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# Secondary Pattern Computation of an Offset Reflector Antenna

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# ERRATA

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Page 2, second paragraph, line 3 (line following eq. (1)): $\vec{f}(\theta,\varphi)$ should be replaced by $\vec{f}$.

Page 2: Equations (1) and (4) should be enclosed in boxes.

Page 3: Equations (5), (6), and (7) should be enclosed in boxes.

Page 3, third paragraph, line 2: $O^r$,• should be $O^r$,.

Page 4, second paragraph from bottom, line 4: Kuffman and Crowell should be Kauffman and Croswell.

Page 4, second paragraph from bottom, line 13: Delete the following sentence: The agreement is also very good for the directivity.

Page 11, reference 10: Clam, Peter T. should read Lam, Peter, T.C.

Figure 6: The legend should read as follows: On-focus case (feed is at focal point).

# SECONDARY PATTERN COMPUTATION OF AN OFFSET REFLECTOR ANTENNA

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## SUMMARY

Reflector antennas are widely used in communications satellite systems because they provide high gain at low cost. In analyzing reflector antennas the computation of the secondary pattern is the main concern. A computer program for calculating the secondary pattern of an offset reflector has been developed and implemented at the NASA Lewis Research Center. The theoretical foundation for this program is based on the use of geometrical optics to describe the fields from the feed to the reflector surface and to the aperture plane. The resulting aperture field distribution is then transformed to the far-field zone by the fast Fourier transform algorithm. Comparing this technique with other well-known techniques (the geometrical theory of diffraction, physical optics (Jacobi-Bessel), etc.) shows good agreement for large (diameter of $100\lambda$ or greater) reflector antennas.

## INTRODUCTION

The accurate prediction of radiation characteristics for a microwave antenna is essential in designing antenna systems. Antenna radiation characteristics such as beam width, gain, aperture efficiency, side-lobe level, and cross polarization are used in analyzing and designing advanced antenna systems. The aperture field method (ref. 1) described in this report is one of several methods for predicting antenna performance characteristics. The method assumes that the tangential electric field on a planar surface in front of the antenna reflector is known. This aperture field distribution is then transformed to the far-field zone of the reflector by using a two-dimensional Fourier transform.

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To compute the tangential electric fields on the aperture surface, the geometrical optics method (ref. 2) is used. In this technique the energy coming from the feed and reradiating from the reflector surface is characterized by an astigmatic tube of rays (ref. 3) that allows calculation of the amplitude and polarization of the electric field. This report also presents a method for computing the secondary pattern by using the aperture field method and a fast Fourier transform (ref. 4). Computation of the tangential electric field in the aperture plane is briefly described. A reflector configuration (fig. 1) is analyzed, and the results are compared with other well-known computational techniques. A description and a copy of the program are included in appendixes A and B, respectively.

The author wishes to thank Dr. Shung-Wu Lee and Dr. Peter T.C. Lam, of the University of Illinois, for their cooperation and recommendations.

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# APERTURE FIELD METHOD APPROACH

## Description of Problem

The geometry of the problem under consideration is shown in figures 2 and 3. A reflector $\Sigma_R$ is illuminated by the energy from a feed source at $F_1$. The problem is defined as (1) to determine the tangential electric field at an observation point on the aperture grid $F_2$, as shown in figure 2, and (2) to determine the secondary pattern from the near-field distribution by using the fast Fourier transform, as shown in figure 3.

## Incident Electric Field on Reflector

The radiated electric field from the feed antenna has the asymptotic form given by equation (1):

$$\boxed{\vec{E} \sim \frac{e^{-jkr}}{r}\,\vec{f}(\theta,\ \varphi)} \qquad (1)$$

where $\vec{f}(\theta, \varphi)$ is the active element pattern, $k = 2\pi/\lambda$ is the wave number, and $r$ is the distance from the source to the reflection point. The vector function in equation (1) can be approximated by equation (2).

$$\vec{f}\,(\theta,\varphi) \cong \hat{\theta}U_E(\theta)(ae^{j\psi}\cos\varphi + b\sin\varphi) + \hat{\varphi}U_H(\theta)(b\cos\varphi - ae^{j\psi}\sin\varphi) \qquad (2)$$

where $U_E(\theta)$ is the E-plane active pattern, $U_H(\theta)$ is the H-plane active pattern, and a, b, and $\psi$ are polarization parameters:

The various feed polarizations are described in the following table:

| | a | b | $\psi$ |
|---|---|---|---|
| Linear x | 1 | 0 | 0 |
| Linear y | 0 | 1 | 0 |
| Right-hand circular polarized (RHCP) | $1/\sqrt{2}$ | $1/\sqrt{2}$ | $\pi/2$ |
| Left-hand circular polarized (LHCP) | $1/\sqrt{2}$ | $1/\sqrt{2}$ | $-\pi/2$ |

Typically these element patterns can be approximated by $(\cos\theta)^q$; that is,

$$U_E(\theta) = (\cos\theta)^{q_E} \qquad (3a)$$

$$U_H(\theta) = (\cos\theta)^{q_H} \qquad (3b)$$

If equations (3a) and (3b) are used to represent the active element patterns, the power radiated (ref. 5) by this source is given by equation (4).

$$\boxed{P_{RAD} = \frac{q_E + q_H + 1}{60(2q_E + 1)(2q_H + 1)}} \qquad (4)$$

Two coordinate systems (fig. 4) are used, the feed coordinate system $(x^F, y^F, z^F)$ and the reflector system $(x, y, z)$. These two coordinate systems are related by Eulerian angles $(\alpha, \beta, \gamma)$ illustrated in figure 5. Reference 6 provides a good description of these angles, which transform the incident field on the feed coordinate system into the reflector coordinate system.

