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FINAL REPORT

to

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

AN ADAPTIVE LEARNING CONTROL SYSTEM

FOR

LARGE FLEXIBLE STRUCTURES

Research Grant NAG-1-6

11/1/79 - 8/31/84

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## INTRODUCTION AND SUMMARY

This report is a summary of research activities performed under grant NAG-I-6. The objective of the research has been to study the design of adaptive/learning control systems for the control of large flexible structures. In 1 an adaptive/learning control methodology for flexible space structures was described. The approach was based on using a modal model of the flexible structure dynamics and an output-error identification scheme to identify modal parameters. The identification scheme was tested on simulated data characteristic of the solar electric propulsion (SEP) array. In 2 a least-squares identification scheme was proposed for estimating both modal parameters and modal-to-actuator and modal-to-sensor shape functions. The technique was applied to experimental data obtained from the NASA Langley beam experiment. In 3 a separable nonlinear least-squares approach was described for estimating the number of excited modes, shape functions, modal parameters, and modal amplitude and velocity time functions for a flexible structure. A digital computer program was delivered to the NASA-Langley Research Center and was used to process experimental flexible beam data to obtain estimates of the parameters and functions mentioned above. Because least-squares residuals are computed during the identification procedure , the method provides its own monitoring of identification quality. In 4 a dual-adaptive control strategy was developed for regulating the modal dynamics and identifying modal parameters of a flexible structure. It is well known that input signals that are optimum for identifying system parameters often yield a dynamic response characterized by large ampli-tude modal variations. Hence a min-max approach was used for finding an input to provide modal parameter identification while not exceeding reasonable bounds on modal displacement. The approach was tested using simulated beam data.

## FUTURE PLANS

Our future research plans include the following: (1) simplification of the separable nonlinear least squares computational procedure to obtain a "real-time" identification algorithm that will run on the same time scale as the process to be controlled; (2) derivation of a convergence proof for the nonlinear least-squares identification algorithm that uses a "moving data window;" (3) generalization of the nonlinear dual-adaptive control method and derivation of asymptotic stability conditions for the resulting closed-loop system.

# REFERENCES

1.  Montgomery, R.C. and Thau, F.E., "Adaptive and Learning Control of Large Space Structures," AIAA Guidance and Control Conference, June, 1980.

2.  Thau, F.E., Montgomery, R.C., and Horner, G.C., "On-line Structural Parameter Identification," Proc. AIAA Guidance and Control Conference, August, 1981, Albuquerque, New Mexico, pp. 530-539.

3.  Thau, F.E., Eliazov, T., and Montgomery R.C., "Least-Squares Sequential Parameter and State Estimation for Large Space Structures," Proceedings of the 1982 American Control Conference, June 14-16, 1982, pp. 16-21.

4.  Thau, F.E., "A Nonlinear Dual-Adaptive Control Strategy for Identification and Control of Flexible Structures," Proceedings of the Workshop on Identification and Control of Flexible Space Structures, Vol. 3, pp. 117-125, July 4-6, 1984.

APPENDIX 1

ADAPTIVE AND LEARNING CONTROL OF LARGE SPACE STRUCTURES

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## Abstract

For large complex structures, knowledge needed for control system design cannot be determined analytically at present. Also, ground testing is not possible because of the size of the objects and the fact that they are not designed to be self-supporting in Earth gravity. The adaptive/learning system for space operations assumes that structural testing must be conducted during deployment or assembly. Testing is conducted only when necessary to insure adequate control performance. Required design information is analytically extrapolated and monitored during other times. This report describes the adaptive/learning system proposed and presents simulation results using the SEP (solar electric propulsion) array and a novel remote sensor which involves raster scan television coverage of the motions of the array from four cameras on the corners of the space shuttle payload bay. This report includes a detailed description of the simulation, the filtering algorithm for processing the TV data, the parameter extraction algorithm, and simulation results gathered thusfar.

## I. Introduction

Because of the desire for efficiency, the structural designs of large space systems are driven to low weight per unit area of surface. This necessarily results in low effective stiffness of the structures making them very flimsy and flexible. For such structures, operations involving barging, towing, and assembly require precise knowledge of the structural dynamics of the objects involved to accomplish adequate attitude and configuration control. At present, for complex structures, the knowledge required cannot be determined analytically. Also, ground testing is not possible because of the size of the objects and the fact that they are not designed to be self-supporting in Earth gravity. Thus, it may be that an adaptive control system is required to deal with the problem. Unfortunately, the control excitation levels usually required by adaptive systems (e.g. Ref. 1) are unacceptable for many precision space operations. An alternate procedure is to schedule the adaptation process during those times when it is consistent with operational requirements and to analytically extrapolate and monitor the system during other times. An adaptive/learning system that operates in this manner was proposed and studied in an aircraft application (Ref. 2) and was later modified for use with structural systems (Ref. 3). The distinguishing feature of this system lies in the extrapolation and monitoring processes and the proposed system should not be confused with learning systems as described in reference 4 which deals with pattern classification. In the next section, the adaptive/learning concept is explained in the context of an example of the deployment of the solar electric propulsion

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array (SEP) from the space shuttle payload bay. Alternatives are discussed for the testing, extrapolation, and monitoring subsystems. The estimation and parameter identification subsystems for which simulation results are available are then mathematically outlined. This is followed by a description of the simulation example and by presentation and discussion simulation results.

## II. Adaptive/Learning System Description

For the purpose of explaining the functions of the learning system, consider the problem of deploying a large solar array from its stowed configuration while attached to the space shuttle (Fig. 1). In the upper part of figure 2 a simplified model is shown where the shuttle is represented by the mass m and the solar array, fully deployed, in an undeflected state by the bar and a typical deflected state by the curved line. F represents the force resulting from the reaction jets on the shuttle. The lower part of the figure shows two variations of the plots of the first fundamental frequency of the solar array as a function of the deployment length $\ell$ which is, in this case, the configuration variable. As indicated, the actual variation (shown dashed) might differ from that determined analytically. The system must initially use the analytic variation to schedule the feedback gains during deployment. It is assumed that, at the start of the deployment, information is available through analysis or ground testing that adequately defines the dynamics, and, hence, produces adequate stability and control margins. In other words, the error between the actual and analytical frequency curves of figure 2 is acceptable for small values of $\ell$. Also, if considered necessary, an in-flight test in the stowed configuration could be used to further reduce errors for $\ell = 0$. Semi-empirical methods must be used to incorporate the information obtained during the test into the analytic extrapolation used by the learning system. This is illustrated in figure 3 where the initial error ($\ell = 0$) has been reduced (Test 1) and the analytic extrapolation has been adjusted as a result of Test 2. During further deployment, the error between the actual and the model frequencies may be expected to become large enough that further testing is required (test point 3). The decision to conduct another test can be put on an analytical basis by monitoring sensor signals during deployment and comparing them to the output of an analytic model based on the extrapolation. When the error grows to an unacceptable level, a new test is needed to insure favorable control system margins.

For the adaptive/learning system discussed above, estimation and parameter identification theories are required together with an extrapolation of the system model, a method for evaluating the extrapolation to determine the need for testing, a method of designing the test inputs, and a method of refining the extrapolation process to bring it into agreement with the evolution of the

system. During testing for parameter identification a large signal to noise environment is provided by designed test inputs. In this case, speed of testing is important because parameter identification of ultra low frequency systems - characteristic of large space structures - will require considerable time. During other times, no specific test inputs are available. The only system inputs are those resulting from spacecraft operations, hence, there is a low signal to noise environment for parameter identification. In this case, parameter identification in as low a signal to noise environment as possible is important assuming small deviations from the extrapolation model. For this case sequential parameter identification as employed in reference 2 or a moving window Newtom-Raphson parameter identification may be desirable. The latter offers considerably more tolerance to noise but is also considerably more complex than the former. It also has monitoring information since estimates of the parameter variances are available by processing the information matrix of the data base (Refs. 5 and 6). The former is simpler to implement, requiring considerably less computation and is amenable to parallel processing using a bank of second order parameter estimators. For that reason the former technique has been selected for development here.

The technique used here was applied in reference 1 to the dynamics of a closed ring. The motion of the ring was assumed to be described by a Fourier series in the spatial variable with time varying coefficients. The coefficients were obtained using a fast-Fourier transform (FFT) of noise free measurements at sample times and their time series were used in parallel to identify frequency and damping characteristics of each coefficient. In that case a sine-cosine series was appropriate because of the closed nature of the ring. Generally, however, a sine-cosine approximation may not yield a bank of uncoupled second order dynamic systems that describe the time evolution of the coefficients of the FFT. In that case, the approximation functions of the series used to represent the dynamics must be "tuned" to effect decoupling of the bank of second order systems. The remainder of this section will deal with implementing the bank of sequential parameter estimators using arbitrary approximation functions and "tuning" the approximation functions to decouple the bank of parallel second order estimators. The general philosophy is to assume a set of approximation functions in the spatial and time domain (to account for rotating dynamics, e. g. Ref. 1), to obtain the "best fit" coefficients of a linear representation of the measurements at a sample time using the approximation functions, to select parameters of a bank of uncoupled linear second order difference equations to minimize the error between the model output and the observed coefficient time series, and finally, to "tune" the approximation functions to render the parameters time invariant.

Analytically the motion, w, of the system can be represented by

$$w(s,t,\ell) = \sum_{i=1}^{NM} \psi_i(t,p_i(\ell))\xi_i(s,c_i(\ell)) \qquad (1)$$

where $\xi_i$ are the NM approximation functions to be used to represent w; $\ell$ is a measured configuration variable; $\psi_i$ are the time varying coefficients of $\xi_i$; s is the spatial variable; t is time; $p_i$ is the parameter vector of the $i^{th}$ linear second order difference equations modeling $\psi_i$ and $c_i = \{c_{ij}\}$ is the coefficient vector that relates the $i^{th}$ function $\xi_i$ to a constant set of $N_{\sigma_i}$ basis functions $\sigma_{ij}(s,\ell)$ as follows:

$$\xi_i(s,c) = \sum_{j=0}^{N_{\sigma_i}} \sigma_{ij}(s,\ell)c_{ij}(\ell) \qquad (2)$$

Thus, for each mode number, i, there is a set of basis functions $\{\sigma_{ij}(s,\ell), j=0, 1,,,N_{\sigma_i}\}$ which, when summed using the weights $c_i = \{c_{ij}, j=0, 1,,, N_{\sigma_i}\}$ according to equation (2), produces the $i^{th}$ mode shape approximation function $\xi_i$. The $c_i$ vectors will be adjusted, thus tuning the approximation functions, only if monitoring indicates a deviation of the actual response from the extrapolation (1): One method for selecting the tuning functions $\sigma_{ij}$ is illustrated in figure 4 where the mode shape $\xi_1$ is represented by a table of values at discrete values of s. The function $\sigma_{10}$ is the best ground analysis estimate of the $1^{st}$ mode shape. Remaining $\sigma_{1j}$ functions are selected to allow for adjustment of the value of $\xi_1$ corresponding to the $j^{th}$ table point. Initially, the values of $N_{\sigma_i}+1$ dimensional $c_i$ vectors are $[1,0,,,0]$ which corresponds to using the best ground based analysis to start the system. Updating $c_i$ and monitoring will require time series analysis of the parameters $p_i$.

The parameters $p_i$ are selected to minimize the error between the output of each model,

$$\psi_i(k) = A_{1i}\,\psi_i(k-1) + A_{2i}\,\psi_i(k-2)$$
$$+ B_{1i}\,F_i(k-1) + B_{2i}\,F_i(k-2) \qquad (3)$$

and the "best" fit of the sensor data to the approximation function representation $\psi_i(k)$ at sample time $t_k$. In (3) the parameter vector $p_i$ is $p_i = [A_{1i}, A_{2i}, B_{1i}, B_{2i}]$ and $F_i$ is the time series force command inputs which are known to the controller. Thus $\psi_i$ is, in effect, the input sequence for a second order parameter estimator. In this report, this sequence is obtained using least squares fit of position measurements, y, which can be sequentially implemented. The form of the estimation problem is

$$y(k) = \sum_{i=1}^{NS} H_i(k)\bar{\psi}_i(k) + v(k) \qquad (4)$$

where y is the measurement vector, $H_i$ is a known operator algebraically related to $\xi$, $v(k)$ is the error in the fit and $\bar{\psi}_i$ is the estimate of $\psi_i$. The least squares solution to estimate $\bar{\psi}_i$ is well known (Ref. 7) and will not be duplicated here. The solution may be sequentially processed but one must take care not to propagate the estimated variance of $\bar{\psi}_i$ since this is equivalent to averaging the measurements at different times and will eventually produce an insensitive estimation of $\bar{\psi}_i$. The technique used here to avoid this difficulty is to set the a priori variance estimate to a constant at each k to start the sequential estimation.

Subscripts i will now be dropped for simplicity since processing for each approximation function is parallel and identical. The parameter vector p can be selected iteratively using an output error formulation (Ref. 8) wherein the measurement sequence is $\bar{\psi}_k$ (k indicating the sample time index) and the model output error is

$$e_{k-1} = \bar{\psi}_{k-1} - A_1\bar{\psi}_{k-2} - A_2\bar{\psi}_{k-3}$$
$$- B_1 F_{k-2} - B_2 F_{k-3} \qquad (5)$$

The iteration of p for the output error formulation is given by

$$p_k = p_{k-1} + e_{k-1} \cdot \begin{bmatrix} W_1\bar{\psi}_{k-2} \\ W_2\bar{\psi}_{k-3} \\ W_3 F_{k-2} \\ W_4 F_{k-3} \end{bmatrix} \qquad (6)$$

As indicated in reference 1, the weights $W_1,,W_4$ should be selected to be consistent with

$$W_1\bar{\psi}_{k-2}^2 + W_2\bar{\psi}_{k-3}^2 + W_3 F_{k-2}^2 + W_2 F_{k-3}^2 < 2$$

and the inputs, $\bar{\psi}_k$ and $F_k$, must be sufficiently varying and large if convergence of the sequence $p_k$ to some value say $p_c$ is to be assured.

After convergence, the parameters track a sequence of converged parameters $p_c$ which satisfy (3) locally (for the sample sequence when the parameters were identified.) If the sequence $p_c$ is constant our job is done. However, if the sequence varies, "tuning" is needed for the approximation functions. The basic task is to adjust c so that,

for all approximation functions, the identity e = 0 (Eq. 5) holds for a constant $p_c = p^*$. Consider then the variation of the NM dimensional error vector $F(p_c,c) = \{e^i\}$ where $e^i$ is the $i^{th}$ error function taken locally and $p_c = \{p_{c_i}\}$. The terms involving c appear explicitly in F only through the least square estimation process of $\bar{\psi}$ and hence we may calculate the change in F caused by a change in c by first calculating the change in $\bar{\psi}$ given the measurement y (locally) and the change in c. This may be accomplished analytically (using the formulas of Ref. 7) or numerically. Similarly a change in $p_c$ results in a change in F which is linear (through e) and is easily calculated. Hence, we may construct the equation

$$\Delta F = \nabla_c F \, \Delta c + \nabla_{p_c} F \, \Delta p_c$$

where $\nabla_{p_c} F = [\nabla_{p_{c_1}} F,,,\nabla F_{p_{c_{NM}}}]$ and $\Delta p_c = [\Delta p_c^1,,\Delta p_c^{NM}]$. If a constant value $p^* = [p_i^*]$ exists then we may set $\Delta p_c^i = p_i^* - p_{c_i}$, i = 1, 2,,,NM, and, hence,

$$(\nabla_c F, \, \nabla_{p_c} F)\begin{bmatrix} c \\ p^* \end{bmatrix} + \nabla_{p_c} F \, p_c = \Delta F$$

In the last equation $p_c$ is the parameter vector of all modes identified locally assuming convergence of the output error algorithm. The solution of $\Delta F = 0$ for $p^*$ and c provides, in the variational sense, the value of c that will result in a constant $p^*$ solution of the output error parameter identification problem for each mode taken over the data base. The next two sections concern the application of the methods presented here to a large space structure control problem. The only results gathered thusfar pertain to the least square estimation problem which serves as the input driver to the parallel output error identifiers and the output error identification process itself.

### III. Simulation of the SEP Array

A simulation of a large space structure has been developed to test the algorithms presented here as well as others. The structure used is the solar electric propulsion (SEP) array which is scheduled for deployment tests in orbit attached to the space shuttle as shown in figure 1. This section is a discussion of the finite element modeling of the array, the simulation, and the sensors and actuators simulated.

The SEP array is illustrated in figure 5. It consists of a canister, an extendable mast, a containment box with cover, and the solar array. The array is folded in the containment box and is lifted during deployment by the mast and box cover. The canister and box are attached to the shuttle payload bay using a carriage assembly not shown. First, the mast will be deployed without the array to test the mast deployment machanism. Then, deployment of the complete configuration will be

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tested. The deployment mechanism is designed so that it may be stopped at 3/4 of full extension or at full extension. At these points dynamics testing may be accomplished. Here, we assume that such tests will be conducted by firing the space shuttle reaction control jets which produce three axis force and moment control at the base of the canister.

