NASA Technical Memorandum 87188

# SCARE—A Post-Processor Program to MSC/NASTRAN for the Reliability Analysis of Structural Ceramic Components

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(NASA-TM-87188) SCARE: A POST-PROCESSOR PROGRAM TO MSC/NASTRAN FOR THE RELIABILITY ANALYSIS OF STRUCTURAL CERAMIC COMPONENTS (NASA) 24 p HC A02/MF A01 CSCL 20K
N86-146[illegible]
Unclas
G3/39 05056

Prepared for the
31st International Gas Turbine Conference and Exhibit
sponsored by the American Society of Mechanical Engineers
Dusseldorf, West Germany, June 8-12, 1986

NASA

# SCARE - A POST-PROCESSOR PROGRAM TO MSC/NASTRAN FOR THE RELIABILITY ANALYSIS OF STRUCTURAL CERAMIC COMPONENTS

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## SUMMARY

A computer program is developed for calculating the statistical fast fracture reliability and failure probability of ceramic components. The program includes the two-parameter Weibull material fracture strength distribution model, using the principle of independent action for polyaxial stress states and Batdorf's shear-sensitive as well as shear-insensitive crack theories, all for volume distributed flaws in macroscopically isotropic solids. Both penny-shaped cracks and Griffith cracks are included in the Batdorf shear-sensitive crack response calculations, using Griffith's maximum tensile stress or critical coplanar strain energy release rate criteria to predict mixed mode fracture. Weibull material parameters can also be calculated from modulus of rupture bar tests, using the least squares method with known specimen geometry and fracture data. The reliability prediction analysis uses MSC/NASTRAN stress, temperature and volume output, obtained from the use of three-dimensional, quadratic, isoparametric, or axisymmetric finite elements. The statistical fast fracture theories employed, along with selected input and output formats and options, are summarized. An example problem to demonstrate various features of the program is included.

E-2654

## INTRODUCTION

The attractive physical and mechanical properties of modern ceramics - high temperature strength, light weight, excellent erosion, corrosion and oxidation resistance, low thermal conductivity, low cost, and wide availability - have made ceramics an increasingly important structural material. The potential of ceramics in demanding structural applications is especially attractive when resistance to high temperatures, such as in heat engines, is the main concern. With today's needs for more fuel efficient transportation, multifuel engine capability and reduced emissions, advanced engines, operating at much higher temperatures with ceramic components, appear to be mandatory to economically meet these national objectives.

However, ceramics, like all other brittle materials, display linear stress-strain behavior from zero to fracture. The lack of ductility and yielding capability give ceramic materials their most undesirable characteristics such as low strain tolerance, low fracture toughness, and large variation in observed fracture strength. This wide variation of material strength is due to the nature and distribution of intrinsic microscopic flaws, which are unavoidably present as a result of materials processing operations. Failure in ceramics usually initiates at a single weakest flaw when the local stress there reaches a critical value. Because of the large scatter in strength, designers today use statistics and reliability analysis for the failure prediction of brittle material components, which may be subject to arbitrary loadings and multidimensional stress states.

The first probabilistic approach used to account for the scat r in fracture strength of brittle materials was introduced by Weibull (ref. .). His analysis was based on the weakest link theory (WLT) and assumed a unique strength cumulative distribution for the uniaxial fracture data obtained from simple specimen tests. Weibull also proposed a method for calculating the failure probability in multidimensional stress fields when using material parameters obtained from uniaxial tests. His approach basically involves calculating the risk of rupture by averaging the tensile stress in all directions. This is intuitively plausible but not rigorous, and consequently other models were introduced. The most widely used among them is the assumption that principal stresses act independently (PIA) (ref. 2). This is a very convenient formulation because of its simplicity. However, as shown by several investigators these models can lead to unsafe estimates of failure probability, since they both eglect the shear force, and in case of the PIA hypothesis, the effects of combined local principal stresses (ref. 3).

An important element of failure predictive theories is the crack extension criterion. In the classical Weibull formulation, a normal stress criterion is used, which is likely to be correct when the dominant crack is normal to a uniaxial tensile stress. However, in a multiaxial stress field with flaws orientated at arbitrary angles to the applied stresses both normal tensile stresses and in-plane shear stresses will influence the deformation and fracture processes (refs. 4 to 6), and lead to a different fracture response than that of the uniaxial case. Several fracture criteria have been proposed (refs. 7 to 9), with the critical coplanar strain energy release rate, $G_c$, criterion, among those available in this study, leading to the best agreement with available brittle material experimental data.

The primary objective of this report is to develop a public domain computer program which will be coupled to a general purpose finite element code, such as MSC/NASTRAN (ref. 10), to predict the fast fracture failure probability of ceramic components due to the presence of volume type flaws. The user is given various options to select currently available fracture models in addition to calculating statistical material parameters. Two versions of the program are presently available which are designated as SCARE1 (Structural Ceramics Analysis and Reliability Evaluation) and SCARE2, respectively (ref. 11). In SCARE1, the finite element centroidal principal stresses are taken as constant throughout the element volume and the convergence of the mesh for accurate stress analysis leads to convergence for volume type flaw reliability analysis. However, previous results from higher order isoparametric finite elements with permissible internal stress gradients showed that the accuracy of failure predictions is significantly improved when the finite element volumes are further subdivided. In the SCARE2 version, all 6-sided HEXA MSC/NASTRAN elements are further discretized into 27 subelements, which are then used with interpolated principal stresses to perform all analysis.

## PROGRAM CAPABILITY AND DESCRIPTION

The basic computational elements of the post-processor program for the reliability analysis of structural ceramic components using the SCARE2 version of the code are shown in figure 1. Clearly the NASTRAN part is totally independent of SCARE, and output data from other general purpose analysis programs could also be used as long as similar elements are available for the thermal and stress analysis of the structure. For computational efficiency, all the experimental fracture stresses, if used to calculate material parameters, as well as the elemental principal stresses are normalized. Initially, the normalizing stress used is the average of all experimental fracture data when specimen test results are available. When material parameters are known, an appropriate value of the Weibull scale parameter, $\sigma_0$, is used to normalize all stresses.

As shown in figure 1, executing the SCARE program requires FORTRAN logical units 1, 3, 4, 5, and 7 in addition to those generally used in performing NASTRAN analysis (such as units 5, 6, and 14). The program uses these tape drives for intermediate storage of large amounts of data, so that the transfer of information from NASTRAN to SCARE is done internally rather than manually. MSC/NASTRAN Bulk Data including required nodal temperatures, and element centroidal as well as selected corner node stresses, are stored on logical unit 7 where NASTRAN punch files are saved for access and further processing in SCARE. Since element volumes are obtained through a NASTRAN Parameter call, the required volume data is taken from the printout files on unit 6 and stored on logical unit 3. The input to SCARE is handled through logical unit 5, but the output from SCARE had to be placed on unit 1 to avoid potential terminal problems. In addition to NASTRAN output files and analysis data, input to SCARE includes control indices specifying various fracture models, temperature dependent material parameters if available, specimen geometry and ordered (in ascending order for a given temperature) fracture strength data when required statistical parameters are internally calculated.

In order to use WLT, no principal compressive stress is permitted to exceed three times the maximum principal tensile stress in absolute value. If this criterion is violated in an element, compressive stress state predominates and the corresponding reliability is set equal to unity. Additionally, when using the PIA model in conjunction with Weibull statistics, only tensile principal stresses can contribute to failure and fracture due to compression is inadmissible.