## Reflected Electric Field

For a given feed point $F_1$ and an observation point $F_2$ (fig. 2), a reflection point $O^r$ may exist on the reflector $\Sigma_R$. This point is called the specular point. This type of ray reflection satisfies Snell's law of reflection. Reference 7 describes a search procedure to obtain the specular point. The reflected field at the aperture point $F_2$ is given by equation (5).

$$\boxed{\vec{E}^r_{(F_2)} = DF \cdot e^{-jkd_2}[2(\hat{n} \cdot \vec{E}^i)\hat{n} - \vec{E}^i]} \tag{5}$$

where $d_2$ is the distance from $O^r$ to $F_2$.

Equation (5) is given in terms of the incident field $\vec{E}^i$ at the reflection point $O^r$, the surface unit normal $\hat{n}$ of the reflector at $O^r$, and a divergence factor DF. The divergence factor in equation (5) is given by

$$\boxed{DF = \frac{1}{\sqrt{1 + \frac{d_2}{R^r_1}}} \; \frac{1}{\sqrt{1 + \frac{d_2}{R^r_2}}}} \tag{6}$$

where $(R^r_1, R^r_2)$ are the principal radii of curvature of the reflected wavefront passing through $O^r$. Reference 7 gives an expression for these parameters as a function of the principal radii of curvature of a parabolic reflector.

## Secondary Pattern

From the Hygens theorem (ref. 1) a solution for the far-field zone may be obtained if the tangential fields at $\Sigma_A$ (fig. 3) are known. The aperture plane $\Sigma_A$ is taken to be perpendicular to the z-axis.

The field in the far-field zone is given in terms of the tangential electric field at $\Sigma_A$, which is denoted by $\vec{E}_a$ (ref. 8). Equation (7) relates the aperture field $\vec{E}_a$ to the far-field zone electric field.

$$\boxed{\vec{E}(\vec{r}) \sim jk_0 \frac{e^{-jkr}}{2\pi r} [\hat{\theta}E_\theta + \hat{\varphi}E_\varphi]} \tag{7}$$

$$E_\theta = F_Y \cos\varphi \cos\theta - F_X \sin\varphi \cos\theta \tag{7a}$$

$$E_\varphi = F_X \cos\varphi + F_Y \sin\varphi \tag{7b}$$

$$\vec{F} = \iint_{\Sigma_A} \vec{E}_a e^{jk(ux+vy)}\, dx\, dy = F_x \hat{a}_x + F_y \hat{a}_y \tag{7c}$$

$$u = k \sin\theta \cos\varphi \tag{7d}$$

$$v = k \sin\theta \sin\varphi \tag{7e}$$

and $\theta$ and $\varphi$ are the spherical coordinates of far-field point. The integral in equation (7c) is a double Fourier transform. The computer program developed uses a fast Fourier transform (FFT) algorithm to accomplish this task. References 9 and 10 describe in detail the use of the FFT algorithm to solve for the double Fourier transform.

The aperture field theory used to determine the secondary pattern is exact if the tangential fields are known everywhere on the aperture plane $\Sigma_A$. When employing the FFT, $\Sigma_A$ is truncated. To minimize the amount of computer time, $\Sigma_A$ should be as small as possible while capturing almost all of the field. The number of grid points should satisfy the Nyquist (sampling) theorem. The secondary pattern can be reconstructed from a discrete set of near-field values if the spacing is $\lambda/2$ or less. Reference 11 describes sample spacing for high-gain reflector antennas.

This method of calculating the secondary pattern is accurate in cases where the antenna diameter is of the order of $100\lambda$ or more. If the antenna diameter is less than $100\lambda$, the accuracy is reduced, specifically in the sidelobe region. This approach was originally used by ~~Kuffman and Crowell~~ Kaufmann & Croswell (ref. 4) and developed extensively by many others (e.g., Hwang, et al., ref. 12). The reflector configuration described in figure 1 was analyzed by using various methods (physical optics (Jacobi-Bessel) (refs. 13 and 14), the geometrical theory of diffraction (ref. 15), and geometrical optics). The calculated patterns and directivities are shown in figures 6 and 7. Figure 6 shows an on-focus case; the directivity and the far-field pattern are in very good agreement. Figure 7 shows an off-focus (or scan) case; the agreement is still very good. The computer program given in appendix B was used to analyze this configuration. ~~The agreement is also very good for the directivity~~. Appendix A contains a detailed description (users guide) of all the input parameters, including those for the configuration shown in figure 1.

## Directivity

The far-field zone is usually divided into two orthogonal polarizations. Following Ludwig's definition 3 (ref. 16), the following unitary polarization vectors are introduced:

$$\hat{R} = \hat{\theta}(ae^{j\psi} \cos\varphi + b \sin\varphi) + \hat{\varphi}(-ae^{j\psi} \sin\varphi + b \cos\varphi) \tag{8a}$$

$$\hat{C} = \hat{\theta}(ae^{-j\psi} \sin\varphi - b \cos\varphi) + \hat{\varphi}(-ae^{-j\psi} \cos\varphi + b \sin\varphi) \tag{8b}$$

If the secondary pattern can be expressed as

$$\vec{E} = \frac{e^{-jkr}}{r} [\hat{\theta} E_\theta + \hat{\varphi} E_\varphi] \tag{9}$$

the reference-polarization expression for $\vec{E}$ is

$$\vec{E} \cdot (\hat{R}^*)^* \tag{10a}$$

and the cross-polarization expression is

$$\vec{E} \cdot (\hat{C}^*)^* \tag{10b}$$

The directivity for the reference polarization is defined by

$$D_R(\theta,\varphi) = \frac{4\pi \,|\vec{E} \cdot \hat{R}|^2/Z_0}{P_{rad}} \tag{11a}$$

