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MULTIPLE-SCALE TURBULENCE CLOSURE MODELING OF CONFINED RECIRCULATING FLOWS

By C. P. Chen
Resident Research Associate
National Research Council
Science and Engineering Directorate
Systems Dynamics Laboratory

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16. ABSTRACT

A multiple-scale turbulence closure scheme is developed for the numerical predictions of confined recirculating flows. This model is based on the multiple-time-scale concepts of Hanjalic et al. (1980) and takes into account the non-equilibrium spectral energy transfer mechanism. Problems concerning new formulation of energy transfer rate equations and subsequent model coefficient redefinition and energy spectrum partition are discussed. Comparisons are made with several experiments of internal recirculating flows for the purpose of model validation. Numerical results using the present model show significant improvement of predictive capability over that obtained with the single-scale k-ε model and show promising potential for complex turbulent flow predictions.

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INTRODUCTION

It has long been recognized that the currently widely used two-equation and Reynolds stress turbulence closure models suffer a serious drawback: a single set of scales is used to characterize the whole spectrum of turbulent motion [1]. Under this simplification, for example, if only a single length scale is used it must be a macro or integral length scale. Inherently, the equilibrium spectral energy transfer assumption has to be made for the turbulence to be characterized by the scales of energy containing eddies. This implies that the set of characteristic scales is much smaller than the set of scales characterizing the mean flow. Thus, strictly speaking, this level of model should be limited to cases where the mean flows are slowly evolving. It is not surprising to see why the $k-\varepsilon$ turbulence model and Reynolds stress model seem to do well in certain types of free shear flows and geophysical flows [2,3] within their own general limitations. This is almost never the case, however, for the complex confined recirculating flows where the energy spectrum is not in equilibrium and the mean flow is changing rapidly.

Over the past few years, several turbulence models have been devised to take into account the non-equilibrium spectral energy transfer. Full spectral modeling such as the ones devised by Cambon et al. [4] and Bertoglio [5] involves different scales at each wave number ot the equations for the energy spectrum. Another approach undertaken by Launder and coworkers [6,7,8] and Birch [9] is to use the multiple-time-scale concepts to characterize the different scales in turbulent flows.

In this study, a multiple-scale turbulence model will be developed based on the simplified spectral model of Hanjalic et al. The theoretical foundation and model constants establishment will be described in the following section. This report concludes with some elementary tests by comparing the numerical predictions using the proposed model with experiments of several confined recirculating flows, leaving the more practical applications such as swirling flow predictions for future communication.

MULTIPLE-SCALE TURBULENCE MODELING

The theoretical basis of the approach used here for the model development is described in detail in Hanjalic et al. [8]. By recognizing that the turbulence in recirculating flow is highly non-spectral-equilibrium and that the different energy transfer rates for different eddies should be modeled separately, the fluctuating parts of the velocity u_i can be decomposed into low-wave-number and high-wave-number parts, u_{il} and u_{ih}. Defining $\overline{u_{il}^2}/2 \equiv k_p$, $\overline{u_{ih}^2}/2 \equiv k_t$, the kinetic energies of turbulence of large-scale energy-containing eddies and high-wave-number transfer eddies respectively, the transport equations for k_p and k_t can be obtained by multiplying the instantaneous momentum equation by u_{il} and u_{ih} and taking Reynolds averaging.

For example, at high Reynolds number these equations for homogeneous flows are:

$$\frac{D\,k_p}{Dt} = -\overline{u_{il}\,u_{jl}}\,\frac{\partial U_i}{\partial x_j} - \varepsilon_p \tag{1}$$

$$\frac{D\,k_t}{Dt} = \varepsilon_p - \varepsilon_t \tag{2}$$

where

$$\varepsilon_p = -\overline{u_{il}\,\frac{\partial u_{ih}\,u_{jh}}{\partial x_j}} + \overline{u_{ih}\,\frac{\partial u_{il}\,\partial u_{jl}}{\partial x_j}} \quad . \tag{3}$$

The essential idea of arriving at equations (1) and (2) is the partition of the kinetic energy spectrum. In the simplest sense, one can imagine that the energy spectrum of turbulence is divided into roughly three regions: large-scale energy production, intermediate energy transfer, and small-scale dissipation eddies. The energy content of the dissipative eddies is negligible at high Reynolds number, thus $k \equiv k_p + k_t$, where k is the kinetic energy of turbulence. Another assumption involved in deriving equation (2) is that the spectral partition has to be at sufficiently high wave number such that $\overline{u_{ih}\,u_{il}}$ is isotropic. The significance of this assumption is that the dynamics of transfer eddies is disconnected from mean field and is solely controlled by the energy transfer rate ε_p and ε_t.

Conceptions of ε_p and ε_t based on their definitions [ε_p is given by equation (3) and $\varepsilon_t = \nu\overline{(\partial u_{ih}/\partial x_j\;\partial u_{jh}/\partial x_j)}$] are not very useful. As pointed out by Launder [8], the triple correlation ε_p seems to function as the energy transfer rate from one fluctuating field to another. The dissipation ε_t should also be regarded as the flux of energy through wave number space which k_t occupies [10]. Following the commonly used balance equation for the energy dissipation rate, and with the simplification that the energy transfer rate across the transfer range and dissipative range is equal, the following forms are proposed for ε_p and ε_t to close off equations (1) and (2): (in homogeneous high Reynolds number flow)

$$\frac{D\,\varepsilon_p}{Dt} = \frac{1}{\tau_p}\left[C_{p_1}\left(-\overline{u_i u_j}\,\frac{\partial U_i}{\partial x_j}\right) - C_{p_2}\,\varepsilon_p\right] \tag{4}$$

$$\frac{D\,\varepsilon_t}{Dt} = \frac{1}{\tau_p}\,[C_{t_1}\,\varepsilon_p - C_{t_2}\,\varepsilon_t] \tag{5}$$

where

$$\tau_p = \frac{k_p}{\varepsilon_p} \quad .$$

These two equations have similar form to those proposed by Hanjalic et al. [8], however, the time scale used in equation (5) is different from the previous proposal. This time scale is obtained by requiring that the source-sink imbalance of the right hand side of equation (5) should have the same order of magnitude as the right hand side "advective term." In general, the advection term, $\partial/\partial t + U_j\, \partial/\partial x_j$, would have the macro-time scale. The difference of this time scale is by no means trivial. It will be shown that the time scale used in equation (5) has the property that in the limiting case when $\varepsilon_p = \varepsilon_t = \varepsilon$, the multiple-scale model should recover the single-scale k-ε model.

The key feature of the multiple-scale model is the partition of the energy spectrum. This would allow the energy transfer of large-scale eddies to be related directly to the mean strain, while the energy transfer rate of small eddies to be related to its own action rather than to the large eddy motion in the low wave number region of the spectrum. The partition between the production and transfer region of the spectrum is characterized by the model coefficients introduced. For example, in the downstream of a grid-generated homogeneous turbulent flow, equations (1), (2), (4) and (5) [exclude the first term on the right hand side of equations (1) and (4)] can be solved analytically:

$$k_p = k_{po} \left[1 + \frac{t}{t_o} \right]^{-n} \tag{6}$$

$$\varepsilon_p = \varepsilon_{po} \left[1 + \frac{t}{t_o} \right]^{-(n+1)} \tag{7}$$

$$\frac{k_t}{k_p} = \frac{C_{t_1} + C_{p_2} - C_{t_2}}{C_{t_2} - C_{p_2}} \tag{8}$$

$$\frac{\varepsilon_t}{\varepsilon_p} = \frac{C_{t_1}}{C_{t_2} - C_{p_2}} \tag{9}$$

where

$$t_o = \frac{n\, k_{p_o}}{\varepsilon_{p_o}} \quad , \qquad n = \frac{1}{C_{p_2} - 1} \, .$$

and k_{p_o} and ε_{p_o} are the initial values.