The program has broad capabilities by allowing the user to specify temperature dependent statistical material parameters, several crack configurations and four fracture criteria. Uniaxial fracture data along with specimen geometry from four point modulus of rupture (MOR) bar tests can be used to calculate Weibull parameters and the Batdorf crack density coefficient. Figure 2 contains the flowchart summarizing the available options is fracture criteria and flaw configurations used to model the volume imperfections. Note that two of the failure criteria are for shear-insensitive cracks, even in polyaxial stress states. The other two criteria are used for the more general shear-sensitive model. The available three crack configurations include the spherical void, which is isotropic or direction independent, and is inherently assumed in the Weibull PIA and normal stress failure theories. However, imperfections in high density, sintered ceramics are best represented by the structural response of penny-shaped and Griffith cracks. Among the available criteria and crack configurations shown in figure 2, the penny-shaped crack (PSC) with the $G_c$ criterion gives the highest failure probability for a given case, while the PIA approach yields consistently the lowest failure estimate. It should also be noted that the Batdorf shear-insensitive fracture model, although in slightly different form, gives identical results to the originally proposed Weibull normal stress averaging method.

## INPUT INFORMATION

NASTRAN (NASA Structural Analysis) is a large, comprehensive, general purpose finite element computer code for structural analysis, which was developed under NASA sponsorship to fill the need for a universally available analysis program. In addition to the government supported version, there are several, greatly enhanced, proprietary versions of this program, the most widely known of which is called MSC/NASTRAN. This program is used throughout the world in large corporations, government laboratories, and most commercial data centers. The SCARE program utilizes results from only a very small fraction of available NASTRAN analysis capability. Since fast fracture mechanical design of ceramic components requires only the temperature and stress distributions, static analysis results from rigid formats 61 and 47 (in case of cyclic symmetry) are most often used. Ceramics are also extremely sensitive to geometric discontinuities, requiring the use of isoparametric three-dimensional and quadratic axisymmetric finite elements. Within MSC/NASTRAN, these elements are denoted as HEXA, PENTA, and TRIAX6, respectively. Although the midedge nodes of HEXA elements in NASTRAN are optional, their use when analyzing with SCARE is required. It is assumed here that analysts using the SCARE program would be fully familiar with MSC/NASTRAN, and its input requirements in creating the Executive Control, Case Control, and Bulk Data decks. Preceding the entire NASTRAN input file is the system operating JCL (job control language) set of commands, which usually identify the job, user, set time and memory requirements, and define NASTRAN input, punch, plot, and printout files. If the self-contained, solid modeling processor, called MSGMESH, is used to discretize the structure, the SCARE program includes a number of sorting routines to permit arbitrary numbering of elements and nodes. Figure 3 shows the arrangement of a typical NASTRAN input file.

The MSC/NASTRAN program at the Lewis laboratory runs on the CRAY 1-S/2200 computer in a batch-mode. Input and output to and from the CRAY is handled through an IBM 370 mainframe computer, which serves as the front-end processor for the CRAY. Consequently, an additional set of JCL commands is required to handle the involved data sets, compiler, FORTRAN logical units, and execution commands. Both versions of the SCARE source program are permanently stored on the IBM in the user's library, where all reliability calculations are eventually made. Details of the system JCL for executing MSC/NASTRAN first, and then the SCARE program can be found in reference 11. These instructions are unique to the computer system existing at Lewis, but are representative of the required commands in performing reliability analysis at other installations.

SCARE input requirements can be grouped into three categories. The first category, called Master Control Deck, defines control indices, information on the finite element mesh and some integer data describing crack configurations, fracture criteria and material parameter format. Figure 4 shows the details of the required information, with explanatory notes and size limitations available in the program user's manual (ref. 11). The second category, called the Specimen Deck, summarizes fracture specimen results needed in calculating statistical fracture parameters, or direct material properties, including Poisson's ratio, when available. There are five different entries required in this category. The first entry includes the material Poisson's ratio, which is used in the reliability calculations when PSC's are selected for volume imperfections. The second entry defines the MOR specimen geometry, which was used in generating fracture data. The third entry includes experimental, extreme fiber fracture stresses, arranged in ascending order for a given temperature. For multiple temperature tests, the temperatures must also be ordered according to ascending values, since calculated material parameters are interpolated within SCARE. At a specified temperature, fracture readings must be unique and multiple values of identical magnitudes are not permitted. The number of available fracture readings for all temperature tests must be the same. The fourth entry is used when material statistical parameters are directly available as a function of temperature. The three required parameters are the Weibull modulus or shape parameter, m, the Batdorf normalized crack density coefficient or flaw parameter, $k_B$, and the Weibull scale parameter, $\sigma_0$. These material parameters must be so arranged that they correspond to ascending order of discrete temperatures. The last entry in the Specimen Deck category lists the discrete temperatures at which fracture data or material parameters are known. Additional explanation of the required input, including size limitations, can be found in reference 11.

The last SCARE input category, called the Structures Deck, contains results of the finite element structural analysis needed for failure probability predictions. These include element volume, element or nodal temperatures, and element principal stresses along with the appropriate identification numbers. In the present version of SCARE, which relies on MSC/NASTRAN output files, all of this data is internally manipulated through subroutine ELEM, and the Structures Deck requires no specific input by the user. It is this input data, however, that has to be carefully catalogued if another general purpose program were to be used or element data were to be directly read, instead of using temporary storage devices.

## OUTPUT INFORMATION

The results of MSC/NASTRAN thermal analysis are the grid point temperatures, which can be obtained at transient or steady-state conditions. After solution of the component temperature distribution, the most severe thermal gradients can be selected and combined with the mechanical loads to obtain an elastic solution (rigid formats 47 or 61). The usual output from these rigid formats includes the nodal displacements along the global coordinate directions and the element stresses. Depending on the element type, normal and shear stresses in the local element, or material coordinate system are always available at element centroids. Additionally, element principal stresses are calculated there for the HEXA and PENTA elements. Corner node stresses are also printed for these elements. For available stress recovery options, users should consult reference 10 and the appropriate program manuals. In addition to the displacements and stresses, useful parameters such as element volumes, element areas, component center of gravity, moments of inertia, etc. can be calculated through the Parameter call feature of the program. For volume flaw reliability analysis, the element volumes are essential, since in the weakest link model, integration of the stress distribution over the material volume is needed.

The first part of all SCARE output data contains an echo of important NASTRAN finite element analysis results. Identifying labels, element type, and number of elements in the model are noted. A table of element centroidal principal stresses with appropriate element identification numbers is given. For the SCARE2 version, the 27 centroidal subelement principal stresses within each HEXA element are listed. Element volumes and calculated element temperatures are summarized in another output table. Next, the selected fracture model is identified and the room temperature (70 °F) statistical fracture parameters are listed. Additionally, a table of discrete temperatures with corresponding material parameters, which were either internally calculated or directly supplied, is printed. For the shear-sensitive fracture models, the crack shape is identified along with a more specific description of the fracture criterion. The last table in the SCARE output file contains an element results summary, listing the element number, corresponding element survival probabili ies and material parameters. Finally, the overall component probability of failure as well as the component probability of survival are printed.

## THEORY

The statistical nature of fracture in engineering materials can be viewed from two distinct and extreme models. The first was presented by Weibull and is termed the weak link model. With it a structure is characterized as a series of links connected in such a manner that the structure fails whenever any of the links fractures. In contrast the second model is referred to as the bundle or parallel model for which failure is defined only when all links in parallel have fractured. Structural ceramics have been observed to approach the weakest link hypothesis and fail when the stress intensity factor at any one flaw reaches a critical value. In view of its pessimism, WLT design is in most cases conservative. Other important features of WLT are that it predicts size effect and that failure of a complex component may not be initiated at the point of highest nominal stress. A particularly severe flaw may be located at somewhat less highly stressed point and may still be the first crack to become critical. It is for this reason that the entire field solution of the stresses must be obtained and examination of the most highly stressed point, as in ductile materials, is no longer adequate.