The simulation is developed as shown in figure 6. First, a finite element model of the physical structure is developed (Fig. 7) in a form required by the SPAR computer program (Ref. 9). The finite element model involves 436 joints with 1705 interconnecting elements. The elements are bars, tubes, and beams which are included in the SPAR element repetoire. A total of 1350 degrees of freedom (including translation and rotational joint motion components as appropriate) are involved in the SPAR model. A modal simulation was selected because of the large order of the state required to simulate the 1350 coupled degrees of freedom. Mode-frequency data required for the simulation is also provided by the SPAR program (Fig. 6). This mechanization also provides for the use of flight test data reduced to mode-frequency data (dashed blocks of Fig. 6). At present, a maximum of 10 modes can be used in the simulation. However, that limitation can be removed without serious computational penalty.

The simulation also assumes a novel remote optical sensing concept that involves raster scan television coverage of the array (Fig. 8). Optical targets are distributed on the SEP structure and can be identified in the TV raster and registered in digital form at a given sample frequency. The sensors measure target motion but do not include components along the line of sight of each camera to the associated target. A total of 23 targets are assumed placed on the SEP structure which are observed with four cameras thus producing 184 sensor components at each sample time. The location of the cameras is shown in figure 8. (The targets on the containment box are not shown.) All sensor data is generated during the simulation but, for the studies undertaken here, only 16 sensor camera-target combinations are used. These 16 have been selected to render some of the 10 modes essentially unobservable.

In summary, the simulation to be used in evaluating algorithms for the adaptive/learning system is a modal one which uses up to 10 modes of six degree-of-freedom motion of the SEP array. The simulation inputs are the forces and moments applied to the base of the SEP canister. Sensor information is the motion of spots located on the SEP structure as perceived by four cameras located in the space shuttle payload bay.

## IV. Simulation Results

The simulation results gathered thusfar pertain to the first test--mast deployment without the attached array--at 3/4 and full extension. The least squares estimation of modal amplitudes has been developed and tested, in simulation, assuming that $\sigma_{i0}$ functions are the ones computed from SPAR. The sensor targets were three equally spaced along the containment box cover (at the top of the mast) and one spot halfway down the extension axis on the mast. All four cameras were used. This spot-

camera combination renders the seventh SPAR mode essentially unobservable since it is predominantly a torsional excitation of the containment box, the targets on which were not processed. The assumed a priori variance of $\bar{\psi}$ was proportional to the square of the product of frequency of the associated mode and the sample time, $\tau = .1$ sec. For the array simulated the lowest model frequency was .109 Hz and the highest frequency (mode 10) was 7.0 Hz. (This will obviously challenge the sample data parameter identification process). The modes are ordered in increasing frequency and Shannon's sampling theorem is violated for modes 8, 9, and 10. Mode 7 is, as mentioned earlier, unobservable. Also, modes 1 and 2 are very close, frequency-wise, but correspond to vibrations which are totally different spatially. Measurement noise was assumed to be independent of other measurements and Gaussian with zero mean. Studies have been made with measurement noise standard deviation of .03 in. to .003 in. dependent on the range of the target to the camera. For the results, herein, the sensor standard deviation was .01 in. for the closest target which was on the mast and .003 in. for the targets on the containment box cover. Thus, the sensor processing was essentially noise free with respect to motions of about six inches.

Figure 9 shows the spatial distribution of the mast motion along its deployment axis at selected times during a 10 sec. simulation at 3/4 extension. Only the $x_2$ component is shown which is normal to the space shuttle longitudinal axis for $\theta=0$ (Fig. 8). The simulation resulted from a unit excitation of all 10 modes.

The measurements taken from cameras 1 and 2 ($x_1$ and $x_2$ raster components) for the target on the mast are shown in figure 10. Figures 11, 12, and 13 are results taken from the estimation and identification algorithms of the learning system during the simulation. Figure 11 corresponds to controller mode 2, figure 12 to mode 6, and figure 13 to mode 10. In each figure we have the sequence of $\bar{\psi}$ estimates and the identifiers iterative estimate of $A_1$ and $A_2$. Figure 13 also includes the modal estimation error which was the largest of all 10 modes. The length of time for convergence is substantially larger for mode 2 parameters than for mode 6. This is, of course, to be expected since the frequency of mode 2 is much lower than that of 6. Mode 7 results are not presented since $\bar{\psi}_7$ cannot be estimated. For this mode the parameter identification algorithm does not converge using the $\bar{\psi}_7$ sequence as input since it is uncorrelated with the motion. Tests as recommended in reference 5 should be made on the estimated variance of $\bar{\psi}_7$ before passing it to the sequential identification algorithm. The mode 10 results are somewhat unexpected since convergence to the actual parameters does occur although Shannon's sampling theorem is violated. Table 1 shows the actual, initial, and final $A_1$ parameter values for all modes. The B parameter values were not identified since there was no force or moment inputs. Note that under the assumed structure the unobservability of mode 7 does not affect other modes.

## V. Concluding Remarks

This paper has presented an adaptive/learning control system concept for large space structures. The concept employs a priori ground tests and analysis initially to model the structure. The learning process requires in-flight testing to refine the model whenever control system performance degrades to an unacceptable level. Subsystems required for learning involve distributed sensor processing and parameter identification--algorithms which have been proposed in this report. A simulation of a large space structure has been developed to study the performance of the proposed algorithms. The structure simulated was the solar electric propulsion array which is scheduled to be flight tested for deployment and structural dynamics. To this time the only identification performed has been the parameters of a bank of second order difference equations (one for each mode in the learning system model). For the simulations conducted the parameter estimator behaved as expected--converging to the actual parameter values. The behavior of the system with realistic noise levels in the sensors is yet to be evaluated as is the approximation functions "tuning" algorithm which was presented.

## References

1. Montgomery, R. C.; and Johnson, C. R.: The Dual Momentum Control Device for Large Space Systems: An Example of Distributed System Adaptive Control. Proceedings of the 12th Annual Asilomar Conference on Circuits, Systems and Computers, November 1978.

2. Mekel, Ralph; Montgomery, R. C.; and Dunn, H. J.: An Adaptive Learning Control System for Aircraft. Presented at the Eighth Asilomar Conference on Circuits, Systems, and Computers, Pacific Grove, CA, December 3-5, 1974.

3. Hall, Kenneth R.: A Learning Control System Extension to the Modal Control of Large Flexible Rotating Spacecraft. Proceedings of the AIAA Guidance and Control Conference, August 6-8, 1979, Boulder, CO, pp. 549-557.

4. Tsypkin, Y. A.: Self-Learning - What is it? IEEE Transactions of Automatic Control, Vol. AC-13, No. 6, December 1968, pp. 608-612.

5. Dunn, H. J.; and Montgomery, R. C.: A Moving Window Parameter Adaptive Control System for the F-8 DFBW Aircraft. IEEE Transactions on Automatic Control, Vol. AC-22, No. 5, October 1977.

6. Montgomery, R. C.; Mekel, R.; and Nachmias, S.: A Learning Flight Control System for the F-8 FDBW Aircraft. AIAA Guidance and Control Conference Proceedings, Palo Alto, CA, August 1978.

7. Gura, I. A.: An Algebraic Solution of the State Estimation Problem. AIAA Journal, Vol. 7, July 1969, pp. 1242-1247.

8. Mendel, J. M.: Discrete Techniques of Parameter Estimation: The Equation Error Formulation. New York, Marcel Dekker, Chapter 4, 1973.

9. Whetstone, W. D.: SPAR Structural Analysis System Reference Manual, Volumes 1, 2, and 3. NASA CR 158970-1, December 1978.

Table 1  Initial, Final, and Actual $A_1$ Parameter Values for the Simulation

| Mode | Initial | Final | Actual |
|------|---------|-------|--------|
| 1 | .9953 | 1.9953 | 1.9953 |
| 2 | .9952 | 1.9902 | 1.9952 |
| 3 | .9855 | 1.9852 | 1.9855 |
| 4 | .3470 | 1.3443 | 1.3470 |
| 5 | .2637 | 1.2637 | 1.2637 |
| 6 | -1.5482 | -.5482 | -.5482 |
| 7* | -1.9849 | 1.8561 | -.9849 |
| 8 | -2.8423 | -1.8412 | -1.8423 |
| 9 | -2.9973 | -2.0291 | -1.9973 |
| 10 | .7520 | 1.7413 | 1.7520 |

*Unobservable
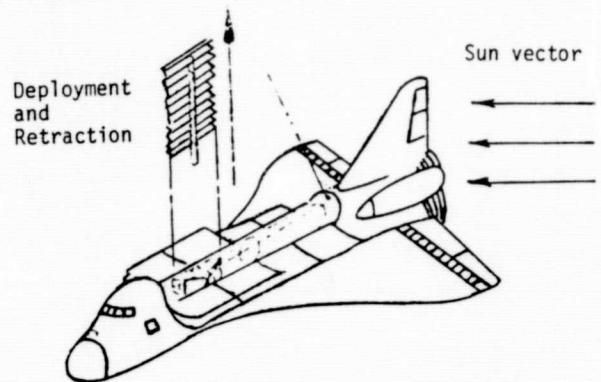


Figure 1.- Solar Array Deployment from the Space Shuttle.
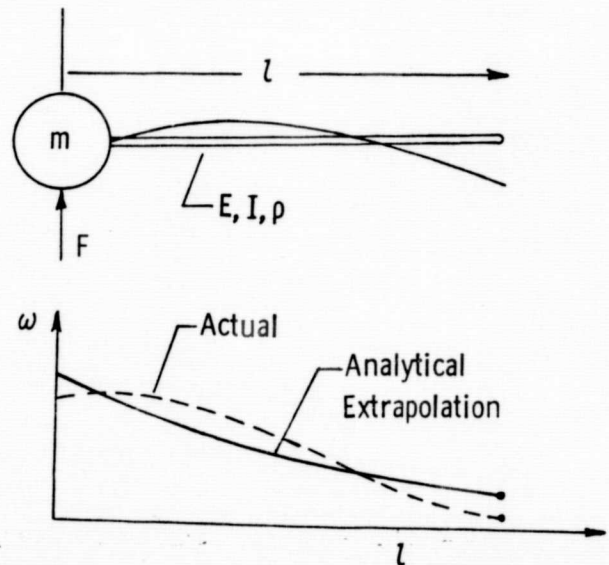


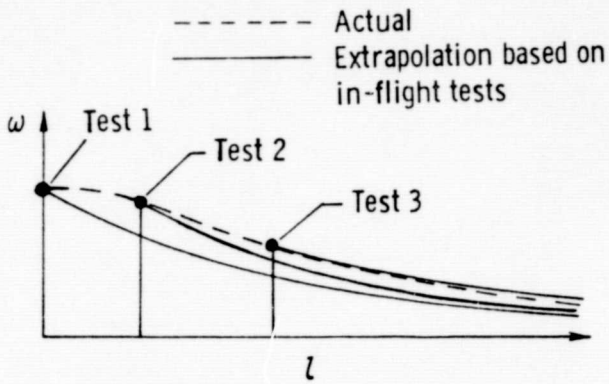Figure 2.- Simplified model of solar array deployment showing the variation of frequency with array length.

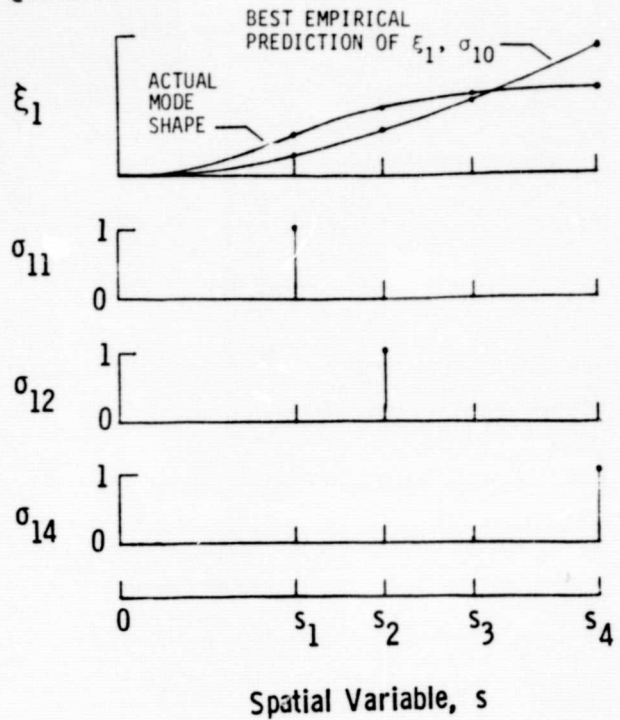Figure 3.- Learning system operation.



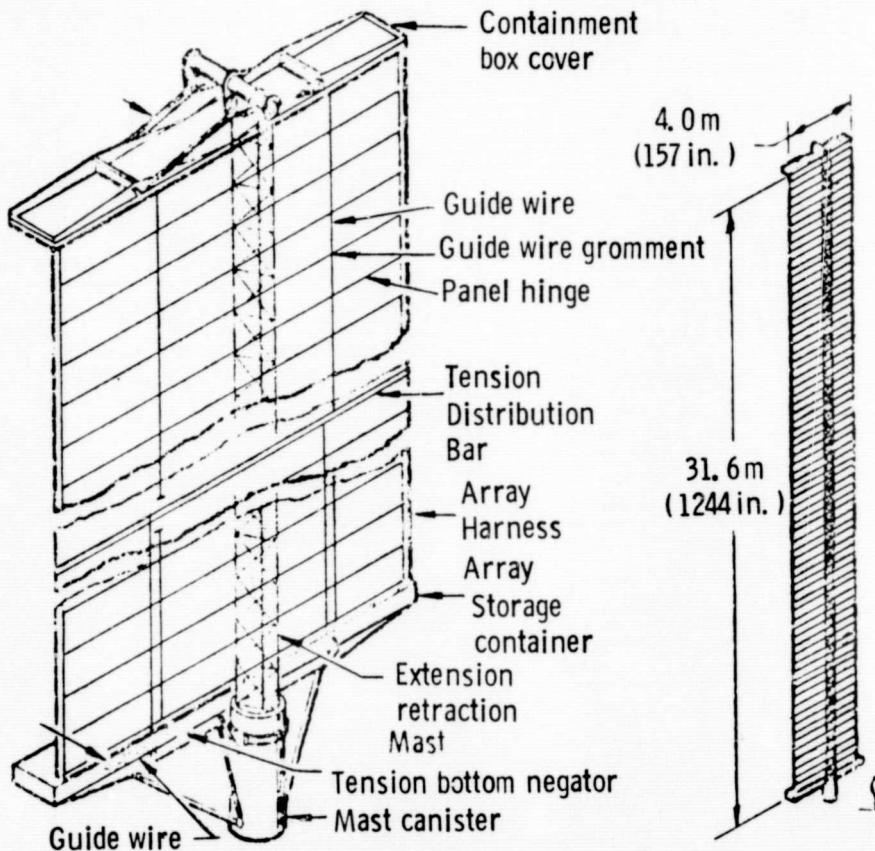Figure 4.- Sketch illustrating the selection of the $\sigma_{ij}$ functions.
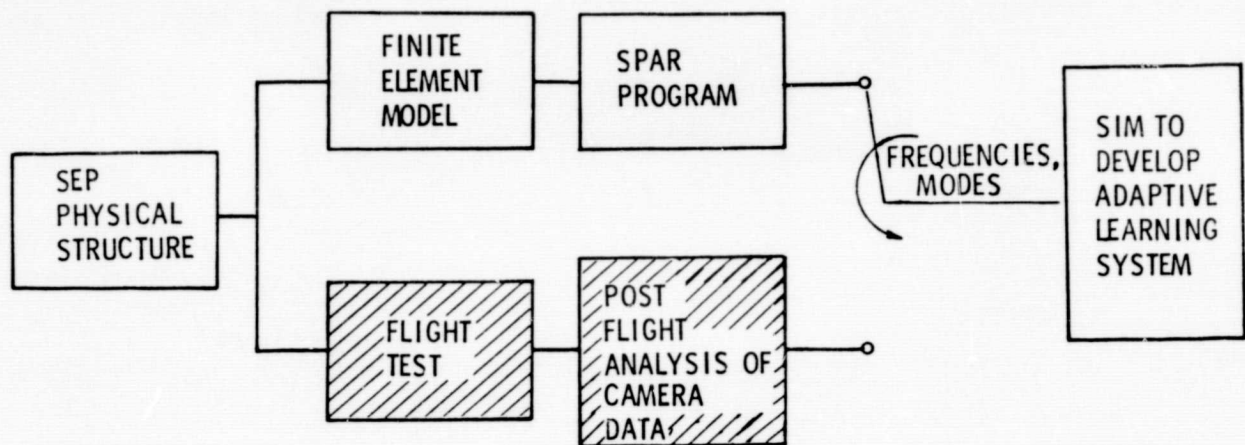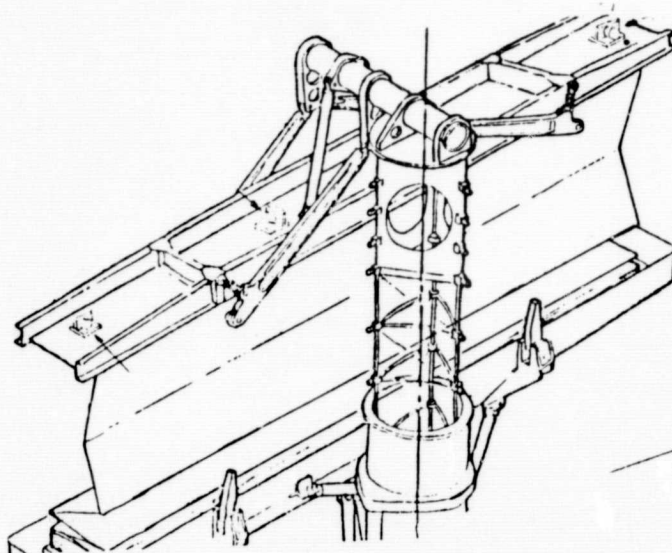


Figure 5.- Solar array geometry.

