NSG 1185

KINEMATICS OF FOLDABLE DISCRETE SPACE CRANES

by

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ABSTRACT

Exact kinematic description of a NASA proposed prototype foldable-deployable discrete space crane are presented. A computer program is developed which maps the geometry of the crane once controlling parameters are specified. The program uses a building block type approach. In which it calculates the local coordinates of each repeating cell and then combines them with respect to a global coordinates system.

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Introduction

The coming years will see an increasing utilization of space through the use of very large, lightweight structures constructed in orbit for such purposes as antennas, energy collection platforms and work stations. With the recent successful flights of the Space Shuttle, such projects are one step closer towards becoming reality. It will be most desirable that these structures be made of repeating sub-assemblies in order to aid in the ease of construction, minimize fabrication costs, and reduce complications in analysis.

Under our NASA Grant NSG 1185, we have been investigating various methods of designing and analyzing the geometry of discrete large space structures. Our previous extensive work has dealt with both the areas of design and analysis. Most recently, we completed and published results on establishing the dynamic behavior of large space structures in the forms of periodic trusses and frames.

For the purposes of in-space construction and subsequent reparis, large space tools are required. It is most desirable for these tools to be kept packed and to be deployed only whenever they are needed. An important example of such tools are the deployable crane (large arm) and telescopic supports as illustrated in Figure 1. This tool has the mobility of full or partial deployment and to move in any direction in order to reach desired points in the host structure.

The mechanism and control of the deployment of this tool is very essential. This specific NASA crane-prototype can be deployed and maneuvered in various directions by changing the lengths of members in its bays. To know how much change in these lengths is required to achieve the

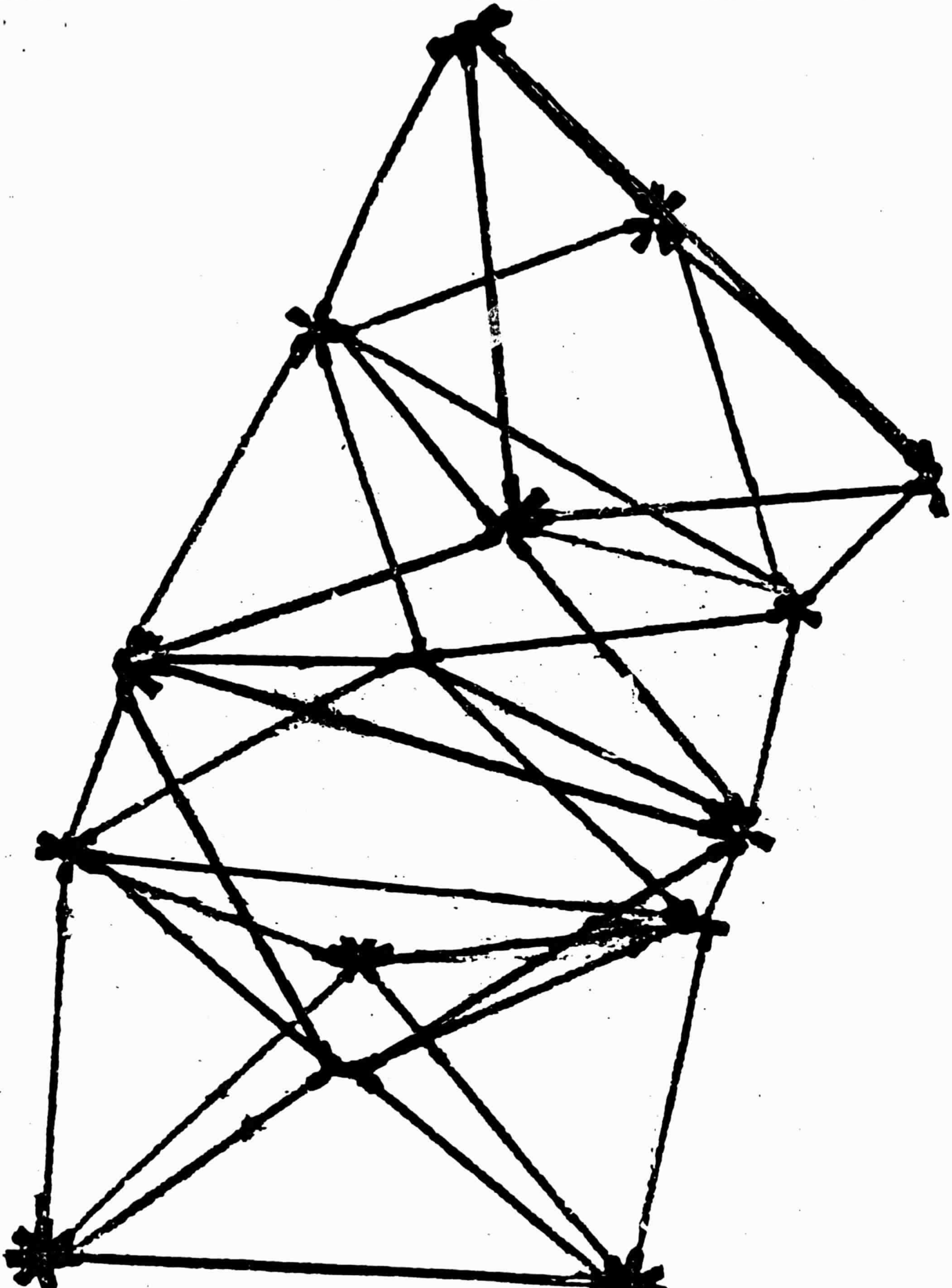

Figure 1 Crane Geometry

required configuration is essential. This is also important to know for the control of the vibrations of the crane.

In this report we analyze the kinematics of the crane configuration which result from the changes of the lengths of the bay members. Specifically, we developed a computer program capable of calculating the coordinates of each node of the crane resulting, from member length changes. This information is very essential in the utility of the crane. The new configuration kinematics can also be fed to a vibration program to calculate the dynamic characteristics of the new configuration, if found necessary.

Structures Description

The specific crane structure under consideration consists of repeating unit cells as shown in Figure 2. Each unit cell constitutes a truss substructure with nine nodal points as illustrated in Figure 3. In this unit cell the lengths of the rods connecting the nods 4, 5 and 6 are variables and hence control the overall shape of the cell and ultimately the total structure. On the other hand all remaining rod members have constant lengths. Specifically, members (1,2), (1,3), (2,3), (7,8), (7,9) and (8,9) have constant equal lengths denoted by G, and aside from the variable member lengths (4,5), (4,6) and (5,6), all remaining members have the constant equal lengths B.

Summary of the Analysis Procedure

In order to give complete description of the kinematics of the crane with arbitrary number of unit cells, we use a building block type approach consisting of the following steps:

(1) By choosing a local coordinate system (on the single unit cell level) we calculate the coordinates of its' nine nodes.