Experimental fracture strength data obtained from uniaxially loaded simple specimens, when arranged in ascending order, can be represented in two different forms. The probability density function of a random variable (fracture strength) is a mathematical function that best represents the data in a relative frequency histogram, that is failure stress versus number of failures. The result, typically, would be a bell shaped curve (ref. 12). Alternatively, fracture data can be plotted as stress versus failure probability which leads to an S-shaped curve, called the cumulative distribution function of this random variable. Various distribution curves have been used to characterize the material's fracture property. The two most commonly used distribution functions are the Gaussian (normal) distribution and Weibull's distribution. The Weibull distribution is selected to characterize ceramic strength variations, since the Gaussian distribution is intrinsically associated with the bundle model and is incompatible with WLT. Consequently, the uniaxial fracture data is approximated by the 3-parameter Weibull distribution, defined by

$$P_f = 1 - \exp\left[-\int_V \left(\frac{\sigma - \sigma_u}{\sigma_0}\right)^m dV\right] \qquad (1)$$

where $P_f$ is the probability of failure, $\sigma_0$ is the scale parameter with dimensions of stress x $(\text{volume})^{1/m}$, $\sigma_u$ is the threshold stress which is usually taken as zero, m is the Weibull modulus which measures the degree of strength variability, $\sigma$ is the applied tensile stress and V the stressed volume.

In the analysis of failure of brittle materials subject to multiaxial stress states, the Weibull model, when combined with the PIA hypothesis, yields

$$P_f = 1 - \exp\left\{-\int_V \left[\left(\frac{\sigma_1}{\sigma_0}\right)^m + \left(\frac{\sigma_2}{\sigma_0}\right)^m + \left(\frac{\sigma_3}{\sigma_0}\right)^m\right] dV\right\} \tag{2}$$

where $\sigma_1$, $\sigma_2$, and $\sigma_3$ are the principal stresses and it was assumed that $\sigma_u = 0$. Equation (2) has been widely used in the past to estimate failure probabilities of ceramic structures (ref. 13). The failure probability using the normal tensile stress averaging method, as proposed early by Weibull (ref. 1), and described later through an integral formulation (ref. 14) can be calculated from

$$P_f = 1 - \exp\left\{-\int_V \left[k_{wp} \int_A \sigma_n^m \, dA\right] dV\right\} \tag{3}$$

where $k_{wp}$ is the polyaxial Weibull crack density coefficient given by

$$k_{wp} = \left(\frac{2m + 1}{2\pi}\right)\left(\frac{1}{\sigma_0}\right)^m \tag{4}$$

This constant can be obtained by making the result of integrating equation (3), using the normal stress $\sigma_n$ distribution on an arbitrary plane, obtained from the Cauchy infinitesimal tetrahedron in principal stress space as shown in figure 5, for uniaxial stress cases, agree with the results obtained from the uniaxial, 2-parameter Weibull equation. The area integration is performed on the surface of the unit sphere where the normal stress is tensile and neglecting regions where the normal stress is compressive. The crack-like flaws can then be regarded as located in these arbitrary planes which are tangent to the sphere and are acted upon by $\sigma_n$ which is induced by the principal stresses $\sigma_1$, $\sigma_2$, and $\sigma_3$. Since equation (3) is just the shear-insensitive case of the more general Batdorf (ref. 15) polyaxial stress fracture model, its SCARE implementation follows a somewhat different format. The polyaxial Weibull equation has also been extensively used in the past (ref. 14), but since it neglects the effects of shear loads, it also underestimates failure for the more general loading condition.

In the previously described two multi-dimensional stress fracture models no direct use was made of the hypothesis that fractures are due to crack growth. In references 4 and 15, attention is focused on the cracks and their failure under stress. Since there is not as yet a consensus regarding how to treat mixed mode fracture, even in ductile materials, the SCARE program includes several fracture criteria and flaw shapes. Rufin et al. (ref. 16) recently compared results obtained from various fracture models and experimental tests, with similar work being reported in reference 5.

Consider now a small uniformly stressed material element of volume $\Delta V$. The probability of failure under an applied state of stress can be written as (ref. 16)

$$P_f = P_1 P_2 \tag{5}$$

where $P_1$ is the probability of existence in $\Delta V$ of a crack having a critical stress in the range of $\sigma_{cr}$ to $\sigma_{cr} + d\sigma_{cr}$, and $P_2$ denotes the probability that a crack of critical stress $\sigma_{cr}$ will be oriented in a direction such that an effective stress $\sigma_e$ equals or exceeds $\sigma_{cr}$. $\sigma_{cr}$ is defined as the remote, uniaxial, normal fracture stress of a given crack. Failure will occur when the effective stress (a function of chosen crack configuration and fracture criterion) exceeds $\sigma_{cr}$ for a particular crack. $P_1$ has the form

$$P_1 = \Delta V \frac{dN}{d\sigma_{cr}} (\sigma_{cr})\, d\sigma_{cr} \tag{6}$$

and

$$P_2 = \frac{\Omega}{4\pi} \tag{7}$$

where $N(\sigma_{cr})$ is the crack density function (the density of cracks having a critical stress $\le \sigma_{cr}$) and $\Omega$ is the solid angle in principal stress space containing all the crack orientations for which $\sigma_e \ge \sigma_{cr}$. Using the weakest link theory, the overall failure probability can be calculated from (ref. 15)

$$P_f = 1 - \exp\left[-\int_V dV \int_0^{\sigma_1} \left(\frac{\Omega}{4\pi}\right) \frac{dN}{d\sigma_{cr}}\, d\sigma_{cr}\right] \tag{8}$$

The crack density function $N(\sigma_{cr})$ is a material constant and is independent of stress state. It is usually expressed as a power function of $\sigma_{cr}$, that is $N(\sigma_{cr}) = k_B \sigma_{cr}^m$, where the flaw distribution parameters $k_B$ and $m$ can be evaluated from experimental data using uniaxial or equibiaxial tension specimens. Batdorf (ref. 15) initially proposed a Taylor series expansion for $N(\sigma_{cr})$, but this method had computational difficulties. Recently, a more convenient integral equation approach was formulated and extended to the use of data from 4-point MOR bar tests (ref. 16).

The statistical analysis of fracture is greatly simplified by assuming that cracks are shear-insensitive. For this case fracture occurs when $\sigma_e = \sigma_{cr} = \sigma_n$ and there is no need to specify the crack shape or the material's Poisson's ratio. Note that the crack size is never used in statistical fracture theories and is always eliminated from the analysis. Since for uniaxial loading shear-insensitive cracks are assumed to dominate the fracture process, $\sigma_e$ is defined such that in the absence of shear on the crack plane, $\sigma_e = \sigma_n$. In a similar manner, when the $G_c$ criterion is used, we define $\sigma_e$ as the uniaxial normal stress that would induce the same energy release rate as the actual stress. The same ideas can be extended to noncoplanar crack growth criteria, such as the maximum G or strain energy density, to define $\sigma_e$ for those applications. In any event, for polyaxial stress states, the effective stress $\sigma_e$ is a function of both $\sigma_n$ and $\tau$, where $\tau$ is the shear stress in the crack plane. Similarly to calculating $\sigma_n$ at any point, the Cauchy infinitesimal tetrahedron of Fig. 5 can also be used to obtain $\tau$ on the same arbitrary crack plane. Reference 4 gives effective stress expressions for two crack shapes using the maximum tensile stress and $G_c$ fracture criteria. The same four options for shear-sensitive cracks are available in SCARE, as can be seen in figure 2. The best choice among them is $\sigma_e$ for PSC's, given by

$$\sigma_e = \sqrt{\sigma_n^2 + \tau^2/(1 - 0.5\ \nu)^2} \qquad (9)$$

where $\nu$ is Poisson's ratio.

