IMPROVING THE ACCURACY OF THE BOUNDARY INTEGRAL METHOD BASED ON THE HELMHOLTZ INTEGRAL

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Several recent papers in the literature have been based on various forms of the Helmholtz integral to compute the radiation fields of vibraing bodies. The surface integral has the form

$$P(R_0)=\frac{1}{2\pi}\oint [i\omega\rho G(R,R_0)\, V(R_0)-\frac{\partial G}{\partial n}(R,R_0)P(R_0)]\, dS_0$$

where the symbols $P, R_0, w, \rho, G, R, V,$ and S_0 are acoustic pressure, source coordinate, angular frequency, fluid density, Green function, field coordinate, surface velocity and body surface respectively. A discretized form of the surface integral is

$$\begin{bmatrix} \partial_{ij} - D_{ij} \end{bmatrix} \begin{bmatrix} P_j \end{bmatrix} = \begin{bmatrix} M_{ij} \end{bmatrix} \begin{bmatrix} V_j \end{bmatrix}$$

where D and M are the dipole and monopole coefficients and i and j are the field and source coordinates. Solutions to the above surface integral are complicated with the singularity of the Green function at $R=R_0$ and with the uniqueness problem at interior eigen frequencies of the enclosed space. The use of the interior integral circumvents the the singularity problem since the field points are chosen in the interior space of the vibrating body where a zero pressure condition exists. The interior integral has the form

$$\oint \frac{\partial G}{\partial n}(R,R_0)P(R_0)\ \ dS_0 = \oint\ i\omega\rho G(R,R_0)\, V(R_0)dS_0$$

The discretized version of the integral relates the surface pressure to the surface velocity through a transfer function, T_{ij} as

$$\begin{bmatrix} P_j \end{bmatrix} = \begin{bmatrix} T_{ij} \end{bmatrix} \begin{bmatrix} V_j \end{bmatrix}$$

In the above form, the field points are located in the interior space enclosed by the surface of the body. In general, we have found that T_{ij} is not invariant with choice of interior points, i.e., different sets of interior field points produce different sets of surface pressures. (It can be shown that the same problem exists for the surface integral applications). T_{ij} can be made invariant (or nearly so) by placing a stronger condition on the pressure field in the interior space. With a finite set of of interior points, the requirement that the zero pressure condition exists everywhere in the interior volume is not met. To satisfy this requirement, the interior equation can first be integrated over an incremental interior volume ΔV_k as

$$\int_{\Delta V_k} \left[\oint \left\{ \frac{\partial G}{\partial n}(R,R_0)P(R_0) - i\omega\rho G(R,R_0)\, V(R_0) \right\} dS_0 \right] dV$$

which, after the volume integral integration gives

$$\oint \left\{ \frac{\partial G_k}{\partial n}(R_0)\, P(R_0) - i\omega\rho G_k(R_0)\, V(R_0) \right\} dS_0$$

Breaking up the interior space into the same number of elemental volumes as surface elememts produces a discretized form of the interior equation as

$$\begin{bmatrix} P_j \end{bmatrix} = \begin{bmatrix} T_{Kj} \end{bmatrix} \begin{bmatrix} V_j \end{bmatrix}$$

An added advantage of satisfying the field pressure condition in the interior volume is that the uniqueness problem associated with the interior eigen modes is eliminated since the interior pressure is necessarily zero at all frequencies. Examples of the above method will be presented for a variety of radiators.