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An Investigation of Adaptive Controllers for Helicopter Vibration and the Development of a New Dual Controller

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SUMMARY

An investigation of the properties important for the design of stochastic adaptive controllers for the higher harmonic control of helicopter vibration is presented. Three different model types are considered for the transfer relationship between the helicopter higher harmonic control input and the vibration output; 1) Nonlinear, 2) linear with slow time varying coefficients, and 3) linear with constant coefficients. The stochastic controller formulations and solutions are presented for a dual, cautious, and deterministic controller for both linear and nonlinear transfer models.

Extensive simulations are performed with the various models and controllers. It is shown that even for a linear model the cautious adaptive controller can sometimes result in unacceptable vibration control including an apparent controller divergence. This is found to occur for both constant parameter conditions representative of steady flight and time varying parameter conditions representative of maneuvering flight conditions.

A new second order dual controller is developed which is shown to modify the cautious adaptive controller by adding numerator and denominator correction terms to the cautious control algorithm. The new dual controller is simulated on a simple single-control vibration example and is found to achieve excellent vibration reduction and significantly improves upon the cautious controller.

Nonlinear, time varying coefficient and constant coefficient systems are each found to exhibit distinctive characteristics using the adaptive controllers. Simulation and analysis are presented in an attempt to further understand the closed loop behavior.

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LIST OF SYMBOLS

a	constant coefficient used in the parameter variation model in Appendix A
A	constant matrix used in the parameter variation model in Eq. (2)
b	control coefficient used in the vibration model in Appendix A
$\hat{b}$	estimated control coefficient
B	control matrix used in the vibration model in Eq. (1)
$\hat{B}$	estimated control matrix
c	uncontrolled vibration in Eq. (1), and Appendix A
$\hat{c}$	estimated uncontrolled vibration
$C(k)$	cost function from time step k to N
d,e	parameters involved in the nonlinear model in Eq. (43)
$E\{\ \}$	denotes expected value
$E(\cdot\,\|\,\mathcal{I}^k)$	denotes conditional expected value based upon information $\mathcal{I}^k$ of time step k
f	parameter involved in the nonlinear model in Eq. (43)
$f_1(\ ),f_2(\ )$	general nonlinear functions
f_ℓ	vector of sensitivities used in the new dual solutions of Eq. (30)
F_ℓ	a matrix of sensitivities used in the new dual solution of Eq.(30)
H	measurement matrix used in Eq. (9)
I	iteration step
$\mathcal{I}^k$	cumulative information at time step k
J	expected value of the cost function
J^*	optimal expected value of the cost
k	time step number
$K_\ell(1)$	Kalman gain for the parameters involved in the row ℓ of the matrix B
ℓ	row number of matrix B
$L[\ ]$	gain factor in the controller design in Eq. (15)
N	N step horizon
P	parameter covariance matrix used in Eq. (19)
P_B	covariance matrix associated with the parameters of matrix B
P_c	covariance matrix associated with the parameters of vector c

P_{Bc}	cross covariances between the elements of B and c
$P^{\ell}_{i,j}$	i-j th element of the covariance matrix associated with the elements of the ℓ th row and the matrix B and vector c
$P_{i,j}$	i-j th element of the covariance matrix P
$\bar{P}$	nominal covariance matrix
Q	weighting matrix on the state (vibration amplitude)
q_{ℓ}	individual element of the weighting matrix Q relevant to the row ℓ
r	scalar control weights
R	weighting matrix on the controls
$\tilde{R}$	rate weighting matrix on the incremental control in Eq. (23)
S,T	parameters involved in the nonlinear model in Eq. (7)
u	control vector
U^{N-1}	set of all the controls $\{u(i)\}_{i=0}^{N-1}$ from step 0 to step N-1
u^I	control at iteration step I
u^D	deterministic controller
u^S	stochastic controller
u^{HCE}	Heuristic Certainty Equivalence controller
u^{STURE}	Self Tuning Regulation Controller
$u^{CAUTIOUS}$	Cautious Controller
u^{CL}	Closed loop controller
$u^{*(1)}$	Dual Controller based upon first order Taylor series expansion
$u^{*(2)}$	Dual Controller based upon second order Taylor series expansion
u^*	Dual Controller
u_1	Cosine Component input
u_2	Sine Component input
v	parameter process noise
V	process noise covariance
$V_{i,j}$	i-j th element of the process noise covariance V

w	measurement noise
W	measurement noise covariance
x	state vector (vector of vibration sine or cosine amplitude component)
x_1	longitudinal hub cosine component of vibration
x_2	longitudinal hub sine component of vibration
X^N	set of all the states $\{x(i)\}_{i=0}^{N}$ from step 0 to step N
$\hat{x}$	estimated state vector
y	measurement vector
y_1	measurement of longitudinal hub cosine component of vibration
y_2	measurement of longitudinal hub sine component of vibration
α	scalar step-size parameter used in Eq. (33)
α_1	scalar parameter used in Eq. (41)
ν_ℓ	innovation associated with the elements of row ℓ
θ	parameter vector in the model
$\hat{\theta}$	estimated parameter vector
$(\ )'$	denotes transpose
$(\ )^{-1}$	denotes matrix inverse

1. INTRODUCTION

The use of adaptive controllers for higher harmonic control (HHC) to eliminate helicopter vibration has been investigated in numerous analytical studies [1, 2, 3, 4, 5], wind tunnel tests [1, 7, 8] and flight tests [9, 10]. The effectiveness of higher harmonic control has been experimentally verified and found to work exceptionally well in most cases. However, specific cases have been found where the use of an adaptive controller either failed to converge to the true minimizing solution, failed to simultaneously reduce several vibration components, or have shown a tendency for divergence. This has been reported in analytic studies [17, 22], wind tunnel tests [1, 7] and flight tests [9, 10].

The adaptive closed loop controllers can be considered to have attained the proof of concept status. However, there is a lack of understanding of the intrinsic controller properties. Specifically, there does not exist at this time a stability analysis for the closed loop system. Cases exist where the controller fails to fully minimize vibrations and this is not fully understood. The problem is further complicated by the various model descriptions of the helicopter HHC input to vibration output. The helicopter vibration model which is assumed to be in quasi-steady state can exhibit nonlinear, linear time varying, or linear constant coefficient behavior. In addition, the vibration model is in a sense stochastic since the transfer matrix varies in an unknown manner to a certain degree with flight condition and since the measurements are contaminated by random noise.

Because of the diversity of helicopter vibration model types and the stochastic nature of the environment, it is difficult to generalize the properties of the closed loop system. A solution which shows excellent convergence during one flight condition may be unacceptable during another condition. The difficulty is further complicated by the fact that the adaptive control algorithm is itself nonlinear and its properties cannot be analyzed by linear methods.

There are several variations for the implementation of the adaptive controllers used for helicopter vibration minimization. These have been referred to as local or global models. They can be implemented with and without stochastic properties (such as cautious and probing) and can be designed to minimize various optimization criteria to include specific vibration states, HHC control inputs and rate of change of HHC input. Extensive simulations have been done using the various controllers as reported in detail in [1-8]. Although the various controllers can be implemented differently they all contain an adaptive parameter identification algorithm followed by a controller which utilizes the identified parameters.

It is of importance to further understand the closed loop adaptive system properties. The adaptive control implementation can result in a divergence. In addition, depending upon the nature of the random disturbance the controller can fail to minimize vibrations under certain conditions. This behavior would certainly be unacceptable for use on production helicopters.

This report presents results for the three helicopter vibration model descriptions; 1) nonlinear, 2) linear time varying and 3) linear constant coefficient. Simulations are presented which show specific conditions when the closed loop adaptive controller exhibits divergence, or fails to fully minimize vibrations. An investigation of the properties of the closed loop controllers is presented and a new dual controller is developed.

A stochastic adaptive control formulation is presented for nonlinear and linear model descriptions. A new second order dual control solution is developed in an attempt to improve upon the non-dual controller (i.e. cautious or deterministic). The new dual controller is such as to modify the cautious adaptive controller by adding numerator and denominator correction terms to the cautious control algorithm. Conditions are simulated where the cautious controller fails to minimize vibration and the new dual controller is found to achieve excellent vibration reduction and convergence. Although, the new dual controller is developed for a multi-input/multi-output vibration problem, the simulations are presented for a scalar example.

The theoretical background and HHC problem formulation is presented in section 2. Stochastic control solutions are presented for both the linear and nonlinear problem descriptions in section 3. The simulation models used are discussed in section 4 and the details of the various closed loop simulation results are presented in section 5. The derivation of the new second order dual controller is presented in the appendicies.

2. THEORETICAL BACKGROUND

Helicopter vibration can be reduced by active blade control using controllers employing stochastic control theory. Various control policies are available and research is on going to extend these ideas for further improvement in vibration reduction. A summary of the vibration problem and various control approaches is provided here.