The solid angle $\Omega$ depends on the fracture criterion selected, the assumed crack configuration and on the applied stress state. Closed form expressions for $\Omega$ can be derived for analytically simple fracture criteria in uniaxial and balanced biaxial stress states (ref. 15). Assuming a uniaxial stress, $\sigma$, and the normal stress (shear-insensitive) fracture criterion, we obtain (ref. 5)

$$P_2 = \frac{\Omega}{4\pi} = \left[1 - \left(\frac{\sigma_{cr}}{\sigma}\right)^{1/2}\right] \qquad (10)$$

Note that when a shear-sensitive fracture criterion is used, the crack shape must also be specified. In general for three-dimensional stress states, $\Omega$ must be determined numerically. Using the shear-insensitive case as an example, we obtain at fracture (ref. 15)

$$\sigma_e = \sigma_n = \sigma_{cr} = \sigma_3 + (\sigma_1 - \sigma_3)\cos^2\alpha + (\sigma_2 - \sigma_3)\cos^2\beta\sin^2\alpha \qquad (11)$$

where from figure 5 angles $\alpha$ and $\beta$ define the crack plane in principal stress space, on which $\sigma_n$ and $\tau$ act. Using direction cosines l, m, and n, the equilibrium of forces on the Cauchy tetrahedron yields values of $\sigma_n$ and $\tau$ in terms of the principal stresses and the angles $\alpha$ and $\beta$ (ref. 11). It is computationally convenient to define $\varphi = \cos^2\beta$. Then equation (11) can be rewritten as

$$a_1\varphi^2 + a_2\varphi + a_3 = 0 \qquad (12)$$

where $a_1 = 0$, $a_2 = (\sigma_2 - \sigma_3)\sin^2\alpha$ and $a_3 = (\sigma_1 - \sigma_3)\cos^2\alpha + \sigma_3 - \sigma_{cr}$. Solving for $\varphi$ gives

$$\varphi = \frac{-a_3}{a_2} = \frac{\sigma_{cr} - \sigma_3 - (\sigma_1 - \sigma_3)\cos^2\alpha}{(\sigma_2 - \sigma_3)\sin^2\alpha} \qquad (13)$$

If we define $\overline{\beta} = \cos^{-1}\sqrt{\varphi}$, then $P_2$ can be calculated from (ref. 15) results from equation (1) are compared to the expression given in equation (15), and the relationship between $\sigma_0$ and C is derived by making $P_f$ from the two equations agree. For a rectangular beam of width w, length $L_1$ between symmetrically placed outer loads and length $L_2$ between inner loads, the result is

$$\sigma_0 = \left[\left(\frac{wh}{2}\right) \frac{(L_1 + mL_2)}{C(m + 1)^2}\right]^{1/m} \tag{16}$$

In addition to obtaining $\sigma_0$ and m, the SCARE program requires knowledge of $k_B$. We can evaluate $k_B$ from 4-point flexure data by substituting equation (10) into equation (8). Integration by parts of the results gives

$$P_s = \exp\left[-\int_V\int_0^{\sigma_1} \frac{N(\sigma_{cr})}{2\sqrt{\sigma_{cr}\sigma_1}}\, d\sigma_{cr}\, dV\right] \tag{17}$$

We again utilize the power function form of $N(\sigma_{cr})$ in equation (17) and carry out the stress integration. Similarly to the Weibull analysis, $\sigma_1$ is expressed in terms of $\sigma_f$ and the beam height. The volume integration is then performed over the tensile portion of the beam, including effects of changing $\sigma_1$ along the beam length. Results from this integration are compared to equation (15) and the relationship between $k_B$ and C is derived by setting $P_f$ equal from the two equations. For a 4-point loaded beam specimen having a rectangular cross section, we obtain

$$k_B = (2m + 1)\left[\frac{2C(m + 1)^2}{wh(L_1 + mL_2)}\right] \tag{18}$$

By comparing equations (16) and (18), we conclude that when using the normal stress failure criterion, $k_B$ and $\sigma_0$ are related by

$$k_B = (2m + 1)\left(\frac{1}{\sigma_0}\right)^m \tag{19}$$

However, equation (19) changes when the $G_c$ criterion is used (ref. 5).

## EXAMPLE

In order to validate SCARE, a number of example problems were analyzed from the open literature (ref. 11). Among them failure probability predictions were made, using Batdorf's shear-insensitive fracture model, for a silicon nitride disk rotating at various angular velocities (ref. 14). The dimensions of the disk along with appropriate material statistical parameter data are given in figure 6. Because of the simple geometry, only eight HEXA elements were used in one 15° sector MSC/NASTRAN model of the disk. The calculated NASTRAN stresses and volumes both were within approximately 1 percent of the available closed form answers. Both SCARE1 and SCARE2 predictions were generated and the results were compared to those listed in reference 14. Reliability calculations were also made at various speeds using other fracture theories. Selected results from these analyses are shown in figure 7 and results from equation (1) are compared to the expression given in equation (15), and the relationship between $\sigma_0$ and C is derived by making $P_f$ from the two equations agree. For a rectangular beam of width w, length $L_1$ between symmetrically placed outer loads and length $L_2$ between inner loads, the result is

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In order to validate SCARE, a number of example problems were analyzed from the open literature (ref. 11). Among them failure probability predictions were made, using Batdorf's shear-insensitive fracture model, for a silicon nitride disk rotating at various angular velocities (ref. 14). The dimensions of the disk along with appropriate material statistical parameter data are given in figure 6. Because of the simple geometry, only eight HEXA elements were used in one 15° sector MSC/NASTRAN model of the disk. The calculated NASTRAN stresses and volumes both were within approximately 1 percent of the available closed form answers. Both SCARE1 and SCARE2 predictions were generated and the results were compared to those listed in reference 14. Reliability calculations were also made at various speeds using other fracture theories. Selected results from these analyses are shown in figure 7 and table II. For a given speed, the Weibull PIA model yields clearly the lowest failure estimate, while the shear-sensitive PSC with the $G_c$ criterion gives the highest. The agreement between SCARE2 results and those in reference 14 was within 10 percent, with the difference probably due to the different stress-volume data used in solving the reliability problem. Laboratory measurements agree best with the selected shear-sensitive fracture model, as can be noted, especially in the high failure probabilities range. Since only seven disks were fracture tested compared to 85 MOR specimens, there is some concern about the accuracy of the experimental disk Weibull modulus of 4.95, which causes the greater difference between experimental and predicted $P_f$ at lower failure probabilities.

## CONCLUSIONS

A general purpose, statistical, fast fracture failure probability code has been generated, which is coupled with MSC/NASTRAN, and can be used to design structural ceramics components. The program includes a number of widely used polyaxial fracture models, appropriate extreme value statistics and the ability to calculate material failure distribution parameters, all for volume distributed flaws. Current work includes extension of this same capability to bimodal flaw populations, where failure due to extrinsic defects is a concurrent possibility. The addition of more advanced failure criteria which permit out-of-plane crack extension is also planned. Finally, the problem of a transversely loaded circular plate will be investigated, both analytically and experimentally, to resolve some of the contradictory trends reported in references 5 and 16.

## REFERENCES


1. Weibull, W.: A Statistical Theory of the Strength of Materials. Ingeniors Vetenskaps Akademien Handlinger, No. 151, 1939.

2. Freudenthal, A.M.: Statistical Approach to Brittle Fracture. Fracture, Vol. 2: An Advanced Treatise, Mathematical Fundamentals, H. Liebowitz, ed., Academic Press, 1968, pp. 591-619.

3. Batdorf, S.B.: Some Approximate Treatments of Fracture Statistics for Polyaxial Tension. Int. J. Fract., Vol. 13, No. 1, Feb. 1977, pp. 5-11.

4. Batdorf, S.B.; and Heinisch, H.L., Jr.: Weakest Link Theory Reformulated for Arbitrary Fracture Criterion. J. Am. Ceram. Soc., Vol. 61, No. 7-8, July-Aug. 1978, pp. 355-358.

5. Shetty, D.K.; Rosenfield, A.R.; and Duckworth, W.H.: Statistical Analysis of Size and Stress State Effects on the Strength of an Alumina Ceramic. Methods for Assessing the Structural Reliability of Brittle Materials, ASTM-STP-844, S.W. Freiman and C.M. Hudson, eds., American Society for Testing and Materials, 1984, pp. 57-80.