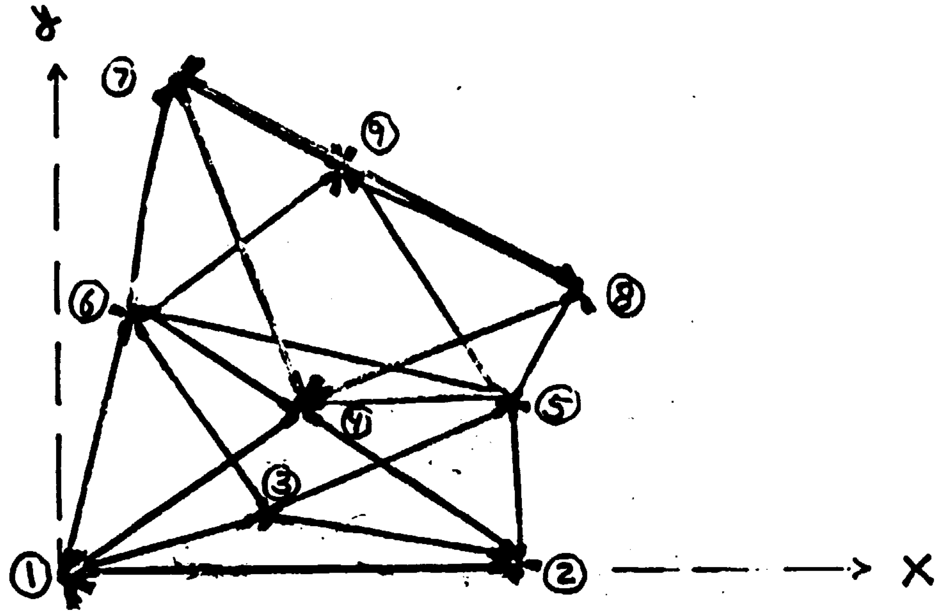

Figure 2 Repeating Unit Cell

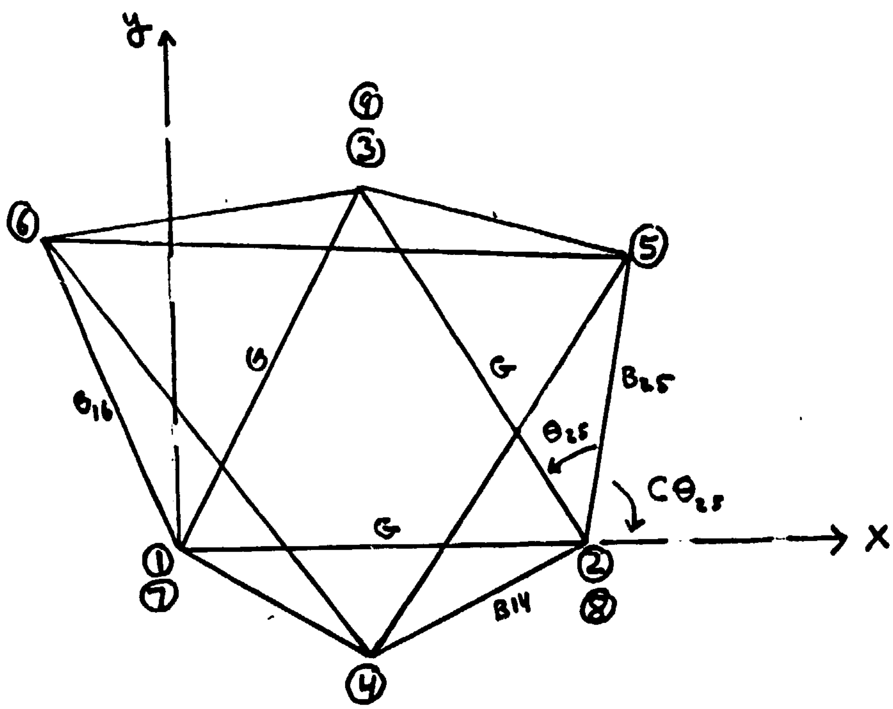

Figure 3 Projected Geometry

(2) These coordinates are then referred to a pre-assigned global coordinate system (for the total structure).

(3) If the nodal global coordinates of the (n-1)th cell are known, then the plane of its' top three nodes (7, 8 and 9) will define the base for the nth cell. This suggests the use of coordinate transformations to define such a base.

(4) Orthogonal coordinate transformations are constructed for the top plane of each cell as a base for the next cell.

<u>Note</u>: A major fact in constructing the local coordinates of each unit cell is that the plane of its 7, 8 and 9 nodes is a mirror image of the plane of the nodes 1, 2 and 3; the mirror being the plane of the nodes 4, 5 and 6.

The floor of this unit cell is the equilateral triangle with side lengths G and having the nodal coordinates:

Node 1: (0,0,0) (1)

Node 2: (G,0,0) (2)

Node 3: $(G/2, \frac{\sqrt{3}}{2} G, 0)$ (3)

Now, let us arbitrarily assume that points 4, 5 and 6 have the vertical heights (coordinates) $Z_4 = H_4$; $Z_5 = H_5$ and $Z_6 = H_6$. If we now designate the projected lengths of the members 2-5, 1-6 and 1-4 by B_{25}, B_{16} and B_{14}, respectively, then one concludes that

$$B_{25} = (B^2 - H_5^2)^{1/2} \tag{4}$$

$$B_{14} = (B^2 - H_4^2)^{1/2} \tag{5}$$

$$B_{16} = (B^2 - H_6^2)^{1/2} \qquad (6)$$

where B is a constant.

Furthermore, by inspection one can immediately deduce the coordinates of the point 4 as

$$\text{Node 4:} \quad (\frac{G}{2}, -[B_{14}^2 - (.5G)^2]^{1/2}, H_4) \qquad (7)$$

Let us now concentrate on finding the coordinates of point 5. From Figure 3 we see these to be

$$\text{Node 5:} \quad (G + B_{25}\cos C\theta_{25}, B_{25}\sin C\theta_{25}, H_5) \qquad (8)$$

using the trigonometric manipulations

$$\sin C\theta_{25} = \sin(\frac{2\pi}{3} - \theta_{25})$$

$$= \sin\frac{2\pi}{3}\cos\theta_{25} - \cos\frac{2\pi}{3}\sin\theta_{25}$$

$$= \frac{\sqrt{3}}{2}\cos\theta_{25} + 1/2\sin\theta_{25}$$

yields

$$\sin C\theta_{25} = \frac{\sqrt{3}}{2}\ \frac{G}{2B_{25}} + \frac{1}{2}\ \frac{[B_{25}^2 - (G/2)^2]^{\frac{1}{2}}}{B_{25}} \qquad (9)$$

Hence,

$$Y_5 = B_{25}\sin C\theta_{25}$$

$$= \frac{\sqrt{3}}{4} G + \frac{1}{2}(B_{25}^2 - (G/2)^2)^{\frac{1}{2}} \qquad (10)$$

and by similar arguments we see that

$$\cos C\theta_{25} = \cos(\frac{2\pi}{3} - \theta_{25})$$

$$= \cos\frac{2\pi}{3}\cos\theta_{25} + \sin\frac{2\pi}{3}\sin\theta_{25}$$

$$= -\frac{1}{2}\cos\theta_{25} + \frac{\sqrt{3}}{2}\sin\theta_{25}$$

$$= -\frac{1}{2}\frac{G}{2B_{25}} + \frac{\sqrt{3}}{2}\frac{[B_{25}^2 - (G/2)^2]^{\frac{1}{2}}}{B_{25}} \tag{11}$$