2.1. Higher Harmonic Control Problem Formulation

The linear multivariable model representing the higher harmonic control (HHC) input to output vibration is

$$x(k+1) = c(k) + B(k)\,u(k) \tag{1}$$

where c(k) is an unknown vector and B(k) is a matrix of unknown parameters. The unknown elements of c(k) and B(k) are denoted as θ(k) with covariance matrix P(k). These unknown parameters are time varying and their variations are modelled as

$$\theta(k+1) = A\,\theta(k) + v(k) \tag{2}$$

with

$$E\{v(k)\} = 0 \qquad \text{and } E\{v(k)\,v'(j)\} = V\,\delta_{kj} \tag{3}$$

In equilibrium flight the steady outputs (vibrations) at zero control is c(k). This is a static model where it is required to find the control u(k) which reduces the uncontrolled vibration c(k).

The measurement equation is

$$y(k) = x(k) + w(k) \tag{4}$$

where

$$\begin{aligned} &E\{w(k)\} = 0 \quad ; \quad E\{w(k)w'(j)\} = W\,\delta_{kj} \\ &E\{w(k)v'(j)\} = 0 \end{aligned} \tag{5}$$

and x(k), y(k) being n dimensional vectors. The general control criterion to be minimized is the expected value of the cost from step 0 to N

$$J(0) = E\{C(0)\} = E\{\sum_{k=1}^{N} x'(k)\,Qx(k) + u'(k-1)\,Ru(k-1)\} \tag{6}$$

The nonlinear HHC problem formulation is given next. The nonlinear relationship between HHC input and vibration output can be approximated by the Volterra harmonic series developed in [17]. The input-output nonlinear relationship including up to third order terms is,

$$x(k+1) = c(k) + B(k)u(k) + \frac{1}{2} u'(k) S(k)u(k) + u'(k) T(k) u^2(k) \qquad (7)$$

This relationship represents both the steady and the varying flight conditions. During varying flight conditions the coefficients $c(k)$, $B(k)$, $S(k)$ and $T(k)$ vary according to Equations 2 and 3. The measurement equation and the control criterion are the same as before.

The solutions to these problems are given in Section 3.

2.2 Stochastic Control Theory Background

In most real world systems there are inherent uncertainties that prevent the use of deterministic control theory. These uncertainties, either in the system itself or in the measurements made on the system, can be appropriately modelled as stochastic processes. An open-loop controller does not require any measurement of the system. It is a function of the initial state and time. The feedback controllers utilize real-time observations.

In a stochastic environment the control has a dual effect [11, 12]: it affects the system's state as well as the uncertainty in the estimated parameters. This dual effect can be utilized for designing good controllers for the vibration reduction in a helicopter. It enhances estimation of the unknown parameters but keeps the output at a recommended level.

Let us consider a stochastic system with unknown parameters θ where the state of the system at time k, $x(k)$ evolves according to the equation

$$x(k+1) = A(\theta)\, x(k) + B(\theta)\, u(k) + v(k) \qquad (8)$$

where $u(k)$ is the control applied at time k and $v(k)$ is the process noise.

The measurement is given by

$$y(k) = H\,x(k) + w(k) \tag{9}$$

where $w(k)$ is the measurement noise.

We are interested in a fixed end-time problem and the performance index to be minimized is

$$\min_{U^{N-1}} J = \min_{U^{N-1}} E\{\, C(0, X^N, U^{N-1}) \,\} \tag{10}$$

where

$$C(0, X^N, U^{N-1}) = x'(N)\,Qx(N) + \sum_{k=0}^{N-1} x'(k)\,Q(k)\,x(k) + u'(k)R(k)\,u(k) \tag{11}$$

$$U^{N-1} \triangleq \{\, u(i) \,\}_{i=0}^{N-1} \tag{12}$$

$$X^N \triangleq \{\, x(i) \,\}_{i=0}^{N} \tag{13}$$

In an equivalent deterministic situation, (known θ and no noises) the parameters and the state variables are known exactly and we are interested in

$$\min_{U^{N-1}} C(0, X^N, U^{N-1}) \tag{14}$$

The deterministic controller is

$$U^D(k) = L[k,\theta]\,x(k) \tag{15}$$

If the parameters were known, but the noise existed, then the stochastic controller for this problem (LQ) has the Certainty Equivalence (CE) property [12,20,21]

$$U^S(k) = L[k,\theta]\,\hat{x}(k|k) \tag{16}$$

For a plant with unknown parameters the stochastic controllers are adpative in nature and the parameter θ is estimated in real-time.

A stochastic controller that ignores the uncertainties in the parameter estimates is the Heuristic Certainty Equivalence controller

$$u^{HCE}(k) = L[k, \hat{\theta}(k)]\ \hat{x}(k|k) \qquad (17)$$

In the Self Tuning Regulator (STURE) perfect state observations are available and unknown parameters are estimated in real time. The controller is

$$u^{STURE}(k) = L[k, \hat{\theta}(k)]\ x(k) \qquad (18)$$

A cautious controller belongs to the feedback class [12,20,21] and makes use of the uncertainties in the parameter estimates. It is given by

$$u^{CAUTIOUS}(k) = L[k, \hat{\theta}(k), P(k)]\ \hat{x}(k|k) \qquad (19)$$

assuming

$$P(j) = P(k) \qquad \forall \quad j > k \qquad (20)$$

All the above mentioned controllers exhibit dual effect in a stochastic environment (non-neutral) but none of the above uses the dual effect in its design. The dual controller incorporates this and thus belongs to the closed-loop class and anticipates future learning. It takes into account the functional dependence of future covariances on the current control u(k). It is given by

$$u^{CL}(k) = \underset{u(k)}{\arg\min}\ J^{*}[k, \hat{x}(k|k), \hat{\theta}(k), P(j|j), \quad j \geq k] \qquad (21)$$

This often leads to a heavy computation load [13, 14, 15, 16]. Our idea is to get a control of the form

$$u(k) = L[k, \hat{\theta}(k), P(k), \frac{\partial p(j)}{\partial u(k)}, \quad j > k]\ \hat{x}(k|k) \qquad (22)$$

which is simpler to handle and analyse.

3. STOCHASTIC CONTROL SOLUTION

This section presents the control solutions for the deterministic, the cautious, the first order dual and the second order dual controller based upon a linear transfer matrix vibration description. In addition, the nonlinear stochastic formulation and solution is also presented. These controllers are evaluated by simulation in subsequent sections.

The general linear and the nonlinear problems are formulated in previous sections and the various stochastic control policies are discussed in Section 2. The different solutions to the linear and the nonlinear problems are given here. Three objective functions for the design of the controllers are:

$$\text{Deterministic:}\quad J_{DET} = \{x'(k)Qx(k) + u'(k-1)Ru(k-1) + \Delta u'(k-1)\,\tilde{R}\,\Delta u(k-1)\} \tag{23}$$

$$\text{Cautious:}\quad J_{CAUT} = E\{J_{DET}\} \tag{24}$$

$$\text{Dual:}\quad J_{DUAL} = E\{\sum_{i=1}^{N} J_{DET}\} \tag{25}$$

A dual solution is presented for N = 2 based upon an approximate linearization.

Solution to the Linear HHC Problem

For the linear HHC problem with $\tilde{R} = 0$ we have the following control solutions derived in [3,4] and Appendix A:

$$u^{D}(0) = -[B'(k)Q\,B(k) + R]^{-1}[B'(k)\;Q\;c(k)]; \tag{26}$$

$$u^{CAUT}(0) = -[\hat{B}'(0)Q\hat{B}(0) + \sum_{\ell=1}^{n} q_\ell\, P_B^{\ell}(0) + R]^{-1}\,[\hat{B}'(0)Q\hat{c}(0) + \sum_{\ell=1}^{n} q_\ell\, P_{Bc}^{\ell}(0)]\;; \tag{27}$$

$$u^{*(1)}(0) = -[\hat{B}'(0)Q\hat{B}(0) + \sum_{\ell=1}^{n} q_\ell\, P_B^{\ell}(0) + R]^{-1}\,[\hat{B}'(0)\;Q\hat{c}(0) + \sum_{\ell=1}^{n} (q_\ell\, P_{Bc}^{\ell}(0) + f_\ell)] \tag{28}$$

where the vector f_ℓ is

$$f_{\ell} = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} \frac{\partial J^*(1)}{\partial P^{\ell}_{i,j}(1)} \cdot \frac{\partial P^{\ell}_{i,j}(1)}{\partial u(0)} \tag{29}$$