Similarly directivity for the cross polarization is defined by

$$D_C(\theta,\varphi) = \frac{4\pi \,|\vec{E} \cdot \hat{C}|^2/Z_0}{P_{rad}} \tag{11b}$$

## CONCLUDING REMARKS

A method, called the aperture field technique, has been developed for calculating the secondary pattern of an offset reflector illuminated by a feed with arbitrary polarization. By using the fast Fourier transform the far-zone electric field is computed very efficiently. The results for the secondary pattern are in good agreement with those obtained by other well-known techniques. This method can be conveniently extended to secondary pattern computation for multiple and numerically specified reflectors.

The computer program based on the aperture field technique is one of the main research tools used at the NASA Lewis Research Center for analyzing advanced antenna systems.

# APPENDIX A

## DESCRIPTION OF PROGRAM

A computer program was designed to calculate the antenna far-field performance characteristics. The method of analysis is geometrical optics. The output (unit FT07) is interfaced with a plotting program to display the far-field values. Any of the available plotting routines in the computer library may be used. The main features in this program (SOURCE • RAP$11) are the calculation of directivity, feed efficiency, and plots of E or H far-field plane cuts.

### Input (Unit FT05) Parameters to RAP$11

| | |
|---|---|
| F | focal length, m |
| F1 | fraction of wavelength that defines surface sample spacing |
| F2 | fraction of wavelength that defines aperture sample spacing |
| DIA | diameter of reflector (if circular projection), m |
| XF,YF,ZF | coordinates of center (origin) of feed coordinate system on reflector coordinate system |
| DELTX | offset of center of reflector relative to x-axis, m |
| DELTY | offset of center of reflector relative to y-axis, m |
| FREQ | frequency |
| QE | E-plane cosine pattern exponent |
| QH | H-plane cosine pattern exponent |
| POL | 1, Y-POL; 2, X-POL; 3, RHCP; 4, LHCP |
| ZETA | maximum far-field angle for which pattern will be calculated |

The input is described in figure 8.

The input for the case described in figures 1, 6, and 7 (to be read in FT05) is as follows:

F = 2.43
F1 = 0.5
F2 = 0.5
DIA = 2.754
XF = YF = ZF = 0.0
DELTX = -1.377
DELTY = 0.434
FREQ = 11.74E9
QE = 3.2
QH = 2.6
POL = 3
ZETA = 4.0