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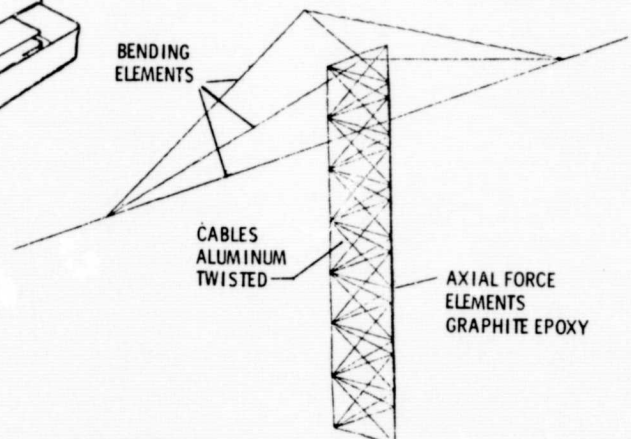
Figure 6.- Simulation Data Flow.

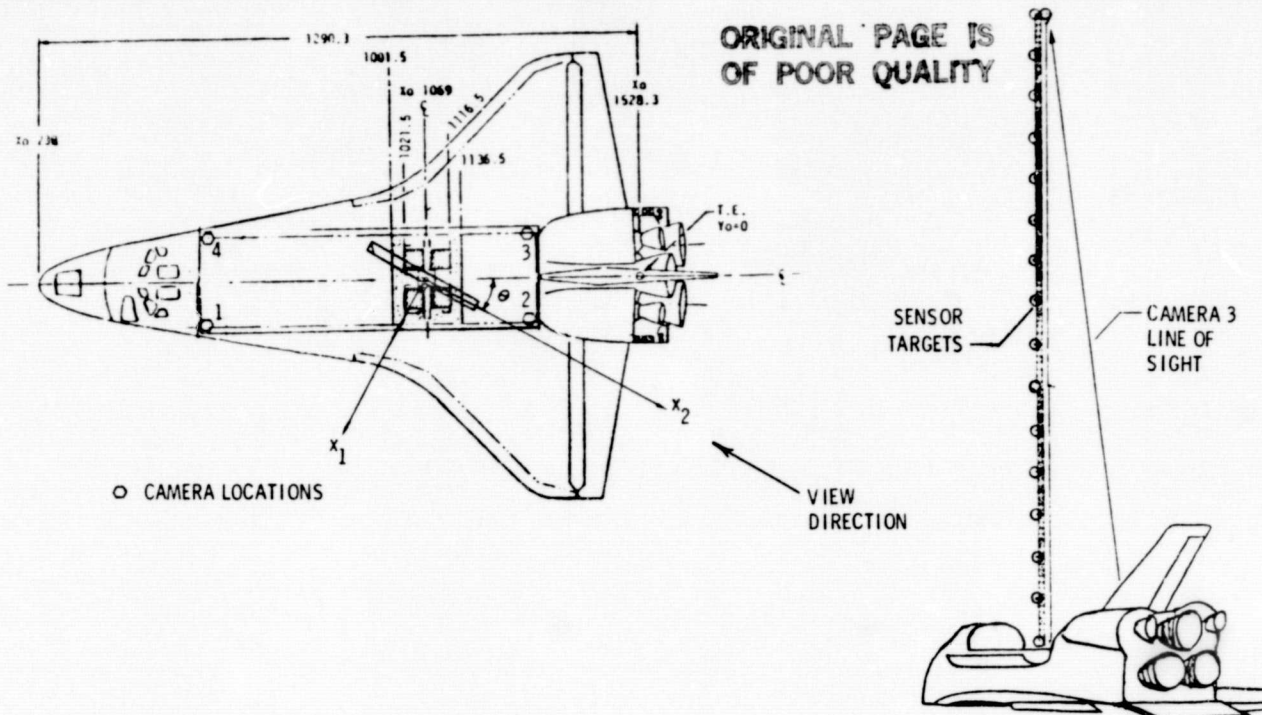## PHYSICAL MODEL

## SPAR MODEL



Figure 7.- SEP Modeling.

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ORIGINAL PAGE IS
OF POOR QUALITY

O CAMERA LOCATIONS

SENSOR TARGETS

CAMERA 3 LINE OF SIGHT

VIEW DIRECTION

Figure 8.- SEP Remote Sensing Concept.

$X_2$, IN
TIME= 0.0

$X_2$, IN
TIME= 2.5

$X_2$, IN
TIME= 5.0

$X_2$, IN
TIME= 7.5

$X_2$, IN
TIME= 10.0

DEPLOYMENT AXIS X3 - IN.
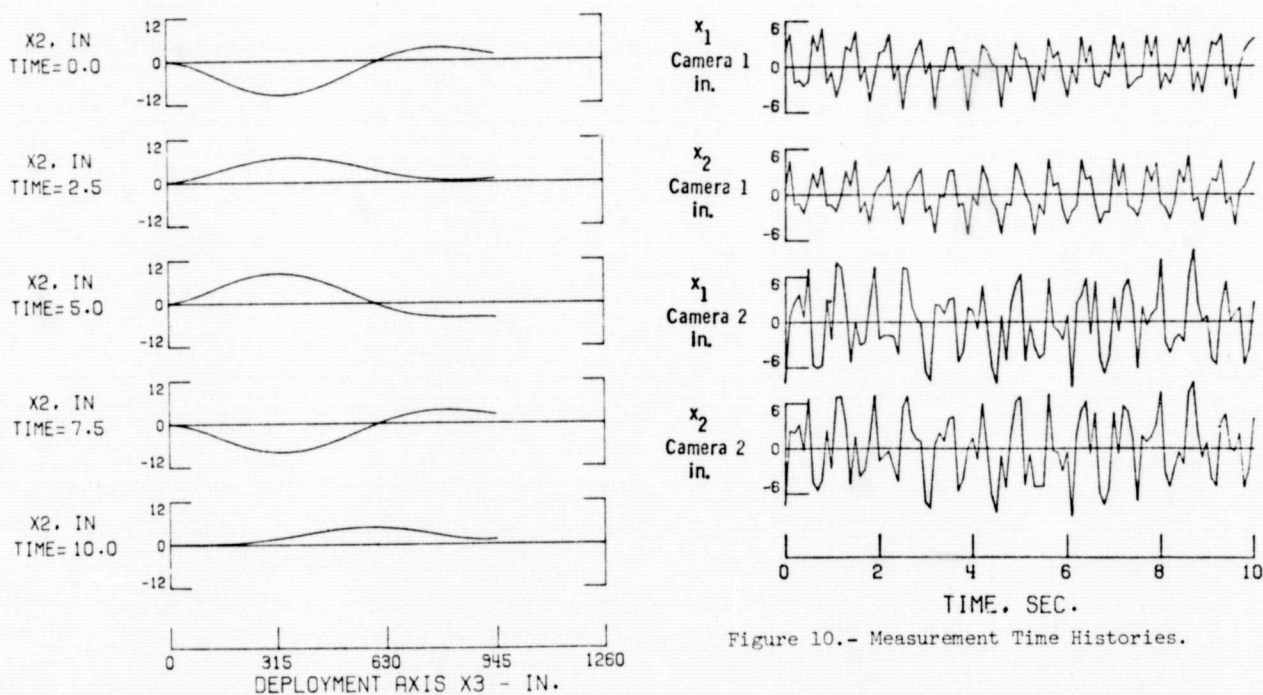
Figure 9.- Mast Deflection Along the
Deployment Axis.

$x_1$ Camera 1 in.

$x_2$ Camera 1 in.

$x_1$ Camera 2 in.

$x_2$ Camera 2 in.

TIME, SEC.

Figure 10.- Measurement Time Histories.

8
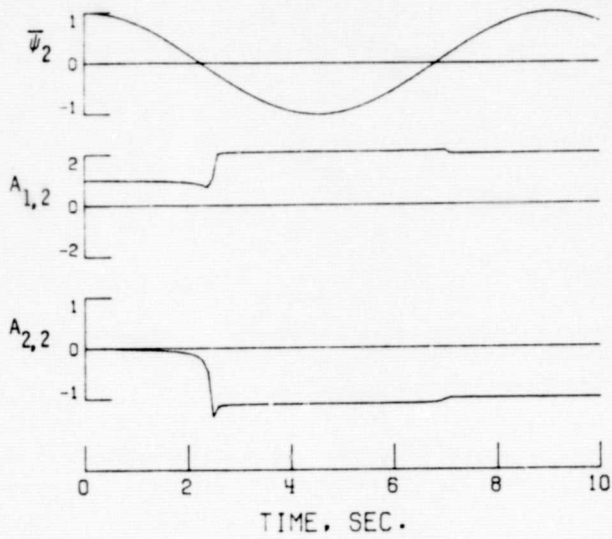
Figure 11.- Mode 2 Estimation and
Identification Results.

Figure 13.- Mode 10 Estimation and
Identification Results.

Figure 12.- Mode 6 Estimation and
Identification Results

9

# APPENDIX 2

## ON-LINE STRUCTURAL PARAMETER IDENTIFICATION

ON-LINE STRUCTURAL PARAMETER IDENTIFICATION

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## Abstract

Algorithms are presented for on-line parameter identification of structural dynamic systems. As an example, they are used to calculate the parameters of a modal model of a flexible beam. The algorithms are tested using hardware consisting of a 12 ft. beam with four voice coil actuators and nine noncontacting displacement sensors. They are programmed in a CDC Cyber 175 digital computer which provides input command signals for the actuators, reads the sensor data, and processes the algorithm to calculate consistent estimates of the modal parameters of the beam. Experimental results are compared with those of simulation analysis.

## I. Introduction

The control of large flexible space structures will depend on knowledge of the structural dynamics of the objects involved. Since it is unlikely that this information can be determined analytically or from ground testing, adaptive control schemes have been proposed [1-3]. One of these schemes [3], an indirect approach to adaptive control, is shown in the block diagram of Figure 1. This approach is based upon a modal decomposition of the dynamic response of the flexible structure and was designed to make use of the parallel processing features of modern microcomputers. One of the significant problems involved in the implementation of this indirect approach to the adaptive control of large space structures is that of real-time, or on-line, identification of the parameters of the large, flexible structural components. Satisfactory performance of the parallel structure identification technique indicated in Figure 1 can be achieved only when the approximation functions noted in the Figure correspond to the natural modes of the flexible structure [3]. This paper presents a new technique for using on-line measurements to improve estimates of the approximation functions, and combines this approximation function up-dating approach with the modal parameter identification algorithm presented in [3].

Time series analysis for the purpose of parameter identification for multi-input, multi-output systems has received a good deal of attention [4-7] in recent years under various assumptions regarding the parameter matrices appearing in these models and on the nature of assumed random disturbances.

Reference [4] contains a thorough treatment of least-squares and instrumental variable approaches to the problem of generating consistent estimates of parameters in univariate and multivariate models. The instrumental-variable approach to system identification is examined in [5] for a single-input,

single-output system where measurements of the system input and noisy measurements of the system output are available. It is shown following [4] that the instrumental variable method yields consistent parameter estimates. In [6] a review of a number of methods of estimation of parameters in multivariate autoregressive moving average models is presented. Among the techniques considered are generalized least-squares, instrumental variable methods, and maximum likelihood estimation procedures. A "limited information" estimation is proposed in [6] and, for autoregressive models, is shown to produce parameter estimates asymptotically identical to conditional maximum likelihood estimates while requiring reduced computational effort. In [7] a statistically efficient least-squares procedure is developed for estimating the natural frequencies and damping parameters of a system characterized by a lumped mass-spring-damper model under stationary random forcing functions. The only available measurement is a sequence of displacements subject to additive noise. Although the formulation in [7] does not include the treatment of known forcing functions nor the inclusion of multi-output measurements, numerical results presented demonstrate that the derived least-squares estimates are a good approximation to maximum-likelihood estimates.

In the following section the structural parameter identification problem is formulated as a problem of calculating the parameters of a linear discrete-time multi-input, multi-output system using noisy on-line measurements. In section 3 we combine the identification algorithm of [3] with the new approach for updating the approximation functions. The NASA-Langley structural dynamics and control test facility is then described in Section 4. Experimental results presented in Section 5 are obtained by applying the proposed on-line identification procedure to a flexible beam assuming that the approximation functions are those taken from an a priori simulation or from a ground test of the flexible structure. A comparison with the results of digital computer simulation studies is also included.

## II. Problem Formulation

The scope of the identification problem can be examined from the following formulation: the motion $w(s,t)$ of a flexible structure may be represented by

$$w(s,t) = \sum_{i=1}^{NM} q_i(t, p_i)\, \xi_i(s) + v(s,t) \qquad (1)$$

where s denotes the spatial variable and t represents time. $\xi_i$ is a finite set of NM spatial functions, for example, the first NM eigenfunctions in a modal expansion, $q_i(t, p_i)$ are time-varying coefficients of $\xi_i$, or modal amplitudes, and $v(s,t)$ represents those higher-order terms not appearing in the finite sum. The parameter vector $p_i$

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represents a set of parameters in a difference
equation governing the time-evolution of the
$\hat{q}_i(t_k, p_i)$ , for example

$$\hat{q}_i(k+1) = A_{1i}\hat{q}_i(k) + A_{2i}\hat{q}_i(k-1) + \tilde{B}_{1i}\hat{F}_i(k) + \tilde{B}_{2i}\hat{F}_i(k-1) \quad (2)$$

where $\hat{q}_i(k) = \hat{q}_i(t_k, p_i)$ and
$p_i = [A_{1i}, A_{2i}, \tilde{B}_{1i}, \tilde{B}_{2i}]$ . For later
reference we designate the components of $p_i$ as
follows: $p_i^1 = A_{1i}$, $p_i^2 = A_{2i}$, $p_i^3 = \tilde{B}_{1i}$
$p_i^4 = \tilde{B}_{2i}$. If there are NA actuators applying
the set of force inputs $U_i$, $i = 1, \ldots,$ NA,
to the flexible structures, then the modal forces
$F_i$ appearing in (2) are given by

$$F_i(k) = \sum_{j=1}^{NA} m_{ij} U_j(k) \quad (3)$$

The measurements available to the
controller are a set of sampled values of the
motion at the points $S_j$, $j = 1, 2, \ldots, NS$.

$$y_j(k) = \sum_{i=1}^{NM} \xi_i(s_j) q_i(t_k, p_i) + v(s_j, t_k) + \tilde{e}_j(k) \quad (4)$$

where NS is the number of sensor locations and
$\tilde{e}_j$ represents the measurement error.

If we write the set of measurements as the
vector $y(k)$, the set of modal amplitudes as the
vector $q(k)$, the set of actuator and modal forces
as the vectors $U(k)$ and $F(k)$, respectively,

$$y(k) = \begin{bmatrix} y_1(k) \\ \vdots \\ y_{NS}(k) \end{bmatrix} \quad q(k) = \begin{bmatrix} q_1(k) \\ \vdots \\ q_{NM}(k) \end{bmatrix} \quad F(k) = \begin{bmatrix} F_1(k) \\ \vdots \\ F_{NM}(k) \end{bmatrix} \quad U(k) = \begin{bmatrix} U_1(k) \\ \vdots \\ U_{NA}(k) \end{bmatrix} \quad (5)$$

then we have the following model

$$q(k+1) = A_1 q(k) + A_2 q(k-1) + \tilde{B}_1 M U(k) + \tilde{B}_2 M U(k-1)$$

$$y(k) = H q(k) + w(k) \quad (6)$$

The identification problem is the on-line calcul-
ation of the elements of the matrices appearing
in (6), where $A_1, A_2, \tilde{B}_1, \tilde{B}_2$ are diagonal mat-
rices with main diagonal elements $A_{1i}, A_{2i}, \tilde{B}_{1i}, \tilde{B}_{2i}$,
$i = 1, \ldots,$ NM respectively, M is an NM × NA
matrix and H is an NS × NM matrix whose columns
are linearly independent in the modal model. The
error vector $w(k)$ appearing in (6) is an NS-vector
whose components $w_i(k)$ are the sum of the measure-
ment errors $\tilde{e}_i(k)$ and the model errors $v(s_i, t_k)$

$$w_i(k) = \tilde{e}_i(k) + v(s_i, t_k), \quad i = 1, \ldots, NS \quad (7)$$

If we define $\tilde{B}_i M = B_i, i = 1, 2$, then the model (6)
simplifies to

$$q(k+1) = A_1 q(k) + A_2 q(k-1) + B_1 U(k) + B_2 U(k-1)$$

$$y(k) = H q(k) + w(k) \quad (8)$$

The available measurements are the series of act-
uator forces $\{U(k)\}$ and sensor measurements
$\{y(k)\}$.