6. Lamon, J.; and Evans, A.G.: Statistical Analysis of Bending Strengths for Brittle Solids: A Multiaxial Fracture Problem. J. Am. Ceram. Soc., Vol. 66, No. 3, Mar. 1983, pp. 177-182.

7. Petrovic, J.J.: Mixed-Mode Fracture of Hot-Pressed $Si_3N_4$. J. Am. Ceram. Soc., Vol. 68, No. 6, June 1985, pp. 348-355.

8. Alpa, G.: On a Statistical Approach to Brittle Rupture for Multiaxial States of Stress. Eng. Fract. Mech., Vol. 19, No. 5, 1984, pp. 881-901.

9. Batdorf, S.B.: Comparison of the Best Known Fracture Criteria. Proceedings of an International Symposium on Absorbed Specific Energy and/or Strain Energy Density Criterion, G. Sih, E. Czoboly, and F. Gillemot, eds. Martinus Nijhoff Publishers, 1982, pp. 243-251.

10. Schaeffer, H.G., MSC/NASTRAN Primer: Static and Normal Modes Analysis, rev. ed., Schaeffer Analysis Inc., Mount Vernon, NH, 1982.

11. Gyekenyesi, J.P.: SCARE - A Post-Processor Program to MSC/NASTRAN for the Reliability Analysis of Structural Ceramic Components. NASA TM-87188. 1986.

12. Shih, T.T.: An Evaluation of the Probabilistic Approach to Brittle Design. Eng. Fract. Mech., Vol. 13, No. 2, 1980, pp. 257-271.

13. Wertz, J.L.; and Heitman, P.W.: Predicting the Reliability of Ceramic Turbine Components. Advanced Gas Turbine Systems for Automobiles, SAE-SP-465, Society of Automotive Engineers, 1980, pp. 69-77.

14. Swank, L.R.; and Williams, R.M.. Correlation of Static Strengths and Speeds of Rotational Failure of Structural Ceramics. Am. Ceram. Soc. Bull., Vol. 60, No. 8, Aug. 1981, pp. 830-834.

15. Batdorf, S.B.; and Crose, J.G.: A Statistical Theory for the Fracture of Brittle Structures Subjected to Nonuniform Polyaxial Stresses. J. Appl. Mech. Vol. 41, No. 2, June 1974, pp. 459-464.

16. Rufin, A.C.; Samos D.R.; and Bollard, R.J.H.: Statistical Failure Prediction Models for Brittle Materials. AIAA J., Vol. 22, No. 1, Jan. 1984, pp. 135-140.

TABLE I. - FORMS OF $P_2$ FOR VARIOUS SHEAR-SENSITIVE FRACTURE CRITERIA AND SELECTED CRACK CONFIGURATIONS ($\sigma_2 \neq \sigma_3$)

| Fracture criterion | Crack configuration | $P_2$ |
|---|---|---|
| Maximum tensile stress | Griffith crack (G.C.) | $\frac{\Omega}{4\pi} = \frac{1}{\pi} \int_0^{\pi} \cos^{-1} \sqrt{\varphi} \sin \alpha \, d\alpha$<br>where<br>$\varphi = \cos^2 \beta = \frac{-a_2 \pm \sqrt{a_2^2 - 4a_1 a_3}}{2a_1}$<br>and<br>$a_1 = (\sigma_2 - \sigma_3)^2 \sin^4 \alpha$<br>$a_2 = (\sigma_2 - \sigma_3) \sin^2 \alpha \{2(\sigma_1 \cos^2 \alpha + \sigma_3 \sin^2 \alpha) - 4\sigma_{cr} - \sigma_3 - \sigma_2\}$<br>$a_3 = (\sigma_1 \cos^2 \alpha + \sigma_3 \sin^2 \alpha)^2 - 4\sigma_{cr}(\sigma_1 \cos^2 \alpha + \sigma_3 \sin^2 \alpha) - (\sigma_1^2 \cos^2 \alpha + \sigma_3^2 \sin^2 \alpha) + 4\sigma_{cr}^2$ |
| | Penny-shaped crack (PSC) | $a_1 = D_1(\sigma_2 - \sigma_3)^2 \sin^4 \alpha$<br>$a_2 = D_1(\sigma_2 - \sigma_3) \sin^2 \alpha \{2(\sigma_1 \cos^2 \alpha + \sigma_3 \sin^2 \alpha) - \frac{4}{D_1} \sigma_{cr} - \sigma_3 - \sigma_2\}$<br>$a_3 = D_1(\sigma_1 \cos^2 \alpha + \sigma_3 \sin^2 \alpha)^2 - 4\sigma_{cr}(\sigma_1 \cos^2 \alpha + \sigma_3 \sin^2 \alpha) - D_1(\sigma_1^2 \cos^2 \alpha + \sigma_3^2 \sin^2 \alpha) + 4\sigma_{cr}^2$ |
| Strain energy release rate | Griffith crack (G.C.) | $\varphi = \frac{\sigma_{cr}^2 - \sigma_1^2 \cos^2 \alpha - \sigma_3^2 \sin^2 \alpha}{(\sigma_2^2 - \sigma_3^2) \sin^2 \alpha}$ |
| | Penny-shaped crack (PSC) | $a_1 = D_2(\sigma_2 - \sigma_3)^2 \sin^4 \alpha$<br>$a_2 = D_1(\sigma_2^2 - \sigma_3^2) \sin^2 \alpha + 2D_2(\sigma_2 - \sigma_3) \sin^2 \alpha \cdot (\sigma_3 \sin^2 \alpha + \sigma_1 \cos^2 \alpha)$<br>$a_3 = D_1(\sigma_1^2 \cos^2 \alpha + \sigma_3^2 \sin^2 \alpha) + D_2(\sigma_1 \cos^2 \alpha + \sigma_3 \sin^2 \alpha)^2 - \sigma_{cr}^2$<br>where<br>$D_1 = \frac{1}{(1 - 0.5\nu)^2}, \quad D_2 = \frac{-\nu(1 - 0.25\nu)}{(1 - 0.5\nu)^2}$ |

TABLE II. - EXAMPLE 1 FAILURE PROBABILITIES AS A FUNCTION OF ROTATIONAL SPEED FOR VARIOUS FRACTURE MODELS

[$m = 7.65$; $\sigma_0 = 74.82$ MPa $(m)^{0.3922}$; $k_B\sigma_0^m = 16.30$; NGP = 5]

| Angular speed (rpm) | SCARE2 (HEXA elements) | | | SCARE1 (HEXA elements) | Ford (Ref. 14) (axisymmetric elements) | Ford (Ref. 14) |
|---|---|---|---|---|---|---|
| | Weibull PIA | Batdorf shear-insensitive | Batdorf shear-sensitive PSC $G_c$ criterion | Batdorf shear-insensitive | Shear-insensitive | Experimental |
| 70 000 | 0.0021 | 0.0026 | 0.0078 | 0.0022 | 0.0023 | 0.0583 |
| 75 000 | .0061 | .0075 | .0222 | .0064 | .0067 | .1121 |
| 80 000 | .0163 | .0201 | .0584 | .0170 | .0179 | .2017 |
| 85 000 | .0412 | .0505 | .1426 | .0426 | .0446 | .3367 |
| 93 000 | .1530 | .1850 | .4549 | .1579 | .1650 | .6321 |
| 100 000 | .3954 | .4623 | .8410 | .4074 | .4223 | .8714 |
| 104 000 | .6000 | .6763 | .9649 | .6124 | .6321 | .9514 |
| 110 000 | .8847 | .9301 | .9996 | .8931 | .9055 | .9949 |
| 114 000 | .9736 | .9900 | 1.0000 | .9792 | .9830 | .9994 |