# APPENDIX B

## COMPUTER PROGRAM

```
C******THIS PROGRAM IS REFLECTOR ANALYSIS THIS THE MOST GENERAL VERSION
C******THE METHOD OF ANALYSIS IS APERTURE INTEGRATION.

      DIMENSION PNUM(500,500)
       DIMENSION X(500),Y(500),ERX(500,500),ERY(500,500)
      DIMENSION PH(500,500),EXR(5000),EXI(5000),EYR(5000),EYI(5000)
       DIMENSION FIELDX(1000),FIELDY(1000),FIELD(1000),ANG(1000)
         DIMENSION RTNARR(2)
C*****INPUT PARAMETERS TO THIS PROGRAM***************************
C*****F : FOCAL LENTGTH M
C*****F1 : FRACTION OF WAVELENTGH (SURFACE SAMPLE SPACING)
C*****F2 : FRACTION OF WAVELENTGH (APERTURE SAMPLE SPACING)
C*****DIA : DIAMETER M
C*****XF,YF,ZF : ORIGIN OF FEED COORDINATE SYSTEM.
C*****DELTX,DELTY : OFFSET X-AXIS,Y-AXIS RESP.
C*****FREQ : FREQUENCY HZ
C*****QE,QH,Q :COSINE PATTERN EXPONENT
C*****POL : 1 Y-POL, 2 X-POL, 3 RHCP, 4 LHCP
C*****ZETA : MAX FAR-FIELD ANGLE.
       INTEGER*2 N1
       NAMELIST/INPUT/F,F1,F2,DIA,XF,YF,ZF,DELTX,DELTY,FREQ,Q,ZETA,QH,
       QE,POL
       READ(5,INPUT)
C*****ALL CONSTANT PARAMETERS IN THE PROGRAM**************************
      PINT=0.
      WRITE(9,997)
997   FORMAT(5X,'THIS IS RFLECTOR ANALYSIS PROGRAM. BY R.J. ACOSTA')
       PI=4.*ATAN(1.)
       AWAVE=3E8/FREQ
       SSP=F1*AWAVE
       NSP=(DIA/SSP)+1
       SAP=F2*AWAVE
       NAP=(DIA*1.5/SAP)+1
       RMAX=0.5*DIA
       XC=DELTX+RMAX
       YC=DELTY+RMAX
      ZC=-(XC**2+YC**2)/(4.*F)+F
       XSMAX=XC+RMAX
       YSMAX=YC+RMAX
       XAMAX=XC+RMAX*1.5
       YAMAX=YC+RMAX*1.5
      ALIM=.5*SAP
      FACT=((AWAVE**2)*(120.*PI))
      FACT=1/FACT
C*****POWER RADIATED BY THE FEED SOURCE.******************************
      PRAD=60.*(2.*Q+1)
      PRAD=1/PRAD
C*****THIS CALCULATES THE SUBSTENTED ANGLE BY REFLECTOR REL TO FEED*****
     XU=XC
     YU=YC+RMAX
     ZU=-(XU**2+YU**2)/(4.*F)+F
     XL=XC
     YL=YC-RMAX
     ZL=-(XL**2+YL**2)/(4.*F)+F
     A117=(XU-XF)*(XL-XF)+(YU-YF)*(YL-YF)+(ZU-ZF)*(ZL-ZF)
     A115=(XU-XF)**2+(YU-YF)**2+(ZU-ZF)**2
     A115=SQRT(A115)
     A116=(XL-XF)**2+(YL-YF)**2+(ZL-ZF)**2
     A116=SQRT(A116)
     SI=ARCOS(A117/(A115*A116))
```

```
C*****CALCULATION OF POWER INTERCEPTED***********************************
      EXP=2.*Q+1
      P11=-COS(.5*SI)**EXP
      P12=P11/EXP+1/EXP
      PINT=P12/60
      PEFFR=PINT/PRAD
      TAPPER=20.*ALOG10(COS(SI*.5)**Q)
      WRITE(9,998)PEFFR,TAPPER
998   FORMAT(/////,5X,'EFFICIENCY OF FEED',F10.3,5X,'TAPPER OF
      FEED =',E15.5)
      DO 400 I=1,NAP
      DO 410 J=1,NAP
      PNUM(I,J)=0.
      ERX(I,J)=0.
      ERY(I,J)=0.
 410  CONTINUE
 400  CONTINUE
C*****NUMERICAL INTEGRATION OF FEED POWER******************************
      DELTAP=PI/NSP
      DELTA=SI/NSP
      NSP1=NSP*.5
      DO 201 I=1,NSP1
      ZETA1=DELTA*(I-1)
      A145=2.*Q
      FACT1=COS(ZETA1)**A145
      FACT1=FACT1*SIN(ZETA1)*DELTA
      PINTN=PINTN+FACT1
201    CONTINUE
      PINTN=PINTN/60
      PNUR=PINTN/PINT
      WRITE(9,993)PNUR
993   FORMAT(/////,5X,'THIS NUMERICAL TO EXACT RATIO =',F10.3)




C******SET SURFACE COORDINATES****************************************

       DO 10 J=1,NSP
       DO 11 I=1,NSP
       XS=XSMAX-SSP*(I-1)
       YS=YSMAX-SSP*(J-1)
       RS=SQRT((XS-XC)**2+(YS-YC)**2)
       IF(RS.LE.RMAX)GO TO 12
       GO TO 11
12     CONTINUE
       ZS=-(XS**2+YS**2)/(4.*F)+F
C******CALCULATE THE NORMAL TO SURFACE*******************************

       AMAGN=SQRT(XS**2+YS**2+(4.*F**2))
       ANX=-XS/AMAGN
       ANY=-YS/AMAGN
       ANZ=-2.*F/AMAGN
C******CALCULATE INCIDENT RAY****************************************

       AIMAGN=SQRT((XS-XF)**2+(YS-YF)**2+(ZS-ZF)**2)
       SIX=(XS-XF)/AIMAGN
       SIY=(YS-YF)/AIMAGN
       SIZ=(ZS-ZF)/AIMAGN
C******CALCULATE REFLECTED RAY***************************************

       SRX=SIX-2.*(SIX*ANX+SIY*ANY+SIZ*ANZ)*ANX
       SRY=SIY-2.*(SIX*ANX+SIY*ANY+SIZ*ANZ)*ANY
       SRZ=SIZ-2.*(SIX*ANX+SIY*ANY+SIZ*ANZ)*ANZ