Hence,

$$X_5 = G + B_{25}\cos C\theta_{25}$$

$$= \frac{3G}{4} + \frac{\sqrt{3}}{2}[B_{25}^2 - (G/2)^2]^{\frac{1}{2}} \tag{12}$$

By symmetry and inspection of the coordinates of point 5 we can deduce the coordinates of point 6 as

Node 6:

$$X_6 = \frac{G}{4} - \frac{\sqrt{3}}{2}[B_{16}^2 - (G/2)^2]^{\frac{1}{2}} \tag{13}$$

$$Y_6 = \frac{\sqrt{3}}{4}G + \frac{1}{2}[B_{16}^2 - (G/2)^2]^{\frac{1}{2}} \tag{14}$$

and

$$Z_6 = H_6$$

<u>Remarks</u>

Notice that from (10) and (12) that

$$X_5 = \sqrt{3}\quad Y_5 \tag{15}$$

and from (13) and (14)

$$X_6 - 1 = -\sqrt{3}\quad Y_6 \tag{16}$$

Calculations of the Lengths D_{45}, D_{46} and D_{56}:

In what follows we shall normalize our length G to be unity. Accordingly, by inspection we have the further relations

$$X_4^2 + Y_4^2 + Z_4^2 = B^2 \tag{17a}$$

$$X_6^2 + Y_6^2 + Z_6^2 = B^2 \tag{17b}$$

$$(X_5 - 1)^2 + Y_5^2 + Z_5^2 = B^2 \tag{17c}$$

We also have

$$D_{45}^2 = (X_5 - X_4)^2 + (Y_5 - Y_4)^2 + (Z_5 - Z_4)^2 \tag{18a}$$

$$= X_5^2 + Y_5^2 + Z_5^2 + X_4^2 + Y_4^2 + Z_4^2$$

$$- 2(X_4X_5 + Y_4Y_5 + Z_4Z_5)$$

$$= (X_5 - 1)^2 + 2X_5 - 1 + Y_5^2 + Z_5^2 - 2(X_4X_5 + Y_4Y_5 + Z_4Z_5) + B \tag{18b}$$

$$= 2B^2 + 2X_5 - 1 - 2(X_4X_5 + Y_4Y_5 + Z_4Z_5)$$

where we used (17c), hence

$$D_{45}^2 - 2B^2 + 1 = 2[X_5 - X_4X_5 - Y_4Y_5 - Z_4Z_5] \tag{19a}$$

$$= 2[X_5(1 - X_4) - Y_4Y_5 - Z_4Z_5] \tag{19b}$$

which from (7), where now $X_4 = \frac{1}{2}$,

$$= 2[\frac{X_5}{2} - Y_4Y_5 - Z_4Z_5] \tag{19c}$$

Hence,

$$\frac{1}{2}[D_{45}^2 - 2B^2 + 1] = \frac{X_5}{2} - Y_4Y_5 - Z_4Z_5$$

which, by using the relation (15) can eliminate X_5 to get

$$\frac{1}{2}[D_{45}^2 - 2B^2 + 1] = Y_5(\frac{\sqrt{3}}{2} - Y_4) - Z_4Z_5$$

which can be rewritten as

$$B^2 - (\frac{1+D_{45}^2}{2}) = Y_5(Y_4 - \frac{\sqrt{3}}{2}) + Z_4Z_5 \tag{20}$$

Let now

$$A_{45} = B^2 - (\frac{1 + D_{45}^2}{2}) \tag{21}$$

Then we have

$$A_{45} = Y_5(Y_4 - \frac{\sqrt{3}}{2}) + Z_4Z_5 \tag{22}$$

Let further

$$Y_4^* = Y_4 - \frac{\sqrt{3}}{2} \tag{23}$$

Then

$$A_{45} = Y_5Y_4^* + Z_4Z_5 \tag{24}$$

By similar arguments one can replace subscript 5 by 6 and immediately obtain the relation

$$A_{46} = Y_6Y_4^* + Z_4Z_6 \tag{25}$$

where

$$A_{46} = B^2 - \frac{(1 + D_{46}^2)}{2} \tag{26}$$

Finally from the relation

$$D_{56}^2 = (X_5 - X_6)^2 + (Y_5 - Y_6)^2 + (Z_5 - Z_6)^2$$

$$= X_5^2 + Y_5^2 + Z_5^2 + B^2 - 2(X_5X_6 + Y_5Y_6 + Z_5Z_6)$$

one gets,

$$D_{56}^2 = 2B^2 + 2X_5 - 1 - 2(X_5X_6 + Y_5Y_6 + Z_5Z_6)$$

thus, we define

$$A_{56} = B^2 - \frac{(1 + D_{56}^2)}{2} \tag{27a}$$

and get

$$A_{56} = X_5(X_6 - 1) + Y_5Y_6 + Z_5Z_6 \tag{27b}$$

which, if the relation (16) is used, finally yields

$$A_{56} = - \sqrt{3} \quad X_5Y_6 + Y_6Y_5 + Z_5Z_6 \tag{27c}$$

Now, if the relation (15) is used to eliminate X_5 we finally get

$$A_{56} = Z_5Z_6 - 2Y_5Y_6 \tag{28}$$

Method of Solution (The Inverse Problem)

We indicated earlier that if H_4, H_5 and H_6 are known then all of the coordinates of the points 1, 2, 3, 4, 5 and 6 will be known. Unfortunately, what is known a priori are not the H_4, H_5 and H_6, but D_{45}, D_{46} and D_{56}. In what follows we shall develop solution methods whereby if the lengths D_{45}, D_{46} and D_{56} are given then we can solve the inverse problem and find H_4, H_5 and H_6 and hence all the coordinates of the nodes. This will be done by simple iteration on the value of H_4 as will be demonstrated below.

Let us assume that D_{45}, D_{46}, D_{56} and H_4 are given. Then in the relations (24), (26) and (28), the parameters A_{45}, A_{46}, A_{56}, Y_4, Y_4^* and ($Z_4 = H_4$) will be known. Furthermore, since from (10), Y_5 can be written in terms of $Z_5 = H_5$ and from (14) Y_6 can be written in terms of $Z_6 = H_6$, we can easily see that we have only the two unknowns H_5 and H_6.

Specifically, we now use the relations (24) and (10) and get

$$A_{45} = Y_4^*[\frac{\sqrt{3}}{4} + \frac{1}{2}(B_{25}^2 - .25)^{\frac{1}{2}}] + Z_4Z_5$$

or

$$A_{45} - \frac{\sqrt{3}}{4} Y_4^* = \frac{Y_4^*}{2}(B_{25}^2 - .25)^{\frac{1}{2}} + Z_4Z_5$$

Now, from (4)

$$B_{25}^2 = B^2 - H_5^2 = B^2 - Z_5^2$$

we get

$$4(A_{45} - \frac{\sqrt{3}}{4} Y_4^* - Z_4Z_5)^2 = Y_4^{*2}[B^2 - Z_5^2 - .25]$$

which can be further rearranged in the form of a quadratic equation for Z_5 is

$$(4Z_4^2 + Y_4^{*2})Z_5^2 - 8F_{45}Z_4Z_5 + 4F_{45}^2 - Y_4^{*2}(B^2 - .25) = 0 \tag{29}$$

where

$$F_{45} = A_{45} - \frac{\sqrt{3}}{4} Y_4^* \tag{30}$$

Notice in (29) that if $Z_4 = H_4$ is known and since B is known then Z_5 is the only unknown. Hence, given H_4 there is a unique value for Z_5.