## III. On-Line Parameter Identification

The adaptive control system in Figure 1 was
designed to take advantage of the parallel pro-
cessing capability of modern microcomputers. Unfor-
tunately, the parallel architecture is correct only
in case the approximation functions in the Figure
are the natural modes of the structural system.
Because of the immense computational problem posed
by computing the mode shapes, it is unrealistic
to assume that the control outputs, pole-zero
parameters, and mode shapes can be computed at the
same frequency. The philosophy adopted herein is to
assume that the control outputs are computed at the
highest frequency, parameters that define the pole-
zero characteristics at a lower frequency, and the
mode shapes are updated when possible. This sec-
tion first summarizes an algorithm, taken from
reference[3], for identifying the parameters that
define the pole-zero characteristics of the system.
Then an original method for updating the approx-
imation functions in Figure 1 is presented.

### 1. Identification of the Pole-Zero Characteristics

Here it assumed that the approximation functions
are given and that the parallel architecture of
Figure 1 is valid. Under these assumptions the
problem is reduced to one of identifying
$p_i = (A_{1i}, A_{2i}, \tilde{B}_{1i}, \tilde{B}_{2i})$ of equation (2). The
technique of reference[3] is followed here. It
is an output error formulation wherein the error
is given by

$$e_i(k-1) = q_i(k-1) - A_{1i} q_i(k-2) - A_{2i} q_i(k-3) - \tilde{B}_{1i} F_i(k-2) - \tilde{B}_{2i} F_i(k-3) \quad (9)$$

and the formula for updating the estimates of the
vector $p_i$ is

$$p_i(k) = p_i(k-1) + e_i(k-1) \cdot \begin{bmatrix} W_1 q_i(k-2) \\ W_2 q_i(k-3) \\ W_3 F_i(k-2) \\ W_4 F_i(k-3) \end{bmatrix} \quad (10)$$

In equation (10) the weights W should be selected
to satisfy the requirement that

$$W_1 q_i^2(k-2) + W_2 q_i^2(k-3) + W_3 F_i^2(k-2) + W_4 F_i^2(k-3) < 2 \quad (11)$$

to insure stability of the identification algorithm.

### 2. Updating of the Approximation Functions

The updating approach is based upon rewriting
the second equation in (8) as

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$$\underset{\sim}{g}(k) = H^{+} y(k) - H^{+} w^{-}(k) \qquad (12)$$

where

$$H^{+} = (H^{T}H)^{-1} H^{T} \qquad (13)$$

and $(\cdot)^{T}$ denotes matrix transpose. The inverse appearing in (13) exists since the columns of $H$ are linearly independent. Combining (12) with the first equation in (3) yields

$$y(k+1) = M_{1} y(k) + M_{2} y(k-1) + N_{1} U(k) + N_{2} U(k-1) + n(k+1) \qquad (14)$$

where

$$M_{i} = H A_{i} H^{+} , \quad i = 1,2 \qquad (15a)$$

$$N_{i} = H B_{i} , \quad i = 1,2 \qquad (15b)$$

$$n(k+1) = w^{-}(k+1) - M_{1} w^{-}(k) - M_{2} w^{-}(k-1) \qquad (15c)$$

It is interesting to note that if NS=NM, ie. if we have selected a number of measurements equal to the order of the modal model to be identified, then (15a) may be written as

$$M_{i} [h_{1}, \ldots, h_{NM}] = [h_{1}, \ldots, h_{NM}] A_{i}, \quad i = 1,2 \qquad (16)$$

where $h_{j}$ denotes the $j$th column vector of $H$. Since in the modal model $A_{i}, i = 1,2$, is a diagonal matrix with main diagonal elements $a_{ji}, j = 1, \ldots,$ NM, it is clear from (16) that $a_{ji}$ is an eigenvalue of $M_{i}$ and that column vector $h_{j}$ is the corresponding eigenvector.

Hence, a procedure for determining the modal parameters is as follows: 1) Use measurement data to determine the matrices $M_{1}$, $M_{2}$, $N_{1}$, $N_{2}$ in (14). 2) Calculate the eigenvalues and eigenvectors of $M_{1}$ and $M_{2}$ to determine $A_{1}, A_{2},$ and $H$. 3) Solve for $B_{1}$ and $B_{2}$ from (15b). Since the eigenvectors of $M_{i}$ are linearly independent but are, in general, non-unique, the identification of $H$ and hence, of $B_{1}$ and $B_{2}$ is also non-unique. This, however, is not surprising given the form of the modal model (3).

The matrices in (3) may be determined by taking a sequence of measurement data $y(k+1), y(k+2),$ $\ldots, y(k+N+2)$ and ordering these measurements as follows:

$$y^{T}(k+3) = \left[ y^{T}(k+2) \vdots y^{T}(k+1) \vdots U^{T}(k+2) \vdots U^{T}(k+1) \right] \begin{bmatrix} M_{1}^{T} \\ M_{2}^{T} \\ N_{1} \\ N_{2} \end{bmatrix} + n^{T}(k+3)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$y^{T}(k+N+2) = \left[ y^{T}(k+N+1) \vdots y^{T}(k+N) \vdots U^{T}(k+N+1) \vdots U^{T}(k+N) \right] \begin{bmatrix} M_{1}^{T} \\ M_{2}^{T} \\ N_{1}^{T} \\ N_{2}^{T} \end{bmatrix} + n^{T}(k+N+2) \qquad (17)$$

Equation (17) may be written in more compact notation as

$$Y(N) = S(N) M^{T} + V(N) \qquad (18)$$

where

$$S(N) = \left[ Y(N-1) \vdots Y(N-2) \vdots U(N-1) \vdots U(N-2) \right]$$

$$Y(K) = \begin{bmatrix} y^{T}(K-(N-3)) \\ \vdots \\ y^{T}(K+2) \end{bmatrix} \qquad U(L) = \begin{bmatrix} U^{T}(L-(N-3)) \\ \vdots \\ U^{T}(L+2) \end{bmatrix} \qquad (19)$$

$$K = k+N, k+N-1, k+N-2 \qquad L = k+N, k+N-1$$

and

$$V(N) = \begin{bmatrix} n^{T}(k+3) \\ \vdots \\ \vdots \\ n^{T}(k+N+2) \end{bmatrix} \qquad M^{T} = \begin{bmatrix} M_{1}^{T} \\ M_{2}^{T} \\ N_{1}^{T} \\ N_{2}^{T} \end{bmatrix} \qquad (20)$$

Note that the $Y(K)$ are NxNM matrices and that the $U(L)$ are NxNA matrices. The $j$th column $y^{j}(K)$ of matrix $Y(K)$ comprises a sequence of measurements taken at the $j$th measurement point

$$y^{j}(K) = \begin{bmatrix} y_{j}(K-(N-3)) \\ \vdots \\ y_{j}(K+2) \end{bmatrix} \qquad (21)$$

Now, if $N > 2(NM + NA)$, a least-squares solution $\hat{M}^{T}$ to (18) may be obtained as follows:

$$\hat{M}^{T} = S^{+} Y(N) \qquad (22)$$

where $S^{+}$ denotes a pseudo-inverse that is readily computed [8,9].

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However, since the disturbances $V(N)$ are contemporaneously correlated with the elements of $S(N)$ the ordinary least-squares procedure will yield asymptotically biased estimates [-]. A technique for providing consistent estimates is the instrumental variable method [4,5]. If the lagged output and input measurements $y(k), y(k-1), U(k), U(k-1),\ldots, y(k+N-1), y(k+N-2), U(k+N-1), U(k+N-2)$ are chosen as candidate instrumental variables, we form the array $Z^T(N)$,

$$Z^T(N) = \begin{bmatrix} y(k) & \cdots & y(k+N-1) \\ y(k-1) & \cdots & y(k+N-2) \\ U(k) & \cdots & U(k+N-1) \\ U(k-1) & \cdots & U(k+N-2) \end{bmatrix} \quad (23)$$

From the definition of the instrumental variable matrix $Z^T(N)$ in (23) and measurement error matrix $V(N)$ in (20) and (15) it is clear that the corresponding columns of $Z(N)$ and rows of $V(N)$ are uncorrelated.

If we premultiply (18) by $Z^T(N)$ we obtain

$$Z^T(N)Y(N) = Z^T(N)S(N)M^T + Z^T(N)V(N) \quad (24)$$

The instrumental variable estimate has the form

$$\hat{M}^T = \left[ Z^T(N)S(N) \right]^+ Z^T(N)Y(N) \quad (25)$$

A computational procedure based upon the foregoing discussion is as follows:

1. Assume the model to be identified is of order NM and, from a set of NM measurements, form the $N \times 2(NA + NM)$ matrix $S(N)$ in (18).

2. Form the instrumental variable matrix $Z(N)$ and the solution (25).

3. Compute the eigenvalues of the $\hat{M}_1, \lambda_{1i},\ldots,\lambda_{NMi}$ and the eigenvectors of the $\hat{M}_2, v_{1i},\ldots v_{NMi}, i=1,2$

4. Compute the solution $\hat{B}_i^j$, to

$$H\hat{B}_i^j = \hat{N}_i^j, \quad \begin{array}{l} i = 1,2 \\ j = 1,\ldots, NA \end{array} \quad (26)$$

From these steps the identified modal model for mode $j$ is

$$\hat{q}_{cj}(k+1) = \lambda_{j1}\hat{q}_{cj}(k) + \lambda_{j2}\hat{q}_{cj}(k-1) + \hat{B}_{j1}U(k) + \hat{B}_{j2}U(k-1) \quad (27)$$

$$y(k) = [v_1, v_2,\ldots, v_{NM}]\hat{q}_c(k)$$

The solution (25) can also be expressed in an iterative fashion [5,6].

It is interesting to note that this procedure also provides an independent estimate of the modal pole-zero parameters. However, because the foregoing procedure requires the computation of a matrix pseudo-inverse and eigenvectors and eigenvalues of the computed matrices $M_1$ and $M_2$ it is anticipated that the updating of the approximation functions will take place at a lower frequency than the computation of the modal parameters from the output error algorithm.

### IV. Experimental Test Facility

The algorithms presented above have been tested by digital computer simulation and by application to the NASA-Langley structural dynamics and control experimental apparatus. The experimental test facility is shown in the photograph of Figure 2 and in the diagram of Figure 3. The apparatus comprises a uniform rectangular-section 12 ft. long flexible beam, four electromagnetic actuators, nine non-contacting sensors to measure deflection of the beam at various locations, and four strain gage load cells at the actuator attachment points. Although the sensors and actuators can be placed at arbitrary stations along the beam, the locations of the sensors and actuators used in the experiments reported below are those shown schematically in Figure 5.

The beam is supported using two cables that are attached to the building. These are used to off load the bearings of the four actuators shown in the photograph of Figure 2. The actuators are Unholtz-Dickie Model 1 Shakers which have a stroke of 1" and a maximum force output of 50 lbf. They are attached to the beam with spring steel flexure couplers that permit transverse vibrations in the horizontal plane while suppressing torsional motions. The couples are attached to strain gage load cells which were manufactured in-house at LaRC. The load cells are then hard mounted to the beam. An additional purpose of the cables is to maintain a horizontal loading condition by preventing droop of the end of the actuator attachment assembly. On the side of the beam opposite the actuators are noncontacting deflection sensors. These are Kaman KD-2300 series system sensors. The range of the Kaman sensors is 2 in. The resolution of the sensors is .001 in.

The components mentioned above are interfaced to the LaRC Cyber 175 real time computer system as indicated in Figure 4. The Cyber computer reads the sensor data, processes the algorithm to calculate estimates of the parameters of the beam, and provides input command signals for the actuators. A high gain analog electrical circuit is used to assure that command forces from the control system resident on the Cyber computer are realized at the load cells on the beam. For that reason the force commands from the Cyber computer are sent to the EAI 680 analog computer where the circuit adjusts the current delivered to the actuators so that the commanded load occurs at the load cells. The maximum commanded load from the Cyber is set to 10 lbf whereas the maximum value

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This indicates a need for a set of convergence criteria tests, similar to those of 11 , to indicate when a set of calculated modal parameters have been identified to sufficient accuracy. Thus far the parameter identifiers have used a set of approximation functions obtained from the SPAR program. The performance of the approximation function updating algorithm and the performance of the indirect adaptive closed-loop feedback system are currently under evaluation.

#### References

1. Montgomery, R. C. and Johnson, C. R.: "The Dual Momentum Control Device for Large Space Systems: An Example of Distributed System Adaptive Control," Proceedings of the 12th Annual Asilomar Conference on Circuits, Systems and Computers, November 1978.

2. Hall, Kenneth R., "A Learning Control System Extension to the Modal Control of Large Flexible Rotating Spacecraft," Proceedings of the AIAA Guidance and Control Conference, August 6-8 1979, Boulder, CO, pp. 549-557

3. Montgomery, R. C. and Thau, R. E., "Adaptive and Learning Control of Large Space Structures," Presented at the 1980 AIAA Guidance and Control Conference, June, 1980.

4. Goldberger, A. S., Econometric Theory, John Wiley and Sons, Inc. 1964, Chapter 7.

5. Wong, K. Y. and Polak, E., "Identification of Linear Discrete Time Models Using the Instrumental Variable Method," IEEE Trans. Vol. AC-12, pp. 707-718, Dec. 1967.

6. Kashyap, R. L. and Nasburg, R. E. "Parameter Estimation in Multivariate Stochastic Difference Equations," IEEE Trans. Vol. AC-19, No. 6, December 1974, pp. 784-797.

7. Gersch, W. and Foutch, D. S., "Least Squares Estimates of Structural System Parameters Using Covariance Function Data," IEEE Trans. on Automatic Control, Vol. AC-19, No. 6, Dec. 1974, pp 898-903.

8. Armstrong, E. S., "ORACLS-A System for Linear-Quadratic Gaussian Control Law Design," NASA Technical Paper 1106, April 1978.

9. Wilkinson, J. H. and Reinsch, C., Handbook for Automatic Computation, Vol. II Linear Algebra, Springer-Verlag, 1971.

10. Whetstone, W. T., SPAR Structural Analysis System Reference Manual. Volumes 1, 2, and 3. NASA CR-158970-1. December 1978.