Here, if one assumes that the energy transfer rate is constant from small wave length range through the whole cascade down to the high-wave number region, $\varepsilon_p = \varepsilon_t = \varepsilon$ implies $C_{t_1} = C_{t_2} - C_{p_2}$ and only a negligible portion of the total energy is contained in k_t. The partition thus moves towards the very high wave number region and the commonly used k-ε model is recovered.

For practical applications, this report follows the gradient formulation of Launder and Spalding [12], the model equations become

$$\frac{\partial}{\partial t} k_p + \frac{\partial}{\partial x_i} (U_i k_p) = \frac{\partial}{\partial x_i} \left(\frac{\nu_t}{\sigma_{k_p}} \frac{\partial k_p}{\partial x_i} \right) + P_k - \varepsilon_p \qquad (10)$$

$$\frac{\partial}{\partial t} k_t + \frac{\partial}{\partial x_i} (U_i k_t) = \frac{\partial}{\partial x_i} \left(\frac{\nu_t}{\sigma_{k_t}} \frac{\partial k_t}{\partial x_i} \right) + \varepsilon_p - \varepsilon_t \qquad (11)$$

$$\frac{\partial}{\partial t} \varepsilon_p + \frac{\partial}{\partial x_i} (U_i \varepsilon_p) = \frac{\partial}{\partial x_i} \left(\frac{\nu_t}{\sigma_{\varepsilon_p}} \frac{\partial \varepsilon_p}{\partial x_i} \right) + \frac{\varepsilon_p}{k_p} (C_{p_1} P_k - C_{p_2} \varepsilon_p) \qquad (12)$$

$$\frac{\partial}{\partial t} \varepsilon_t + \frac{\partial}{\partial x_i} (U_i \varepsilon_t) = \frac{\partial}{\partial x_i} \left(\frac{\nu_t}{\sigma_{\varepsilon_t}} \frac{\partial \varepsilon_t}{\partial x_i} \right) + \frac{\varepsilon_p}{k_p} (C_{t_1} \varepsilon_p - C_{t_2} \varepsilon_t) \qquad (13)$$

where U_i denotes the mean parts of the velocity and P_k is the production of kinetic energy by mean-velocity gradients. The suggested form of eddy viscosity ν_t is

$$\nu_t = C_\mu (k_p + k_t) \frac{k_p}{\varepsilon_p} \quad . \qquad (14)$$

The various model coefficients introduced above have yet to be determined. Guided by Hanjalic et al., these coefficients were assigned by calibrating against simple homogeneous flow experiment [13], by inferring from limiting single-scale model results and by requiring that the turbulence quantities k_p, k_t, ε_p, and ε_t should stay positive during evolution. They are

$C_\mu = 0.09$

$$C_{p_1} = 1.6$$

$$C_{p_2} = 1.8 - 0.3 \left[\frac{1 - k_t/k_p}{1 + k_t/k_p}\right]$$

$$C_{t_1} = 1.15$$

$$C_{t_2} = 1.8 \frac{\varepsilon_t}{\varepsilon_p} \quad .$$

The dependence of C_{p_2} and C_{t_2} on the two parameters, k_t/k_p characterizing the shape of the spectrum and $\varepsilon_t/\varepsilon_p$, characterizing the degree of spectral imbalance, allow the "inter-talk" between two different range eddies [8]. The Prantl numbers for turbulence quantities are assigned by requiring the same propagation rate of these quantities [14], and they are:

$$\sigma_{k_p} = 1. \qquad \sigma_{k_t} = 1.$$

$$\sigma_{\varepsilon_p} = 1.22 \qquad \sigma_{\varepsilon_t} = 1.22 \quad .$$

NUMERICAL EXAMPLES

The previous multiple-scale model of Hanjalic et al. was tested extensively in various boundary layer and free shear flows [8,15,16]. This version, however, does not show any improvement in the confined recirculating flow prediction [17]. The comparisons shown in this paper thus were made primarily for confined recirculating flows. The comparison for other flows such as free shear flows and swirling flows are given in References 18 and 19. Recently the sensitivity of the numerical prediction to the boundary conditions has gained increased concern [20,21]. The specification of boundary conditions will be described here. Mean quantities of inlet fluid are taken from experimental data directly, turbulence quantities k_p, k_t, ε_p, and ε_t have to be estimated in terms of the assumed shape of energy spectrum. Since the total turbulent kinetic energy is $k = k_p + k_t$ and ε_t is equal to the dissipation rate ε, the "unequilibrium spectrum" shape at the inlet plane can be used, i.e., $k_p = k_t = 0.5\ k$ and $\varepsilon_p/\varepsilon_t = 0.5$ [equations (8) and (9)]. A sensitivity study has been carried out in Reference 19 and it is shown that the assumed shape of inlet spectrum does not have significant effect on the mean field predictions. However, the "equilibrium" shape, $k_p \cong k$, $k_t \cong 0$ and $\varepsilon_p / \varepsilon_t = 1$ is not recommended. Along the solid wall, the dependent variables are matched to the usual wall functions [22] and equilvalent local-equilibrium expressions for the turbulence quantities. Specifically, the near wall energy transfer rates ε_p and ε_t are set equal to one another.

In Table 1, the predicted reattachment of the flow over a backward-facing step, 2-D sudden expansion and axisymmetric expansion are compared with those of the single-scale k-ε model. The same backstep flow is also studied extensively in the 1980-81 AFOSR-HTTM Stanford Conference [25]. It was found that the reattachment length is underpredicted by various groups using the same k-ε model, ∿ 5.5 step heights (±1) predicted versus experimental value of 7±1. The calculated reattachment lengths using the present model show good agreement with the data. The global features of the flow such as pressure coefficients C_p's $[C_p = (P-P_{ref})/\frac{1}{2} \rho U_{ref}^2]$ on both step-side wall and no-step side wall of the backstep flow and pipe expansion flow are shown in Figure 1. The multiple-scale model shows significant improvement in the prediction of backstep flow. In the pipe expansion, the multiple-scale model improves the prediction in the recirculating region, however, underpredicts the experimental results in the range x = 9 - 15 H, indicating a slower recovery in this region.

Figure 2 shows the predicted locus of flow reversal downstream of the expansion of step flow and axisymmetric pipe flow. The k-ε does not account for the time lapse between extra energy being supplied and the effect being felt in the dissipation motions, thus the predicted rate of jet spread is too high, i.e., the thickness of the recirculation zone is too small. The sluggish response of the multiple-scale model after the initial region slows down the jet spread and improves the prediction of recirculation zone thickness. It is interesting to note that the predicted recirculation zone width is thinner even at the reattachment point further downstream than is experimentally observed for the axisymmetric expansion case. The reduction of the recirculation zone thickness of the step flow is due to the stabilizing effect of the opposite wall [26].