TABLE III. - EXAMPLE 1 FAILURE PROBABILITIES AS A FUNCTION OF APPLIED PRESSURE FOR VARIOUS FRACTURE MODELS

[$m = 28.53$; $\sigma_0 = 36\ 200$ psi $(in)^{0.105}$; $k_B\sigma_0^m = 58.06$; NGP = 15]

| Pressure, MPa (psi) | SCARE 2 (three-dimensional elements) | | | SCARE 1 (three-dimensional elements) | | | SCARE 1 (axisymmetric elements) |
|---|---|---|---|---|---|---|---|
| | Weibull PIA | Batdorf shear-insensitive | Batdorf shear-sensitive PSC en. re. ra. cr. | Weibull PIA | Batdorf shear-insensitive | Batdorf shear-sensitive PSC en. re. ra. cr. | Batdorf shear-sensitive PSC en. re. ra. cr. |
| 1.31 (190) | ------ | 0.0039 | 0.0065 | ------ | 0.0030 | 0.0049 | 0.0064 |
| 1.38 (200) | 0.0037 | .0168 | .0279 | 0.0029 | .0128 | .0211 | .0275 |
| 1.45 (210) | .0149 | .0658 | .1078 | .0115 | .0504 | .0821 | .1061 |
| 1.52 (220) | .0552 | .2265 | .3495 | .0425 | .1770 | .2760 | .3448 |
| 1.59 (230) | .1828 | .5987 | .7832 | .1430 | .4996 | .6827 | .7775 |
| 1.66 (240) | .4934 | .9538 | .9942 | .4054 | .9028 | .9790 | .9937 |
| 1.72 (250) | .8869 | .9999 | 1.0000 | .8110 | .9994 | 1.0000 | 1.0000 |
| 1.79 (260) | .9987 | 1.0000 | 1.0000 | .9939 | 1.0000 | 1.0000 | 1.0000 |

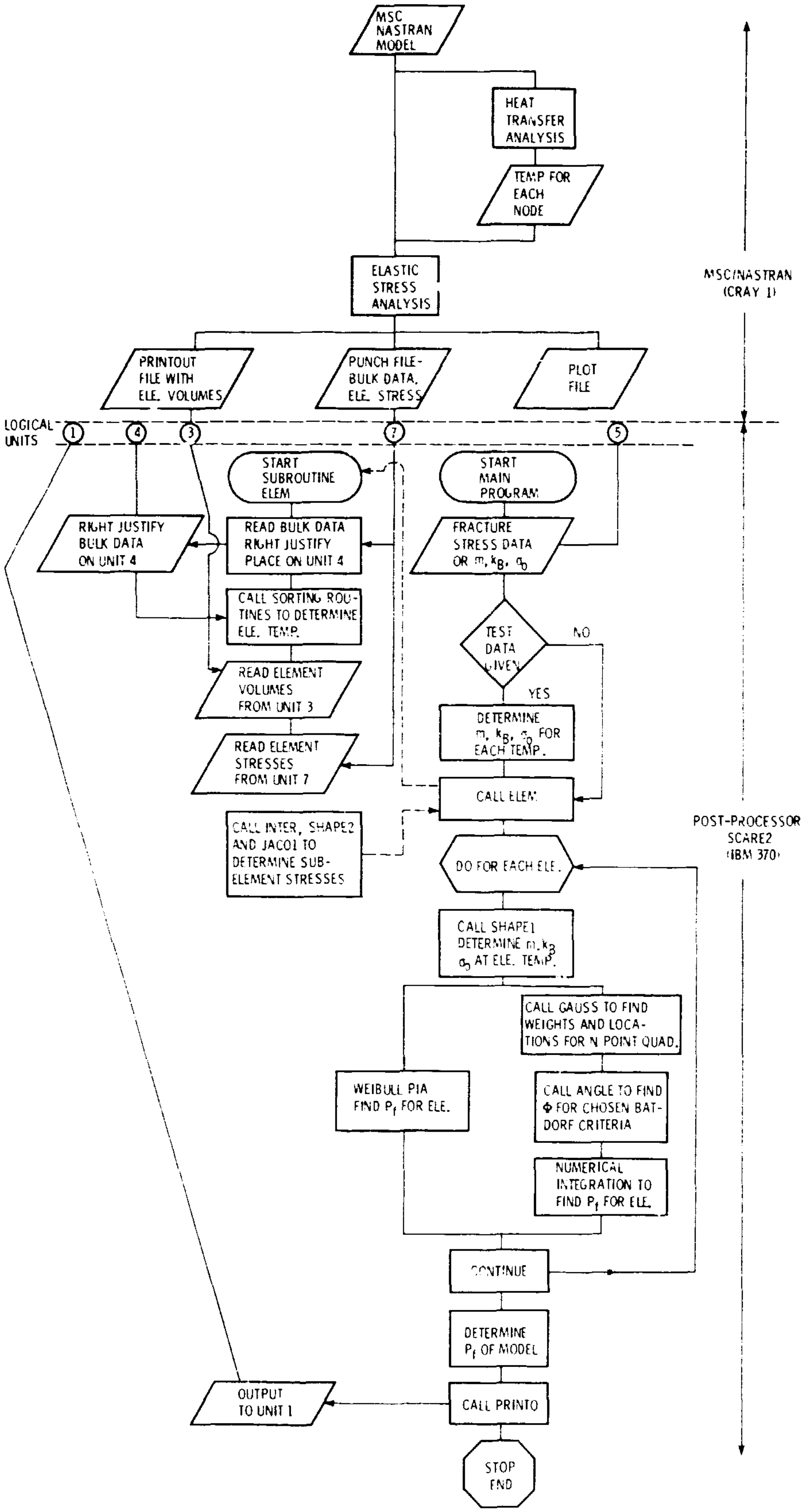



Fig. 1 Computational elements of the SCARE2 reliability analysis program.

