EPSILON AURIGAE IN AN EVOLUTIONARY CONTEXT

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ABSTRACT. Basic observational data of Epsilon Aurigae are summarized and used as the basis of a discussion of possible evolutionary states of the system. Constraints posed by the presence of a cold disk surrounding the secondary star are also outlined.

Possible evolutionary models of the F0 Ia supergiant range from pre-main-sequence contraction through shell hydrogen burning, core helium burning, to shell helium burning, depending on the absolute luminosity of the system, for models in which no mass transfer has taken place. Models invoking binary interaction include core and shell helium-burning stars, and pre-white dwarfs, again depending on the absolute luminosity of the system. A massive shell helium burning star or a pre-white dwarf mass transfer remnant would appear the most likely of these models at present. Observational tests of these models are briefly outlined.

1. INTRODUCTION

With an orbital period of 9885 days, Epsilon Aurigae (ε Aur) lies at the long-period extreme of binary star systems which could conceivably undergo mass transfer (see, e.g., Webbink 1979b). That the enigmatic dark secondary in this supergiant (F0 Iap) binary must be highly flattened in profile has long been recognized (Kopal 1954; Hack 1961; see also Ludendorff 1924), a circumstance reminiscent of the accretion disks which commonly occur among interacting binaries (see, e.g., Pringle 1981). In this short paper, we shall therefore explore what evolutionary paths may give rise to an F0 Ia supergiant in a long-period binary.

The basic observational data concerning ε Aur are summarized in Table 1. The effective temperature quoted here for the F0 Iap primary is the mean of the values determined by Castelli (1978) from a fine analysis of its optical spectrum, and by Hack and Selvelli (1979) from

TABLE 1. BASIC DATA

Sp	= F0 Iap	
$T_{eff}^{(1)}$	= 7650 ± 150 K	Castelli 1978; Hack and Selvelli 1979
$T_{eff}^{(2)}$	= 480 ± 45 K	Backman 1985
V	= 3.00	Gyldenkerne 1970
E_{B-V}	= 0.30 ± 0.05	Hack and Selvelli 1979
P	= 9885^d	Huang 1974
r_1	= 0.052	Huang 1974
r_2	= 0.178	Huang 1974
f(m)	= 3.25 ± 0.38 $M_\odot$	Wright 1970
$A_1 \sin i$	= 13.37 ± 0.53 AU	Wright 1970
e	= 0.200 ± 0.034	Wright 1970
π_{rel}	= $-0\overset{''}{.}0010 \pm 0\overset{''}{.}0010$	van de Kamp 1978
a_1	= $0\overset{''}{.}0227 \pm 0\overset{''}{.}0010$	
i	= 89° ± 3°	

a model atmosphere fitted to its ultraviolet continuum. The temperature attributed to the cool secondary is that characterizing the infrared excess of this system. It is, of course, a well-known quandary that no contribution from the secondary is detected anywhere in the optical or ultraviolet flux distribution of this system (except possibly at wavelengths $\lesssim$1600 Å -- Hack and Selvelli 1979; Parthasarathy and Lambert 1983; Boehm, Ferluga, and Hack 1984). Finally, it should be noted that the fractional radii for the two components quoted here from Huang (1974) are those for his thick-disk model for the cool secondary. Somewhat different values, most notably smaller r_1, were deduced by Wilson (1971) from a thin-disk model.

By equating the spectroscopic and astrometric orbital semimajor axes of the primary, a distance to ε Aur can be derived (van de Kamp 1978), and the absolute parameters of the primary component thereby fixed. The resultant values are listed in Table 2. The mass limit for the primary quoted here follows from an upper limit to its surface gravity (log g < 1.5) dictated by its Ia luminosity classification.

TABLE 2. DERIVED DATA

d	$= 578 \pm 51$ pc
M_V	$= -6.74 \pm 0.30$
$\log L_1/L_\odot$	$= 4.54 \pm 0.12$
$\log R_1/R_\odot$	$= 2.02 \pm 0.06$
$\log M_1/M_\odot$	$\lesssim 1.11 \pm 0.13$

2. THE COOL DISK

Although very little is known at present about the properties of the disk-like eclipsing object, its mere existence as well as its state could, in principle, place serious constraints on the evolutionary states of the binary. A few of these are summarized here.