TABLE 1. REATTACHMENT LENGTH PREDICTIONS

Flow	Single-Scale (k-ε)	Multiple-Scale	Experiment
Back Step (2:3)	5.2 H	6.7 H	∿ 7 H [23]
2-D Expansion	6.3 H	7.3 H	∿ 7^+ H
Axisymmetric Expansion (1:2)	8.0 H	9.6 H	∿ 9 H [24]

In Figure 3, the mean velocity profiles are plotted at x/H = 5.2, 10.7, and 16. The improvement of the multiple-scale model predictions is obvious in the recirculation region (x = 5.2 H). The apparent good agreement observed for k-ε model at x = 10.7 H and 16 H is superficial since a too short reattachment length is calculated by the k-ε model. If the reattachment point had been used as the adjusted reference point, both models would show the comparable results and the agreement with experiment data is fair. The recovery of the mean velocity profiles is thus underpredicted by both models.

The turbulent kinetic energy k (= $k_p + k_t$) and the turbulent shear stress profiles at three downstream locations are shown in Figures 4 and 5. The location of the peaks and magnitudes of these two quantities are much improved by the multiple-scale turbulence model. The underprediction of the magnitudes of $\overline{u'v'}$ implies that the k-ε model connects the energy dissipation rate ε too strongly to the mean flow field. The multiple-scale model has the tendency to reduce this strong connection, thus giving more realistic length scales in the redeveloping region.

The lingering response of the turbulence field represented by the multiple-scale turbulence model can be investigated more vividly by observing the evolution of the kinetic energy level along the plane of re-entrant corner. Experimental observation indicates that k rises along the shear layer from the step edge [27]. Because of the stabilizing effect of the wall opposite to the step in the back-step flow, the potential core region issuing from upstream persists and the maximum k shifts quickly towards the wall after the recirculating region. In the pipe sudden-expansion flow, the potential core is considerably shorter than the reattachment length and maximum k migrates towards the central region of the pipe. Due to the dissimilar physics of the two flows, the kinetic energy profiles evolve quite differently downstream of the step corner. In the backstep flow, the k-ε model shows monotonous decay of k downstream of the initial peak at 1 step height. The multiple-scale model shifts the initial peak about 1 H downstream and reaches a plateau at about the value of x which roughly corresponds to reattachment. The experimental data of Kim et al. [23] seems to reflect this feature. In the pipe expansion flow, the k-ε model gives an overshoot before reaching the second peak of kinetic energy at about the reattachment length x_r. It is shown analytically by smith [28] that, in the initial part of the shear layer behind the re-entrant corner, large localized peaks of turbulence quantities are a necessary feature of the k-ε model. The slower response of the multiple-scale model reduces this overshoot and moves the peak value of k more downstream and reproduces the experimental trend faithfully.

CONCLUSION

A multiple-scale (two-scale) turbulence model developed based on the Hanjalic et al. multiple-time-scale concepts is described in this study. The preliminary success of predictions of several confined recirculating flows using this model stems from recognizing that the turbulence in complex recirculating flows is not in spectral equilibrium, and that the different energy transfer rates for different eddies should be modeled separately. It has been demonstrated in this report that the present model has excellent potential for complex turbulent flow predictions. Work in progress indicates that the present model shows excellent predictive capability for confined swirling flow calculations.

Further research concerning the establishment of the empirical coefficients which control the division of the turbulent energy spectrum is desirable. For example, by making the model coefficients C_{p_2} and C_{t_2} depend on k_t/k_p and $\varepsilon_t/\varepsilon_p$ significantly enhances the flow recovery after the reattachment. When compared to the rather detailed study of the turbulence model, the treatment of boundary conditions, especially turbulence quantities at the solid wall, seems rather crude presently. Further development in this area is obviously required.

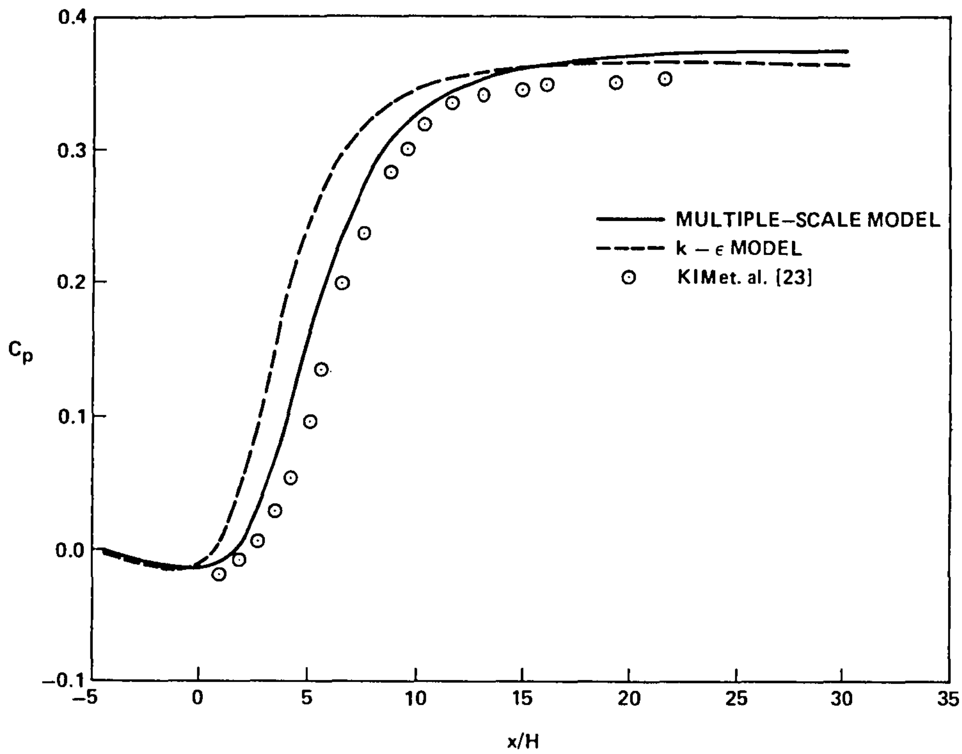

Figure 1a. Pressure distribution on the no-step side wall of the backstep flow.