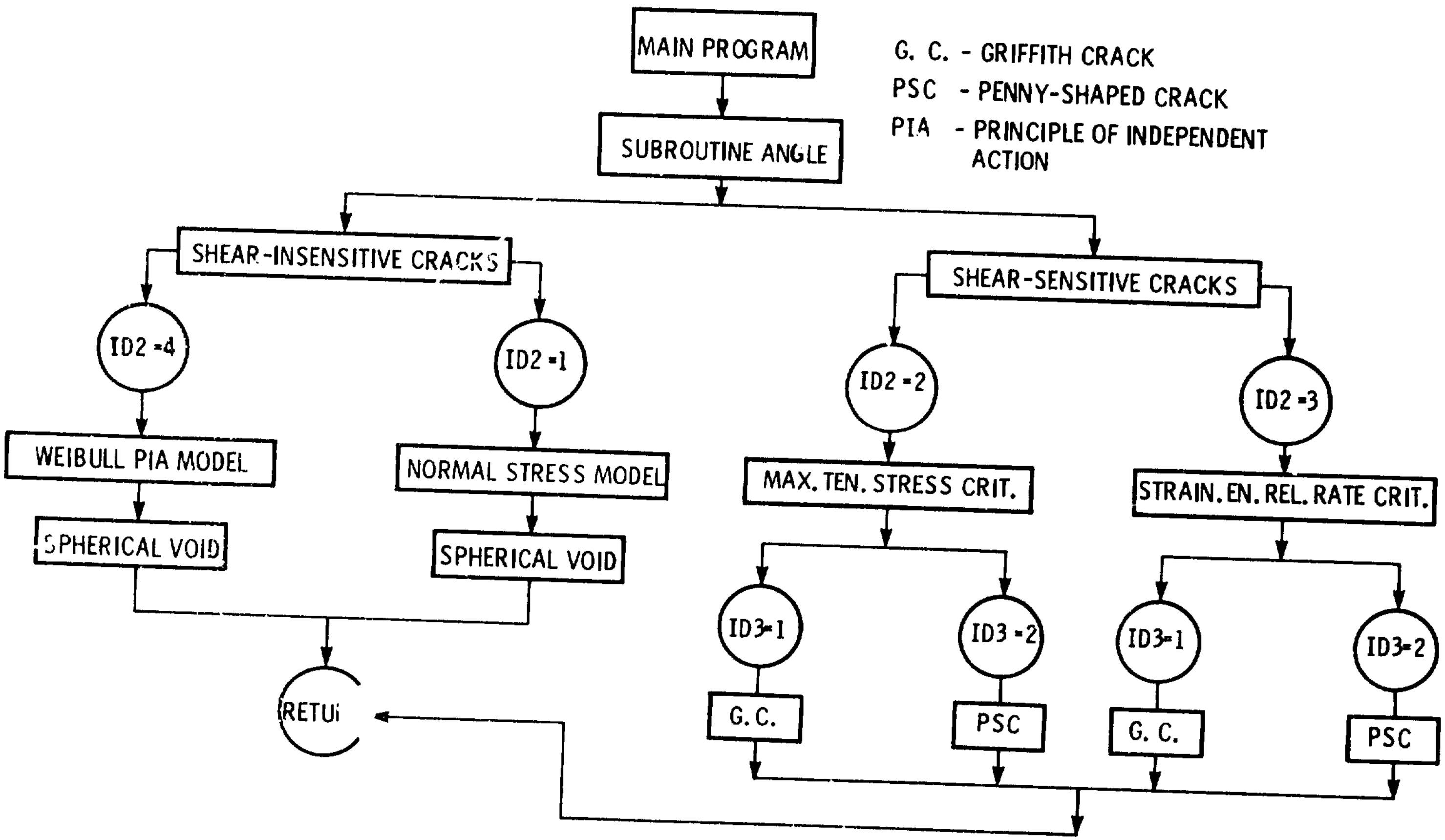



Fig. 2 Flowchart for subroutine ANGLE.

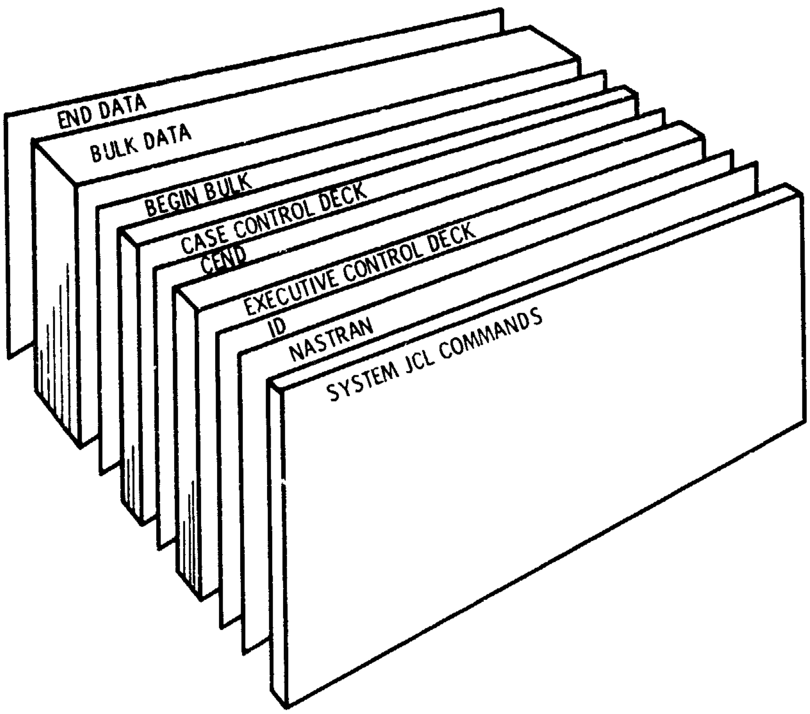



Fig. 3 NASTRAN input file arrangement.

| COLUMNS | VARIABLE | ENTRY (FORMAT - 11I5) |
|---|---|---|
| 1 - 5 | ID1 | CONTROL INDEX FOR EXPERIMENTAL DATA<br>1: PURE BENDING TEST DATA<br>2: 4-pt BENDING TEST DATA<br>3: ALL THE MATERIAL PARAMETERS VMT, VKT AND VSPT ARE KNOWN AS INPUT |
| 6 - 10 | ID2 | CONTROL INDEX FOR FRACTURE CRITERIA<br>1: SHEAR-INSENSITIVE, NORMAL STRESS CRITERION<br>2: MAXIMUM TENSILE STRESS CRITERION<br>3: ENERGY RELEASE RATE CRITERION<br>4: WEIBULL PIA SHEAR-INSENSITIVE MODEL |
| 11 - 15 | ID3 | CONTROL INDEX FOR CRACK SHAPES<br>1: GRIFFITH TYPE CRACK<br>2: PENNY-SHAPED CRACK |
| 16 - 20 | NE | TOTAL NUMBER OF ELEMENTS IN MODEL |
| 21 - 25 | NH | NUMBER OF HEXA ELEMENTS IN MODEL |
| 26 - 30 | NP | NUMBER OF PENTA ELEMENTS IN MODEL |
| 31 - 35 | NA | NUMBER OF TRIAX6 ELEMENTS IN MODEL |
| 36 - 40 | NT | TOTAL NUMBER OF SPECIMENS IN EACH SET AT A GIVEN TEMPERATURE |
| 41 - 45 | NGP | NUMBER OF GAUSSIAN QUADRATURE POINTS |
| 46 - 50 | NS | NUMBER OF SEGMENTS IN CYCLIC SYMMETRY PROBLEMS |
| 51 - 55 | JT | NUMBER OF TEST TEMPERATURES AT WHICH MATERIAL DATA IS SPECIFIED |

Fig. 4 SCARE master control deck data requirements.

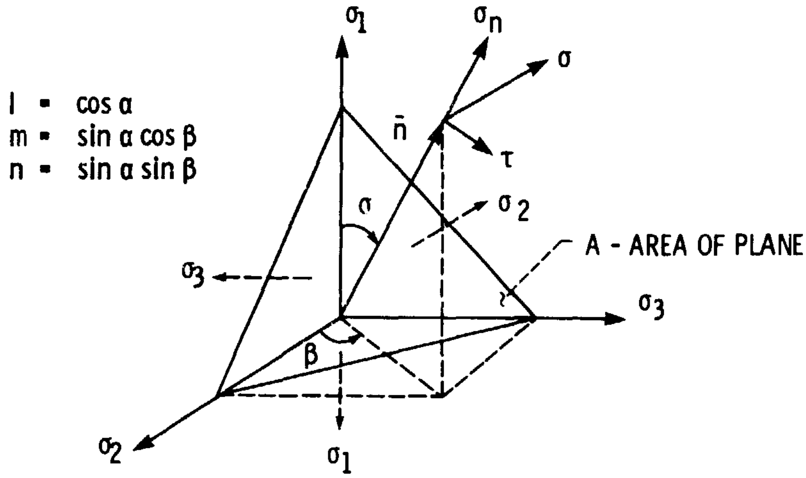
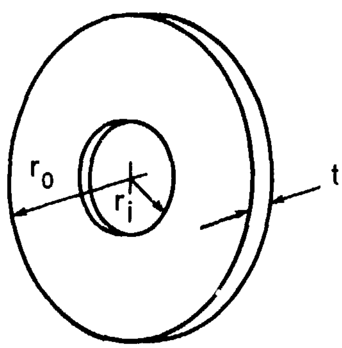



Fig. 5 Stresses on Cauchy infinitesimal tetrahedron in principal stress space.




DATA:

NC - 132 HOT PRESSED $Si_3N_4$

$m = 7.65$

$\sigma_0 = 74.82$ MPa $m^{.3922}$

$\bar{k}_B = 16.30$

$r_i = 6.35$ mm (.25 in)

$r_o = 41.275$ mm (1.625 in)

$t = 3.80$ mm (.15 in)

RPM RANGE - 70K TO 114K

Fig. 6 Example 1 - rotating annular disk (Ref. 14).