11. Dunn, H. J. and Montgomery, R. C., "A Moving Window Parameter Adaptive Control System for the F8-DFBW Aircraft," IEEE Trans. Vol. AC-22, No. 5, October, 1977.
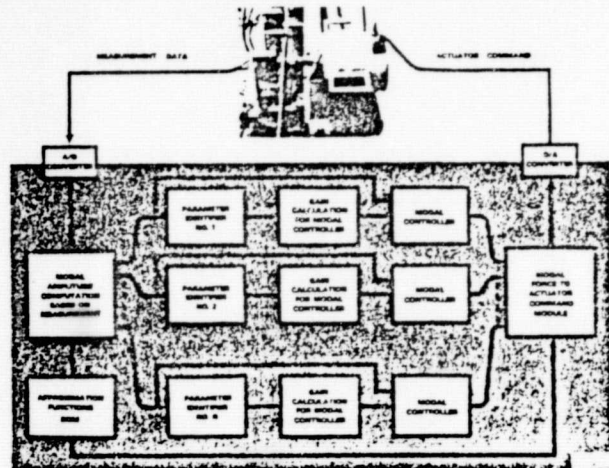
DISTRIBUTED ADAPTIVE CONTROL



Figure 1.- Schematic diagram of distributed adaptive control system.
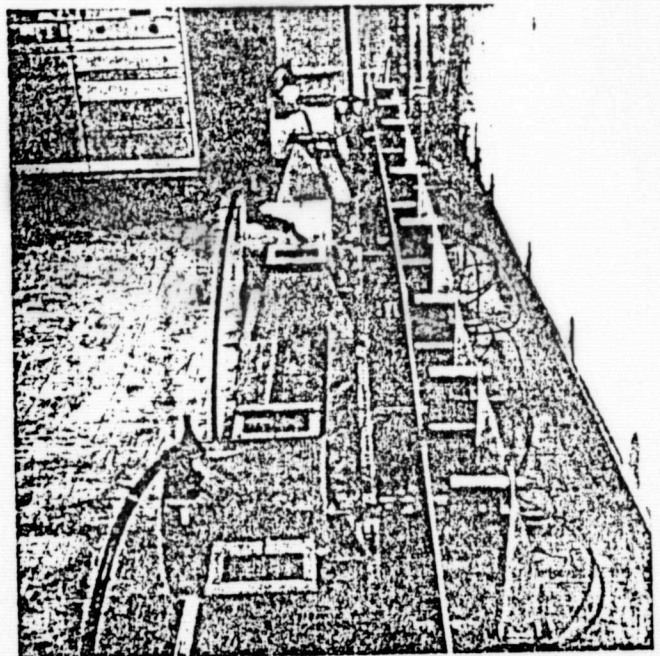


Figure 2.- Experimental beam facility.
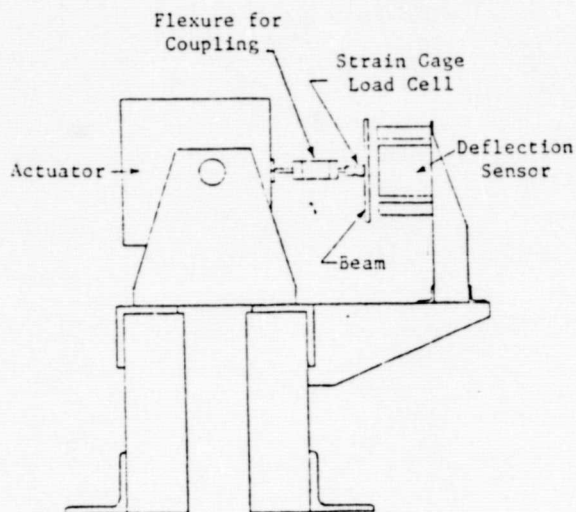
Figure 3.- End view of experimental apparatus.

SIGNAL DISTRIBUTION
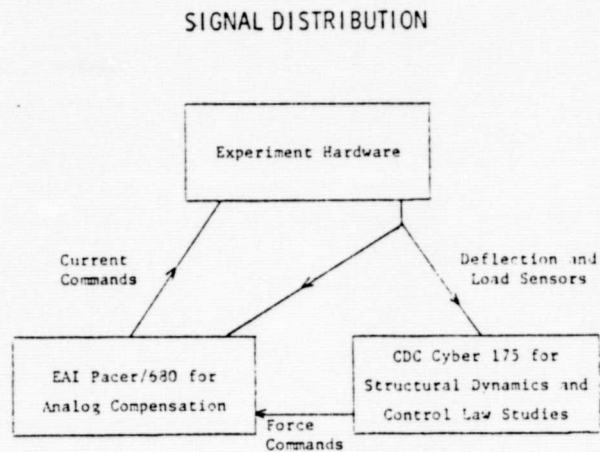


Figure 4.- Experimental signal distribution.
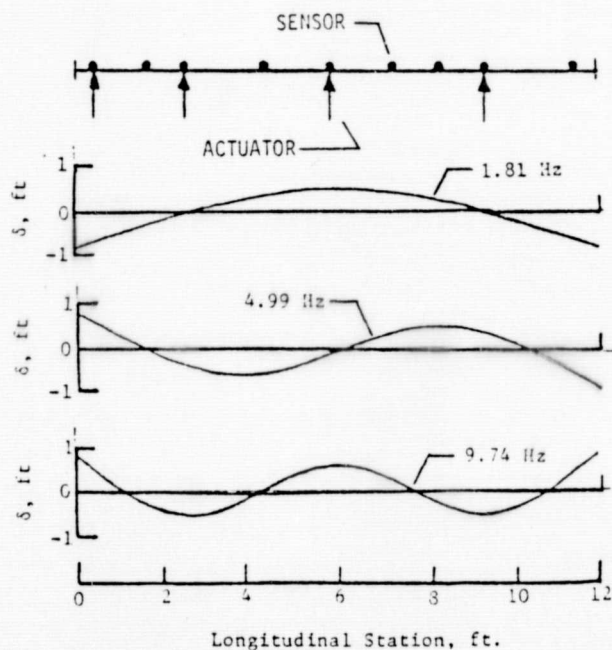


Figure 5.- First, second, and third flexible
modes of the beam.

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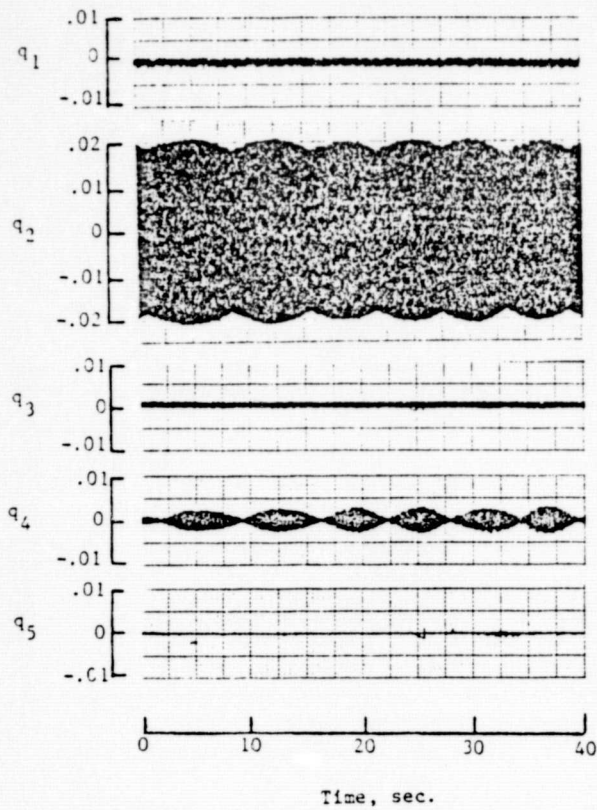
Figure 6.- Modal decomposition of second
flexible mode--unforced response.



Figure 7.- Modal decomposition of third
flexible mode--unforced response.



Figure 8.- Simulated sensor response.
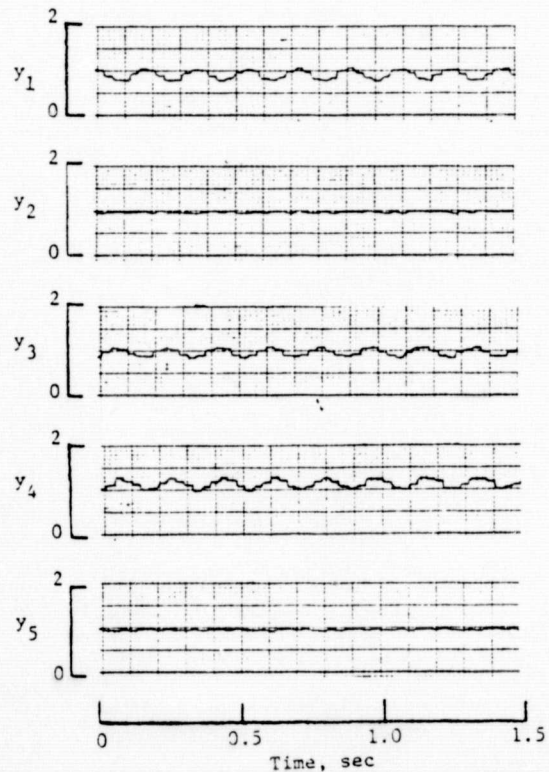


Figure 9.- Experimental sensor response.

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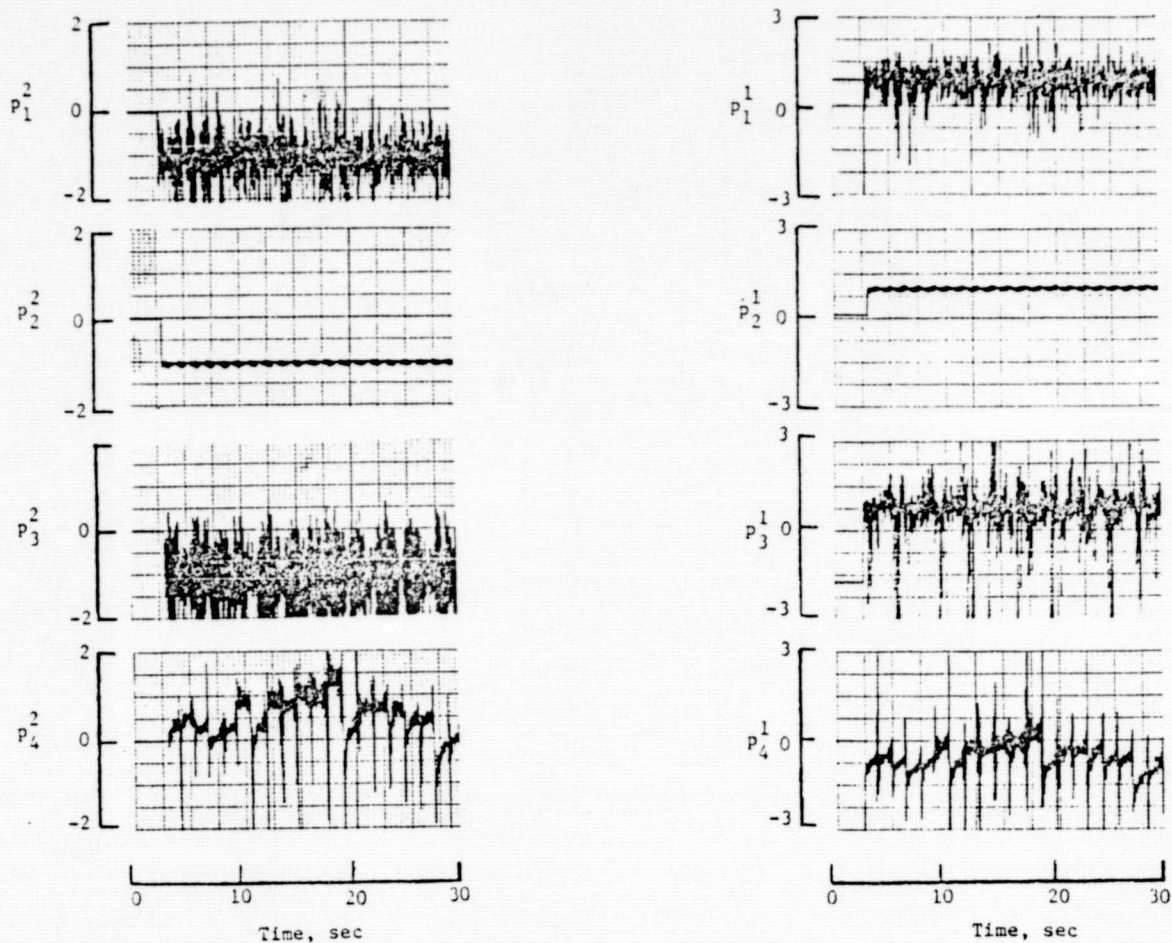
Figure 10.- Output of parameter identifiers, mode 2 excited, (experimental).
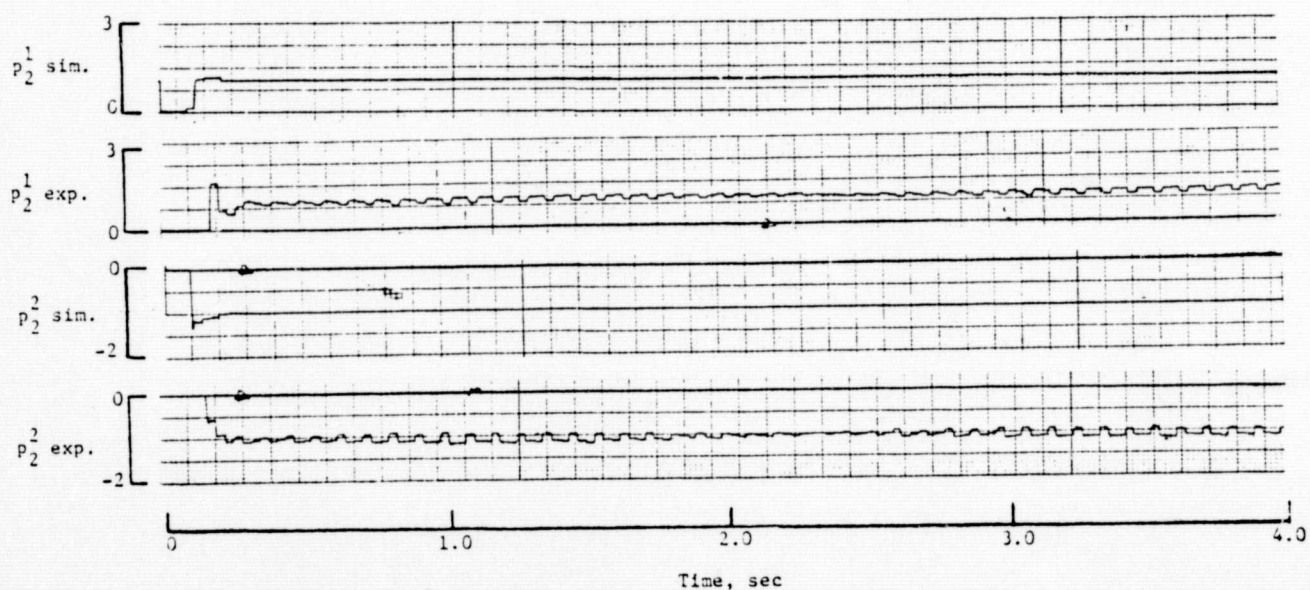


Figure 11.- Identification of mode 2 parameters (simulated and experimental).

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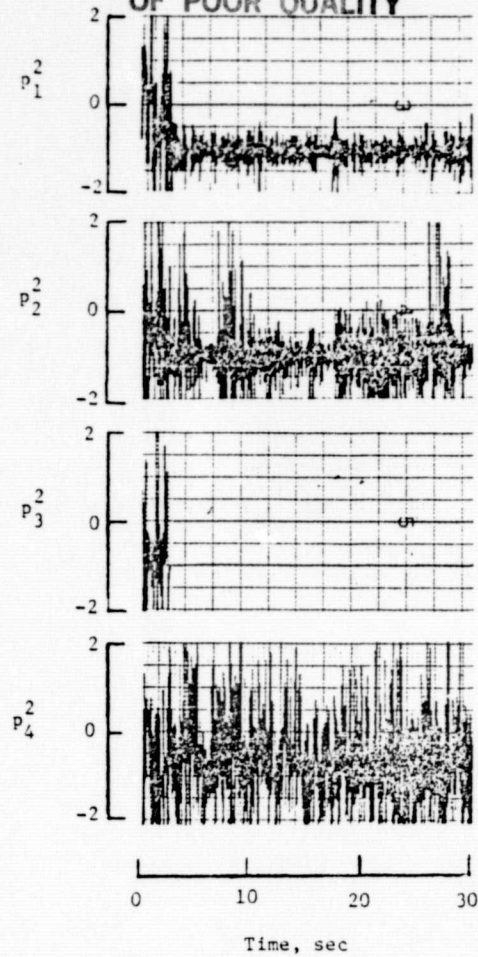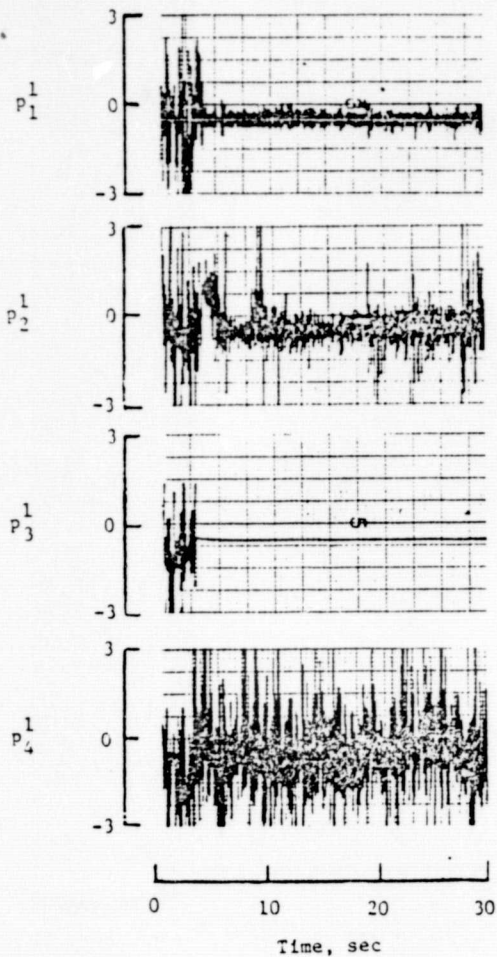
Figure 12.- Output of parameter identifiers, mode 3 excited (experimental).
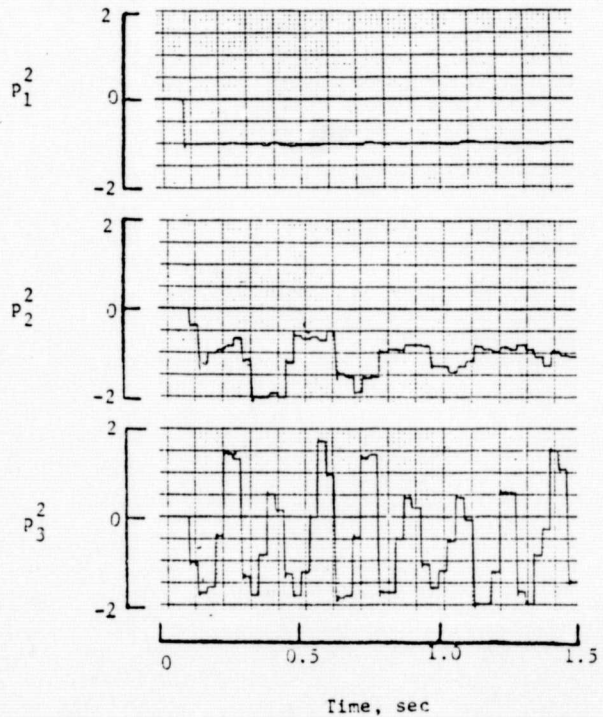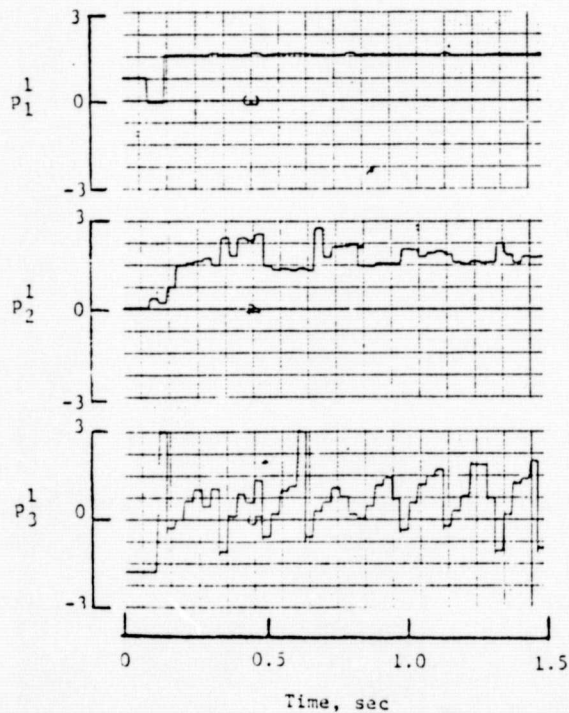
Figure 13.- Output of parameter identifiers--actuator 1 excitation.