The superscript (1) in $u^{(1)}_{DUAL}(0)$ indicates the dual solution is derived based upon a first order Taylor series expansion as developed in [3,4]. The partials are evaluated at the nominal cautious control $u(0)$, the parameter $\hat{\theta}(0)$ and the nominal covariance $\bar{P}(1)$. The new dual control (Appendix A) is

$$u^{*(2)}(0) = -[\hat{B}'(0)\ Q\hat{B}(0) + \sum_{\ell=1}^{n} (q_{\ell}\ P^{\ell}_{B}(0) + F_{\ell}) + R]^{-1}\ [\hat{B}(0)\ Q\hat{c}(0)$$

$$+ \sum_{\ell=1}^{n} (q_{\ell}\ P^{\ell}_{Bc}(0) + f_{\ell})] \tag{30}$$

where the matrix F_{ℓ} and the vector f_{ℓ} are

$$F_{\ell} = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} \left(\frac{\partial J^*(1)}{\partial P^{\ell}_{i,j}(1)} - \frac{1}{2} \frac{\partial}{\partial \hat{\theta}^{\ell}_{i}(1)} \cdot \frac{\partial J^*(1)}{\partial \hat{\theta}^{\ell}_{j}(1)} \right) \frac{\partial}{\partial u(0)} \frac{\partial P^{\ell}_{i,j}(1)}{\partial u(0)} \Bigg|_{u^I(0),\hat{\theta}(0),\bar{P}(1)} \tag{31}$$

$$f_{\ell} = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} \left(\frac{\partial J^*(1)}{\partial P^{\ell}_{i,j}(1)} - \frac{1}{2} \frac{\partial}{\partial \hat{\theta}^{\ell}_{i}(1)} \cdot \frac{\partial J^*(1)}{\partial \hat{\theta}^{\ell}_{j}(1)} \right) \left(\frac{\partial P^{\ell}_{i,j}(1)}{\partial u(0)} \right.$$

$$\left. - \frac{\partial}{\partial u(0)} \frac{\partial P^{\ell}_{i,j}(1)}{\partial u(0)}\ u^I(0) \right) \Bigg|_{u^I(0),\hat{\theta}(0),\bar{P}(1)} \tag{32}$$

where $\ell = 1, 2, \ldots, n$ row of B and $P_{i,j}$ is the i,jth element of the covariance matrix P.

The superscript (2) in $u^{(2)}_{DUAL}(0)$ indicates the dual solution is derived based upon a second order Taylor Series expansion as developed in Appendix A.

3.2 Solution to the Nonlinear HHC Problem

The deterministic nonlinear problem is to determine the HHC which minimizes the quadratic criterion of Eq. (23) subject to the nonlinear model of Eq. (7). The stochastic one-step problem uses the criterion of Eq. (24) and the stochastic multi-step problem uses the criterion of Eq. (25).

The nonlinear equation shown in Eq. (7) is a global nonlinear model (using the definition of [3,4]). Taking the difference between two successive time points results in a local nonlinear model (this was formally done in [22] for the linear case). Ignoring nonlinear terms in Eq. (7) results in the global linear model. Ignoring nonlinear terms in the local nonlinear model results in the local linear model. Thus, four possible representations can be derived from Eq. (7) ; 1) global nonlinear, 2) local nonlinear, 3) global linear, and 4) local linear.

The development to follow will only consider the global nonlinear model of Eq. (7) and deterministic criterion of Eq. (23). Although it is possible to treat the stochastic criteria and local nonlinear models, this will not be included here. A subsequent section discusses a nonlinear stochastic solution.

3.3 Open Loop and Adaptive Higher Harmonic Control

The minimization of the deterministic cost function Eq. (23), subject to the global nonlinear model Eq. (7) is a problem in nonlinear programming. This is equivalent to open-loop (or off-line) optimization. There are many optimization techniques available for optimization of a nonlinear function of many variables and [24] provides a detailed examination of such techniques. Two optimization techniques which require explicit calculation of the gradient of the cost function are the gradient method and Newton's method [24]. Two optimization methods which do not require explicit gradient calculation are Powell's method and Rosenbrock's method [24].

Newton's method is presented here as an example of an open loop optimization method. It will also be shown that under suitable approximations this method leads to the open loop global adaptive controller of [1, 3, 4, 22].

Newton's method [24] for minimization of the cost function J of Eq. (23) subject to the nonlinear model (Eq. 7) is

$$u^{I} = u^{I-1} - \alpha[\nabla^2 J]^{-1}_{I-1} \nabla J_{I-1} \qquad (33)$$

where I represents the iteration number, α is a scalar step length parameter ($0 \leq \alpha \leq 1$), ∇J represents the gradient of J with respect to u^{I-1} and $\nabla^2 J$ represents the second gradient (Hessian).

The gradient ∇J and Hessian $\nabla^2 J$ are computed at I-1 from Eq. (23) which yields

$$\nabla J = \frac{\partial x'}{\partial u} Q\ x + R\ u + \tilde{R}\ \Delta u \tag{34}$$

$$\nabla^2 J = \frac{\partial}{\partial u}\left(\frac{\partial x'}{\partial u} Q\right) x + \frac{\partial x'}{\partial u} Q\ \frac{\partial x}{\partial u} + R + \tilde{R} \tag{35}$$

where the vector x represents the vector of vibration components of Eq. (7).

In order to use the open loop Newton control of Eq. (33), the gradients $\frac{\partial x'}{\partial u}$ and $\frac{\partial}{\partial u}\left(\frac{\partial x'}{\partial u}\ Q\right) x$ in Eq. (34) and (35) must be explicitly determined.

A second algorithm is developed using an approximation to $\nabla^2 J$. A simplification to Eq. (35) results if the second gradient terms are ignored. Thus an approximation form for $\nabla^2 J$ is

$$\nabla^2 J \approx \frac{\partial x'}{\partial u}\ Q \frac{\partial x}{\partial u} + R + \tilde{R} \tag{36}$$

Thus, an approximate Newton algorithm using Eq. (33), Eq. (34) and Eq. (36) is

$$u^I = u^{I-1} - \alpha\left[\frac{\partial x'}{\partial u} Q \frac{\partial x}{\partial u} + R + \tilde{R}\right]^{-1} \cdot$$

$$\left[\frac{\partial x'}{\partial u} Q\ x + R\ u^{I-1} + R\ \Delta u^{I-1}\right] \tag{37}$$

If we assume a linear transfer matrix for Eq. (7) (i.e. S = T = 0, in Eq. (7) then

$$x = c + Bu \tag{38}$$

where B represents the linear transfer matrix.

Substitution of Eq. (38) into Eq. (37) and assuming $\tilde{R} = 0$ yields

$$u^I = u^{I-1} - \alpha[B'QB + R]^{-1}[B'Q(c + Bu^{I-1}) + R\ u^{I-1}] \tag{39}$$

If we assume $\alpha = 1$, Eq. (39) reduces to

$$u^I = -\ [B'QB + R]^{-1}[B'Qc] \tag{40}$$

which has the identical form as the global linear adaptive controller with on-line identification of B and c. For on-line identification the iteration number I is replaced with time step number i.

Thus, the global linear adaptive controller of [1,3,4] can be viewed as a on-line version of a Newton method of optimization using an approximation to the Hessian $\nabla^2 J$ and assuming a linear transfer matrix. It is also possible to start with the local nonlinear model formulation and under suitable approximation develop a on-line Newton type algorithm which uses the local linear transfer matrix approximation.

The above example development of a Newton type adaptive controller provides insight into convergence behavior since off-line Newton algorithms have been extensively researched. In [25] (pp. 392-400) a further comparison of two iterative methods for solving the roots of a nonlinear equation is discussed. The first method called the Method of Chords is analogous to the global linear model assumption. The second method called the Method of Tangents is analogous to the local linear model assumption. Convergence behavior of the adaptive controllers is further understood with comparison to these two iterative methods.

The global linear cautious controller of [1,7,22,23] is found to exhibit good convergence to a local minimum solution for a nonlinear transfer relationship. The caution property is a result of including the parameter covariance matrix in the control solution [1]. The improved convergence with the caution property is similar to the off-line nonlinear optimization method referred to as the Marquardt method in [26]. The Marquardt method [26] is similar to Newtons method where the control is iterated using

$$u^I = u^{I-1} - [\nabla^2 J + \alpha_1 I]^{-1} \nabla J \tag{41}$$

where α_1 is a scalar parameter multiplied by the identity matrix I. Eq. (41) is identical to the Newton algorithm of Eq. (33) with $\alpha=1$ if $\alpha_1=0$ in Eq. (41).

The term $\alpha_1 I$ in Eq. (41) is used to improve convergence. When the control is far removed from the minimum solution α_1 is set to a very large number then the matrix inverse in Eq. (41) is a small value approximately equal to I/α_1. Thus the algorithm is similar to the gradient method

$$u^I = u^{I-1} - 1/\alpha_1 \, \nabla J \tag{42}$$

The gradient method is known to converge better than the Newton's method when far removed from the minimum solution (i.e. when the control is in a very non-quadratic region). As the minimum solution is approached α_1 is then set to a small number approaching zero and Eq. (41) behaves like Newton's method of Eq. (31).

Newton's method has superior convergence to the gradient method in the vicinity of the minimum solution (it has quadratic convergence).