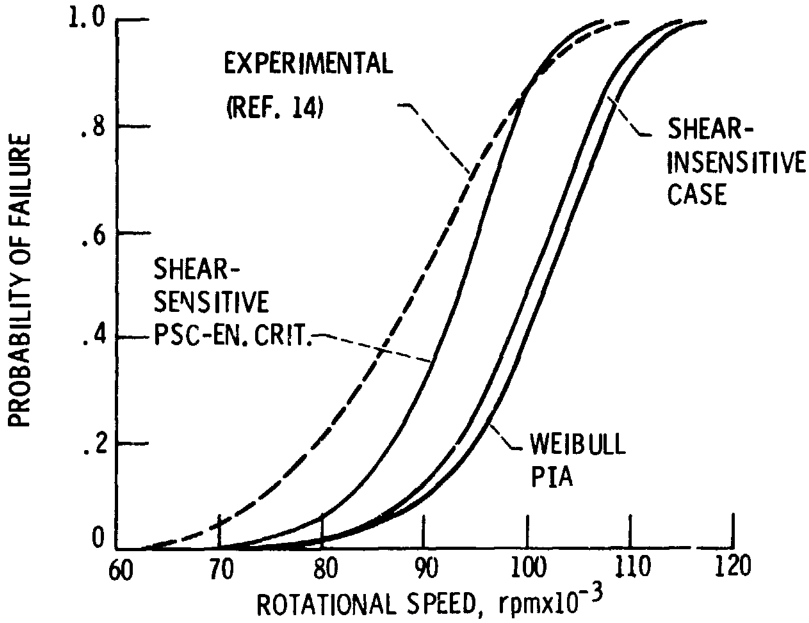



Fig. 7 Example 1 probability of failure vs disk rotational speed for various fracture models (SCARE2 data).

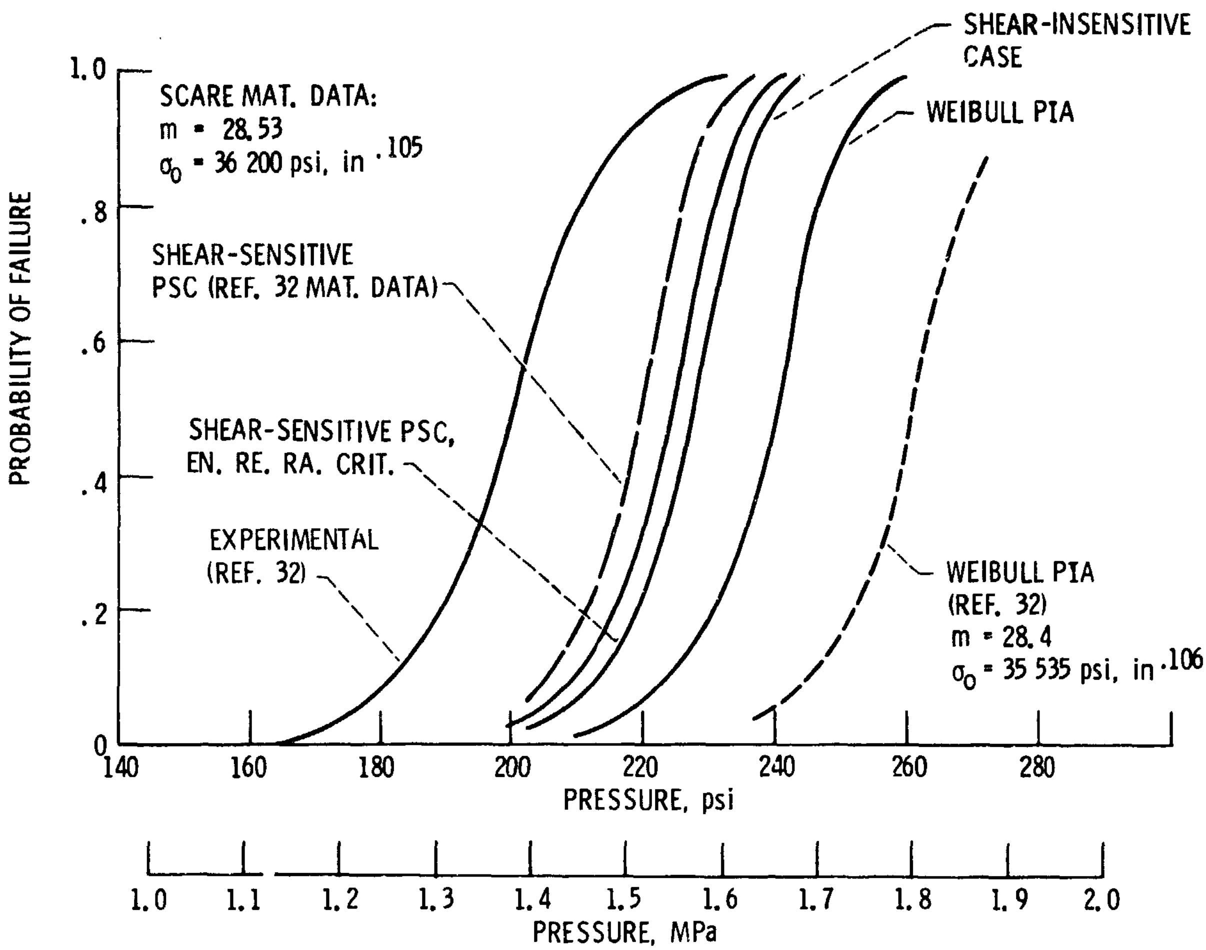



Figure 8. - Example 1 probability of failure vs applied pressure for various fracture models (SCARE2 data).

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|---|---|---|
| 1. Report No. NASA TM-87188 | 2. Government Accession No. | 3. Recipient's Catalog No. |
| 4. Title and Subtitle SCARE - A Post-Processor Program to MSC/NASTRAN for the Reliability Analysis of Structural Ceramic Components | | 5. Report Date |
| | | 6. Performing Organization Code 533-05-11 |
| 7. Author(s) John P. Gyekenyesi | | 8. Performing Organization Report No. E-2654 |
| | | 10. Work Unit No. |
| 9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135 | | 11. Contract or Grant No. |
| | | 13. Type of Report and Period Covered Technical Memorandum |
| 12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546 | | 14. Sponsoring Agency Code |

15. Supplementary Notes

Prepared for the 31st International Gas Turbine Conference and Exhibit, sponsored by the American Society of Mechanical Engineers, Dusseldorf, West Germany, June 8-12, 1986.

16. Abstract


A computer program is developed for calculating the statistical fast fracture reliability and failure probability of ceramic components. The program includes the two-parameter Weibull material fracture strength distribution model, using the principle of independent action for polyaxial stress states and Batdorf's shear-sensitive as well as shear-insensitive crack theories, all for volume distributed flaws in macroscopically isotropic solids. Both penny-shaped cracks and Griffith cracks are included in the Batdorf shear-sensitive crack response calculations, using Griffith's maximum tensile stress or critical coplanar strain energy release rate criteria to predict mixed mode fracture. Weibull material parameters can also be calculated from modulus of rupture bar tests, using the least squares method with known specimen geometry and fracture data. The reliability prediction analysis uses MSC/NASTRAN stress, temperature and volume output, obtained from the use of three-dimensional, quadratic, isoparametric, or axisymmetric finite elements. The statistical fast fracture theories employed, along with selected input and output formats and options, are summarized. An example problem to demonstrate various features of the program is included.


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|---|---|---|---|
| 17. Key Words (Suggested by Author(s)) Ceramics; Cracks; Fracture mechanics; Finite elements; Stress; Temperature; Statistics; Weibull; Failure theories | 18. Distribution Statement Unclassified - unlimited STAR Category 39 | | |
| 19. Security Classif. (of this report) Unclassified | 20. Security Classif. (of this page) Unclassified | 21. No. of pages | 22. Price* |

*For sale by the National Technical Information Service, Springfield, Virginia 22161