By interchanging 5 and 6 similar relation to (29) can be constructed for point 6 as

$$(4Z_4^2 + Y_4^{*2})Z_6^2 - 8F_{46}Z_4Z_6 + 4F_{46}^2 - Y_4^{*2}(B^2 - .25) = 0 \tag{31}$$

where

$$F_{46} = A_{46} - \frac{\sqrt{3}}{4} \; Y_4^* \tag{32}$$

Thus, given $Z_4 = H_4$ then there is a unique solution for Z_6.

Since, if Z_4 is known, we can calculate Z_5 and Z_6 uniquely, the condition that fixes the exact value on Z_4 is the relation (28). Specifically one has to iterate of Z_4 until the calculated values Z_5 and Z_6 via (29) and (31) satisfy (28).

Now equations (29) and (31) admit the two solutions

$$Z_{5(1,2)} = [T_2 \pm (T_2^2 - T_1T_3)^{\frac{1}{2}}]/T_1 \tag{33}$$

$$Z_{6(1,2)} = [W_2 \pm (W_2^2 - W_1W_3)^{\frac{1}{2}}]/W_1 \tag{34}$$

where

$$T_1 = W_1 = 4Z_4^2 + Y_4^{*2}$$

$$T_2 = 4F_{45}Z_4$$

$$T_3 = 4F_{45}^2 - Y_4^{*2}(B^2 - .25)$$

$$W_2 = 4F_{46}Z_4$$

$$W_3 = 4F_{46}^2 - Y_4^{*2}(B^2 - .25) \tag{35}$$

Corresponding to the values (33) and (34) we can calculate new values for Y_5 and Y_6 which we shall refer to as K_5 and K_6, respectively. Thus, we have (see (10) and (14) with $G = 1$)

$$K_5 = \frac{\sqrt{3}}{4} + \frac{1}{2}(V_{25}^2 - .25)^{\frac{1}{2}} \tag{36a}$$

$$K_6 = \frac{\sqrt{3}}{4} + \frac{1}{2}(V_{16}^2 - .25)^{\frac{1}{2}} \tag{36b}$$

where (see also (4) and (6))

$$V_{25} = (B^2 - Z_{5(1)}^2)^{\frac{1}{2}} \tag{37a}$$

$$V_{16} = (B^2 - Z_{6(1)}^2)^{\frac{1}{2}} \tag{37b}$$

From (33), (34), (36a) and (36b) we construct the function (see 28)

$$S_{56} = Z_{5(1)}Z_{6(1)} - 2K_5K_6 \tag{38}$$

where the idea here is to iterate on $Z_4 = H_4$ until the value of S_{56} converges to the value of A_{56} as calculated via (27). Once this convergence takes place we will have the right values of Z_5 and Z_6 (i.e. namely H_5 and H_6) and we can then proceed to calculate the correct coordinates of points 4, 5 and 6.

Calculations of the Coordinates of Nodes 7, 8 and 9

By inspection one can easily find that the nodes 7, 8 and 9 are the mirror images of nodes 1, 2 and 3 respectively with respect to the plane of nodes 4, 5 and 6. To this end, using some algebraic manipulation we thus get:

$$X_{i+6} = X_i - 2n_iP_i$$

$$Y_{i+6} = Y_i - 2n_iP_i$$

$$Z_{i+6} = Z_i - 2n_iP_i, \quad i = 1, 2, 3 \tag{39}$$

where

n_i are the components of the unit normal to the plane of the nodes 4, 5, 6 and P_i is the dot product between the normal $\underset{\sim}{n}$ and the vector connecting the nodes 1 and 4. This completes the local coordinates of the repeating unit cell.

Building Block Results

With the coordinates of nodes 7, 8 and 9 now known, their plane defines the base for the next unit cell. This is done by introducing a linear transformation of the original coordinates according to