First, we note that the infrared temperature deduced for the disk is consistent with its having little or no internal energy sources whatever, but radiating primarily reprocessed light from the supergiant primary. For a thin disk of small radius relative to the orbital separation, A, and irradiated by a spherical star of effective temperature T_1 and radius $R_1 = r_1 A$ lying in the plane of the disk, the equilibrium temperature of the disk is

$$T_d = \left(\frac{r_1^3}{6\pi}\right)^{1/4} T_1 \quad .$$

In the case of ε Aur, the data of Table 1 lead to an expected disk temperature of $T_d = 400 \pm 10$ K, which is comparable with the temperature characterizing the infrared excess. From considerations of vertical hydrostatic equilibrium, we would expect the ratio of thickness h to radius r of a gaseous disk should be of the same order as the ratio of sound speed c_s to orbital velocity v_{orb}. At this temperature $c_s \approx 2.5$ km s^{-1}, and at the outer edge of the disk $v_{orb} \approx 25$ km s^{-1}, giving an aspect ratio $h/r \approx 0.1$ which is in good agreement with that deduced by Huang (1974) for his thick-disk model. It should be noted, however, that the grains, which presumably constitute the dominant opacity source for the disk, could nevertheless settle to a thin layer at the midplane of the disk, depending upon their sizes, the gas density of the disk, and the degree of turbulence in the disk.

The low disk temperature also implies a fairly small net accretion rate through the disk. For a steady-state disk (a reasonable approximation for disks older than their various timescales -- see

Lynden-Bell and Pringle 1974),

$$\dot{M} \approx \frac{8\pi \mathfrak{F}(R)\, R^3}{3G\, M_2} \quad ,$$

where $\mathfrak{F}(R)$ is the net surface brightness of the disk (energy radiated minus that intercepted, per unit area). Applying this relationship at one-half the disk radius, with

$$\mathfrak{F}(R) \lesssim \sigma\, T_2^4$$

over most of the disk, we deduce

$$\dot{M} \lesssim 10^{-7}\, M_{\odot}\, yr^{-1} \quad .$$

A similar limit follows from the weakness of the excess ultraviolet emission shortward of 1600 Å (see, e.g., the symbiotic star models of Kenyon and Webbink 1984).

A lower limit to the age of the disk can be derived from standard accretion disk theory (Shakura and Syunyaev 1973; Pringle and Rees 1972). The timescale characterizing decay of the disk is the viscous timescale:

$$\tau_{\nu} \approx \frac{r^2}{\alpha\, \Omega_{orb}\, h^2} \quad ,$$

where r and h are the disk radius and thickness, as above, Ω_{orb} is the orbital angular frequency of the disk, and α is a dimensionless parameter (the ratio of viscous stress to gas pressure) which must be $\lesssim 1$. The mass flux through the inner disk is controlled by conditions at its outer edge, where, for ϵ Aur, we obtain an estimate $\tau_{\nu} \approx 200\, \alpha^{-1}$ yr. Values of α deduced from observations of cataclysmic variable stars ($\alpha \approx 0.1$; Pringle 1981) would suggest that the disk in ϵ Aur must be at least a few millenia old.

A lower limit to the mass of the disk can be estimated from its projected surface area. We assume its opacity arises from grains, which must have dimensions $r_g \gtrsim 10^{-2}$ cm in order to produce the gray eclipses which are observed. For a projected disk area comparable to that of the primary, with grains of density $\rho_g \approx 3$ g cm^{-3}, a total mass in grains of $M_g \gtrsim 5 \times 10^{24}$ g is indicated. At solar abundances, with all refractory materials condensed into grains, this grain mass corresponds to a total disk mass $M_d \gtrsim 10^{27}$ g. This value is within an order of magnitude or so of the product of the limits obtained above for the accretion rate and disk decay timescale.

3. EVOLUTIONARY STATUS OF THE SUPERGIANT

Strictly speaking, the MK spectral classification of the supergiant fixes only its effective temperature and surface gravity within certain limits. The mass of that component is constrained only by the further introduction of information regarding its distance or absolute dimensions, as in Table 2. It is clear from that table that wide latitude remains in the masses which would satisfy available constraints on ε Aur.

