SOME ANTICIPATED CONTRIBUTIONS TO CORE FLUID DYNAMICS FROM THE GRM

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It is broadly maintained that the secular variation (SV) of the large scale geomagnetic field contains information on the fluid dynamics of earth's electrically conducting outer core. The electromagnetic theory appropriate to a simple earth model has recently been combined with reduced geomagnetic data in order to extract some of this information and ascertain its significance [Voorhies, 1984]. The simple earth model consists of a rigid, electrically insulating mantle surrounding a spherical, inviscid, and perfectly conducting liquid outer core. This model has been tested against seismology by using truncated spherical harmonic models of the observed geomagnetic field to locate earth's core-mantle boundary, CMB. Further electromagnetic theory has been developed and applied to the problem of estimating the horizontal fluid motion just beneath CMB. Of particular geophysical interest are the hypotheses that these motions (1) include appreciable surface divergence indicative of vertical motion at depth, and (2) are steady for time intervals of a decade or more. In addition to the extended testing of the basic earth model, the proposed GRM provides a unique opportunity to test these dynamical hypotheses.

Hide's [1978] method for magnetically determining the radius of earth's electrically conducting core rests on the following fact: if the core is, in effect, a perfect liquid conductor, then the number of field line footpoints at CMB must be conserved. The radius b* of a spherical surface at which the absolute flux linkage,

$$\underline{P}(r,t) = \int_0^{2\pi}\int_0^{\pi} \| B_r(r,\theta,\phi;t) \| r^2 \sin\theta \, d\theta \, d\phi, \qquad (1)$$

is the constant, $\underline{P}(b^*)$, may thus be identified as the radius of the core. Here (r,θ,ϕ) are the geocentric spherical polar coordinates and B_r is the radial component of the magnetic flux density.

The assumption of an electrically insulating mantle of vacuum magnetic permeability allows the calculation of $B_r = -\partial V/\partial r$, hence $P(r,t)$, within the mantle by downwardly continuing a model of the scalar magnetic geopotential, $V(r,\theta,\phi;t)$. Conventional spherical harmonic representations of V should be truncated to degree and order $N_B \leq 12$ before downward continuation in order to reduce the effect of crustal magnetic anomalies on estimates of the core field [Langel and Estes, 1982]. The small scale structure of the core field thus lies concealed beneath the crustal field. Yet it is the behavior of the fairly well-known large scales of the core field, characterized by the first 100 or so time-varying Gauss coefficients, which we seek to explain and perhaps predict.

Terrestrial applications of Hide's method have been reported by Hide and Malin [1981], Voorhies and Benton [1982], and Voorhies [1984]. In the latter, the average of 44 magnetic determinations of the core radius is 3506 ± 301 km. Selection of an appropriate subset of field models yields the refined, inverse variance weighted mean value, $b^* = 3485 \pm 35$ km. Both values differ insignificantly from the seismically determined radius $b = 3485$ km. These results, along with those from a study of the global absolute and regional 'patchwise' flux linking CMB, demonstrate the validity of the source-free mantle - frozen-flux core approximation.

This approximation must nevertheless fail over sufficiently long time intervals or on sufficiently small spatial scales. Some evidence of flux diffusion has been found, but it is not yet compelling. This is primarily attributed to a currently inadequate temporal distribution of global vector magnetic data. The data acquired by the lead GRM spacecraft, when suitably reduced, would greatly help delineate the domain of validity of the basic earth model.

It has recently been shown that a steady fluid velocity at CMB, $\underline{v}(b,\theta,\phi) = (u = 0,v,w)$ is uniquely and globally determined by the time-varying radial component of the magnetic flux density, $B_r(b,\theta,\phi;t)$ [Voorhies, 1984; Voorhies and Backus, ms. in preparation). The testable assumption of steady motion thus resolves a fundamental ambiguity in finding $\underline{v}$ from B_r. To see this, note that the radial component of the pre-Maxwell magnetic induction equation evaluated at the top of an ideal liquid core,

$$\frac{\partial B_r}{\partial t} + \frac{v}{b}\frac{\partial B_r}{\partial \theta} + \frac{w}{b\sin\theta}\frac{\partial B_r}{\partial \phi} = B_r \frac{\partial u}{\partial r}, \qquad (2)$$

can be evaluated at three distinct epochs. This provides three distinct equations in the three steady unknowns $(\partial u/\partial r, v, w)$, hence unique solutions for $\underline{v}$ provided certain weak constraints are satisfied. If B_r is known, then (2) can be evaluated at an arbitrarily large number of epochs; the assumption of steady flow thus overdetermines the problem of finding $\underline{v}$ from B_r.

We therefore estimate that steady motion at the top of an ideal liquid core which best fits, in both the spatial and temporal least squares sense, a model of $B_r(b,\theta,\phi;t)$. To do so, set $\underline{v} = \nabla_s T \times r + \nabla_s U$, where T is the streamfunction and U the effective velocity potential, express T and U in terms of a truncated spherical harmonic expansion, and minimize

$$\int_{t_i}^{t_f} \int_0^{2\pi} \int_0^{\pi} \left[\frac{\partial B_r}{\partial t} + \frac{v}{b}\frac{\partial B_r}{\partial \theta} + \frac{w}{b\sin\theta}\frac{\partial B_r}{\partial \theta} - B_r \frac{\partial u}{\partial r}\right]^2 \sin\theta d\theta d\phi dt$$

with respect to the coefficients of T and U. Solutions [Voorhies, 1984] have typically been derived from the $N_B = 8$ GSFC 9/80 of Langel, et al., [1982] with the interval during which the motions are assumed to be steady, $t_f - t_i$, taken to be one or two decades. Such solutions are non-singular, relatively stable against changes in the number of T and U coefficients, and qualitatitively steady in that changes in t_i and t_f do not dramatically alter the derived motional pattern.