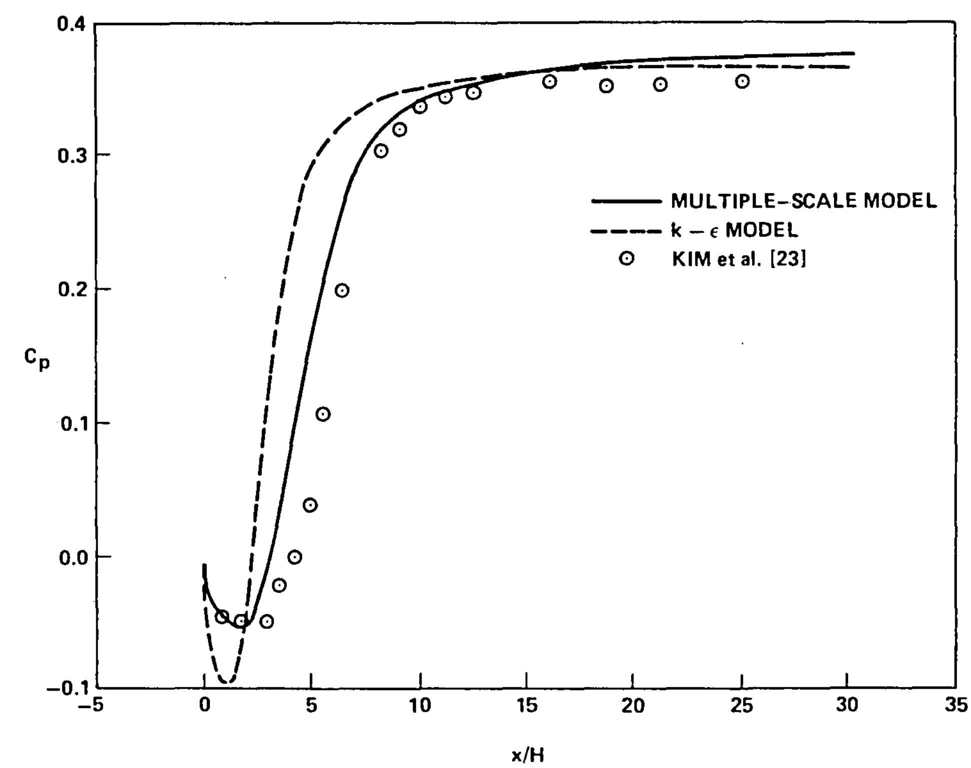

Figure 1b. Pressure distribution on the step side wall of the backstep flow.

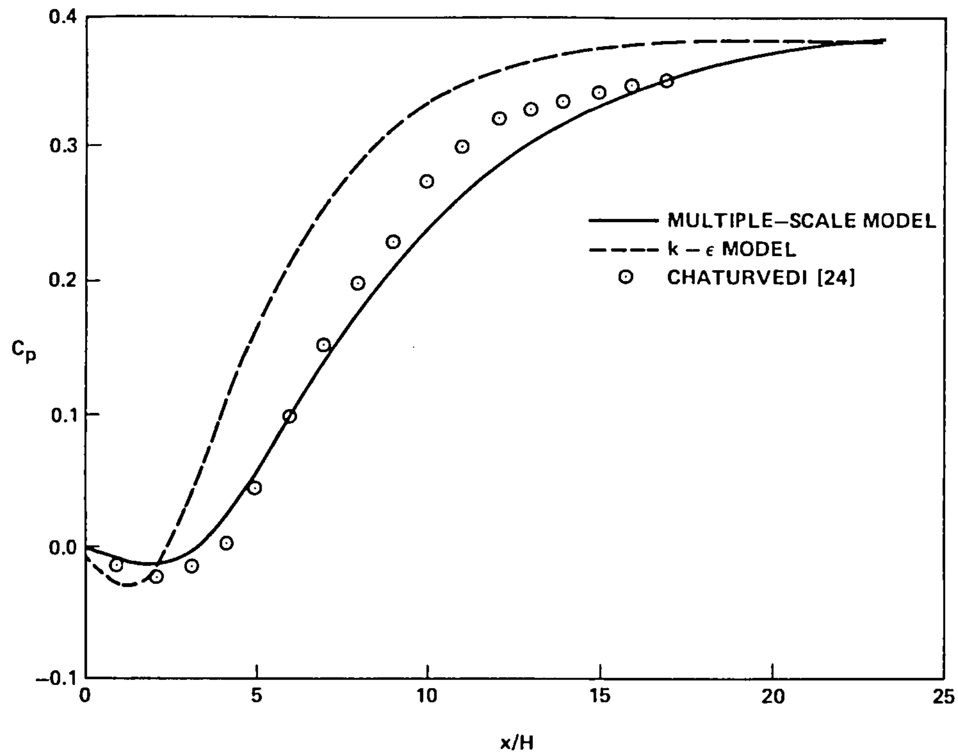

Figure 1c. Pressure distribution on the step side wall of the pipe-expansion flow.

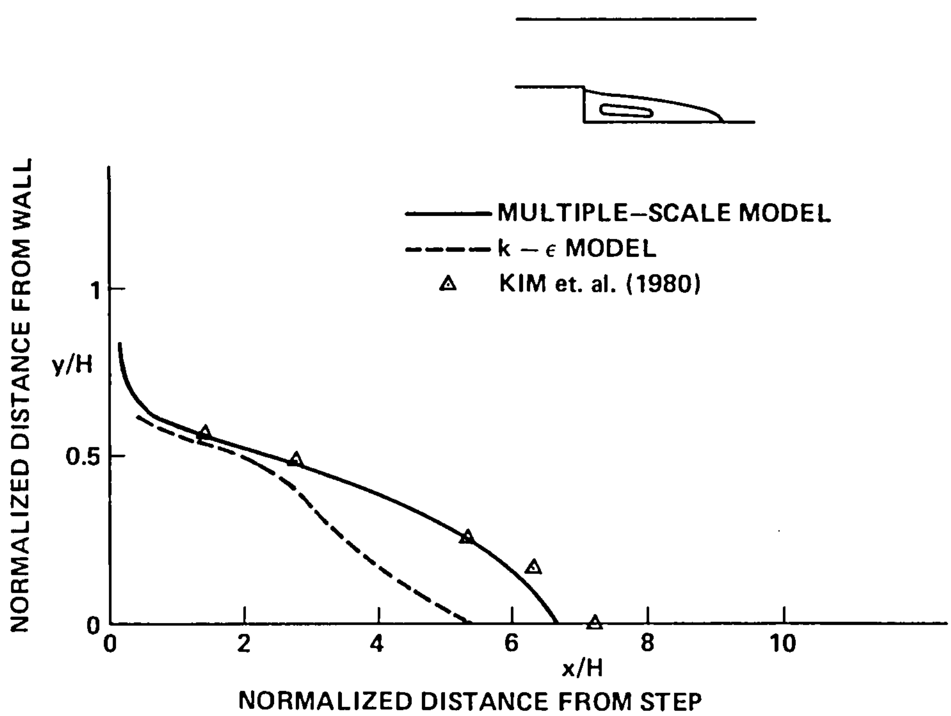

Figure 2a. Locus of flow reversal, backward-facing step.

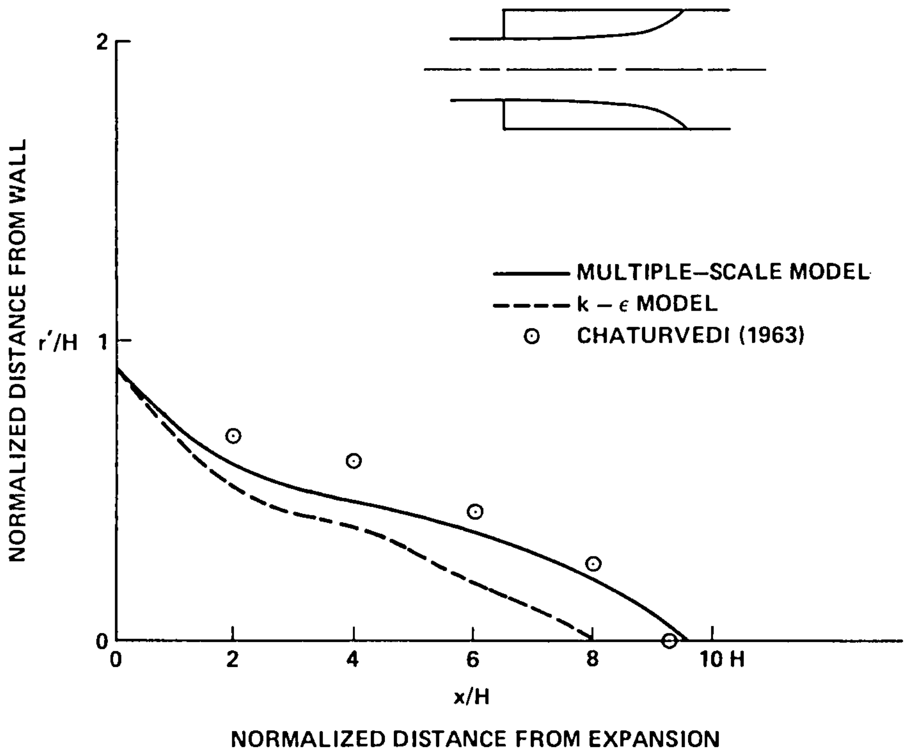

Figure 2b. Locus of flow reversal, pipe expansion.