Eq. (41) is seen to be very similar to the global linear cautious controller where the covariance matrix P_{I-1} is used to replace $\alpha_1 I$ in Eq. (41). The covariance matrix is set to a large number initially and decreases as the parameters are more accurately identified. Thus the cautious controller can be viewed as behaving initially like a gradient algorithm and then like a Newton-type algorithm as the minimum solution is approached. This explains the excellent convergence behavior observed in most cases with the cautious controller.

Adaptive controllers require on-line estimation of the parameters of the model of HHC input to vibration output. If the nonlinear model (Eq. (7)) is used then the parameters are contained in c , B, S, and T. A linear Kalman filter can still be used since the parameters enter linearly.

This section presents a brief treatment of open-loop and adaptive HHC for the nonlinear problem. A large number of both open-loop and closed-loop adaptive algorithms can be developed using the formulation presented. Four categories of plant model are possible:

Model	Criterion	Controller	Estimator
Global Nonlinear	Deterministic	Newton	Nonlinear; c, B, S, T
Local Nonlinear	Cautious	Modified Newton	Linear; c, B
Global Linear	Dual	Other (1)	Other (2)
Local Linear			

(1) Nonlinear Programming Methods

(2) Off-line or on-line based upon Nonlinear Programming Method Chosen

As is apparent from these four categories a large number of combinations exist for both open-loop and adaptive algorithms. Although both the local and global linear adaptive controllers successfully converge to local minimum values of the cost function for a nonlinear vibration model (this is shown in [22, 23]) two specific problems remain due to nonlinearity; 1) multiple minima solutions exist, some of which do not yield sufficiently low vibration levels and 2) adaptive HHC algorithm divergence. The nonlinear formulation presented in this section provides the mathematical framework to further analyze these two specific problems.

3.4 Nonlinear HHC Theory and the Dual Solution

The dual theories developed earlier may be extended in a straightforward manner to a nonlinear model. It is assumed that for at least some flight conditions the mathematical model representing the helicopter vibration is nonlinear. A non-linear model has been proposed in [17, 22]. This provides the motivation to extend existing dual control solutions [13,14,18] to handle a nonlinear model. For simplicity, here a scalar nonlinear model is selected and the algorithmic steps are summarized. There is a distinctive difference in the complexity of this algorithm. This will be indicated in the proper context but the concepts are similar to that of the linear case.

Let us consider a plant

$$x(k+1) = d + eu(k) + fu^2(k) \tag{43}$$

where d, e, f are unknown, but time invariant scalars. These unknown elements d, e, f are denoted as $\theta(k)$ with covariance matrix P(k). Since the parameter set $\{(d,e,f) \varepsilon \theta(k)\}$ is time invariant we represent it as

$$\theta(k+1) = \theta(k) \tag{44}$$

The measurement of x(k) is according to

$$y(k) = x(k) + w(k) \tag{45}$$

where

$$E(w(k)) = 0 \qquad \text{and} \quad E\{w(k)w(j)\} = W\,\delta_{kj} \tag{46}$$

The control criterion to be minimized is the expected value of the cost from step 0 to N

$$J(0) = E\{\,C(0)\,\} = E\{\sum_{k=1}^{N} q\,x^2(k) + r\,u^2(k-1)\} \tag{47}$$

where N = 2 for the two-step ahead criterion.

The minimization of (47) with respect to u(0) and u(1) subject to (43)-(46) is obtained from the Stochastic Dynamic Programming equation [19,20].

$$J^*(k) = \min_{u(k)} \quad E\{C(k) + J^*(k+1)\,|I^k\} \qquad k = N-1, \ldots, 1, 0 \tag{48}$$

where $J^*(k)$ is the "cost-to-go" from k to N and I^k is the cumulated information at time k when the control u(k) is to be determined. For N = 1, Eq. (48) is

$$J^*(0) = \min_{u(0)} \; E\{q\,x^2(1) + r\,u^2(0) + J^*(1)\,|I^o\} \tag{49}$$

where $J^*(1)$ is the optimal cost at the last step and is obtained by minimization of $J(N-1)$ for $N = 2$.

Thus,

$$J^*(1) = \min_{u(1)} E\{q\, x^2(2) + r\, u^2(1) \,|\, I^1\}$$

$$= \min_{u(1)} E\{q\, (\, d + eu(1) + fu^2(1))^2 + r\, u^2(1) \,|\, I^1\}$$

$$= \min_{u(1)} E\{q(d^2 + e^2 u^2(1) + f^2 u^4(1) + 2deu(1) + 2dfu^2(1)$$

$$+ 2ef\, u^3(1) + r\, u^2(1) \,|\, I^1\} \qquad (50)$$

The control u(1) can be computed only by a search procedure on this stochastic surface $J^*(1)$. Here the method differs from that for the corresponding linear plant case and no explicit elimination of u(1) in terms of the plant parameters is possible.

Combining (49) and (50) we get,

$$J^*(0) = \min_{u(0)} E\{qx^2(1) + ru^2(0) + J^*(1) \,|\, I^o\}$$

$$= \min_{u(0)} E\{q(d^2 + e^2 u^2(0) + f^2 u^4(0) + 2deu(0) + 2efu^3(0) + 2dfu^2(0)) + ru^2(0)$$

$$+ \min_{u(1)} E\{q(d^2 + e^2 u^2(1) + f^2 u^4(1) + 2deu(1) + 2efu^3(1) + 2dfu^2(1))$$

$$+ ru^2(1) \,|\, I^1\} \,|\, I^o\} \qquad (51)$$

From (50) and (51) it is evident that

$$J^*(1) = f_1(\hat{\theta}(1),\, P(1),\, u(1)) \qquad (52)$$

from which

$$u^*(1) = f_2(\hat{\theta}(1),\, P(1)) \qquad (53)$$

Combining (52) and (53) we get,

$$J^*(1) = f_1[\hat{\theta}(1),\, P(1),\, f_2(\hat{\theta}(1),\, P(1))] \qquad (54)$$

where $f_1(.,.,.)$ is an explicit function but $f_2(.,.)$ is not one, because $u^*(1)$ can be obtained only by a search if $\hat{\theta}(1)$, P(1) are known. Thus unlike the linear case, u(0) also has to be obtained by a search and no explicit formula can be derived for it. However, an algorithm based on the dual control ideas applied to a linear plant [13,14,18] can be summarized as follows:

(1) use a nominal cautious control $\bar{u}(0)$ obtained by a search method

(2) get a nominal $\bar{P}(1)$

(3) obtain another control $\bar{u}(1) = f_2\,(\hat{\theta}(0),\ \bar{P}(1))$ again by search and

(4) linearize J(1) about $\hat{\theta}(0)$, $\bar{P}(1)$, $\bar{u}(0)$ as

$$J^*(1) = J^*[\hat{\theta}(0),\ \bar{P}(1)]\Big|_{u(1)=\bar{u}(1)} + \frac{\partial J^*(1)}{\partial P(1)}\Big|_{\bar{u}(1),\hat{\theta}(0),\bar{P}(1)} \cdot (P(1) - \bar{P}(1))$$

$$+ \frac{\partial J^*(1)}{\partial \hat{\theta}(1)}\Big|_{\hat{\theta}(0),\bar{P}(1),\bar{u}(1)} \cdot (\hat{\theta}(1) - \hat{\theta}(0))$$

$$+ \frac{1}{2}\,(\hat{\theta}(1) - \hat{\theta}(0))'\ \frac{\partial^2 J^*(1)}{\partial \hat{\theta}^2(1)}\Big|_{\hat{\theta}(0),\bar{P}(1),\bar{u}(1)} \cdot (\hat{\theta}(1) - \hat{\theta}(0)) \tag{55}$$

The covariance P(1) is influenced by u(0) according to the linear Kalman Filter equation. Thus we may linearize P(1) about the control u(0) according to

$$P(1) = \bar{P}(1) + \frac{\partial P(1)}{\partial u(0)}\Big|_{\bar{u}(0),P(0)} \cdot (u(0) - \bar{u}(0))$$

$$+ \frac{1}{2}(u(0) - \bar{u}(0))'\ \frac{\partial^2 P(1)}{\partial u^2(0)}\Big|_{\bar{u}(0),P(0)} \cdot (u(0) - \bar{u}(0)) \tag{56}$$

Using (51),(55), (56) we get u(0) by a final search method. Conceptually, the extension to the nonlinear plant is not difficult but the complexity is increased many fold.

4. SIMULATION MODELS

This section discusses the three distinct mathematical models used to represent the relationship between the helicopter higher harmonic control inputs and its vibration outputs. The model descriptions are:

1) a nonlinear transfer matrix model
2) a linear time varying transfer matrix model, and
3) a scalar model representing a single HHC input to a single vibration output.