```

```
C******APERTURE COORDINATE OF REFLECTED RAY***************************

      XA=XS-ZS*(SRX/SRZ)
      YA=YS-ZS*(SRY/SRZ)
C******LOCATE THIS RAY IN THE APERTURE GRID MATRIX********************




      DO 21 N=1,NAP
      X(N)=XAMAX-SAP*(N-1)
      XDELTA=ABS(X(N)-XA)
      IF(XDELTA.LE.ALIM) GO TO 30
 21   CONTINUE
      GO TO 500
 30   CONTINUE
      DO 20 M=1,NAP
      Y(M)=YAMAX-SAP*(M-1)
      YDELTA=ABS(Y(M)-YA)
      IF(YDELTA.LE.ALIM) GO TO 40
 20   CONTINUE
      GO TO 500
40    CONTINUE
C******STAR THE CALCULATION OF FIELDS AT APERTURE*************

      A11=SQRT((XC-XF)**2+(YC-YF)**2+(ZC-ZF)**2)
      A12=SQRT((XS-XF)**2+(YS-YF)**2+(ZS-ZF)**2)
      ADOT=(XC-XF)*(XS-XF)+(YC-YF)*(YS-YF)+(ZC-ZF)*(ZS-ZF)
      ADOT=ADOT/(A11*A12)

      FPAT=ADOT**Q
C****THIS IS TO CALCULATE THE TOTAL POWER INTERCEPTED BY REFLECTOR****

      IF(FPAT.LT.1E-5)FPAT=1E-5
C*****FINISH FEED PATTERN CALCULATIONS******************************
C*****INCIDENT UNIT VECTOR CALCULATION******************************
      EIX=-(SIX*SIY)
      EIY=(SIX**2+SIZ**2)
      EIZ=-(SIZ*SIY)
      A13=SQRT(EIX*EIX+EIY*EIY+EIZ*EIZ)
      EIX=EIX/A13
      EIY=EIY/A13
      EIZ=EIZ/A13
C*****INCIDENT FIELD AMPLITUDE***************************************
      EINX=EIX*FPAT/A12
      EINY=EIY*FPAT/A12
      EINZ=EIZ*FPAT/A12
C*****REFLECTED FIELD AMPLITUDE CALCULATION**************************
      ERFX=2.*(ANX*EINX+ANY*EINY+ANZ*EINZ)*ANX-EINX
      ERFY=2.*(ANX*EINX+ANY*EINY+ANZ*EINZ)*ANY-EINY
      ERFZ=2.*(ANX*EINX+ANY*EINY+ANZ*EINZ)*ANZ-EINZ
C******THE PHASE CALCULATION ******************************************
      A31=SQRT((XA-XS)*(XA-XS)+(YA-YS)*(YA-YS)+ZS*ZS)
      PHASE=(2*PI/AWAVE)*(A12+A31)
      ERX(N,M)=ERX(N,M)+ERFX
      ERY(N,M)=ERY(N,M)+ERFY
      PH(N,M)=PH(N,M)+PHASE
      PNUM(N,M)=PNUM(N,M)+1
      GO TO 11
500   WRITE(9,300)
300   FORMAT(5X,'CAN NOT LOCATE THIS RAY????????????????')
11    CONTINUE
10    CONTINUE
C*****THIS WILL TAKE THE AVERAGE AMPLITUDE/PHASE******************
       DO 60 I=1,NAP
       DO 61 J=1,NAP
```

```
         IF(PNUM(I,J).EQ.0.) GO TO 61
         ERX(I,J)=ERX(I,J)/PNUM(I,J)
         ERY(I,J)=ERY(I,J)/PNUM(I,J)
         PH(I,J)=PH(I,J)/PNUM(I,J)
    61   CONTINUE
60       CONTINUE
C******COLLAPSE THE FIELD BY ROWS OR IN X,***************************
       WRITE(55)NAP,NAP
      WRITE(55)((ERY(I,J),J=1,NAP),I=1,NAP)
      WRITE(55)((PH(I,J),J=1,NAP),I=1,NAP)
      WRITE(55)((ERX(I,J),J=1,NAP),I=1,NAP)
      WRITE(55)((PH(I,J),J=1,NAP),I=1,NAP)
C******TO GET A COLUMN VECTOR OR Y DISTRIBUTION**********************

        DO 15 I=1,NAP
        DO 16 J=1,NAP


        EXR(I)=EXR(I)+ERX(J,I)*COS(PH(J,I))
        EXI(I)=EXI(I)+ERX(J,I)*SIN(PH(J,I))
        EYR(I)=EYR(I)+ERY(J,I)*COS(PH(J,I))
        EYI(I)=EYI(I)+ERY(J,I)*SIN(PH(J,I))
16      CONTINUE
15      CONTINUE
C*****THIS IS TOTAL POWER IN X AND Y COPONETS OF N.F.*************
C******APLYING THE FFT OR APERTURE INTEGRATION METHOD******************
       NAP=NAP*5

        CALL FFT(EXR,EXI,NAP,NAP,NAP,1)
        CALL FFT(EYR,EYI,NAP,NAP,NAP,1)
        A133=NAP*SAP/AWAVE
        NMAX=A133*SIN(ZETA*PI/180.)
        NMAX=NMAX+1
        DO 70 I=1,NMAX
         J1=NMAX-(I-1)
         A134=J1/A133
        ANG(I)=-ARSIN(A134)*180./PI
        J2=I+NMAX
        A135=(I-1)/A133
         ANG(J2)=ARSIN(A135)*180/PI
70      CONTINUE
        DO 71 J=1,NMAX
         I1=NAP-NMAX-1+1+J
        FIELDX(J)=FACT*(EXR(I1)*EXR(I1)+EXI(I1)*EXI(I1))*COS(ANG(J)
        *PI/180)**2
        FIELDY(J)=FACT*(EYR(I1)*EYR(I1)+EYI(I1)*EYI(I1))
        FIELD(J)=FIELDX(J)+FIELDY(J)
        I2=J+NMAX
        FIELDX(I2)=FACT*(EXR(J)*EXR(J)+EXI(J)*EXI(J))*COS(ANG(J)
        *PI/180)**2
        FIELDY(I2)=FACT*(EYR(J)*EYR(J)+EYI(J)*EYI(J))
        FIELD(I2)=FIELDX(I2)+FIELDY(I2)
71      CONTINUE
        N1=2*NMAX
        CALL SCLBAK(.FALSE.,N1,FIELD,RTNARR)
        GAIN=10.*ALOG10(4.*PI*RTNARR(2)*SAP**4/PRAD)
        WRITE(9,950)GAIN
950     FORMAT(/////,5X,'THIS IS THE GAIN VALUE =',E15.5)
       WRITE(9,965)N1
965    FORMAT(/////,5X,'TOTAL NUMBER OF CALCULATED FAR-FIELD
       VALUES =',I5)
        DO 73 I=1,N1
        FIELD(I)=FIELD(I)/RTNARR(2)
        IF(FIELD(I).LT.1E-6)FIELD(I)=1E-6
        FIELD(I)=10.*ALOG10(FIELD(I))
       WRITE(7,565)ANG(I),FIELD(I)
565     FORMAT(5X,E15.5,5X,E15.5)
 73     CONTINUE
        STOP
        END
```

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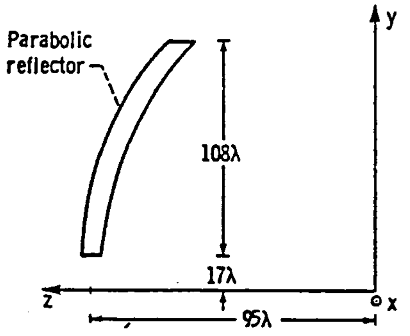
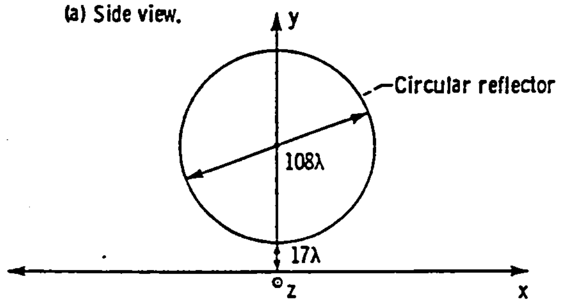


(a) Side view.