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APPENDIX 3

LEAST-SQUARES SEQUENTIAL PARAMETER AND

STATE ESTIMATION FOR LARGE SPACE STRUCTURES

1

LEAST-SQUARES SEQUENTIAL

PARAMETER AND STATE ESTIMATION FOR

LARGE SPACE STRUCTURES

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Abstract

This paper presents the formulation of sim-
ultaneous state and parameter estimation problems
for flexible structures in terms of least-squares
minimization problems. The approach combines an
on-line order determination algorithm, with
least-squares algorithms for finding estimates of
modal approximation functions, modal amplitudes,
and modal parameters. The approach combines pre-
vious results on separable nonlinear least squares
estimation with a regression analysis formulation
of the state estimation problem. The technique
makes use of sequential Householder transforma-
tions. This allows for sequential accumulation
of matrices required during the identification
process. The technique is used to identify the
modal parameters of a flexible beam.

I. Introduction

On line identification of parameters and
states of distributed parameter systems is a
significant problem affecting the design of
adaptive controllers that have been proposed for
large flexible space structures. In [1] we
proposed a technique for identifying the modal
approximation functions and modal parameters of a
finite-dimensional approximate model of a flexi-
ble structure. The technique proposed in [1]
was based upon a priori knowledge of NM, the num-
ber of modes being excited during an identifica-
tion time interval. Because the number of excited
modes is generally unknown we propose here a
procedure for identifying NM on-line. This tech-
nique is based upon using Householder transforma-
tions [2] to determine the rank of a sequential-
ly obtained measurement matrix. Once NM is iden-
tified, the approximation function updating pro-
cedure of [1] is used to determine a set of
normalized approximation functions. The modal
parameters are then determined using a non-linear
least-squares parameter and state estimation al-
gorithm. This algorithm, reported in [3], is
based upon the work of Kaufman [4] and Golub and
Pereyra [5] on separable nonlinear least squares
problems.

In the following section the structural
parameter identification problem is formulated as
a problem of estimating the parameters of a linear
multi-input, multi-output system of unknown order
using noisy on-line measurements. The identifi-
cation algorithms are presented in section 3 and

applied in section 4 to identify the parameters
of a flexible beam from measurements performed at
the NASA-Langley structural dynamics and control
test facility.

II. Problem Formulation

The motion w(s,t) of a flexible structure
is represented by

$$w(s,t) = \sum_{i=1}^{NM} q_i(t,p_i)\xi_i(s) + v(s,t) \quad (1)$$

where s denotes the spatial variable and t repre-
sents time. The $\xi_i(s)$ represent the first
NM eigenfunctions in a modal expansion,
$q_i(t,p_i)$ are modal amplitudes, and
v(s,t) represents those higher-order terms not
appearing in the finite sum. The parameter vec-
tor $p_i$ represents a set of parameters charac-
terizing each mode. For example, if each mode is
characterized by the differential equation

$$\ddot{q}_i + 2\zeta_i\omega_i\dot{q}_i + \omega_i^2 q_i = \alpha_i f_i(t) \quad (2)$$

then parameter vector $p_i$ would comprise the com-
ponents $(\zeta_i, \omega_i, \alpha_i)$. If each mode were
characterized by a difference equation

$$q_i(k+1) = A_{1i}q_i(k) + A_{2i}q_i(k-1) + \tilde{B}_{1i}\tilde{F}_i(k) + \tilde{B}_{2i}\tilde{F}_i(k-1) \quad (3)$$

then parameter vector $p_i$ would comprise the
components $(A_{1i}, A_{2i}, \tilde{B}_{1i}, \tilde{B}_{2i})$. In (2)
and (3) $f_i(t)$ and $\tilde{F}_i(k)$ respectively represent
the modal forces in the continuous and discrete
models.

The measurements available to the controller
are a set of sampled values of the motion at the
points $s_j$, j=1,2,...,NS, and a set of sampled
values of the actuator forces $U_i$, i=1,2,...,NA,
where we have assumed NS sensors and NA actuators
are used. Thus, if $y_j(k)$ denotes a measurement of
the flexible structure motion at point $s_j$, then

$$y_j(k) = \sum_{i=1}^{NM} \xi_i(s_j)q_i(t_k,p_i) + v(s_j,t_k) + \tilde{\xi}_j(k) \quad (4)$$

where $\tilde{\xi}_j(k)$ represents the measurement error and

$j=1,2,\ldots,NS$. If $F_i(k)$ represents the modal force applied to the i-th flexible mode at time $t_k$, then

$$F_i(k) = \sum_{j=1}^{NA} m_{ij} U_j(k) \qquad (5)$$

where $i=1,2,\ldots,NM$ and where the $m_{ij}$ actuator influence parameters are usually unknown.

The parameter identification problem to be treated below is that of on-line processing of the available sequences of actuator force measurements $\{U_j(k)\}$ and sensors measurements $\{y_i(k)\}$ in order to estimate the number of excited modes NM, the approximation functions $\xi_i(s_j)$, the modal parameter vectors $p_i$, and the actuator influence parameters $m_{ij}$.

## III. Identification Algorithms

A set of identification algorithms is described below for performing on-line parameter identification in the following sequence: (1) estimate NM, (2) estimate the $\xi_i(s_j)$, and (3) estimate the $p_i$ and $m_{ij}$. From the description below it will be seen that while the computational procedures for accomplishing these tasks make use of common vectors of stored measurement data, the computational time for each procedure differs from the others so that, in general, these identification subtasks will operate in parallel with transfer of updated estimates occuring as new estimates become available.

### 1. Identification of number of excited modes NM

To examine the nature of this identification subproblem, define the vector of measurements $y(k)$ and modal amplitudes $q(k)$ by

$$y(k) = \begin{bmatrix} y_1(k) \\ \vdots \\ y_{NS}(k) \end{bmatrix}, \quad q(k) = \begin{bmatrix} q_1(k) \\ \vdots \\ q_{NM}(k) \end{bmatrix} \qquad (6)$$

so that from (4) we have

$$y(k) = H q(k) + w(k) \qquad (7)$$

where the NSxNM matrix H has $ij$-th element

$$(H)_{ij} = \xi_j(s_i) \quad \begin{array}{l} i=1,2,\ldots,NS \\ j=1,2,\ldots,NM \end{array} \qquad (8)$$

and the measurement error vector $w(k)$ is an NS-vector whose components are given by

$$w_i(k) = \tilde{e}_i(k) + v(s_i, t_k) \quad i=1,\ldots,NS \qquad (9)$$

If we denote the columns of H by $h^j$, $j=1,2,\ldots,NM$, then (7) may be rewritten as

$$y(k) = h^1 q_1(k) + \ldots + h^{NM} q_{NM}(k) + w(k) \qquad (10)$$

In (10) NM as well as the vectors $h^j$ are unknown and must be determined from the on-line measurements $y(k)$. The procedure described below for finding NM is a generalization and extension of the work of Woodside [6] to multi-dimensional systems.

First consider the case where $w(k) \equiv 0$. The problem then is to determine the number of linearly independent vectors $h^j$ required to represent the measured response vectors $y(k)=z(k)$, where

$$z(k) = \sum_{j=1}^{NM} h^j q_j(k) \qquad (11)$$

Assume that there is some maximum system order $NM^* < NS$ beyond which we will not attempt to model the flexible structure. Form the array

$$Z(M) = [z(k), \ldots z(M-1)], \qquad (12)$$
$$M = 2, 3, \ldots$$

From (11) it is seen that if $M$ is such that $NM^* \geq M > NM$, the M columns of $Z(M)$ will be linearly dependent whereas, the columns of $Z(M)$ be linearly independent if $M \leq NM$. Hence, to determine NM we examine the rank of the $Z(M)$ array for a sequence of $M$ values. If it is found that there is a value $M^*$ such that

$$rank[Z(M^*)] = rank[Z(M^*+1)] \qquad (13)$$

then $NM=M^*$.

A simple rank test is based on the orthogonal decomposition of the data matrix (12). As shown in [2] if A is an mxn matrix $(m > n)$ of rank $k < n$ then there is an mxm orthogonal matrix Q and an nxn permutation matrix P such that

$$QAP = \begin{bmatrix} \overbrace{R_{11}}^{k} & \overbrace{R_{12}}^{n-k} \\ O & O \end{bmatrix} \begin{array}{l} \updownarrow k \\ \updownarrow m-k \end{array} \qquad (14)$$

where $R_{11}$ is a $k$x$k$ upper triangular matrix of rank $k$. A computational algorithm based upon use of Householder transformations for obtaining the decomposition (14) is specified in [2] and is applied in section 4 below where the data

matrix $\hat{\mathcal{J}}(M)$ replaces matrix A. Testing the main diagonal elements of the right hand side of (14) provides a rank test and, hence, an estimate of the number of excited modes.

## 2. Identification of Modal Approximation Functions

Once the number of excited modes NM has been determined, a least-squares procedure can be used to estimate the modal approximation functions. A procedure was presented in [1] for determining the modal approximation functions $k^i, ..., k^{NM}$ for a discrete modal description of the form

$$\dot{q}(k+1) = A_1 q(k) + A_2 q(k-1) + B_1 U(k) + B_2 U(k-1) \quad (15)$$

with available measurements (7). In (15) $A_1$ and $A_2$ are diagonal matrices, $q(k)$ is a vector of modal amplitudes and $U(k)$ is a vector of actuator forces. The procedure consists in finding the eigenvectors corresponding to the NM largest eigenvalues of estimates of matrices $M_1$ and $M_2$ where

$$y(k+1) = M_1 y(k) + M_2 y(k-1) + N_1 U(k) + N_2 U(k-1) + n(k+1) \quad (16)$$

and

$$M_i = H A_i H^+, \quad N_i = H B_i, \quad i = 1, 2 \quad (17)$$

$$n(k+1) = w(k+1) - M_1 w(k) - M_2 w(k-1) \quad (18)$$

$H^+$ in (17) is given by

$$H^+ = (H^T H)^{-1} H^T \quad (19)$$

where $(\cdot)^T$ denotes matrix transpose.

Least-squares estimates of $M_1$, $M_2$, $N_1$, and $N_2$ are obtained by finding the least-squares matrix solution $\hat{M}$ to

$$Y(N) = S(N) M^T + V(N) \quad (20)$$

where

$$S(N) = \left[ Y(N-1) \mid Y(N-2) \mid U(N-1) \mid U(N-2) \right] \quad (21)$$

$$Y(k) = \begin{bmatrix} y^T(k-(N-3)) \\ \vdots \\ y^T(k+2) \end{bmatrix} \quad U(L) = \begin{bmatrix} U^T(L-(N-3)) \\ \vdots \\ U^T(L+2) \end{bmatrix} \quad (22)$$

$$k = k+N, \; k+N-1, \; k+N-2 \qquad L = k+N, \; k+N-1$$

and

$$V(N) = \begin{bmatrix} n^T(k+3) \\ \vdots \\ n^T(k+N+2) \end{bmatrix} \quad M^T = \begin{bmatrix} M_1^T \\ M_2^T \\ N_1^T \\ N_2^T \end{bmatrix} \quad (23)$$

From (17) it is seen that the columns of H are eigenvectors of $M_i$ and the non-zero eigenvalues of $M_i$ are the diagonal elements of the matrices $A_i$. Furthermore the matrices $B_i$ can be determined from (17) once estimates of H and $N_i$ are available. This procedure requires the storage of measurement data over a time-interval that includes at least 2(NM+NA) measurement samples. In situations where more rapid up-dating of parameter estimates is desired, alternate techniques [1], [3] may be used. These are described below.

## 3. Identification of Modal Parameters

In [1] it was assumed that the estimates of modal approximation functions would be calculated at a lower frequency than the computation of the modal parameter estimates. The latter were obtained from an output error algorithm. In this paper we employ a nonlinear least-squares algorithm [3] which could also be used to provide estimates of modal position and velocity. The technique is based upon computing a least-squares estimate of modal amplitudes at each sampling time. For each mode i we use a state description of the form

$$x^i(k+1) = F^i x^i(k) + G^i(p_i) U(k) \quad (24)$$

where

$$F^i = \begin{bmatrix} 0 & 1 \\ a_{2i} & a_{1i} \end{bmatrix} \quad (25)$$

and $G^A$ is a 2x5 matrix whose elements depend on the modal parameters vector $p_i$. From this point on we will drop the superscripts and subscripts "i" since the following algorithmic approach is to be used for each of the NM excited modes.

Since we assume here that the modal approximation functions comprising the H matrix of (7) have already been identified, we now use (7) at each sampling instant to generate an estimate $\hat{q}(k)$,

$$\hat{q}(k) = x_i(k) + n(k)$$
$$= [1, 0] \, x(k) + n(k) \tag{26}$$

of modal position, where $n(k)$ denotes the error in the modal amplitude estimate. A separable nonlinear least-squares formulation of the parameter/state estimation problem is obtained as follows: Solve (24) with respect to $U(k)$ to obtain

$$U(k) = G^+(p) x(k+1) - G^+(p) F(p) x(k)$$
$$= \left[ -G^+(p) F(p), \; G^+(p) \right] \begin{bmatrix} x(k) \\ x(k+1) \end{bmatrix} \tag{27}$$

where $G^+(p)$ denotes the pseudo-inverse of $G(p)$. Combine (26) and (27) as suggested by Duncan and Horn [7] to form the overdetermined system

$$\begin{bmatrix} \hat{u}(1) \\ y(1) \\ U(2) \\ y(2) \\ \vdots \\ U(K) \\ y(K) \end{bmatrix} = F_k(p) \begin{bmatrix} x(1) \\ x(2) \\ \vdots \\ x(K) \end{bmatrix} + G_k \begin{bmatrix} \hat{n}(1) \\ n(1) \\ \vdots \\ n(K) \end{bmatrix} \tag{28}$$

where

$$F_k(p) = \begin{bmatrix} I \\ H \\ -G^+(p)F(p), \; G^+(p) \cdots \\ H \cdots \\ \vdots \qquad \vdots \\ \cdots -G^+(p)F(p), \; G^+(p) \\ \cdots \qquad H \end{bmatrix} \tag{29}$$

and

$$G_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ & & 1 & 0 \\ & & & & \ddots \\ & & & & & 1 \end{bmatrix} \tag{30}$$

The vector on the left-hand side of (28) is a set of input-output measurements taken over an interval of length K. Equation (28) may be rewritten more briefly as

$$Y(K) = F_k(p) X(K) + G_k \, e(k) \tag{31}$$

The simultaneous state/parameter estimation problem is now considered to be the least-squares minimization,

$$\min_{X(K), p} \| F_k(p) X(K) - Y(K) \|^2 \tag{32}$$

Following the work of [4] and [5], (32) is separated into the form

$$\min_{X(K), p} \left\{ \| R X(K) - Q_1 Y(K) \|^2 + \| Q_2 Y(K) \|^2 \right\} \tag{33}$$

where $Q_1$, $Q_2$, and R are defined from the orthogonal decomposition of $F_k(p)$,

$$Q F_k = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} F_k = \begin{bmatrix} R \\ O \end{bmatrix} \tag{34}$$

It is shown in [3,5] that the optimum parameter estimate $\hat{p}$ and optimum state trajectory $\hat{X}(K)$ that minimize (33) can be obtained as solutions to two subproblems: a nonlinear least-squares problem

$$\min_{p} \| Q_2(p) Y(K) \|^2 \tag{35}$$

to yield $\hat{p}$, and a linear least-squares problem based upon

$$R(\hat{p}) X(K, \hat{p}) = Q_1(\hat{p}) Y(K) \tag{36}$$

to yield $\hat{X}(K)$.