4.1 Nonlinear Model

A third order polynomial model has been derived in [23]. The identified longitudinal hub force polynomial model at 120 knots flight condition is shown in Figures 1-3. This longitudinal hub vibration model as a function of the two control inputs θ_{3c} and θ_{3s} is described by the polynomial equations,

$$\begin{aligned} x_1(k+1) = {} & 108.9 - 74.84u_1(k) - 51.04u_2(k) - 6.515u_1^2(k) - 2.825u_2^2(k) \\ & - 1.04u_1(k)u_2(k) + 7.77u_1^3(k) + 22.27u_2(k)u_1^2(k) \\ & + 5.92u_1(k)u_2^2(k) + 4.17u_2^3(k) \end{aligned} \tag{57}$$

$$\begin{aligned} x_2(k+1) = {} & -89.74 + 53.31u_1(k) - 82.56u_2(k) - 7.42u_1^2(k) + 7.34u_2^2(k) + 1.91u_1(k)u_2(k) \\ & + 5.63u_1^3(k) + 26.24u_2(k)u_1^2(k) - 7.09u_1(k)u_2^2(k) + 6.54u_2^3(k) \end{aligned} \tag{58}$$

where

x_1 = longitudinal hub force (cosine component of vibration), LBS.

x_2 = longitudinal hub force (sine component of vibration), LBS.

u_1 = cosine component input, θ_{3c}, deg.

u_2 = sine component input, θ_{3s}, deg.

and measurements according to

$$y_1(k) = x_1(k) + w_1(k) \tag{59}$$

$$y_2(k) = x_2(k) + w_2(k) \tag{60}$$

where it is assumed that

$$E\{w(k)\ w'(j)\} = W\ \delta_{kj} = \text{diag}\ (10.89^2,\ 8.97^2) \tag{61}$$

Figures 1 and 2 show the longitudinal cosine vibration x_1 and the sine vibration x_2 respectively vs. the control inputs θ_{3s} and θ_{3c}. The nonlinearity is clearly shown for $|\theta_{3c}| \geq 1°$ and $|\theta_{3s}| \geq 1°$. The total longitudinal cost function $x_1^2 + x_2^2$ is shown in Figure 3.

4.2 Linear Time - Varying Multivariable Model

The parameters of the helicopter vibration model can be rapidly varying during an acceleration or deceleration maneuver. A simulation of this situation is done using a linear transfer matrix model with varying parameters. The current algorithms are evaluated on this model. The HHC problem formulation for time varying parameters assumes that the transfer matrix elements B(k) and the vibration components without control c(k) are modelled as a random walk

$$\theta(k+1) = \theta(k) + v(k) \tag{62}$$

where

$$\{c,\ B\ \varepsilon\ \theta\} \tag{63}$$

The plant process noise v(k) is zero mean white gaussian of covariance V . The process noise covariance V is assumed a value of 30% of the initial parameter values so that they simulate a rapid maneuver.

The linear vibration model used is

$$x(k+1) = c(k) + B(k)\ u(k) \tag{64}$$

where x(k+1) is the vibration when HHC u(k) is applied. This vibration is measured

$$y(k) = x(k) + w(k) \tag{65}$$

The measurement noise w(k) has a standard deviation of 10% of the uncontrolled vibration at time t=0. The random walk model of (62), (63) can describe the helicopter in any number of transient flight conditions.

(1) Helicopter transition from hover to forward speed (or from forward speed to hover)

(2) Helicopter in a windy environment

(3) Maneuvering flight conditions, and

(4) Nonlinearity (assuming nonlinear effects can be modelled as linearized time-varying parameters).

The simulation model described by (62)-(65) does not describe just one helicopter model. Monte-Carlo simulations with various noise levels on the model of (62)-(65) can describe the helicopter in several different transient modes or perhaps even different helicopters as well in different flight conditions. Thus the model is quite general and can be used to evaluate the existing algorithms under different conditions. The simulation is started with the initial linear plant as

$$x_1(k+1) = 108.9 - 74.84u_1(k) - 51.04u_2(k) \tag{66}$$

$$x_2(k+1) = -89.74 + 53.31u_1(k) - 82.56u_2(k) \tag{67}$$

and the measurements are according to (59), (60).

4.3. Scalar Model

A scalar model is used to evaluate the new dual solution based upon the second order Taylor series expansion and is given by

$$x(k+1) = c + b(k)u(k) \tag{68}$$

$$b(k+1) = a\, b(k) + v(k) \tag{69}$$

and the measurement is according to

$$y(k) = x(k) + w(k) \tag{70}$$

with

$$\begin{aligned} &E(v(k)) = 0 \quad , \quad E(v(k)v'(j)) = V\,\delta_{kj} \\ &E(w(k)) = 0 \quad , \quad E(w(k)w'(j)) = W\,\delta_{kj} \\ &E\{w(k)v'(j)\} = 0 \end{aligned} \tag{71}$$

For the time varying case, the parameters used are

$b(0) = .05$, $P_b(0) = 1.0$, $V = .1$, $c = 1.0$

$W = .01$ and $W = .1$, $a = .9$

For the constant case, the parameters used are

$b(0) = .05$, $P_b(0) = 1.0$, $V = 0$, $c = 1.0$

$W = .01$ and $W = .1$, $a = 1.0$

5. SIMULATION RESULTS

This section presents the results of the cautious and the dual adaptive controllers presented earlier in Sections 2 and 3 using the simulation models presented in Section 4.

5.1 Simulation Model for the Time Varying Multi-variable Linear Model

These two controllers are based on the linear time varying model discussed earlier in Section 4. The process noise covariance of the parameters is selected so that it represents a rapidly varying flight condition. The process noise standard deviation is selected as 10%, 30%, and 50% of the initial parameter values. These three cases are referred to as the 10% noise case, 30% noise case and 50% noise case. Each simulation run is performed for 40 time steps for 100 Monte-Carlo runs. Fig. 4 shows a time history of the cost function using the cautious control for these three noise levels. The 10% noise case (circle in Figure 4) shows excellent vibration reduction. The vibration reduction is poorer with 30% noise (triangle in Figure 4) and the 50% noise (+ symbol). The 30% noise case shows poor vibration reduction for $0 < k < 16$ and $32 < k < 40$ and good reduction for $16 < k < 32$, k being the time step. The 30% noise case is selected for more extensive evaluation. Figures 5 and 6 show the cosine and sine longitudinal components respectively of the amplitudes squared for the 30% noise. The Kalman filter of Figures 4-6 is initialized with parameters taken from a random number generator with a standard deviation equal to the magnitude of the true parameter values. This represents an acceleration run of a helicopter where the parameters are poorly estimated at hover. A second case is also run in which the parameter estimates were initialized as true values with the standard deviation equal to 10% of the true parameters. This represents a transient deceleration run after the parameters have been accurately identified in steady flight conditions. Figures 4 through 6 provide the motivation for studying in depth the effect of the cautious and the first order dual controller [13] on the time varying parameter model. A summary of the results for two cases of initialization is discussed next. A detailed discussion of the second case will be discussed later on.

5.2 Controller Performance for the first case (Kalman Filter Initialization with large Initial Covariance)

Figures 4-6 show that the cautious controller is not able to reduce the vibrations for the regions $7 < k < 16$ and $32 < k < 40$. Referring to Figure 4 (30% noise case) three regions are defined as follows:

Region 1: $k = 7$ to $k = 16$

Region 2: $k = 16$ to $k = 32$

Region 3: $k = 32$ to $k = 40$

The cautious controller demonstrates good performance in Region 2, but poor results in Regions 1 and 3. The dual controller based upon the first order linearization [13] is used next and a performance comparison is plotted in Figures 7-9 for the same Monte Carlo simulations. The design parameter β is set to 1 and 2 [13]. For $\beta = 2$ the results are worse than that for the cautious controller and for $\beta = 1$ they are the same as that for the cautious case. Here it is to be noted that Regions 1 and 3 are uncontrollable i.e., a control which neutralizes the vibrations does not exist within the limits of $\pm 2^0$. However Region 2 is controllable but since the cautious controller performs so well in this region, the dual controller can offer no further improvement. Figures 10-15 show the true parameter variation, the estimated parameters and the associated $\pm$ one standard deviation band from the Kalman filter. Figures 11 and 12 show poor identification of the parameters θ_2 and θ_3 in Region 2 but reasonably good control performance. This is because the true and estimated parameters for θ_2 and θ_3 are far from the zero i.e., still in a controllable mode. A different feature will be demonstrated in the next case.

5.3 Controller Performance for the second case (Kalman Filter Initialization with True Initial Parameters and very small Initial Covariance)

For this run the Kalman filter is initialized with the true parameter values and the initial parameter standard deviation is 10% of the magnitude of the initial parameters. This simulation differs from the previous one only in the Kalman filter initialization.

Figures 16-20 compare the cost using the cautious control (solid line) with the cost using no control (dashed line). The three regions are clearly marked in the figures and the controller demonstrates no improvement over no control in Regions 1 and 3. From examining the determinant, Region 2 is controllable but Regions 1 and 3 are not. Thus Region 2 can be improved upon by a dual controller. Figures 17 and 18 show the cautious controls u_1 and u_2. The controls sometimes go to zero and thus turnoff occurs. Figures 19 and 20 show the contribution from the individual states. Figures 21-26 show the true, estimated parameters and their associated variances. These figures indicate the cause of the poor performance of the cautious controller. The improvement in Region 2 by a dual controller will be discussed in detail in a later sub-section. Figures 27 and 28 show the variation of the determinant of the transfer matrix (measure of controllability) and that of the ratio of its eigenvalues (degree of controllability).