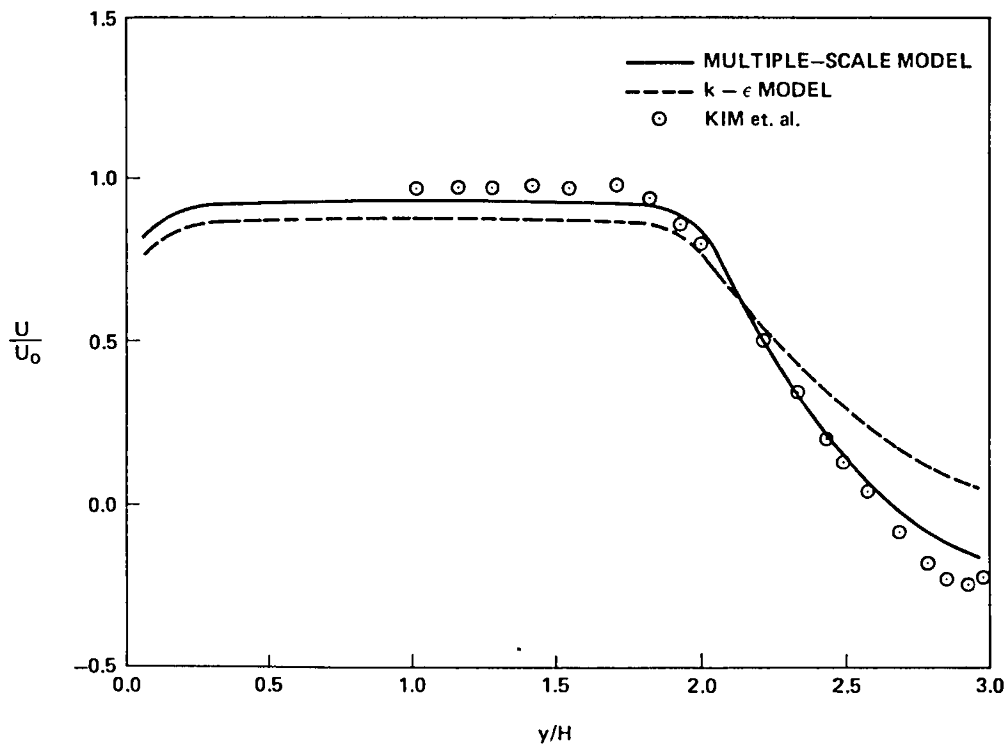

Figure 3a. Mean axial velocity profile at x/H = 5.2, backward-facing step.

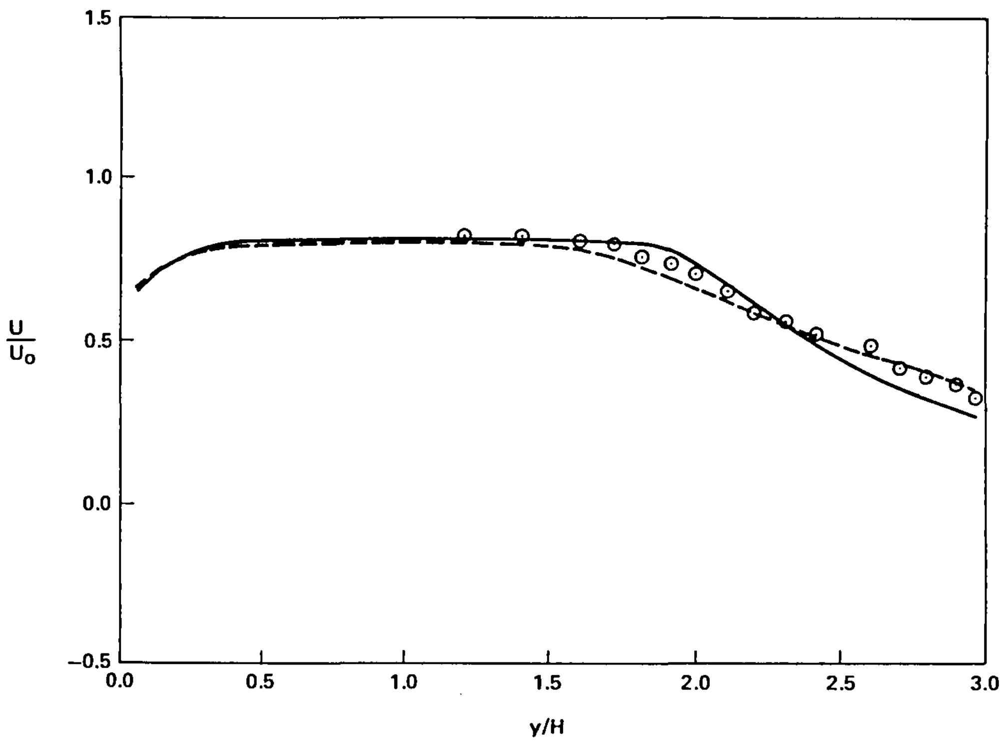

Figure 3b. Mean axial velocity profile at x/H = 10.7, backward-facing step.