$$X'_i = \beta_{ij} X_j \tag{40}$$

where X'_i are the new coordinates and β_{ij} are the direction cosines between X'_i and X'_j. These values of the transformation tensor can be constructed from the normal to the plane of the vortices 7, 8 and 9. Once this is done then a building block approach yields the results for an arbitrary number of cells as is documented by the accompanying computer program. In the program a sample calculation of five unit cells is included and the coordinates of the vortices are listed in local and global coordinate systems.

```
10      DIM XL(9),YL(9),ZL(9),X(200),Y(200),Z(200).B(3,3)
20      D1=D
40      READ B,ERRO,NC
60      FOR II=1 TO NC
100 G=1
120     READ R45,R46,R56
150     A45=B*B-(1+R45*R45)*.5
160     A46=B*B-(1+R46*R46)/2
170     A56=B*B-(1+R56*R56)/2
200     HM=SQR (B*B-.25):STP=.1:HSTRT=0
230 FOR H4=HSTRT   TO HM  STEP STP
240 B14=SQR(B*B-H4*H4)
250 Y4=-SQR(B14*B14-(G/2)*(G/2))
260 Z4=H4
270     Y45=Y4-SQR(3)/2:F45=A45-(SQR(3)/4)*Y45
280 T1=4*Z4*Z4+Y45*Y45
290 T2=4*F45*Z4
300 T3=4*F45*F45-Y45*Y45*(B*B-.25)
310 NA=T2*T2-T1*T3:IF(NA(0)THEN 540
320 Z51=(T2+SQR(NA))/T1
330 Z52=(T2-SQR(NA))/T1
340 F46=A46-(SQR(3)/4)*Y45
350 W1=T1
360 W2=4*F46*Z4
370 W3=4*F46*F46-Y45*Y45*(B*B-.25)
380 MA=W2*W2-W1*W3:IF(MA(0)THEN 540
390 Z61=(W2+SQR(MA))/W1
400 Z62=(W2-SQR(MA))/W1
410 V25=SQR(B*B-Z51*Z51)
420 V16=SQR(B*B-Z61*Z61)
430 K5=(SQR(3)*G+2*SQR(V25*V25-G*G/4))/4
440 K6=(SQR(3)*G+2*SQR(V16*V16-G*G/4))/4
450 S56=Z61*Z51-2*K5*K6:D=S56-A56
460     D=CDBL (D)
461     D1=CDBL (D1)
462     IF (D)(ERRO THEN 590
470     IF D1*D (0 THEN 580
480     D1=D
520 GOTO 550
530 IF(NA(0) THEN 540
540 LPRINT TAB(1);"COMPLEX ROOTS":GOTO 550
550 NEXT H4
560     GOTO 590
590     G=1
591     H5=Z51
592     H6=Z61
600 HM=SQR(B*B-(G*G/3))
610     XL(1)=0
611     YL(1)=0
612     ZL(1)=0
613     XL(2)=0- G
614     YL(2)=G
615     ZL(2)=0
616     XL(3)=G/2
617     YL(3)=G*SQR (3)/2
618     ZL(3)=0
620 B14=SQR(B*B-H4*H4)
630 B25=SQR(B*B-H5*H5)
640 X4=G/2
650 Y4=-SQR(B14*B14-(G/2)*(G/2))
660 Z4=H4
670 X5=3*G/4+(SQR(3)/2)*SQR(B25*B25-(G/2)*(G/2))
680 Y5=SQR(3)*G+2*SQR(B25*B25-(G/2)*(G/2))
```

```
630 Y5=Y5/4
700 Z5=H5
710 B16=SQR(B*B-H6*H6)
720 REM X6=-B16*COS(CTH16)
730 X6=G/4-SQR(3)/2*SQR(B16*B16-(G/2)*G/2)
740 Y6=SQR(3)*G+2*SQR(B16*B16-G/2*G/2)
750 Y6=Y6/4
760 Z6=H6
770      XL(4)=X4
771      XL(5)=X5
772      XL(6)=X6
773      YL(5)=Y5
774      YL(6)=Y6
775      YL(4)=Y4
776      ZL(4)=H4
777      ZL(5)=H5
778      ZL(6)=H6
780      X1=XL(1)
781      Y1=YL(1)
782      Z1=ZL(1)
783      X2=XL(2)
784      Y2=YL(2)
785      Z2=ZL(2)
786      X3=XL(3)
787      Y3=YL(3)
788      Z3=ZL(3)
790 R45=SQR((X5-X4)^2+(Y5-Y4)^2+(H5-H4)^2)
800 R56=SQR((X5-X6)^2+(Y5-Y6)^2+(H5-H6)^2)
810 R46=SQR((X4-X6)^2+(Y4-Y6)^2+(H4-H6)^2)
830      F=((X5-X4)*(X6-X4)+(Y5-Y4)*(Y6-Y4)+(H5-H4)*(H6-H4))/R45/R46
840      P=SQR (1-F^2)
850      N1=((Y5-Y4)*(Z6-Z4)-(Y6-Y4)*(Z5-Z4))/R45/R46
860      N1=N1/P
870      N2=((X6-X4)*(Z5-Z4)-(X5-X4)*(Z6-Z4))/R45/R46
880      N2=N2/P
890      N3=((X5-X4)*(Y6-Y4)-(X6-X4)*(Y5-Y4))/R45/R46
900      N3=N3/P
910      P1=(X1-X4)*N1+(Y1-Y4)*N2+(Z1-Z4)*N3
920      P2=(X2-X4)*N1+(Y2-Y4)*N2+(Z2-Z4)*N3
930      P3=(X3-X4)*N1+(Y3-Y4)*N2+(Z3-Z4)*N3
940      X7=X1-2*N1*P1
950      Y7=Y1-2*N2*P1
960      Z7=Z1-2*N3*P1
970      X8=X2-2*N1*P2
980      Y8=Y2-2*N2*P2
990      Z8=Z2-2*N3*P2
1000     X9=X3-2*N1*P3
1010     Y9=Y3-2*N2*P3
1020     Z9=Z3-2*N3*P3
1030     R78=SQR ((X7-X8)^2+(Y7-Y8)^2+(Z7-Z8)^2)
1040     R79=SQR ((X7-X9)^2+(Y9-Y7)^2+(Z9-Z7)^2):                    R89=SQR ((X8-X9)^2+(Y8-Y9)^2+(Z9-Z8)^2)
1050     XL(7)=X7:XL(8)=X8:XL(9)=X9:YL(7)=Y7:YL(8)=Y8:YL(9)=Y9:ZL(7)=Z7:ZL(8)=Z8:ZL(9)=Z9
1090     X8=X(6*(II-2)+8):               Y8=Y(6*(II-2)+8):               Z8=Z(6*(II-2)+8):               X7=X(6*(II-2)+7):
               Y7=Y(6*(II-2)+7):      Z7=Z(6*(II-2)+7):   X9=X(6*(II-2)+9):   Y9=Y(6*(II-2)+9):   Z9=Z(6*(II-2)+9)
1100     PCOS=((X8-X7)*(X9-X7)+(Y8-Y7)*(Y9-Y7)+(Z8-Z7)*(Z9-Z7))/R78/R79
1110     PSIN=SQR (1-PCOS*PCOS)
1120     N1=((Y8-Y7)*(Z9-Z7)-(Y9-Y7)*(Z8-Z7))/R78/R79
1130     N1=N1/PSIN
1140     N2=((X9-X7)*(Z8-Z7)-(X8-X7)*(Z9-Z7))/R78/R79
1150     N2=N2/PSIN
1160     N3=((X8-X7)*(Y9-Y7)-(X9-X7)*(Y8-Y7))/R78/R79