5.4 Controllability Condition for the Plant

For a plant with known parameters the controllability condition requires that the transfer matrix composed of the parameters θ_2, θ_3, θ_5, θ_6 has an inverse, i.e.,

$$\theta_2\theta_6 - \theta_3\theta_5 \neq 0$$

This condition may be violated due to various reasons:

1) all or some of the parameters individually are simultaneously zero

2) the transfer matrix consists of linearly dependent rows or columns.

When such a situation occurs it is impossible to control the vibrations with only a limited amount of control. For a plant with unknown parameters, the matrix consisting of the estimated parameters $\hat{\theta}_2$, $\hat{\theta}_3$, $\hat{\theta}_5$, $\hat{\theta}_6$ also has a role to play. Sometimes, as in Region 2 of Figure 16, the parameters are poorly estimated as near zero with the wrong sign. In such a case, turn off may occur and the vibration may not at all be reduced even though the true transfer matrix is controllable. Probing helps in such instances and this has been observed using the dual controller. Detailed analysis of these phenomena are discussed next.

5.5 Detailed Discussion of the controller performances for the second case of initialization; Scope of Probing; Uncontrollable regions

Here the Kalman filter is initialized with the true parameter values and the initial parameter standard deviation is 10% of the magnitude of the initial parameters. The simulation is carried out in a Monte Carlo fashion for 100 runs. Both the cautious and the dual controllers are used. Each of these 100 runs is made for 40 time steps. The results of the average cost per run are tabulated in Table 1, using no control, the cautious controller and the dual controller ($\beta = 1$) [13]. The average cost over all runs is given at the bottom of the table. Applying no control, the average cost is 50,531. Applying the one step ahead cautious control, the average cost is 18,051. This is a reduction of 64.3%. The dual controller yields 17,141, a reduction of 66.1%. These figures do not give us any insight into the conditions of the plant when probing is useful. Each run requires individual analysis. Comparative plots of the performance of no control, cautious control, and the dual control for the first 20 runs of this Monte Carlo study are given in Figures 29-48. The symbols in these figures are circle- no control, triangle- cautious control, and plus symbol- dual control ($\beta = 1$). These comparative plots show clearly that there exist some situations when the dual does better than the cautious and when both the dual and the cautious perform well. Four runs are selected from Table 1 and are discussed in detail. The runs and the % cost reductions are shown below.

COST REDUCTIONS FOR 4 SELECTED RUNS
FROM 100 MONTE-CARLO RUNS

Run No.	No Control Cost	Cautious Control		Dual Control	
		Cost	% Reduction	Cost	% Reduction
1	28583	20215	29.2	15764	44.8
2	43381	12067	72.2	12571	71.0
11	28091	3360	88.0	3230	88.5
18	28850	17064	40.8	12511	56.6

From the above table it is clear that dual control sometimes yields larger cost reductions, and at times both the controllers perform equally good. Thus each run needs to be analysed individually in order to discover whether the prevalent conditions are congenial to the use of the dual control. The issues of controllability of the true plant and the controllability of the estimated plant are of concern to the analyst. These are discussed next.

Run Number 1

Referring to Figure 29, one can clearly see that the dual control improves over the cautious control's performance for the region $16 \leq k \leq 28$, by probing effectively during $13 \leq k \leq 15$. The rest of the plot is similar to that of the cautious controller.

Let us first consider the cautious controller. From Figure 27 it is seen that the determinant of the estimated parameter transfer matrix is zero between the time steps 11 and 25. This corresponds to the case of the ratio of smaller to the larger eigenvalues being zero in Figure 28 (also a Run 1 case). The controls in Figures 17 and 18, however, are still bounded because of the caution terms present in the control design. From figures 21-26 we see that in Region 1 θ_2 is estimated with the wrong sign. This leads to a large contribution from the cosine component. In Region 2, θ_3 has a wrong sign. In addition to this the controls are almost turned off and the sine component has a major share in the vibration. In Region 3, θ_3 is estimated well but it is close to zero. Both the sine and the cosine components contribute here.

For the dual we see from Figures 49, 50 that the cosine component contributes in Region 1 and both the sine and the cosine components contribute in Region 3. The controller probes the system and the control values in Figures 51, 52 are non-zero. Next we refer to Figures 53-58. θ_2 is estimated again with the wrong sign in Region 1. θ_3 is estimated correctly between the time steps 10 and 13 and between 34 and 35 but its true value is close to zero. In Region 3, θ_3 has the wrong sign. In Region 3, θ_6 is close to zero again. It is clear from Figures 59, 60 that the determinant of the estimated parameter matrix is not zero between the time steps 11 and 25.

The main improvement is in Region 2 and this leads to a reduction in the average cost from 20215 (Cautious) to 15762 (Dual).

Run Number 2

Referring to Figure 30, one sees clearly that both the dual and cautious controllers perform poorly between the time steps 6 and 8 and between 15 and 25. In these regions the vibrations are more than that obtained by applying no control. For this run, reference is made to Figures 61-68. There is a large contribution from the cosine component between the time steps 15 and 25 (Fig. 61).

It is observed clearly from Figures 69-72 for both the controllers, the true transfer matrix is singular, corresponding to an uncontrollable mode. The parameters θ_2, θ_3, θ_5 are responsible for this. The estimation of the parameters is reasonably good. Under this situation applying no control is the most judicious choice. Any other controller could make the situation worse, which is clearly demonstrated here. At other time steps, when the plant is, in fact, controllable, both the controllers perform well. This uncontrollable situation can be handled by considering a reduced order model or by applying some switches on the state and control weights. This will be discussed later on in detail.

Run Number 11

From Figure 39 it is clearly seen that both the cautious and the dual controllers are successful in bringing down the vibration to a satisfactory level. Both of them reduce the vibration by 88% of that of no control.

Both the controllers perform well because the plant is controllable. The contributions from the sine and the cosine components are reduced substantially. The detailed performances are portrayed in Figures 73-80. The determinants of the matrices composed of the true and the estimated parameters and the ratio of their eigenvalues are plotted in Figures 81-84. They are far from zero.

Run Number 18

A comparative plot showing the performances of the no control, cautious and the dual controller is given in Figure 43. Poor performances are observed between the time steps 9 and 20 and again between the time steps 29 and 33. In the first region, at time step 11, the performances are worse than that of the no control. Beyond time step 11 until time step 16, the dual controller improves the situation, but the cautious still performs poor. The detailed performances are given in Figures 85-92.

The determinants of the matrices composed of the true and the estimated parameters and the ratio of their eigenvalues are plotted in Figures 93-96. For the cautious controller, the estimated parameter transfer matrix has a small value for its determinant between the time steps 11 and 16. The dual controller starts probing earlier around time step 9 (Fig. 87, 88, 91, 92) and its estimated parameter transfer matrix has a non zero value for its determinant during the same time steps. Moreover, during the time steps 16 and 20 the true plant is close to uncontrollability and nothing can be better than no control.

From the detailed study discussed above one may conclude the following:

1) Both the controllers behave poorly whenever the true system is uncontrollable. The controllers have no information, whatsoever, about the present or the future controllability of the plant. In such a situation, the correct diagnostic is to apply no control at all.
2) The cautious controller behaves poorly if the estimated parameters define an uncontrollable situation although the true system is controllable.

In such situations, the dual controller can offer significant improvement over the cautious controller, by probing to better estimate the parameters.

Optimal Control Requirements for the Runs 1, 2, 11, 18

In this section the cautious and the dual controllers are used on the linear model whose unknown parameters are time varying. The stochastic controllers are used in conjunction with an estimator, which supplies the controllers with the parameter estimator. An optimal controller, on the contrary, assumes perfect knowledge of the plant parameters and attempts to control the plant. In this subsection, a study is made on this optimal controller. Plots of the two required optimal controls are given in Figures 97-104. While plotting, the controls are passed through a hard limiter with limits between -10^0 and $+10^0$. When the true plant is uncontrollable, an optimal controller demands an infinite amount of control and thus it hits the boundary of $+10^0$ or -10^0. From these plots, it is evident that for all but Run 11, the plants are uncontrollable at some time during the 40 time steps.

Effect of switches on the control weights R

It has been discussed earlier that in uncontrollable situations the proper diagnostics is to apply no control at all. The control can be switched off by exercising exceedingly large caution or by putting large control weights R in the control design. In Run 2, the region between time steps 15 and 25 is known to be uncontrollable. Thus between these regions the control weights are increased from R = diag (0,0) to R = diag $(10^4,0)$ and R = diag $(10^4,10^4)$, keeping the state weights Q = diag (1,1) as before. The case 'R = diag $(10^4,0)$' switches off the control u_1 and the second case 'R = diag $(10^4,10^4)$' switches off both the controls u_1 and u_2. The total and the individual costs are plotted in Figures 105-107. This switching is quite successful in reducing the vibrations in uncontrollable situations.