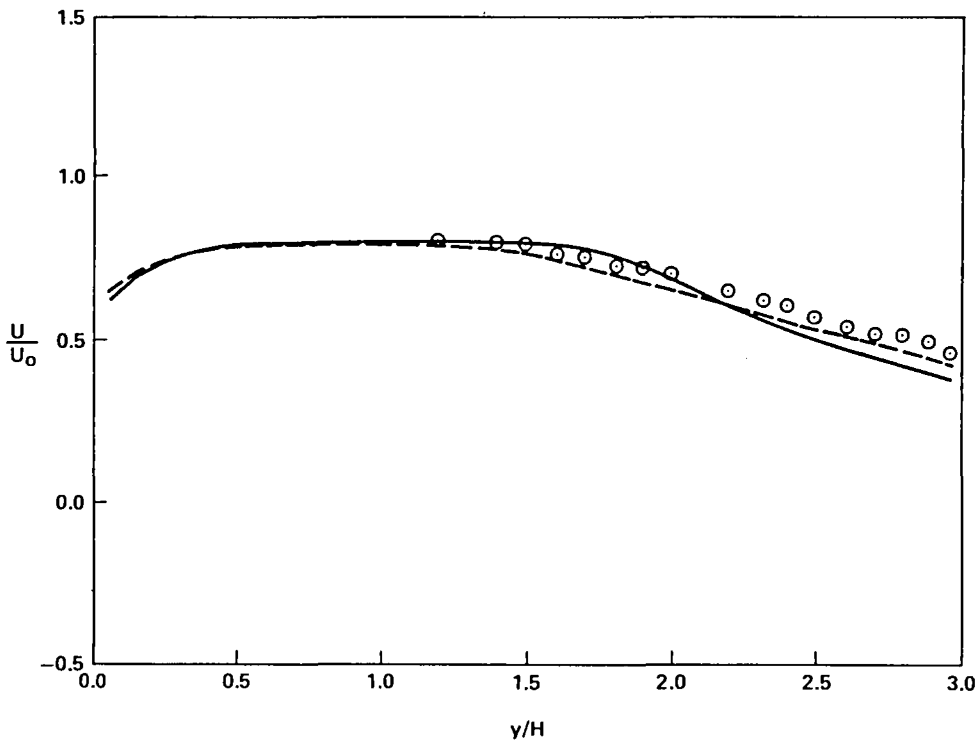

Figure 3c. Mean axial velocity profile at x/H = 16, backward-facing step.

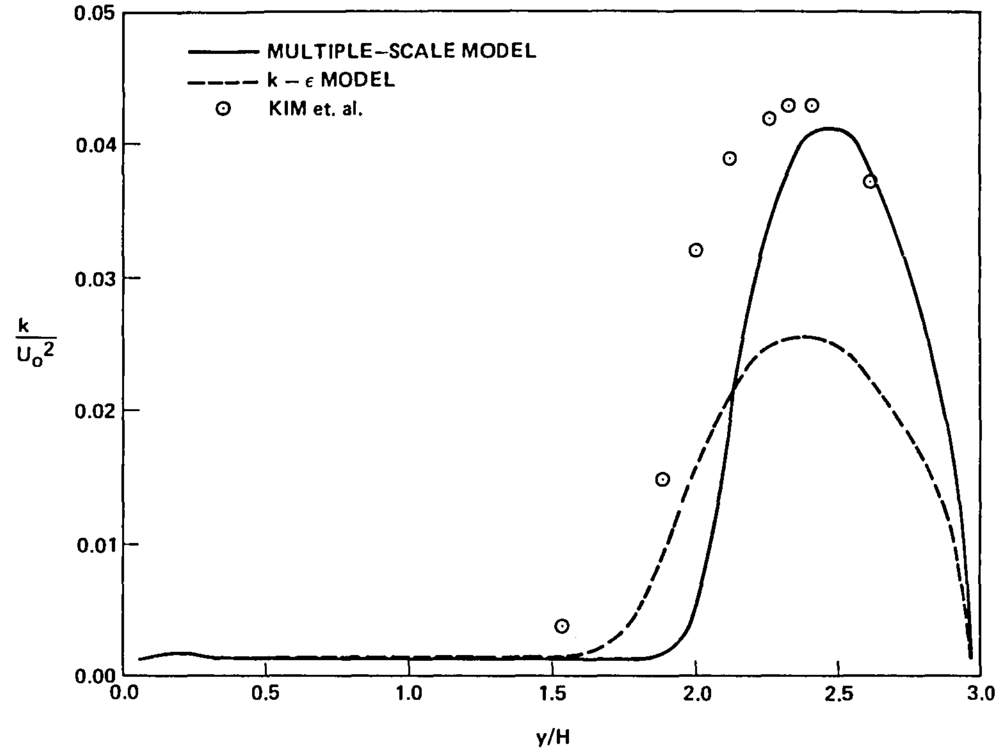

Figure 4a. Turbulent kinetic energy profiles at x/H = 7.7, backward-facing step.

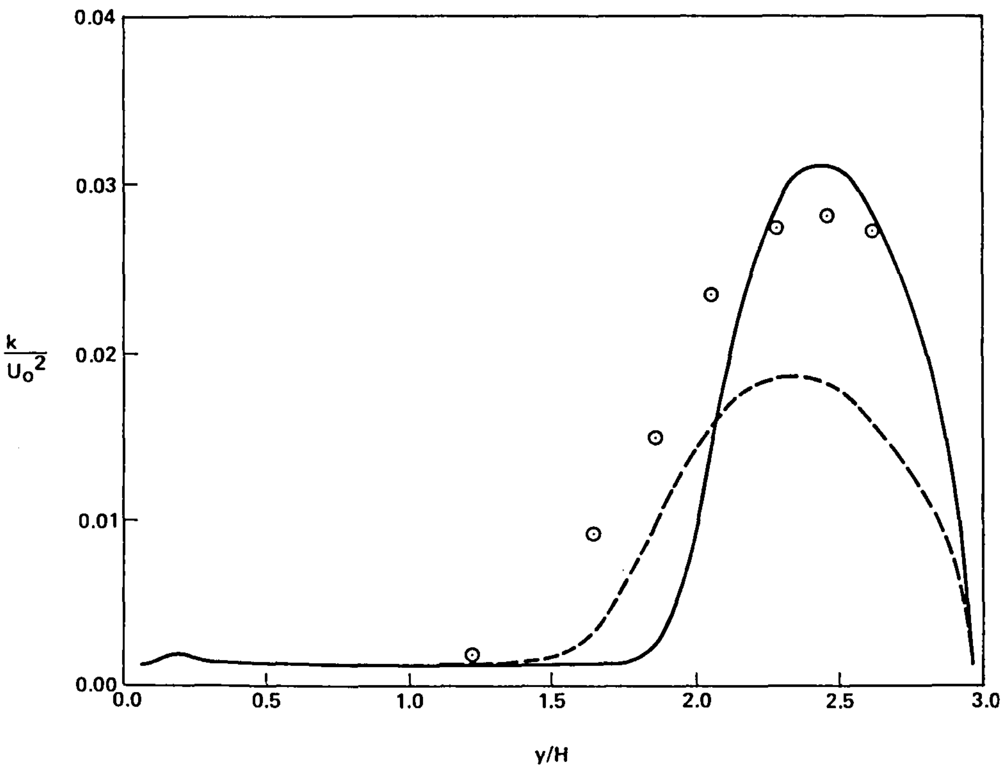

Figure 4b. Turbulent kinetic energy profiles at x/H = 10.3, backward-facing step.