1170     N3=N3/PSIN
1190     B(1,1)=(X8-X7)/R78
1200     B(1,2)=(Y8-Y7)/R78
1210     B(1,3)=(Z8-Z7)/R78
1220     B(3,1)=N1
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```
1230    B(3,2)=N2
1240    B(3,3)=N3
1250    B(2,1)=P(3,2)*B(1,3)-B(1,2)*B(3,3)
1260    B(2,2)=B(1,1)*B(3,3)-B(3,1)*B(1,3)
1270    B(2,3)=B(1,2)*B(3,1)-B(1,1)*B(3,2)
1290                        FOR IU=1 TO 9
1310                        X(6*(II-1)+IU)=X(6*(II-1)+1)+B(1,1)*XL(IU)+B(2,1)*YL(IU)+B(3,1)*ZL(IU):             Y(6*(II-1)+IU)=Y(6*(II-1)+1
)+B(1,2)*XL(IU)+B(2,2)*YL(IU)+B(3,2)*ZL(IU):             Z(6*(II-1)+IU)=Z(6*(II-1)+1)+B(1,3)*XL(IU)+B(2,3)*YL(IU)+B(3,3)*ZL(IU)
1311    NEXT IU
1320    GOTO 1400
1330    REM THIS IS ONLY FOR THE FIRST CELL
1340    FOR FRT=1 TO 9:X(FRT)=XL(FRT):Y(FRT)=YL(FRT):Z(FRT)=ZL(FRT):NEXT FRT
1400    PRINT
1410    LPRINT :LPRINT TAB(50);"ANALYSIS COMPLETE" LPRINT TAB(50);"================="
1420    NEXT II
1430    END
5000    DATA .6667,.01,2
5001    DATA 1,1,1
5002    DATA 1,1,1
```

```
10      DIM XL(9),YL(9),ZL(9),X(200),Y(200),Z(200),B(3,3)
15      INPUT "PLEASE GIVE PROBLEMS NAME";P$:L1=LEN(P$):LPRINT :L=INT((120-L1)/2):LPRINT TAB(L);P$:LPRINT TAB(L);:FOR I=1 TO L1:LPRI
NT "*";:NEXT I
20      D1=D
30      INPUT  "PLEASE GIVE B          ";B
40      INPUT "PLEASE GIVE TOLERRANCE ";ERRO
50      INPUT "PLEASE GIVE THE NUMBER OF CELLS";NC
60      FOR II=1 TO NC
70      PRINT CHR$(27);"Y"0 CHR$(27);"~k"
80      FOR I=1 TO 11:PRINT CHR$(27);"~R":FOR J=1 TO 80:PRINT " ";:NEXT J:NEXT I :PRINT CHR$(27);"~r"
90      PRINT CHR$(27);"Y"0 CHR$(27);"~k"
100 G=1
110     PRINT TAB(5);"FOR CELL ";II:PRINT TAB(5);"============"
115     LPRINT:LPRINT TAB(40);"++++++++++++++++++++++++++++++++++++++++++++++":LPRINT TAB(46);"U N I T    N U M B E R    ";II:LPRINT TA
B(46);"******************************":LPRINT
120 INPUT "R45 :";R45:LPRINT TAB(20),,"R45 = ";R45,
130 INPUT "R46 :";R46:LPRINT "R46 = ";R46,
140 INPUT "R56 :";R56:LPRINT "R56 = ";R56
145     LPRINT:LPRINT
150     A45=B*B-(1+R45*R45)*.5
160     A46=B*B-(1+R46*R46)/2
170     A56=B*B-(1+R56*R56)/2
180 REM FOR I=1 TO 130 :LPRINT"=";:NEXT I
190 REM FOR I=1 TO 130:LPRINT"=";:NEXT I
200     HM=SQR (B*B-.25):STP=.1:HSTRT=0
210     PRINT CHR$(27);"Y"0 CHR$(27);"~k"
220     PRINT CHR$(27);"~B"CHR$(27);"~R";" H4","      H5","      H6","      DIFFERANCE  ";CHR$(27);"~b"CHR$(27)"~r"
230 FOR H4=HSTRT   TO HM  STEP STP
240 B14=SQR(B*B-H4*H4)
250 Y4=-SQR(B14*B14-(G/2)*(G/2))
260 Z4=H4
270     Y45=Y4-SQR(3)/2:F45=A45-(SQR(3)/4)*Y45
280 T1=4*Z4*Z4+Y45*Y45
290 T2=4*F45*Z4
300 T3=4*F45*F45-Y45*Y45*(B*B-.25)
310 NA=T2*T2-T1*T3:IF(NA(0)THEN 540
320 Z51=(T2+SQR(NA))/T1
330 Z52=(T2-SQR(NA))/T1
340 F46=A46-(SQR(3)/4)*Y45
350 W1=T1
360 W2=4*F46*Z4
370 W3=4*F46*F46-Y45*Y45*(B*B-.25)
380 MA=W2*W2-W1*W3:IF(MA(0)THEN 540
390 Z61=(W2+SQR(MA))/W1
400 Z62=(W2-SQR(MA))/W1
410 V25=SQR(B*B-Z51*Z51)
420 V16=SQR(B*B-Z61*Z61)
430 K5=(SQR(3)*G+2*SQR(V25*V25-G*G/4))/4
440 K6=(SQR(3)*G+2*SQR(V16*V16-G*G/4))/4
450 S56=Z61*Z51-2*K5*K6:D=S56-A56
460     D=CDBL (D):D1=CDBL (D1):IF ABS (D)( ERRO THEN 570
470     IF D1*D (0 THEN 580
480     D1=D
490 REM LPRINT TAB(1);H4;TAB(19);Z51;TAB(39);Z52;TAB(59);Z61;TAB(79);Z62;TAB(99);S56;TAB(119);D
500 PRINT H4,Z51,Z61,D
510 REM FOR I=1 TO 130 :LPRINT"-";:NEXT I
520 GOTO 550
530 IF(NA(0) THEN 540
540 LPRINT TAB(1);"COMPLEX ROOTS":GOTO 550
550 NEXT H4
560     GOTO 590
570     PRINT:PRINT CHR$(27);"~B",H4,Z51,Z61;CHR$(27);"~b":GOTO 590
580     PRINT CHR$(27);"Y"0:PRINT CHR$(27);"~k";HSTRT=H4-STP:STP=STP/10:PRINT
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590 G=1:H5=Z51:H6=Z61:PRINT " H5 = ";H5:PRINT "H6 = ";H6:PRINT " H4 = ";H4
600 HM=SQR(B*B-(G*G/3))
610      XL(1)=0:YL(1)=0:ZL(1)=0:XL(2)=G:YL(2)=0:ZL(2)=0:XL(3)=G/2:YL(3)=G*SQR(3)/2:ZL(3)=0
620 B14=SQR(B*B-H4*H4)
630 B25=SQR(B*B-H5*H5)
640 X4=G/2
650 Y4=-SQR(B14*B14-(G/2)*(G/2))
660 Z4=H4
670 X5=3*G/4+(SQR(3)/2)*SQR(B25*B25-(G/2)*(G/2))
680 Y5=SQR(3)*G+2*SQR(B25*B25-(G/2)*(G/2))
690 Y5=Y5/4
700 Z5=H5
710 B16=SQR(B*B-H6*H6)
720 REM X6=-B16*COS(CTH16)
730 X6=G/4-SQR(3)/2*SQR(B16*B16-(G/2)*G/2)
740 Y6=SQR(3)*G+2*SQR(B16*B16-G/2*G/2)
750 Y6=Y6/4
760 Z6=H6