Effect of switches on the state weights Q

During uncontrollable situations, it is impossible for the two controls to affect both the states. Nevertheless, it may be possible to handle one state only at the expense of allowing the other state to run free. This can be done by choosing properly the state weights Q. For Run 2 again, Q is chosen as diag (1,.1) and diag (1, .01) for R = diag (0,0) and R = diag (.01,.01). The total and the individual costs are plotted in Figures 108-113. The cases Q = diag (1,.1) and

and Q = diag (1,.01) put less constraint on the second state. Thus these switches reduce the first state, allowing the second state to go free. The performances hardly differ for the cases R = diag (0,0) and R = diag (.01,.01).

Effect of Initial Covariance of the Parameters on the Controller's Performance (first method of initialization)

Two methods of initialization have been discussed and their performances have been analysed in detail earlier. This study refers to the first method of initialization where the Kalman filter is initialized with parameters taken from a random number generator with an initial covariance. Both the cautious and the dual controllers are used on this plant. The parameters are slowly varying with time (process noise of 10%). A Monte Carlo simulation of 100 runs is made for 40 time steps. The average cost is computed over all runs for each time step. Three choices are made for this initial covariance. The normal initial variance specified on the plots is the square of the magnitude of the true initial parameters. The average performances of the no control, cautious and the dual are plotted in Figures 114-117. The dual controller offers improvement in the case of large initial variance (Figure 116). It has always an initial jump for this large initial variance (Figure 114,116). This is not observed when the initial variance is small (Figure 115). This initial jump can be avoided by using a cautious controller in the beginning and switching to the dual after time step 2 (Figure 117).

5.7 Simulation Results for the Nonlinear Model

The nonlinear model describing the longitudinal hub vibration model is used with the global linear adaptive cautious controller of [12]. The alternate form of the Kalman filter which retains better positive definiteness of the covariance equation is used. The exponential weighting form of the equations is also used with the forgetting factor λ set equal to .99 to discount past data. The initial covariance is taken as a large quantity 250000 (compared to approximately 10000 for the original covariance) to better account for nonlinearity. The graphical descriptions of the cost function of Figures 1-3 indicate the presence of multiple minima solutions. The plots can be broadly divided into the following regions:

Region I : $0 \leq u_1 \leq +2$, $-2 \leq u_2 \leq 0$

Region II : $0 \leq u_1 \leq 2$, $0 \leq u_2 \leq 2$

Region III : $-2 \leq u_1 \leq 0$, $0 \leq u_2 \leq 2$

Region IV: $-2 \leq u_1 \leq 0$, $-2 \leq u_2 \leq 0$

These definitions of the regions provide easier interpretation of the performance of the controllers. Figures 118-122 show the convergence plots of the cautious adaptive controller for 100 time steps. Figures 118-120 are for the case Q = diag $(10^{-5}, 5\times10^{-8})$ and R = diag $(10^{-4}, 10^{-4})$. These weights primarily concentrate on reducing the state x_1 while allowing less restriction on the state x_2. There are more than one occasion before time step 80 when the cost increases sharply. The controls are allowed to move within the limits of ±2. At these points the control is operating in Region II. In this region state x_1 is highly nonlinear and appears to possess a saddle point. At a saddle point the second gradient of a function is zero. This corresponds to an uncontrollable situation as is clear from the equations of [17]. Figures 121-123 discuss the case for which Q = diag (1,1) and R = diag (0,0). Here both the states x_1 and x_2 are equally weighted. In this case convergence occurs rapidly with the controls going to Region I. From figures 1,2 it is clear that the two individual states x_1 and x_2 are well behaved in this region and a fast convergence occurs.

5.8 Simulation of the Scalar Model using the Cautious, Dual and the Dual Adaptive Control based upon Sensitivity Functions (Appendix A and B)

A new adaptive dual control solution based upon the sensitivity functions of the expected future cost is derived and analysed in the Appendices A and B. The controller design and its performance on a scalar model are given in Appendix A. The performance is compared with that of the cautious controller and the first order dual controller. This section discusses in further detail the results shown in Appendices A and B and includes several other figures from the 100 Monte Carlo runs for both the constant parameter and the time varying parameter cases.

Time Varying Parameter Case (Example a)

The details of the simulation model used are presented in Appendix A for the time varying parameter case. Run numbers 7 and 14 are discussed in the Appendix A. Runs 2 and 21 are discussed here. The cautious control is shown by the circle symbol, the first order dual by the triangle, and the second order dual by the plus symbols. Figures 124-126 describe this run. The second order dual performs better than the other two right from the start. Figures 127, 128 gives the time history of the control for runs 7 and 14 respectively. Figures 129-131 describe Run 21. The nature of the performance of all the controllers is the same but in most occasions the second order dual (denoted by the plus symbol) performs better than the other two (see Tables 2, 3, and 6).

Constant Parameter Case (Examble b)

In this case (Section 4) the true parameter is close to zero (i.e., b(0)=.05) but constant. This corresponds to the uncontrollable case. A control magnitude of -20 is required to handle this situation. Runs 26 and 80 are discussed in the Appendix A. Figures 134 and 137 show the time history of the control for Runs 26 and 80. Figures 132 and 133 describe Run 18. Here the first order dual tends to go unstable in the 11 to 15 time step and the cautious is slow in its convergence. Figures 135 and 136 give the cost and the control time history of Run 44. Both the duals work better than the cautious. In all runs of this constant parameter case the new dual shows better results than the other two. It always goes to the right direction of estimation by properly probing from the start (see Tables 4, 5, and 7).

6. CONCLUSIONS

Helicopter vibration can be effectively reduced by applying adaptive control techniques. Although adaptive controllers often show excellent vibration reduction, it has been shown that there exist certain conditions which yield unsatisfactory behavior and thus require improvements. It is shown that the existing cautious controller often exhibits problems like slow convergence, turn off phenomenon, and instability. Also, nonlinearity in the helicopter model is shown to have a significant effect on the HHC convergence behavior. The performance of the cautious controller has been evaluated based upon 100 Monte-Carlo simulations of a linear time varying multivariable model representative of helicopter vibration during maneuvering flight conditions. The cautious controller has been shown to yield unacceptable reduction in vibration whenever the determinant of the estimated parameter model is near zero. This results in an uncontrollable model. The first order dual controller of [13] avoids this situation by probing and estimating the parameters better. It however is still deficient under certain conditions and sometimes yields poor results.

The problems of the previous controllers are overcome by the new adaptive dual control solution developed in Appendix A. The second order dual controller takes into account the dual effect better by performing a second order Taylor series expansion of the expected future cost. It is shown to modify the cautious control solution by introducing numerator and denominator correction terms. These three controllers have been evaluated on both a simple scalar constant parameter model and a time-varying parameter model representative of maneuvering flight. In each case 100 Monte-Carlo simulations are used. The new dual controller improves upon both the constant and the time varying case and provides improved convergence. In the case of the constant parameter model, whenever the unknown parameter is close to zero (i.e. near singular), the cautious and the first order dual controllers have demonstrated slow convergence and turn off. The new second order dual controller consistently demonstrates faster convergence. It avoids problems of turn off, slow convergence and always goes toward the right direction of estimation by properly probing. It has the potential to be used on a multivariable model and the detailed sensitivities have been presented in this report. It is yet to be implemented on this multivariable model. Moreover, its properties and the behavior of the correction terms that enable better performance in the scalar case are not yet fully understood. Computationally, it is complex and may be difficult to use in real time. However, investigation of its properties is thus warranted in order to develop a practical implementation.

In addition to the above linear simulation studies, the global linear adaptive cautious controller has been used on a nonlinear model describing helicopter vibration. Nonlinearity has a significant effect on the convergence behavior. Multiple minima solutions can exist and the algorithms are slower in convergence and can be unstable. To accommodate the nonlinearity, the initial covariance on the parameter estimates is taken as a large quantity. This accounts for the nonlinearity by introducing more caution. Reduction of vibration is possible in most regions of the nonlinear surface. Vibrations cannot be reduced when the nonlinear surface possesses a saddle point. At a saddle point the second gradient of the cost function is zero. This corresponds to an uncontrollable situation and can result in algorithm divergence.

The linear and nonlinear simulation studies investigated in this report clearly demonstrate the need for further research to better understand the convergence properties of the adaptive controller for reduction of helicopter vibration. In the present studies the analytical ground work has been presented. Further analytical work and simulation is required to fully understand the properties of the adaptive controller for helicopter vibration reduction.