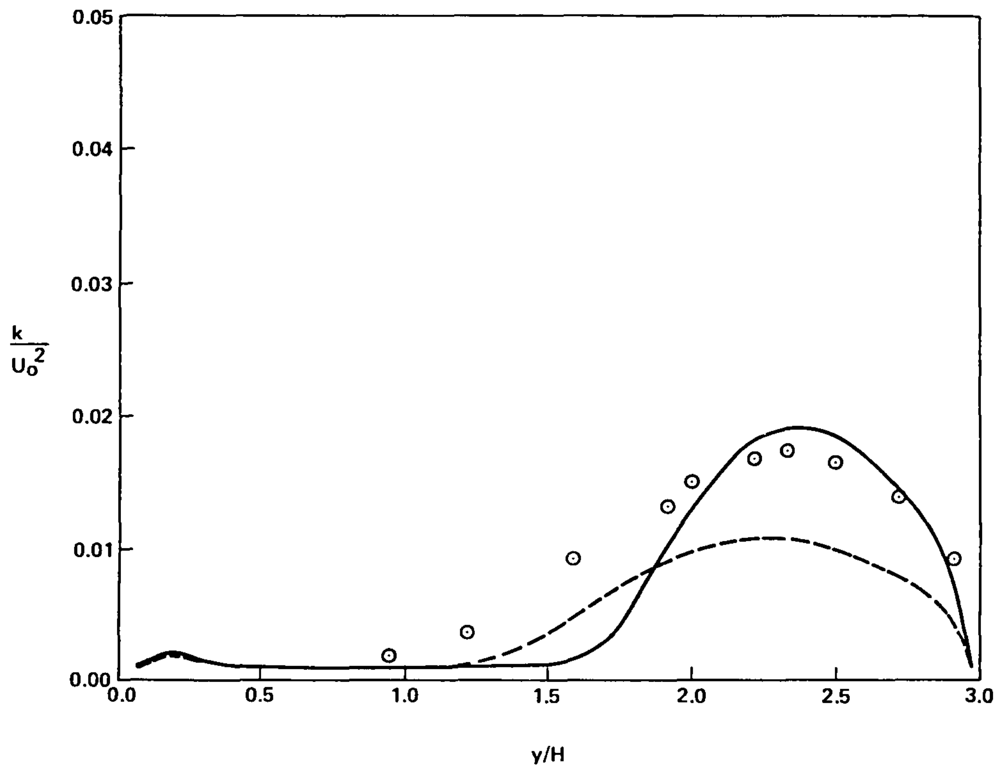

Figure 4c. Turbulent kinetic energy profiles at x/H = 15.7, backward-facing step.

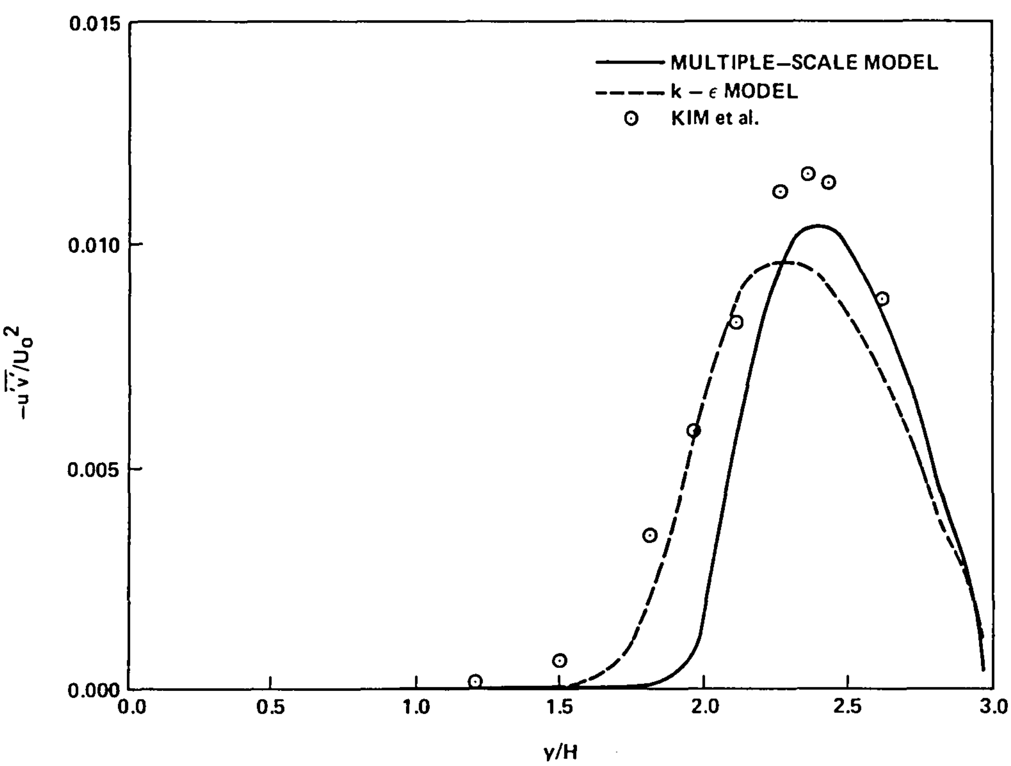

Figure 5a. Shear stress profiles at x/H = 7.7, backward-facing step.

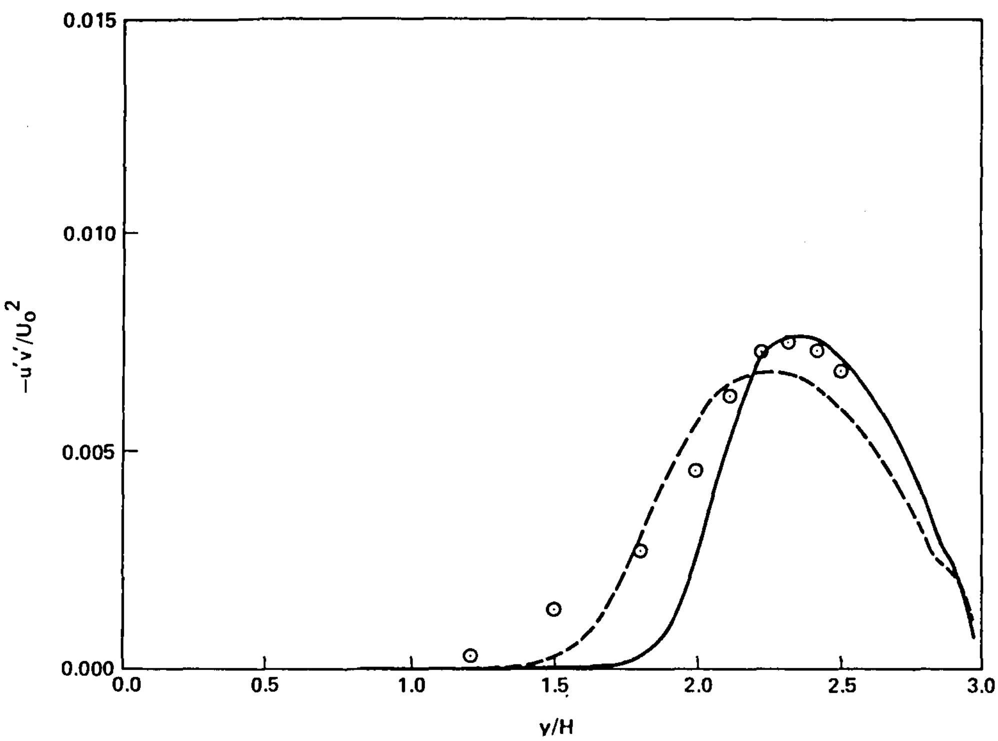

Figure 5b. Shear stress profiles at x/H = 10.3, backward-facing step.