(b) Front view.

Figure 1. - Reflector configuration. Frequency, 11.75 GHz. Polarization of feed: $a = 1/\sqrt{2}$, $b = 1/\sqrt{2}$, and $\psi = 90^\circ$ for right-hand circular polarization; $U_E(\theta) = (\cos\theta)^{3.6}$; $U_H(\theta) = (\cos\theta)^{2.8}$.

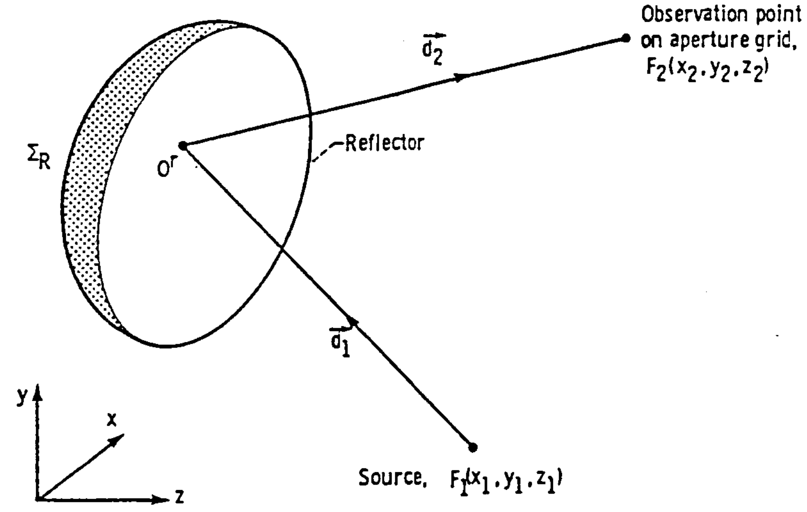


Figure 2. - Reflector $\Sigma_R$ being illuminated by incident field from point source at $F_1$.

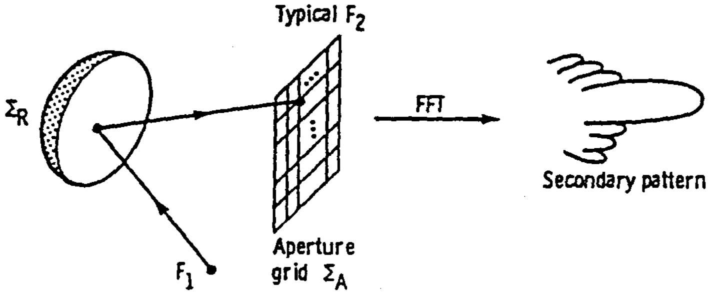


Figure 3. - Secondary pattern using fast Fourier transform (FFT).

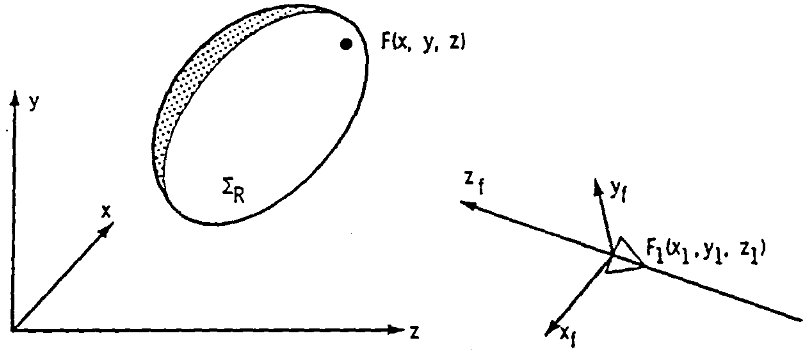


Figure 4. - Feed and reflector coordinate systems.

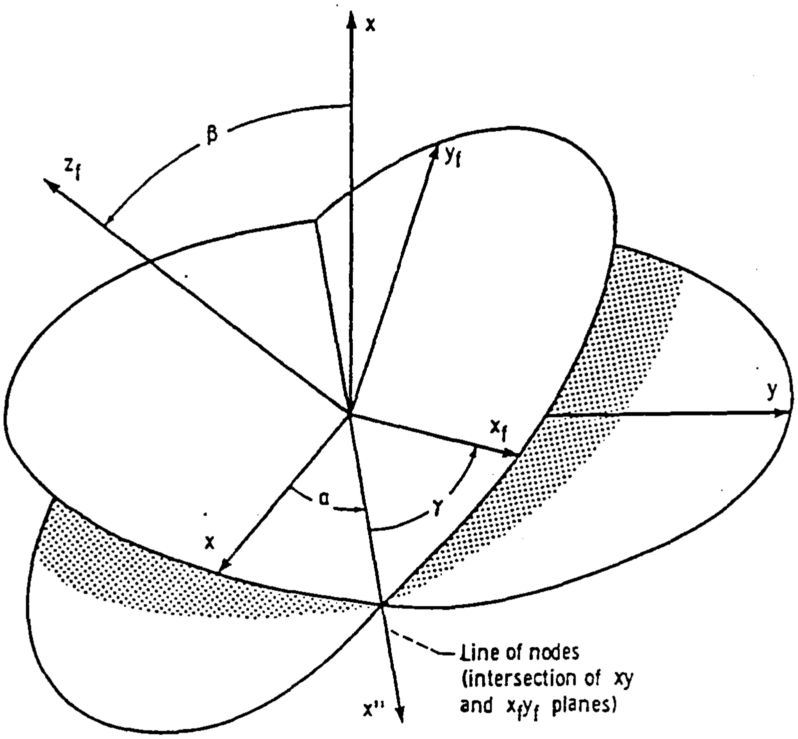


Figure 5. - Eulerian angles.