770      XL(4)=X4:XL(5)=X5:XL(6)=X6:YL(5)=Y5:YL(6)=Y6:YL(4)=Y4:ZL(4)=H4:ZL(5)=H5:ZL(6)=H6
780      X1=XL(1):Y1=YL(1):Z1=ZL(1):X2=XL(2):Y2=YL(2):Z2=ZL(2):X3=XL(3):Y3=YL(3):Z3=ZL(3)
790 R45=SQR((X5-X4)^2+(Y5-Y4)^2+(H5-H4)^2)
800 R56=SQR((X5-X6)^2+(Y5-Y6)^2+(H5-H6)^2)
810 R46=SQR((X4-X6)^2+(Y4-Y6)^2+(H4-H6)^2)
820 PRINT R45,R56,R46
830      F=((X5-X4)*(X6-X4)+(Y5-Y4)*(Y6-Y4)+(H5-H4)*(H6-H4))/R45/R46
840      P=SQR (1-F^2)
850      N1=((Y5-Y4)*(Z6-Z4)-(Y6-Y4)*(Z5-Z4))/R45/R46
860      N1=N1/P
870      N2=((X6-X4)*(Z5-Z4)-(X5-X4)*(Z6-Z4))/R45/R46
880      N2=N2/P
890      N3=((X5-X4)*(Y6-Y4)-(X6-X4)*(Y5-Y4))/R45/R46
900      N3=N3/P
910      P1=(X1-X4)*N1+(Y1-Y4)*N2+(Z1-Z4)*N3
920      P2=(X2-X4)*N1+(Y2-Y4)*N2+(Z2-Z4)*N3
930      P3=(X3-X4)*N1+(Y3-Y4)*N2+(Z3-Z4)*N3
940      X7=X1-2*N1*P1
950      Y7=Y1-2*N2*P1
960      Z7=Z1-2*N3*P1
970      X8=X2-2*N1*P2
980      Y8=Y2-2*N2*P2
990      Z8=Z2-2*N3*P2
1000     X9=X3-2*N1*P3
1010     Y9=Y3-2*N2*P3
1020     Z9=Z3-2*N3*P3
1030     R78=SQR ((X7-X8)^2+(Y7-Y8)^2+(Z7-Z8)^2)
1040     R79=SQR ((X7-X9)^2+(Y9-Y7)^2+(Z9-Z7)^2):R89=SQR ((X8-X9)^2+(Y8-Y9)^2+(Z9-Z8)^2)
1050     XL(7)=X7:XL(8)=X8:XL(9)=X9:YL(7)=Y7:YL(8)=Y8:YL(9)=Y9:ZL(7)=Z7:ZL(8)=Z8:ZL(9)=Z9
1060     LPRINT TAB(48);"LOCAL CO-ORDINATE SYSTEM ":LPRINT TAB(48);"========================":LPRINT :LPRINT TAB(41);"NODE"," X"," Y"
," Z":LPRINT TAB(41);"====","  =","  =","  ="
1070     FOR YG=1 TO 9:LPRINT TAB(41);YG,XL(YG).YL(YG),ZL(YG):NEXT YG:LPRINT:LPRINT
1080     IF II=1 GOTO 1340
1090     X8=X(6*(II-2)+8):Y8=Y(6*(II-2)+8):Z8=Z(6*(II-2)+8):X7=X(6*(II-2)+7):Y7=Y(6*(II-2)+7):Z7=Z(6*(II-2)+7):X9=X(6*(II-2)+9):Y9=Y(
6*(II-2)+9):Z9=Z(6*(II-2)+9)
1100     PCOS=((X8-X7)*(X9-X7)+(Y8-Y7)*(Y9-Y7)+(Z8-Z7)*(Z9-Z7))/R78/R79
1110     PSIN=SQR (1-PCOS*PCOS)
1120     N1=((Y8-Y7)*(Z9-Z7)-(Y9-Y7)*(Z8-Z7))/R78/R79
1130     N1=N1/PSIN
1140     N2=((X9-X7)*(Z8-Z7)-(X8-X7)*(Z9-Z7))/R78/R79
1150     N2=N2/PSIN
1160     N3=((X8-X7)*(Y9-Y7)-(X9-X7)*(Y8-Y7))/R78/R79
1170     N3=N3/PSIN
1175     LPRINT TAB(13);,,,,"NORMAL VECTOR":LPRINT TAB(13);,,,,"============="
1180     LPRINT TAB(18);,,"N1 = ";N1,"N2 = ";N2,"N3 = ";N3
1190     B(1,1)=(X8-X7)/R78
1200     B(1,2)=(Y8-Y7)/R78
1210     B(1,3)=(Z8-Z7)/R78
```

```
1220    B(3,1)=N1
1230    B(3,2)=N2
1240    B(3,3)=N3
1250    B(2,1)=B(3,2)*B(1,3)-B(1,2)*B(3,3)
1260    B(2,2)=B(1,1)*B(3,3)-B(3,1)*B(1,3)
1270    B(2,3)=B(1,2)*B(3,1)-B(1,1)*B(3,2)
1272    LPRINT
1275    LPRINT TAB(53);"TRANSFORMATION MATRIX":LPRINT TAB(53);"====================="
1280    LPRINT:FOR PLO=1 TO 3:LPRINT TAB(42);"I";:FOR LOP=1 TO 3:LPRINT ,B(PLO,LOP);:NEXT LOP:LPRINT TAB(97);"I":LPRINT:NEXT PLO:LPR
INT :LPRINT
1290                    FOR IU=1 TO 9
1310                    X(6*(II-1)+IU)=X(6*(II-1)+1)+B(1,1)*XL(IU)+B(2,1)*YL(IU)+B(3,1)*ZL(IU):Y(6*(II-1)+IU)=Y(6*(II-1)+1)+B(1,2)*X
L(IU)+B(2,2)*YL(IU)+B(3,2)*ZL(IU):Z(6*(II-1)+IU)=Z(6*(II-1)+1)+B(1,3)*XL(IU)+B(2,3)*YL(IU)+B(3,3)*ZL(IU):NEXT IU
1320    GOTO 1350
1330    REM THIS IS ONLY FOR THE FIRST CELL
1340    FOR FRT=1 TO 9:X(FRT)=XL(FRT):Y(FRT)=YL(FRT):Z(FRT)=ZL(FRT):NEXT FRT
1350    LPRINT:LPRINT TAB(47);"GLOOBAL CO-ORDINATE SYSTEM":LPRINT TAB(47);"==========================":LPRINT TAB(33);"LOCAL NO.";TA
B(43);"GLOOBAL NO.";TAB(61);"X-COORD.";TAB(73);"Y-COORD.";TAB(88);"Z-COORD."
1355    LPRINT TAB(33);"=========";TAB(43);"===========";TAB(61);"========";TAB(73);"========";TAB(88);"========"
1357    LPRINT
1359    FOR TY=1 TO 9
1360    LPRINT TAB(33);TY;TAB(43);6*(II-1)+TY;TAB(61);X(6*(II-1)+TY);TAB(73);Y(6*(II-1)+TY)TAB(88);Z(6*(II-1)+TY):NEXT TY:NEXT II:LP
RINT :LPRINT TAB(40);"++++++++++++++++++++++++++++++++++++++++++++"
1370    LPRINT:LPRINT:LPRINT TAB(40);"GLOOBAL CO-ORDINATES OF FINAL CONFIGURATION":LPRINT TAB(40);                          "=============
==============================":LPRINT
1380    LPRINT TAB(20);"LOCAL NO.";TAB(35);"GLOOBAL NO.";TAB(55);"X-COORD.";TAB(75);"Y-COORD.";TAB(95);"Z-COORD."
1390    LPRINT TAB(20);"=========";TAB(35);"===========";TAB(55);"========";TAB(75);"========";TAB(95);"========":LPRINT:FOR I=1 TO
NC:FOR II=1 TO 9
1400    LPRINT TAB(20);II;TAB(35);6*(I-1)+II;TAB(55);X( *(I-1)+II);TAB(75);Y(6*(I-1)+II);TAB(95);Z(6*(I-1)+II):============"
1420    END
```