An algorithm for the solution of these problems based upon the approach of Kaufman [4] has been implemented and used in the simulation study reported in the following section.

## IV. Simulation Results

The algorithms described above have been implemented in a simulation program for identifying the characteristics of a flexible beam used in the NASA-Langley structural dynamics and control test facility. The experimental test facility is described in detail in [1]. Six flexible modes were used in simulating the beam response. Figure 1 shows the results of applying the algorithm of section 3 for testing the number of excited modes when only three modes were actually excited in the simulation. In the figure the magnitude of the main diagonal element $R_{ii}$ of the test matrix on the right hand side of (14) is plotted against the number of assumed excited modes. Results of tests with noiseless measurement data and with two levels of noisy data are presented. It is seen that when the assumed number of modes exceeds the actual number of excited modes (3) the corresponding main diagonal element remains below the test level $\tau$.

Figure 2 shows the result of applying the regression analysis approach of section 3 to identifying the mode shape functions when it was established that only one flexible mode was excited. Test results for two cases are shown in the Figure: (a) a simulation wherein the beam was given an initial displacement corresponding to the first flexible mode and (b) a laboratory experiment in which only the first flexible mode was excited. In the simulation the normalized mode shape obtained agreed with that produced by the SPAR structural analysis program. In the laboratory experiment a distortion in the identified mode shape was observed. In both cases the mode shape was identified on-line after approximately one second of data processing.

Figure 3 presents one illustration to demonstrate the convergence of the nonlinear least squares parameter identification approach of section 3 for identifying the modal parameters of the first flexible mode. Four samples of measurement data are used and convergence of the algorithm is achieved after ten iterations. A study of the algorithm performance with various levels of measurement noise is presented in [3].

## V. Conclusion

In this paper the state/parameter identification problem for large flexible systems is decomposed into a number of simpler subproblems which may be solved by least-squares algorithms. The technique comprises the following steps: (1) estimating the number of excited modes in a set of sequential measurement data, (2) estimating the spatial approximation functions needed in a modal response representation and (3) estimating the pole-zero parameters and modal position and velocity coordinates for each excited mode.

Particular least-squares algorithms for accomplishing the three subtasks above were implemented in a simulation for identifying the parameters of a flexible beam. These algorithms employ Householder transformations thus avoiding the need for matrix inversion and resulting in accurate and rapid identification. Other techniques may also be considered for solving each subproblem. For example, in testing the number of excited modes, identification of the measurement noise covariance matrix could lead to a more accurate setting of the test level used. One subject of current research is a comparison of competing techniques with regard to computational requirements and achievable accuracy.

## VI. References

1. Thau, F. E., Montgomery, R. C., and Horner, G.C., "On-Line Structural Parameter Identification," presented at the 1981 AIAA Guidance and Control Conference, August 1981.

2. Lawson, C.L. and Hanson, R. J., Solving Least Squares Problems, Prentice-Hall, Inc. Englewood Cliffs, New Jersey, 1974.

3. Eliazov, T., Thau, F.E., and Montgomery,R.C., "Least-Squares Sequential Parameter and State Estimation for Multivariate Systems," submitted for publication.

4. Kaufman,L., "A Variable Projection Method for Solving Separable Nonlinear Least Squares Problems," BIT, 15(1975), pp. 49-57.

5. Golub, G. H. and Pereyra, V., "The Differentiation of Pseudo-Inverses and Nonlinear Least Squares Problems Whose Variables Separate," SIAM J. Numer. Anal., Vol. 10, No. 2, April, 1973, pp. 413-432.

6. Woodside, C. M., "Estimation of the Order of Linear Systems,"

7. Duncan, D. B. and Horn, S. D., "Linear Dynamic Recursive Estimation from the Viewpoint of Regression Analysis," J. Amer. Statist. Assoc., 67(1972) pp. 815-821.
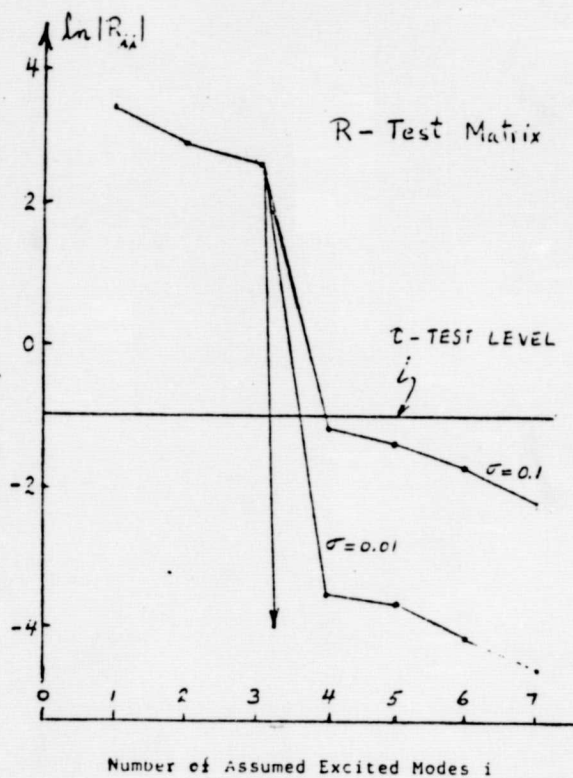
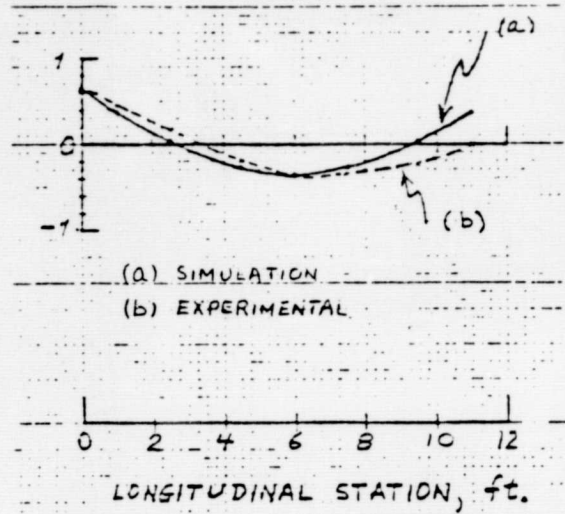Figure 1. Test for Number of Excited Modes



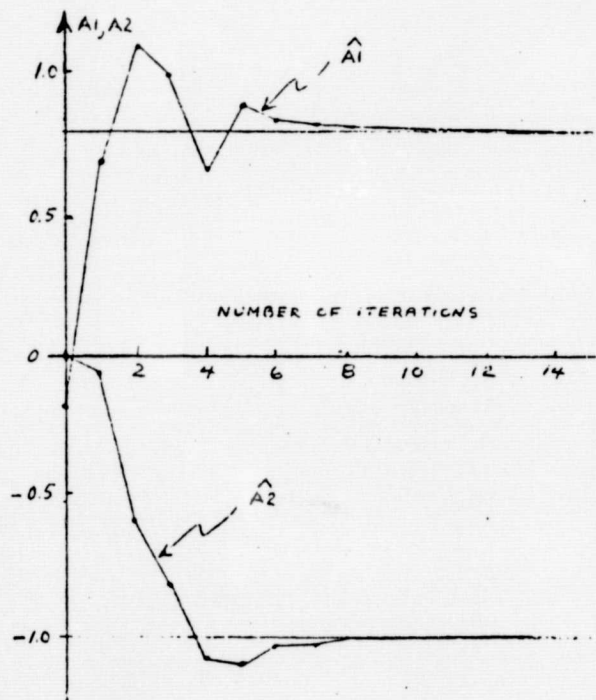Figure 2. Identification of Mode Shape
First Flexible Mode



Figure 3. Convergence of Parameter Estimates

APPENDIX 4

A NONLINEAR DUAL-ADAPTIVE CONTROL STRATEGY

FOR IDENTIFICATION AND CONTROL OF

FLEXIBLE STRUCTURES

# A NONLINEAR DUAL-ADAPTIVE CONTROL STRATEGY
## FOR IDENTIFICATION AND CONTROL OF
## FLEXIBLE STRUCTURES*

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## ABSTRACT

A technique is presented for obtaining a control law to
regulate the modal dynamics and identify the modal parameters
of a flexible structure.  The method is based on using a
min-max performance index to derive a control law which may
be considered to be a best compromise between optimum
one-step control and identification inputs.  Features of the
approach are demonstrated by a computer simulation of the
controlled modal response of a flexible beam.

## I.  INTRODUCTION

A class of indirect adaptive control systems proposed for the
control of large space structures [1] is based on a modal
decomposition of the system dynamics and may incorporate one
or more on-line testing schemes [2] to determine when
successful parameter identification has been achieved.  The
control strategy used in calculating the actuator inputs must
achieve adequate regulation or tracking performance and, at
the same time, provide inputs to allow adequate parameter
identification.  A control system designer is thus faced with
the problem of devising a control strategy to ensure
acceptable system performance even when on-line parameter
identifiability tests have failed because the system

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configuration has changed or the environment in which the system operates has changed.

In this paper we formulate and examine the performance of a nonlinear dual-adaptive control scheme in which a sampled-data controller is designed to select a best compromise between an input signal that is optimum for mean-square system regulation and an input signal that is optimum for parameter identification. Dual control theory, originally formulated by Feldbaum [3,4], has been studied in [5-7] and in the references cited therein. A key concept introduced by Feldbaum is the dual control strategy based on a performance index that takes into account the fact that future observations on the process will be made. A controller may be able to "probe" the system for state and parameter estimation improvement, which then may improve future regulation and tracking performance. In many situations where the dual nature of stochastic control is not taken into account the controller becomes "cautious" [5,6] and tends to "turn-off". This undesirable phenomenon is avoided by the approach described below.

## II. FORMULATION OF AN ADAPTIVE PERFORMANCE INDEX

The discrete-time dynamics for each mode is assumed to be described by the ARMA model

$$y(t)+a_1y(t-1)+a_2y(t-2) = b_1u(t-1)+b_2u(t-2)+e(t) \tag{1}$$

where $y(t)$ denotes modal displacement, $u(t)$ denotes modal force, and $e(t)$ is a sequence of independent, equally-distributed, normal $(0,\sigma^2)$ random variables. It is assumed that $e(t)$ is independent of $y(t-1),y(t-2),...,$ $u(t-1),u(t-2),...$ and that the parameters $a_1,a_2,b_1,b_2$ are unknown constants. If we let $Y_t$ denote the information available to the controller at time t,

$$Y_t = \{y(t),y(t-1),..., u(t-1),u(t-2),...\} \tag{2}$$

$x(t)$ denote the modal parameter vector and $\theta(t)$ denote a modal measurement vector,

$$x^T(t) = (a_1,a_2,b_1,b_2);$$

$$\theta^T(t) = (-y(t-1),-y(t-2),u(t-1),u(t-2)) \tag{3}$$

where $(\,.\,)^T$ denotes vector or matrix transpose, then (1) may be rewritten as

$$y(t) = \theta^T(t)x(t)+e(t) \tag{4}$$

where the constant parameter "dynamics" satisfies

$$x(t+1) = x(t) \tag{5}$$

It can than be shown, following the analysis of [8], that the conditional distribution of $x(t+2)$ given $Y_{t+1}$ is normal with mean $x(t+2)$ and covariance matrix $P(t+2)$ where $x(t)$ and $P(t)$ satisfies the difference equations

$$\hat{x}(t+1) = \hat{x}(t)+K(t)(y(t)-\theta^T(t)x(t)) \tag{6}$$

$$K(t) = P(t)\theta(t)/(\sigma^2+\theta^T(t)P(t)\theta(t)) \tag{7}$$

$$P(t+1) = P(t)-(P(t)\theta(t)\theta^T(t)P(t))/ \\ (\sigma^2+\theta^T(t)P(t)\theta(t)) \tag{8}$$

Furthermore, the control law that minimizes the regulation criterion

$$V_c(u(t)) = E\{y^2(t+1)|Y_t\} \tag{9}$$

is given by

$$u(t) = - \frac{\sum'(\hat{x}_i(t+1)\hat{x}_3(t+1)+P_{3i}(t+1))\theta_i(t+1)}{\hat{x}_3^2(t+1)+P_{33}(t+1)} \tag{10}$$

where $\sum'$ denotes the sum over i = 1 to 4 with the value 3 excluded.

To provide bounded modal inputs that improve parameter identification accuracy while guaranteeing that the modal amplitude will not become excessively large, the controller is designed to optimize, at each sampling instant t, the following performance criterion:

$$\begin{array}{cc} \min & \max [V(\lambda, u(t))] \\ u(t) & \lambda \end{array} \tag{11}$$

subject to the constraints

$$u(t) \leq M, \; 0 \leq \lambda \leq 1 \tag{12}$$

where

$$V(\lambda, u(t)) = \lambda \frac{V_c(u(t))}{V_c^0} + (1-\lambda)\frac{V_I(u(t))}{V_I^0} \tag{13}$$

$\overset{\circ}{V}_c$ denotes an acceptable or desired level of regulation cost. $V_I(u(t))$ denotes and identification cost function of $u(t)$,

$$V_I(u(t)) = \text{trace} \left[ P(t+2) \right] \qquad (14)$$

$\overset{\circ}{V}_I$ denotes and acceptable or desired level of identification cost. The maximization indicated in (11) yields a function $V(u(t))$ which, although not convex, is interpreted as specifying, for each admissible $u(t)$, the most costly linear combination of relative regulation and relative identification cost. Minimization of $V(u)$ thus yields the modal input that minimizes this most costly combination of relative identification and regulation performance.

## III. SIMULATION RESULTS

Since $V_c(u(t))$ and trace $P(t+2)$ are relatively simple functions of $u(t)$ the numerical solution of the one-step optimization problem (11)-(13) at each sampling time is quite feasible. Results of simulation studies described below illustrate an interesting feature of this approach: since the parameters involved in the evaluation of $V_c(u(t))$ and $V_I(u(t))$ depend on system measurements, the optimum distribution of relative cost, $\overset{\circ}{\lambda}(u)$ depends on on-line measurement data and hence, at each sampling instant, the weighting between identification and regulation will change depending on the on-line system performance. This is in contrast to[9] in which a fixed weighting between absolute control and identification cost is used at each sample time.

In the simulation study we compare the performance of three control systems:

   a) A constrained adaptive controller that minimizes (9) subject to the control magnitude constraint.
   b) An optimum identification controller that minimizes (14) subject to the control magnitude constraint.
   c) The one-step dual-adaptive controller based on (11)-(13).

In Figures 1-3 we present simulated modal response data for the first flexible mode of the Langley beam experiment described in [10] where we assume here that a single actuator is used. The accumulated on-line regulation cost, VT, shown in Figure 1 is defined as

$$VT(N) = \sum_{k=1}^{N} y^2(k) \qquad (15)$$

and the on-line identification cost, PT, is defined as

$$PT(N) = \text{trace } [P(N)] \tag{16}$$

where $P(N)$ is calculated on-line using (8). Note that for the first 10 to 15 sampling times the regulation cost of the dual-adaptive controller is close to that of the constrained minimum-variance controller and the identification cost of the dual-adaptive control system is close to that of the constrained one-step optimum identification controller. Figure 2 indicates that the dual-adaptive controller's actuator signals switch between its limits, $\pm 0.5$, more frequently than do the actuator signals of the other controllers. This may be due to the lack of any energy constraint in the above problem formulation.