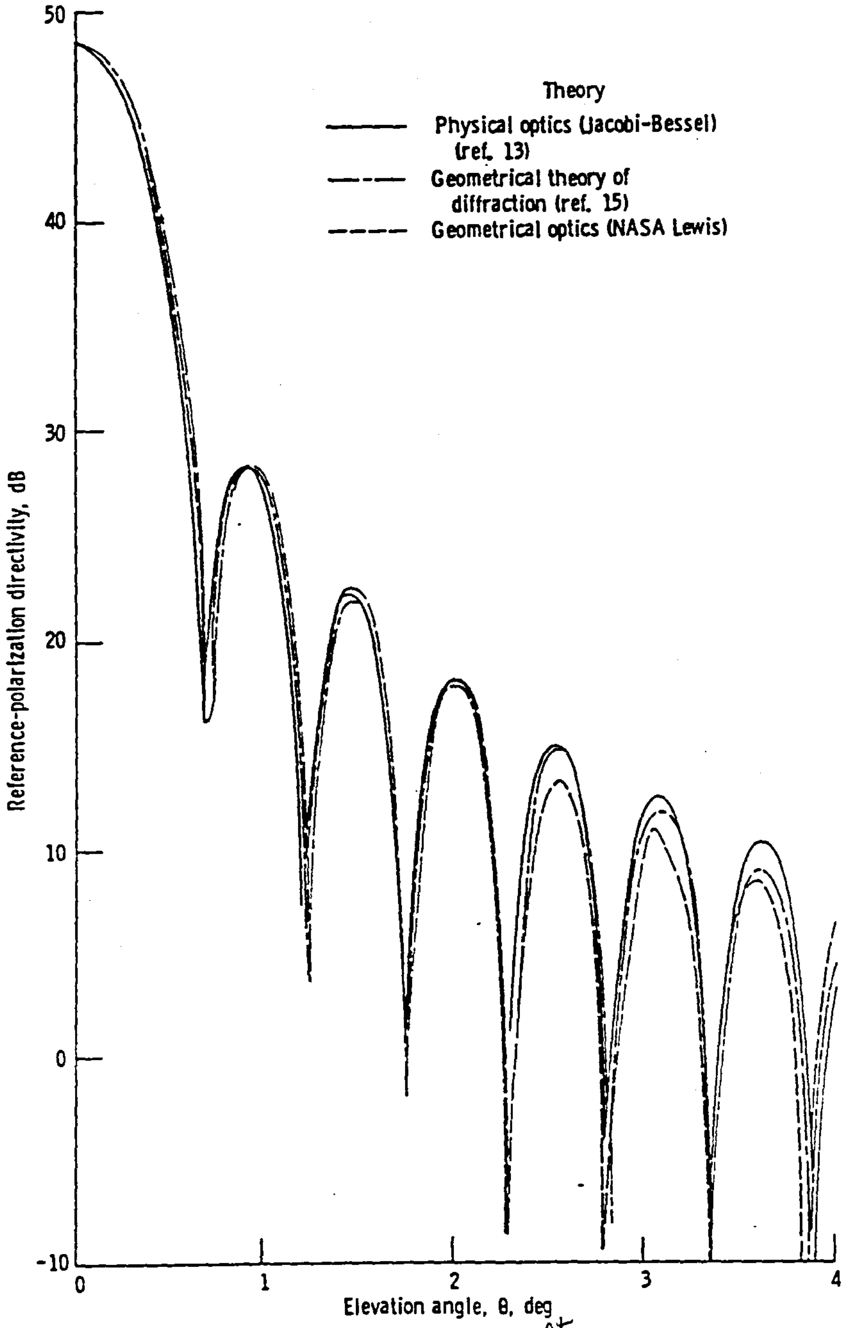


Figure 6. - On-focus case (feed is at focal point).