7

TESTING YOU

UNIT NUMBER 1

LOCAL CO-ORDINATE SYSTEM

NODE	X	Y	Z
1	0	0	0
2	1	0	0
3	.5	.866025	0
4	.5	-.288714	.3333
5	.999967	.577331	.333367
6	3.34829E-05	.577331	.333367
7	0	-5.15475E-05	.666645
8	1	-5.15475E-05	.666645
9	.5	.865974	.666779

GLOOBAL CO-ORDINATE SYSTEM

LOCAL NO.	GLOOBAL NO.	X-COORD.	Y-COORD.	Z-COORD.
1	1	0	0	0
2	2	1	0	0
3	3	.5	.866025	0
4	4	.5	-.288714	.3333
5	5	.999967	.577331	.333367
6	6	3.34829E-05	.577331	.333367
7	7	0	-5.15475E-05	.666645
8	8	1	-5.15475E-05	.666645
9	9	.5	.865974	.666779

UNIT NUMBER 2

LOCAL CO-ORDINATE SYSTEM

NODE	X	Y	Z
1	0	0	0
2	1	0	0
3	.5	.666025	0
4	.5	-.365153	.2472
5	.942442	.544119	.380875
6	-.0324797	.596103	.295735
7	-.0429944	-.0479607	.468538
8	.940477	-.0663983	.64866
9	.432774	.791034	.732607

8

NORMAL VECTOR

N1 = 0   N2 = -1.54651E-04   N3 = 1

TRANSFORMATION MATRIX

I	1	0	0	I
I	0	1	1.54651E-04	I
I	0	-1.54651E-04	1	I

GLOOBAL CO-ORDINATE SYSTEM

LOCAL NO.	GLOOBAL NO.	X-COORD.	Y-COORD.	Z-COORD.
1	7	0	-5.15475E-05	.666645
2	8	1	-5.15475E-05	.666645
3	9	.5	.865974	.666779
4	10	.5	-.365243	.913788
5	11	.942442	.544009	1.0476
6	12	-.0324797	.596005	.963472
7	13	-.0429944	-.0480847	1.13516
8	14	.940477	-.0665502	1.31529
9	15	.432774	.790869	1.39937

++
U N I T   N U M B E R   3
****

LOCAL CO-ORDINATE SYSTEM

NODE	X	Y	Z
1	0	0	0
2	1	0	0
3	.5	.866025	0
4	.5	-.271328	.3476
5	.985023	.568703	.347558
6	.0149769	.568703	.347558
7	0	3.4479E-05	.695173
8	1	3.4479E-05	.695173
9	.5	.86606	.695087

NORMAL VECTOR

N1 = -.180121   N2 = -.201076   N3 = .962873

TRANSFORMATION MATRIX

I	.983472	-.0184655	.180119	I
I	-.0184376	.979402	.201079	I

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I	-.180121	-.201076	.962873	I

9

GLOOBAL CO-ORDINATE SYSTEM

LOCAL NO.	GLOOBAL NO.	X-COORD.	Y-COORD.	Z-COORD.
1	13	-.0429944	-.0480847	1.13518
2	14	.940477	-.0665502	1.31529
3	15	.432774	.790869	1.39937
4	16	.391134	-.392951	1.50537
5	17	.85266	.42083	1.76161
6	18	-.101353	.438742	1.58688
7	19	-.168211	-.187834	1.80455
8	20	.815261	-.206299	1.98466
9	21	.307573	.651137	2.06866

++
U N I T   N U M B E R   4
****

LOCAL CO-ORDINATE SYSTEM

NODE	X	Y	Z
1	0	0	0
2	1	0	0
3	.5	.866025	0
4	.5	-.346446	.2728
5	1.04997	.6062	.27289
6	-.0499691	.6062	.27289
7	0	-5.18601E-05	.545666
8	1	-5.18601E-05	.545666
9	.5	.865974	.54583

NORMAL VECTOR

N1 = -.180123   N2 = -.200979   N3 = .962893

TRANSFORMATION MATRIX

I	.983472	-.0184655	.180119	I
I	-.0184198	.979422	.200984	I
I	-.180123	-.200979	.962893	I

GLOOBAL CO-ORDINATE SYSTEM

LOCAL NO.	GLOOBAL NO.	X-COORD.	Y-COORD.	Z-COORD.
1	19	-.168211	-.187834	1.80455
2	20	.815261	-.206299	1.98466
3	21	[illegible]	[illegible]	[illegible]

5	23	.804084	.331658	2.37826
6	24	-.277674	.351969	2.18015
7	25	-.266497	-.297552	2.32995
8	26	.716975	-.316017	2.51007
9	27	.209258	.541386	2.59423

++

U N I T   N U M B E R   5

****

10

LOCAL CO-ORDINATE SYSTEM

NODE	X	Y	Z
1	0	0	0
2	1	0	0
3	.5	.866025	0
4	.5	-.253369	.3609
5	1.019	.588321	.312989
6	9.67237E-03	.571766	.342687
7	.0209821	.028559	.729007
8	1.01933	.0263093	.671579
9	.518207	.890808	.632599

NORMAL VECTOR

N1 = -.18012 N2 = -.201165 N3 = .962855

TRANSFORMATION MATRIX

I	.983472	-.0184655	.180119	I
I	-.018454	.979383	.201166	I
I	-.18012	-.201165	.962855	I

GLOOBAL CO-ORDINATE SYSTEM

LOCAL NO.	GLOOBAL NO.	X-COORD.	Y-COORD.	Z-COORD.
1	25	-.266497	-.297552	2.32995
2	26	.716975	-.316017	2.51007
3	27	.209258	.541386	2.59423
4	28	.16491	-.62753	2.71654
5	29	.66843	.196861	2.93321
6	30	-.32926	.193311	2.77657
7	31	-.377697	-.41662	3.04141
8	32	.614535	-.425706	3.16549
9	33	.112763	.438064	3.21159

++

GLOOBAL CO-ORDINATES OF FINAL CONFIGURATION

LOCAL NO.	GLOBAL NO.	X-COORD.	Y-COORD.	Z-COORD.
1	1	0	0	0
2	2	1	0	0
3	3	.5	.866025	0
4	4	.5	-.288714	.3333
5	5	.999967	.577331	.333367
6	6	3.34829E-05	.577331	.333367
7	7	0	-5.15475E-05	.666645
8	8	1	-5.15475E-05	.666645
9	9	.5	.865974	.666779
1	7	0	-5.15475E-05	.666645
2	8	1	-5.15475E-05	.666645
3	9	.5	.865974	.666779
4	10	.5	-.365243	.913788
5	11	.942442	.544009	1.0476
6	12	-.0324797	.596005	.963472
7	13	-.0429944	-.0480847	1.13518
8	14	.940477	-.0665502	1.31529
9	15	.432774	.790869	1.39937
1	13	-.0429944	-.0480847	1.13518
2	14	.940477	-.0665502	1.31529
3	15	.432774	.790869	1.39937
4	16	.391134	-.392951	1.50537
5	17	.85266	.42083	1.76161
6	18	-.101353	.438742	1.58688
7	19	-.168211	-.187834	1.80455
8	20	.815261	-.206299	1.98466
9	21	.307573	.651137	2.06866
1	19	-.168211	-.187834	1.80455
2	20	.815261	-.206299	1.98466
3	21	.307573	.651137	2.06866
4	22	.280769	-.59121	2.08765
5	23	.804084	.331658	2.37826
6	24	-.277674	.351969	2.18015
7	25	-.266497	-.297552	2.32995
8	26	.716975	-.316017	2.51007
9	27	.209258	.541386	2.59423
1	25	-.266497	-.297552	2.32995
2	26	.716975	-.316017	2.51007
3	27	.209258	.541386	2.59423
4	28	.16491	-.62753	2.71654
5	29	.66843	.196861	2.93321
6	30	-.32926	.193311	2.77667
7	31	-.377697	-.4[illegible]662	3.04141
8	32	.614535	-.42[illegible]706	3.16548
9	33	.112763	.436[illegible]54	3.21159

ANALYSIS COMPLETE