{mn}(\omega)$ is the modal admittance function which is defined as

$$H_{mn}(\omega) = \{M_{mn}[(\omega^2_{mn}-\omega^2) + i(\omega^2_{mn}\,\eta_{mn}+\omega\omega_{mn}\,\delta_{mn})]\}^{-1} \qquad (6)$$

and where M_{mn} is the mode mass, η_{mn} is the structural modal loss factor, and δ_{mn} is the modal acoustic loss factor.

The most difficult task in the numerical evaluation of flow induced vibration is the uncertainty about the forcing functions, i.e., the wavevector-frequency spectrum. Several theoretical forcing function models have been published in recent years, all of them require an empirical fit with experimental data. Published experimental data are very widely scattered depending on the measuring facility, surface property, the method of scaling the data, etc. Selection of a suitable forcing function model and the experimental data thus depend heavily on experienced engineering judgement and knowledge of how the experimental data are obtained.

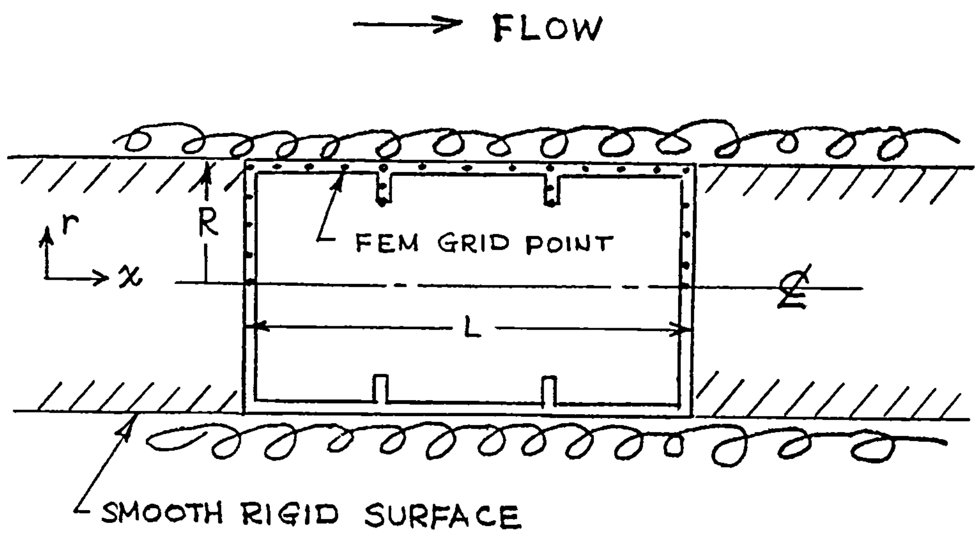

FIG. 1 A CYLINDRICAL SHELL STRUCTURE WITH AN EXTERNAL TURBULENT FLOW