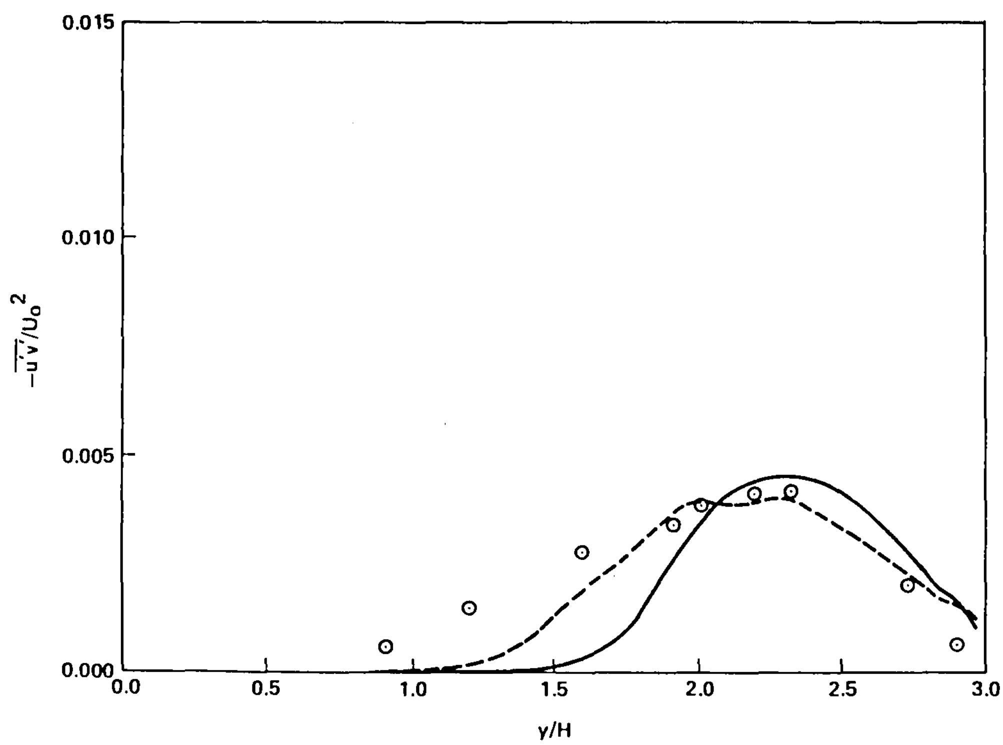

Figure 5c. Shear stress profiles at x/H = 15.7, backward-facing step.

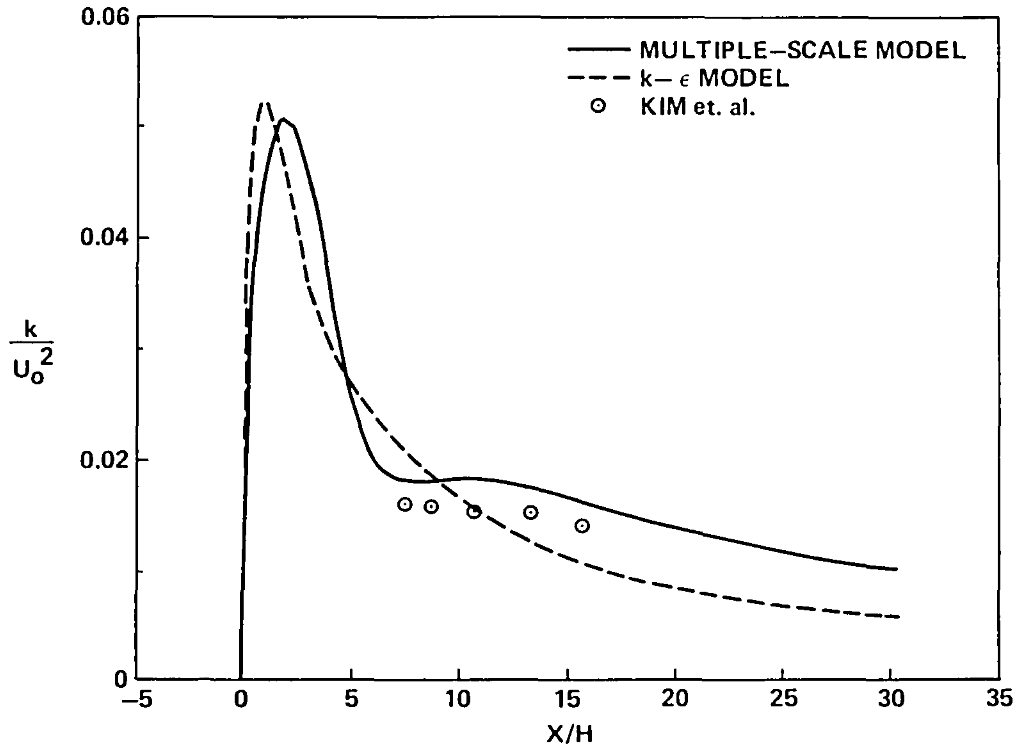

Figure 6a. Development of turbulent kinetic energy in the backward-facing step flow at y/H = 1, y is the distance from wall.

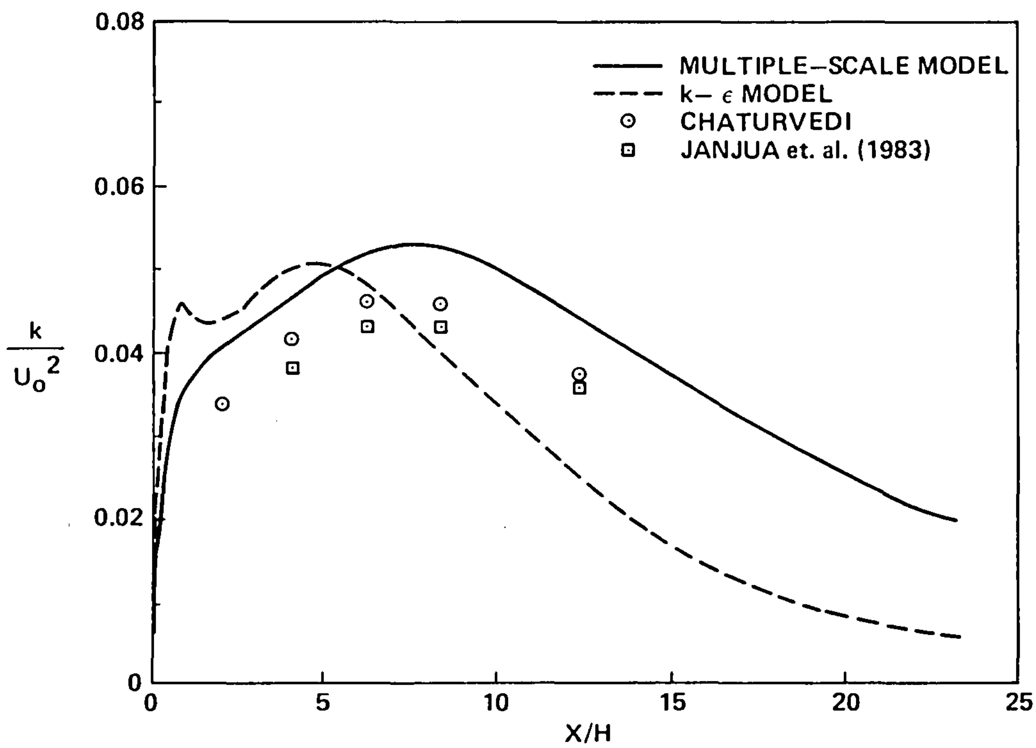

Figure 6b. Development of turbulent kinetic energy in the sudden pipe-expansion at r/R = 1/2, R is the radius of confined chamber.

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APPROVAL

MULTIPLE-SCALE TURBULENCE CLOSURE MODELING OF CONFINED RECIRCULATING FLOWS

By C. P. Chen

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

G. F. McDONOUGH
Director, Systems Dynamics Laboratory

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