There are in fact a number of possible evolutionary phases in which a star, evolving either singly, or as a member of a close binary system, may pass through an F0 supergiant state. These possible evolutionary states are summarized in Table 3 for three assumed values of the luminosity of the supergiant. These luminosities bracket that deduced above in Table 2 and allow for the possibility that the distance to ε Aur deduced there is still significantly in error. Those models corresponding to mass transfer remnants are listed in boldface type. The table lists, for each solution, logarithms of: M_1, the mass of the supergiant (in solar units); g_1, its surface gravity (in cm s^{-2}); $\tau_T \equiv |T_1/\dot{T}_1|$, the timescale (in years) on which its effective temperature

TABLE 3. POSSIBLE EVOLUTIONARY STATES OF ε Aur

State	log M_1	log g_1	log τ_T	log t_d	log M_2	log $L_{2,0}$
			log $L_1/L_\odot$ = 5.00			
1a Pre-ms	1.48	1.41	3.2 (+)	2.90	1.31	4.71
1d He shell	1.24	1.16	4.7 (-)	6.98	1.18	4.32
			log $L_1/L_\odot$ = 4.50			
1a Pre-ms	1.26	1.69	3.6 (+)	3.32	1.19	4.36
1d He shell	1.06	1.49	4.8 (-)	7.23	1.09	4.04
2c Pre-wd	**0.00**	**0.43**	**3.7:(+)**	**3.7:**	**0.68**	**2.70**
			log $L_1/L_\odot$ = 4.00			
1a Pre-ms	1.09	2.02	4.1 (+)	3.76	1.10	4.08
1b H shell	1.01	1.93	5.0 (-)	7.27	1.06	3.96
1c He core	0.94	1.86	5.1 (+)	7.38	1.03	3.85
1d He shell	0.90	1.83	5.1 (-)	7.50	1.01	3.81
2a He ign	**0.24**	**1.17**	**4.5 (+)**	**4.98**	**0.75**	**2.95**
2b He shell	**0.16**	**1.08**	**5.3 (-)**	**6.87**	**0.72**	**2.86**
2c Pre-wd	**-0.14**	**0.79**	**4.5:(+)**	**4.4:**	**0.65**	**2.60**

is evolving, together with an indication whether T_1 is increasing (+) or decreasing (−); t_d, the age of the disk around the secondary component (in years), presumed to be the interval of time since the supergiant component last filled its Roche lobe or since it last reached its maximum radius, as the case may be; M_2, the mass of the secondary (in solar units), as deduced from the spectroscopic mass function and astrometric orbital inclination; and $L_{2,0}$, the luminosity (in solar units) such a secondary would have on the zero age main sequence. The values in this table assume a solar-type composition, and have been interpolated from the calculations of Iben (1965,1966,1972); Lamb, Iben, and Howard (1976); Becker, Iben, and Tuggle (1977); and, for the mass-transfer remnants, Iben and Tutukov (1985). Briefly, the evolutionary stages indicated in the first column of this table are:

(1a) Approach to the main sequence. This interpretation (see, e.g., Kopal 1971) would make ε Aur very young indeed. It may be rejected on several counts: First, ε Aur lies near no known star-forming region. Second, the masses implied for the secondary are all comparable with or slightly smaller than that of the primary. Since the time required to reach the main sequence increases with decreasing mass (e.g., Iben 1965), the secondary could not have contracted to a significantly smaller radius than the primary, as observations appear to demand, even in the event that the secondary is itself double (see below). Finally, except at the highest luminosity (where the age of the disk is unacceptably short), the surface gravities expected for the supergiant would place it in luminosity class Iab or Ib, not Ia as found observationally.

(1b) Shell hydrogen burning. For log $L/L_{\odot} \lesssim 4.3$, a star of intermediate mass (~8–12 $M_{\odot}$) and solar composition passes through the F0 supergiant region prior to helium ignition (e.g., Iben 1966). The expected surface gravity of the supergiant would again place it in luminosity class Iab or Ib, and, as with all models do not invoke mass transfer, the expected luminosity of the secondary is uncomfortably large.

(1c) Core helium burning. Stars which pass through stage (1a) loop backwards to the blue in the Hertzsprung-Russell (HR) diagram following core helium ignition, passing through the Cepheid instability strip as they do so. The evolutionary tracks of these models may be quite complicated and highly sensitive to the initial composition (see, e.g., Becker, Iben, and Tuggle 1977), and they may loop through the F0 supergiant region more than once; but, except for the values of τ_T, all such models have properties (and weaknesses) practically identical to those expected for case (1b) above. In particular, solutions of this type require log $L_1/L_{\odot} \lesssim 4.3$, which would place ε Aur at a distance d $\lesssim$ 440 pc, with a luminosity class Iab or Ib.