A future study will examine the performance of the energy-constrained dual-adaptive controller in comparison with energy-constrained minimum-variance and one-step optimum identification controllers. The relative regulation cost and relative identification cost defined in (13) are plotted in Figure 3 where

$$V_c^{\circ}(N) = \sigma^2 N \tag{17}$$

is the accumulated control cost that would be achieved if the parameters of the system where known precisely and if an unconstrained control law were used; $\sigma^2 = 10^{-4}$ was used in the simulation runs. A constant value $V_I^{\circ} = 10^{-4}$ was chosen as indicating the acceptable level of parameter identification. Figure 3 indicates that, depending on on-line measurements, the one-step identification and regulation cost at one sampling instant can have widely differing shapes from their respective distributions at other sampling times. This leads to the on-line variations in the dual-adaptive control strategy mentioned earlier.

The simulation results indication that the one-step, constrained dual-adaptive controller has the feature of providing, based on measured data, system inputs that result in parameter identification while maintaining bounded modal amplitude response.

# REFERENCES

1. Montgomery, R. C. and Thau, F.E., "Adaptive and Learning Control of Large Space Structures," AIAA Guidance and Control Conference, June, 1980.

2. Sundararajan, N. and Montgomery, R. C., "Adaptive Control of a Flexible Beam Using Least Square Lattice Filters," 1983 Large Space Structures Workshop, Blacksburg, Va.

3. Feldbaum, A. A., "Dual Control Theory I - IV, Automation and Remote Control 1960, vol. 21, pp. 874, pp. 1033; 1902, vol. 22, pp. 1, pp. 109.

4. Feldbaum, A. A., Optimal Control Systems, Academic Press, New York, 1965.

5. Bar-Shalom, Y., "Stochastic Dynamic Programming: Caution and Probing, " IEEE Trans., Vol. AC - 26, No. 5, Oct. 1981, pp, 1184 - 1195.

6. Wittenmark, B., "Stochastic Adaptive Control Methods: A Survey,"Int. J. Control, Vol. 21, No. 5, pp. 705 - 730, 1975.

7. Bar-Shalom, Y. and Tse, E., "Dual Effect, Certainty Equivalence, and Separation in Stochastic Control, "IEEE Trans., Vol. AC - 19, pp. 494 - 500, Oct. 1974.

8. Astrom, K. J. and Wittenmark, B., "Problems of Identification and Control," J. Math. Anal and Appl, Vol. 34, pp. 90 - 113, 1971.

9. Wittenmark, B., "An Active Suboptimal Dual Controller for Systems with Stochastic Parameters," Auto. Contr. Theory Appl., Canada, Vol. e, pp. 13-19, 1975.

10. Thau, F. E., Montogomery, R. C., and Horner, G.C., "On-Line Structural Parameter Identification," Proc. AIAA Guidance and Control Conference, August, 1981, Albuquerque, New Mexico, pp. 530-539.
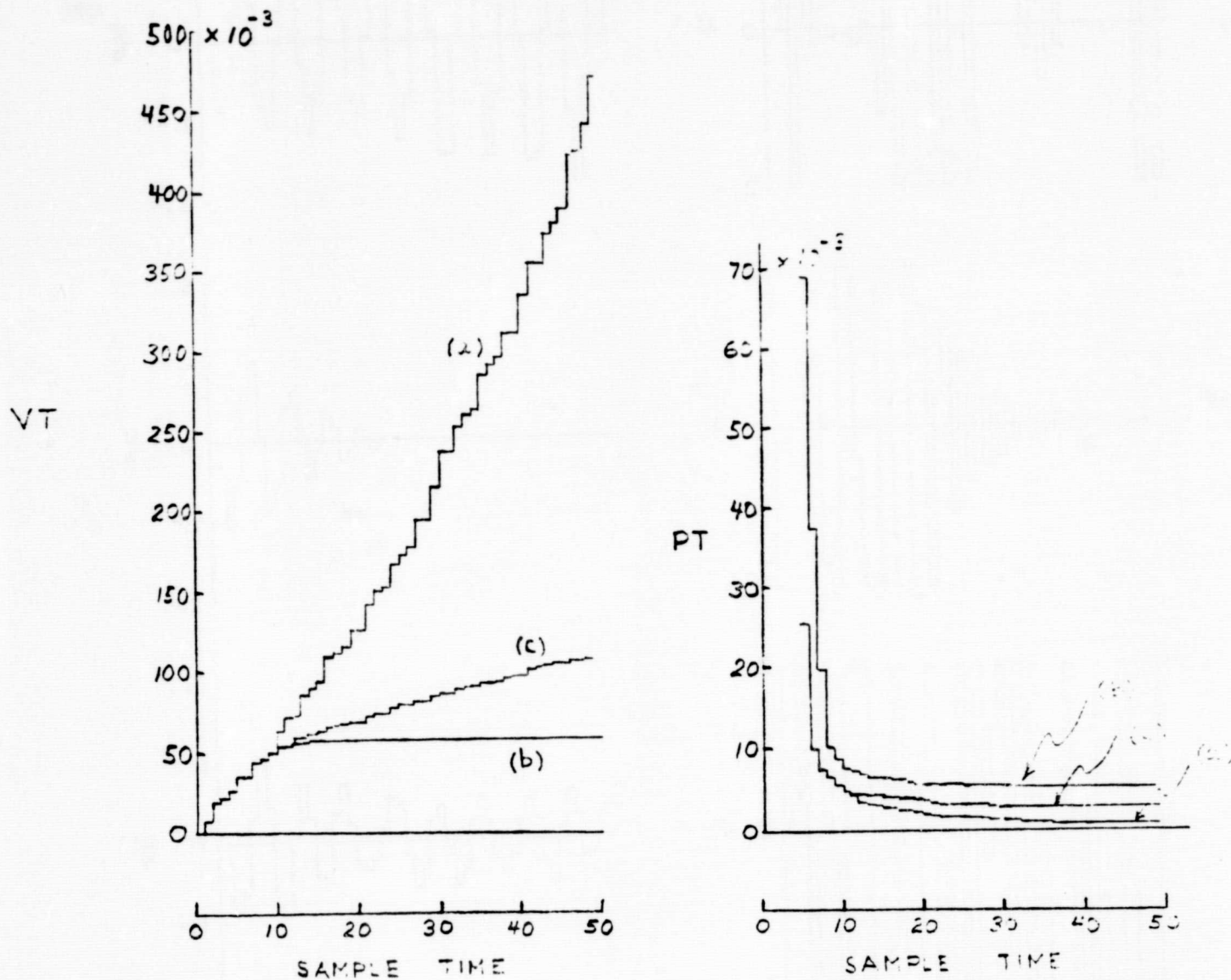
(a) ONE STEP OPTIMUM IDENTIFICATION

(b) MINIMUM-VARIANCE ADAPTIVE

(c) DUAL-ADAPTIVE

FIG. 1    ON-LINE REGULATION AND
IDENTIFICATION COST FOR THREE
FEEDBACK CONTROLLERS
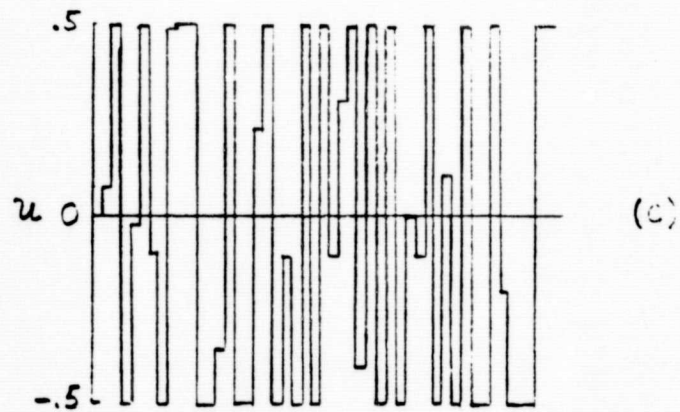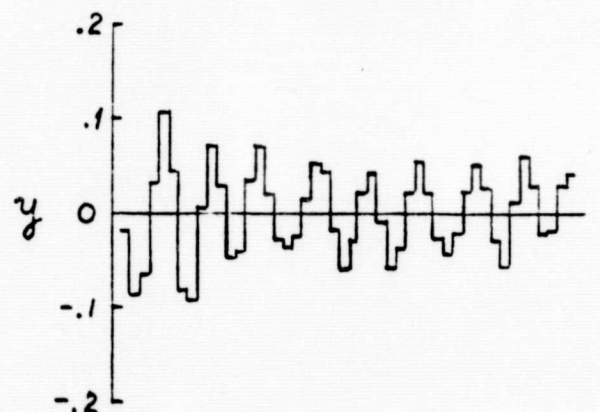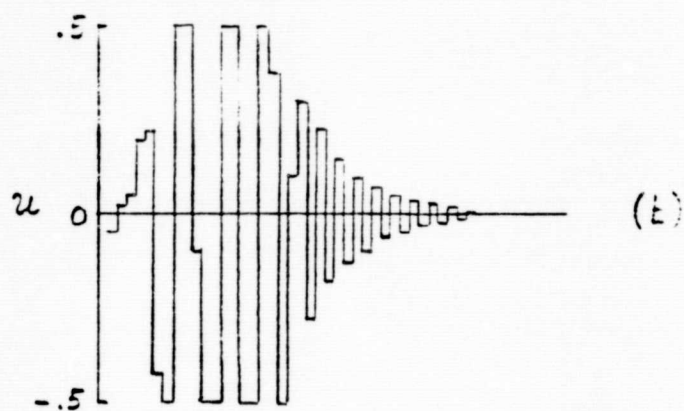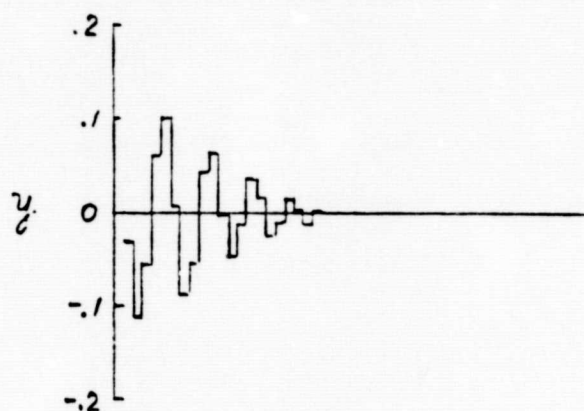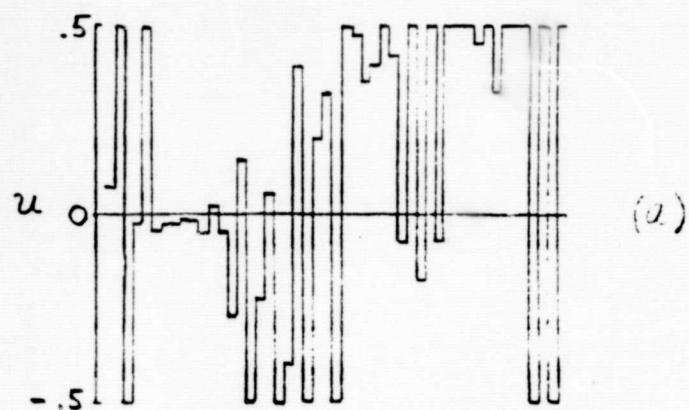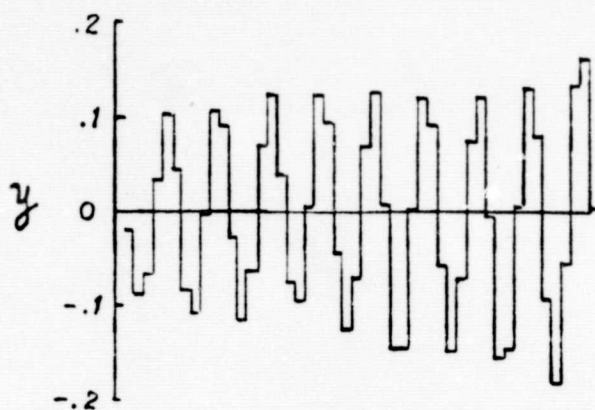
(a) ONE-STEP OPTIMUM IDENTIFICATION

(b) MINIMUM-VARIANCE ADAPTIVE

(c) DUAL-ADAPTIVE

FIG. 2  MODAL DISPLACEMENT AND MODAL FORCE
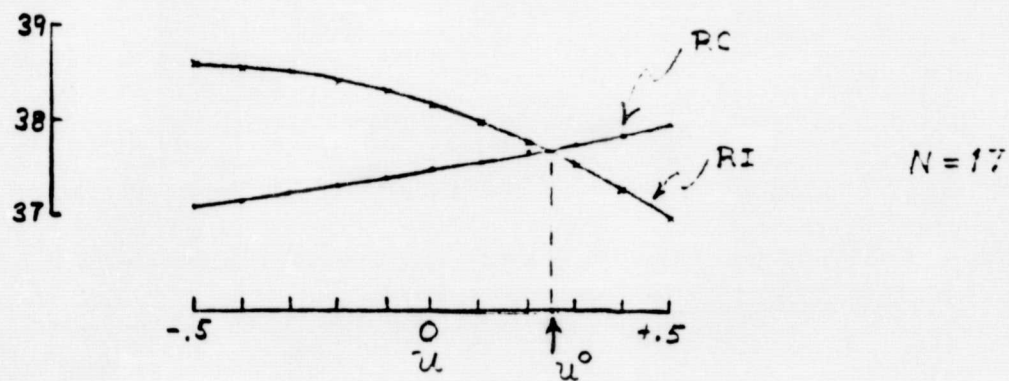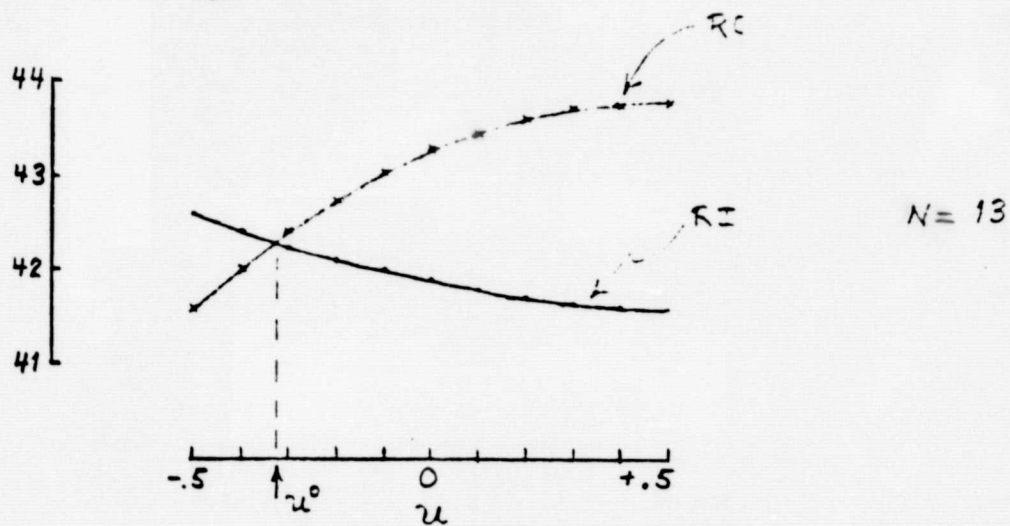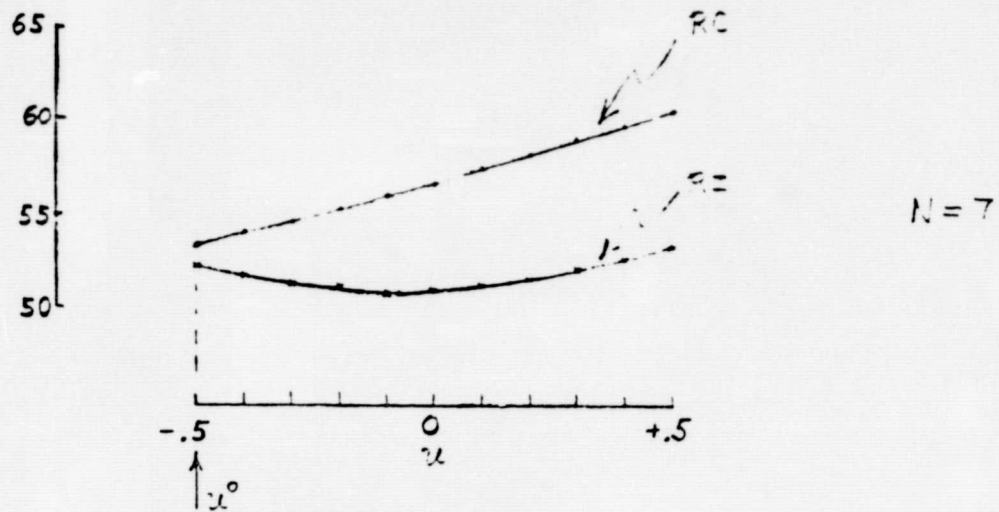FOR FIRST FLEXIBLE MODE

FIG. 3 RELATIVE CONTROL COST $RC = V_c(u)/V_c^o$ AND
RELATIVE IDENTIFICATION COST $RI = V_I(u)/V_I^o$
FOR 3 SAMPLE TIMES