Steady, purely toroidal flows (U=0), suggested by Gubbins [1982], do not fit the input SV models at CMB as well as do steady, combined toroidal-poloidal flows - such as that depicted in Figure 1. This is particularly true when the steady motion hypothesis is relaxed by letting $t_f - t_i$ become small. The normalized root mean square SV residual at CMB, $\delta(\underline{v})$, which measures the difference between the input and 'predicted' SV, is typically 50% for purely toroidal flows and 30% for combined flows. Such results constitute relatively direct evidence of weak, large scale vertical motion near CMB.

The derived flows possess a bulk westward drift of about 0.107^{o}/yr, but are complicated by superimposed jets, vorticies, and upwelling. Typical (rms) speeds and surface divergences are about 5.4×10^{-4} m/s and 2.3×10^{-10} s^{-1}, respectively. It has been found that the residual SV variance, $\delta(\underline{v})^2$, is nearly inversely proportional to both the number of T and U coefficients and to the interval $t_f - t_i$. Moreover, solutions for different intervals, although qualitatively similar, quantitatively differ enough to indicate unsteady flow. The motions are therefore not thought to be steady; <u>however</u>, the appreciable reductions in $\delta(\underline{v})$ obtained for combined flow by decreasing $t_f - t_i$ are not yet warranted by our knowledge of the large scale SV of the core field.

For the first 80 coefficients of the GSFC 9/80 in particular, the recommended 5 conditional standard deviation uncertainty estimates imply an nrms SV residual of less than 17% is unwarranted. Since these are considered minimum estimates of the total uncertainty in the large scale SV at CMB, the values of $\delta(\underline{v})$ obtained with $t_f - t_i = 10$ or 20 years (20% or 30%) are as small as are currently warranted. Thus, over decade intervals and Mm length scales, the reduced geomagnetic data is adequately described by the quasi-steady, large scale, combined toroidal-poloidal circulation of a frozen-flux core beneath an insulating mantle.

Given the basic earth model, the dawn-dusk uncertainties of the GSFC 12/83 [Langel and Estes, 1984] main field coefficients and the GSFC 9/80 SV coefficients have been used to estimate that nrms SV residuals of less than 13% at CMB would be warranted by a GRM-less-MAGSAT data based SV model. The data acquired by GRM should thus be capable of resolving whether or not the fluid motions at the top of the core are indeed time dependent.

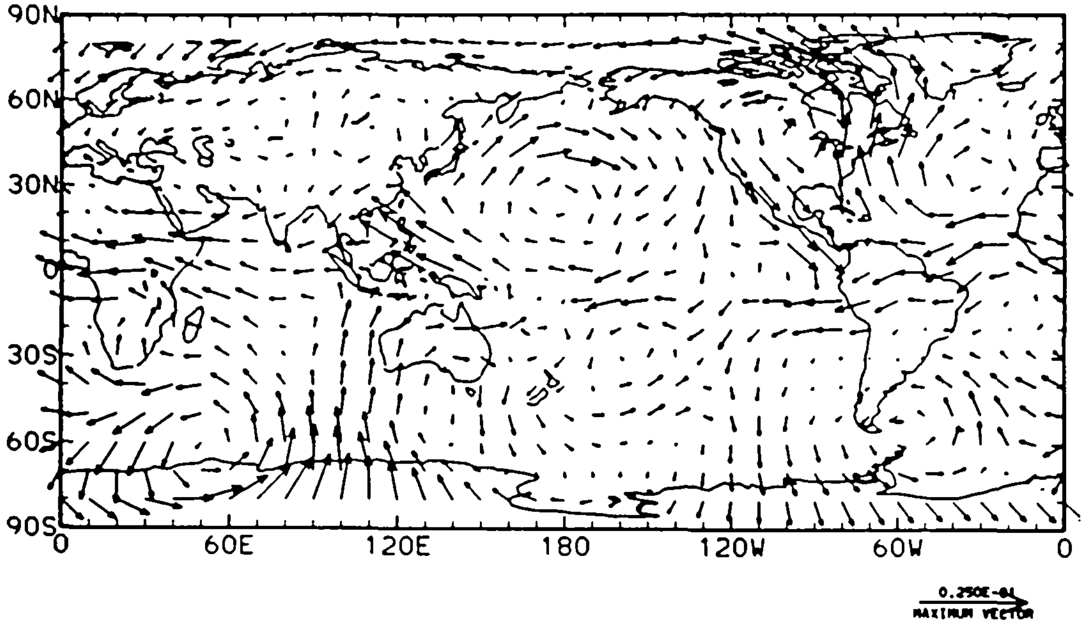

Figure 1. A steady combined flow at the top of Earth's core derived from the GSFC 9/80. Reference vector is 87.125 km/yr. $\underline{v}^{rms}$ = 16.44 km/yr = 5.21x10^{-4} m/s.

References

Gubbins, D., 1982. Phil. Tras. Roy. Soc. Lond., A306:249-256.

Hide, R., 1978. Nature 271:640.

Langel, R.A., Estes, R.H., 1982. Geophys. Res. Lett. 9:350-253.

Langel, R.A., Estes, R.H., Mead, G.D., 1982. J. Geomag. Geoelectr. 34:327.

Langel, R.A., Estes, R.H., 1984. NASA Tech. Mem. #86133.

Voorhies, C.V., 1984. Ph.D. thesis, Univ. of Colorado at Boulder.

Voorhies, C.V., Benton, E.R., 1982. Geophys. Res. Lett. 9:258.