(1d) Shell helium burning. Stars with masses above ~12 $M_{\odot}$ undergo core helium burning before ever reaching F0 supergiant dimensions (Lamb, Iben, and Howard 1976), and those of somewhat smaller masses (as in case 1c above) do so at the high-temperature ends of their blue loops in the HR diagram. They then evolve redward through the F0 supergiant during or following core helium exhaustion and the readjustment to shell helium burning. Such models span the entire range of luminosities of interest, and at higher luminosities (log $L_1/L_\odot \gtrsim 4.5$) have suitably low surface gravities. Their lifetimes in the F0 supergiant band are at least an order of magnitude longer than those characterizing pre-main sequence models of equal luminosity, making this configuration inherently more likely. However, as in the cases discussed above, it is difficult to understand the low relative luminosity of the secondary.

On this last count, those models interpreting ϵ Aur as a mass-transfer remnant fare much better, as they imply much lower masses for the F0 supergiant, and correspondingly lower masses for the secondary, at a given luminosity. The lower mass assigned to the supergiant also removes any difficulty with the luminosity class assigned to ϵ Aur: all post-mass transfer supergiants in the luminosity range of interest have very low (class Ia) surface gravities (see Table 3). The possible models summarized in Table 3 are:

(2a) Core helium ignition. In many ways, this is an analogue to case (1c) above. A star of initial mass 8-12 $M_\odot$ in this case fills its Roche lobe and is stripped nearly to its helium core. The star then contracts toward the helium main sequence (see Iben and Tutukov 1985), passing through the F0 supergiant region. There are, however, a number of difficulties with this interpretation: As in cases (1b) and (1c) above, models of this type are limited to lower luminosities (log $L_1/L_\odot \lesssim 4.3$), again because more massive initial primaries would have passed through core helium burning before ever becoming as red as spectral type F0. Second, single stars in this initial mass range reach maximum radii of ~500 $R_\odot$ at most (Webbink 1979b, Iben and Tutukov 1985), and hence can fill their Roche lobes only for initial orbital periods $P \lesssim 1000^d$; a further increase in orbital period by a factor of 10 would require that the system have lost roughly 70 percent of its total mass in a stellar wind within the ~10^5 years since it last filled its Roche lobe. Mass loss of this magnitude is excluded by the fact that the present secondary component is at least half as massive as the initial primary.

(2b) Shell helium burning. The helium star remnant of case (2a) will, upon core helium exhaustion, once again progress far to the right in the HR diagram (Paczynski 1971; Iben and Tutukov 1985). Aside from the presumed age of the disk, and a much slower traversal of the F0 supergiant band, models of this sort possess the same general properties and difficulties as those of type (2a). They are of course similarly limited to total luminosities log $L_1/L_\odot \lesssim 4.3$.

(2c) Pre-white dwarf. This is the evolutionary state of the supergiant suggested by Eggleton and Pringle (1985), and corresponds to a primary star in a double-shell-burning phase which has been their Roche lobes at 10^4-day orbital periods, so the difficulties concerning excessive mass loss in a stellar wind which appeared in cases (2a) and (2b) do not pertain here. Depending upon the degree of systemic mass loss in this case, significantly lower-mass progenitor stars may be allowed for the primary. An additional virtue of this model is that it requires a lower-luminosity secondary star than any of the others discussed above. It does, however, imply that the F0 supergiant is in a relatively rapidly evolving state, compared, for example, to model (1d) above.

In summary, it appears that the most viable models for the supergiant in ε Aur are, first, the traditional interpretation in terms of a relatively massive, post-main-sequence star in a state of shell helium burning, or, second, one in which the supergiant is contracting toward a white dwarf state, having been stripped of most of its hydrogen-rich envelope by a combination of tidal mass transfer to the secondary component and mass loss in a stellar wind. The latter model accounts much more successfully for the large luminosity difference between the components, but it also implies that ε Aur is in a much more rapidly evolving state than does the former.

4. EPSILON AURIGAE AS A CLOSE TRIPLE SYSTEM

Recently, it has been speculated that the under-luminous secondary component in ε Aur may itself be binary. On the one hand, by subdividing the mass of that component, its expected intrinsic luminosity could be reduced by a factor of as much as 5 (Lissauer and Backman 1984). On the other, a binary central object could also act as an energy and angular momentum source to the disk, strongly inhibiting its decay while at the same time maintaining it at relatively large thickness by gravitational agitation (Eggleton and Pringle 1985).