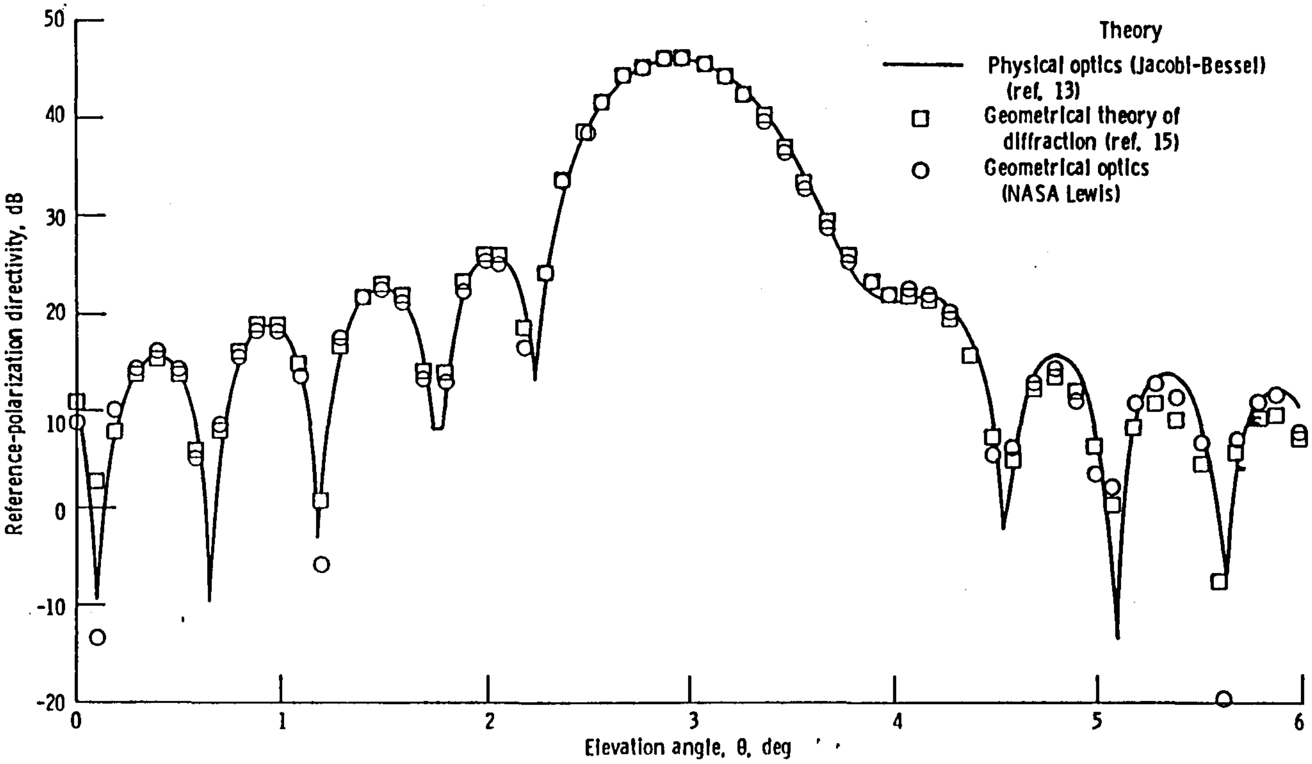
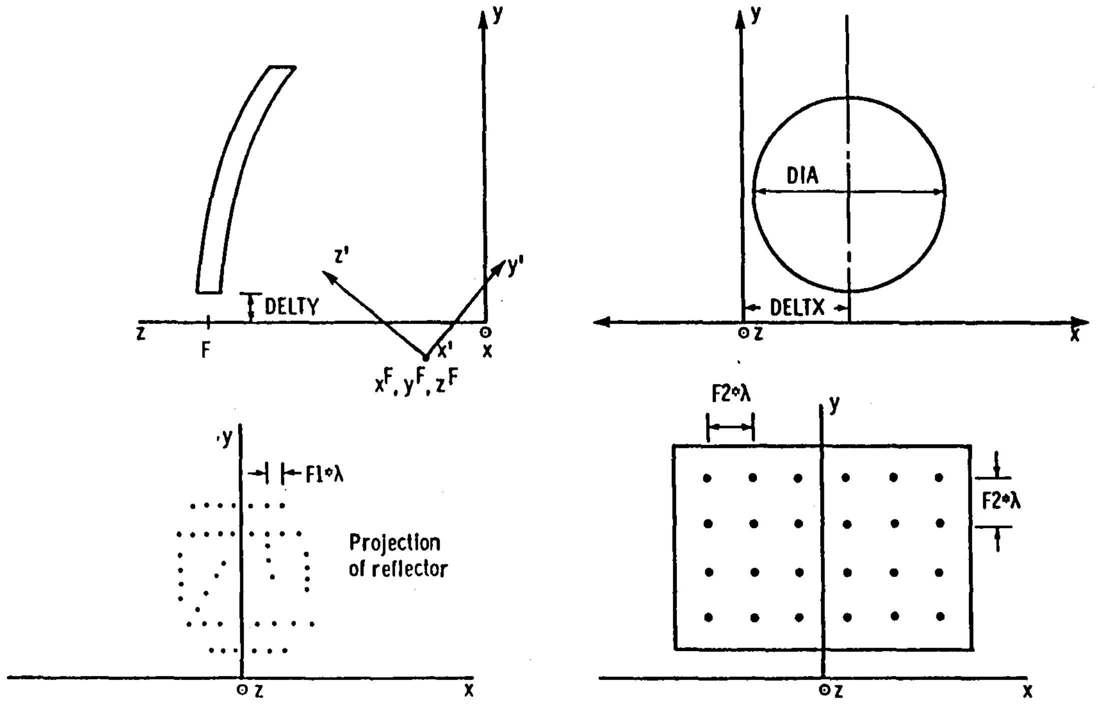


Figure 7. - Scan case (feed moved off focus).



(a) Surface grid.

(b) Aperture grid.

Figure 8. - Description of input.

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16. Abstract

Reflector antennas are widely used in communications satellite systems because they provide high gain at low cost. In analyzing reflector antennas the computation of the secondary pattern is the main concern. A computer program for calculating the secondary pattern of an offset reflector has been developed and implemented at the NASA Lewis Research Center. The theoretical foundation for this program is based on the use of geometrical optics to describe the fields from the feed to the reflector surface and to the aperture plane. The resulting aperture field distribution is then transformed to the far-field zone by the fast Fourier transform algorithm. Comparing this technique with other well-known techniques (the geometrical theory of diffraction, physical optics (Jacobi-Bessel), etc.) shows good agreement for large (diameter of $100\lambda$ or greater) reflector antennas.

17. Key Words (Suggested by Author(s))
Antenna radiation patterns; Near-field analysis; Numerical analysis; Fast Fourier transform

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