The hypothesis of a binary secondary component carries an additional benefit for those interpretations of this system as a post-mass-transfer object. In these models, the primary would have filled its Roche lobe after reaching the giant or asymptotic giant branch, with a deep convective envelope. This tends to be a violently unstable situation. Hydrostatic equilibrium at the base of a deep envelope requires that pressure, P, scale as $P \sim M^2/R^4$. Adiabatic convective equilibrium, on the other hand, gives $P \sim \rho^{5/3}$ (for an ideal gas), or $P \sim M^{5/3}/R^3$. It follows that the adiabatic response of the star to mass is to expand, with $R \sim M^{-1/3}$. At the same time, the Roche lobe radius varies with mass roughly as

$$\frac{d \ \ln R_L}{d \ \ln M} \approx 2\,q - \frac{5}{3} \quad ,$$

where q is the mass ratio (mass of the lobe-filling star divided by that of its companion. It follows that the Roche lobe increases in radius with decreasing mass fast enough to accommodate the hydrostatic response of the lobe-filling star only if $q \lesssim 2/3$, i.e., the lobe-filling star is significantly less massive than its companion. Otherwise mass transfer tends to proceed on a dynamical timescale, and catastrophic losses of systemic mass and angular momentum probably ensue (Meyer and Meyer-Hofmeister 1979; Webbink 1979a). Thus, splitting the secondary's mass between two stars permits the primary star to be at the same time the most massive of the three (and hence first to evolve), and also sufficiently less massive than the total mass of the secondary (and hence stable against dynamical mass transfer).

How common are such triple systems? The system HD 157978/9 which Eggleton and Pringle (1985) chose as their prototype is in fact only one of 26 known close triple systems (i.e., close enough for all three stars to interact during their lifetimes) in the survey by Fekel (1981). Of these, all but five appear in the Bright Star Catalogue, and in at least one-third of these systems the distant member is the most massive component of the system. Considering the profound difficulty in detecting close triple systems, especially among fainter stars, Fekel's list is undoubtedly very incomplete. Considerations of dynamical stability (which limits allowable period ratios in close triple systems), the hierarchical structure of multiple star systems (e.g., Batten 1973), and the distribution of binary and multiple systems in orbital period (Abt and Levy 1976, 1978) yield a very rough estimate that close triple systems occur with perhaps 10 percent of the frequency of close binary systems. Models of ε Aur as a triple system are therefore not at all inherently implausible.

5. OBSERVATIONAL NEEDS

As indicated above, the number and types of possible evolutionary models of ε Aur depend crucially on an accurate estimate of its distance, and hence of its absolute dimensions. The fact that both spectroscopic and astrometric orbits can be measured is an extremely powerful tool for this purpose, but significant differences remain in the orbits obtained by these methods. The radial velocity solutions may be strongly affected by intrinsic pulsations of the supergiant itself, and the reality of the measured orbital eccentricity is open to some doubt on this account. The astrometric orbit is quite small, and vulnerable to a variety of long-term systematic errors, given that it is based on only slightly more than one complete orbit. A simultaneous solution for both spectroscopic and astrometric orbits could provide a more realistic assessment of the uncertainty in the distance and absolute dimensions of the system.

As a rule, post-mass-transfer models of ε Aur predict that its atmosphere should be depleted in hydrogen and oxygen, and enhanced in helium and nitrogen, with carbon possibly either enhanced or depleted (see, e.g., Iben and Tutukov 1985). The fine analyses by Castelli

(1978) and Castelli, Hoekstra, and Kondo (1982) give some evidence of enhancement relative to hydrogen elements from Si to Ba in ε Aur, compared with ϕ Cas, another F0 Ia star. This result is quite uncertain, however, because the hydrogen Balmer lines are themselves contaminated by emission. Modeling of the ultraviolet spectrum at shorter wavelengths could yield abundances for carbon and oxygen, and possibly nitrogen, and infrared spectroscopy of the Paschen or Brackett lines might resolve the uncertainty in the hydrogen abundance itself. It should also be noted that mass-transfer-remnant models also yield surface gravities an order of magnitude smaller than those predicted for massive supergiant models. A quantitative spectroscopic determination of log g could thus prove immensely valuable.

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