APPENDIX A

DUAL ADAPTIVE CONTROL BASED UPON SENSITIVITY FUNCTIONS

A new adaptive dual control solution is presented for the control of a class of multi-variable input-output systems. Both rapidly varying random parameters and constant but unknown parameters are included. The new controller is based upon a on-line Newton type algorithm which is shown to result in a controller which modifies the cautious control design with a numerator and denominator correction. The new controller is shown to depend upon sensitivity functions of the expected future cost. A scalar example is presented to provide insight into the properties of the new dual controller. Monte-Carlo simulations are performed which show improvement over the cautious controller and the Linear Feedback Dual Controller of 10 (13) and (14).

A1. INTRODUCTION

Multi-variable systems which are characterized by uncertain parameters with large random variations are a difficult challenge for most control design techniques. The assumed random nature of the parameter variations often precludes the use of gain scheduling (non adaptive) control design. Stochastic adaptive control theory provides a principal design approach for systems of this type. Exact solution of the stochastic problem with unknown parameters requires solution of the Stochastic Dynamic Programming equation and this is not feasible for practical implementation. The solution is known to have a dual effect [13, 14] that can be used to enhance the real-time identification of system parameters as well as provide good control.

Many suboptimal dual solutions have been suggested [11,13,14,27-32]. The various approaches which have incorporated this dual property can be loosely divided into two classes. In the first class [28-31], the optimal control problem is reformulated to consist of a one-step ahead criterion to be minimized, augmented by a second term which penalizes the cost for poor identification. This approach is attractive due to the analytical tractability of the solution; however, the solution is based on a one-step criterion and does not fully exploit the dual property of a multi-step solution. Padilla and Cruz [34] give a dual control solution for such a plant by minimizing the control objective function subject to an upper bound in the total estimation cost. Their objective function includes a standard control objective function and also a second constraint term which reflects the sensitivity of the parameters to the state of the system. Thus the solution adjusts itself to exercise better estimation for such sensitive parameters within the upper bound. The second class [21,32,33] utilizes the stochastic dynamic programming equation directly and performs linearization of the future cost in order to obtain a solution. Previous control solutions among this second class require a numerical search procedure which poses difficulties for a practical solution for on-line control for multivariable systems.

A linear feedback dual controller was presented in [13,14] based upon a linearization of a two-step criterion and found to offer some improvement over the non-dual cautious control based upon a one-step criterion. The results were based upon a simulation model with constant but unknown parameters. Although the dual control offers some improvement over the cautious controller the improvement is not significant for most practical applications where the system contains constant parameters and the objective is to control in steady state operation. However, for random parameter variations, dual control can sometimes offer significant improvement over non-dual controllers [28,32]. The approximate dual control in [13,14] is attractive due to its simplicity (it is comparable to the cautious control design in algorithm complexity and does not require numerical search). The objective of the present study is to evaluate the cautious controller and the approximate dual controller of [13,14] for large random parameter variations modeled as a random walk. Monte-Carlo simulations are performed and conditions quantified under which the dual controller offers significant improvement over a non-dual cautious controller.

The approximate dual control of [13,14] although offering a reduction in the average cost is found to be unacceptable in many cases. This is attributed to the sensitivity of the expected future cost whenever the system is characterized by limited controllability. An extension of the linearization procedure of [13,14] is presented to account for this sensitivity. The new dual controller inherently includes a robustness property in that the controller accounts for sensitivity of the expected future cost due to parameter estimates and their uncertainty. Simulations are presented which show the improvement of the new dual controller over the cautious controller and the approximate dual controller of [13,14]. The new dual controller uses a Newton type search procedure and is developed for multi-variable systems. One of the main advantages of the new dual control presented herein is that it modifies the cautious controller with a numerator "probing" term and denominator correction term. Although the new dual control is still considered too complex for practical implementation, the structure of the control solution is in a form which permits practical design changes to the cautious controller to include the dual properties.

A2. PROBLEM FORMULATION

The multivariable system under investigation is

$$x(k+1) = c(k) + B(k)\ u(k) \qquad \text{(A2.1)}$$

where $c(k)$ is an unknown vector and $B(k)$ is a matrix of unknown parameters. The unknown elements of $c(k)$ and $B(k)$ are denoted as $\theta(k)$ with covariance matrix $P(k)$. These are represented by a discrete random model

$$\theta(k+1) = A\theta(k) + v(k) \qquad \text{(A2.2)}$$

$$E(v(k)) = 0 \qquad \text{and } E(v(k)v'(j)) = V\ \delta_{kj} \qquad \text{(A2.3)}$$

The measurement equation is

$$y(k) = x(k) + w(k) \qquad \text{(A2.4)}$$

where

$$E(w(k)) = 0 \qquad \text{and} \quad E(w(k)w'(j)) = W\ \delta_{kj} \qquad \text{(A2.5)}$$

$$E(w(k)v'(j)) = 0$$

and $x(k)$, $y(k)$ being n dimensional vectors. The control criterion to be minimized is the expected value of the cost from step 0 to N

$$J(0) = E\{C(0)\} = E\left\{\sum_{k=1}^{N} x'(k)\ Q\ x(k) + u'(k-1)\ R\ u(k-1)\right\} \qquad \text{(A2.6)}$$

where $N = 2$ for the two step ahead criterion.

A3. APPROXIMATE DUAL CONTROLLER FOR TWO STEP CRITERION

The minimization of (A2.6) with respect to $u(0)$ and $u(1)$ subject to (A2.1) - (A2.5) is obtained from the Stochastic Dynamic Programming equation [19,20]

$$J^*(k) = \min_{u(k)} \ E\{C(k) + J^*(k+1)\,|Y^k\} \qquad k = N-1,\ldots,1,0 \qquad \text{(A3.1)}$$

where $J^*(k)$ is the "cost-to-go" from k to N and Y^k is the cumulated information at time k when the control $u(k)$ is to be determined. For $N = 1$, eq. (A3.1) is

$$J^*(0) = \min_{u(0)} \ E\ \{x'(1)Qx(1) + u'(0)\ Ru(0) + J^*(1)\,|Y^o\} \qquad \text{(A3.2)}$$

where $J^*(1)$ is the optimal cost at the last step and is obtained by minimization of $J(N-1)$ for $N = 2$. Assuming diagonal $Q = \mathrm{diag}(q_\ell)$ this results in [13,14]

$$J^*(1) = \hat{c}'(1)Q\hat{c}(1) + \sum_{\ell=1}^{n} q_\ell P_c^\ell(1)$$

$$- [\hat{c}'(1)Q\hat{B}(1) + \sum_{\ell=1}^{n} q_\ell P_{cB}^\ell(1)]\ [\hat{B}'(1)Q\hat{B}(1) + \sum_{\ell=1}^{n} q_\ell P_B^\ell(1) + R]^{-1}\cdot$$

$$[\hat{B}'(1)Q\hat{c}(1) + \sum_{\ell=1}^{n} q_\ell P_{Bc}^\ell(1)] \qquad \text{(A3.3)}$$

and

$$u^*(1) = -[\hat{B}'(1)Q\hat{B}(1) + \sum_{\ell=1}^{n} q_\ell P_B^\ell(1) + R]^{-1}[\hat{B}'(1)Q\hat{c}(1) + \sum_{\ell=1}^{n} q_\ell P_{Bc}^\ell(1)] \quad \text{(A3.4)}$$

where

$$P^\ell(1) = \begin{bmatrix} P_c^\ell(1) & P_{cB}^\ell(1) \\ P_{Bc}^\ell(1) & P_B^\ell(1) \end{bmatrix} \quad \text{(A3.5)}$$

$P(1)$ is the expected value of $(\theta(1))^2$ for time step 2 given measurement $y(1)$ at time step 1. The index ℓ is used to represent the row number in Eq. (A2.1) and $P^\ell(1)$ is the associated parameter covariance.

The parameter estimates $\hat{\theta}(1)$ and covariances $P(1)$ are obtained from the Kalman filter. Since W is diagonal one can decouple the estimation. Then

$$\hat{\theta}^\ell(1) = \hat{\theta}^\ell(0) + K_\ell(1) \quad \nu_\ell(1) \quad \text{(A3.6)}$$

$$K_\ell(1) = P^\ell(0)H'(1) \ [H(1)P^\ell(0)H'(1) + W_\ell]^{-1} \quad \text{(A3.7)}$$

$$\tilde{P}^\ell(1) = P^\ell(0) - K_\ell(1)H(1)P^\ell(0) \quad \text{(A3.8)}$$

$$P^\ell(1) = A\tilde{P}^\ell(1)A' + V \quad \text{(A3.9)}$$

where

$$\nu_\ell(1) = y_\ell(1) - H(1)\hat{\theta}^\ell(0) \quad \text{(A3.10)}$$

$$H(1) = [1 \ u^T(0)] \quad \text{(A3.11)}$$

$$\theta^\ell(1) = [c_\ell(1) \ B_\ell(1)]^T \quad , \quad \ell = 1,2,\ldots n \ \text{row of B} \quad \text{(A3.12)}$$

As discussed in [13,14] $J^*(1)$ is a nonlinear function of the parameter estimates $\hat{\theta}(1)$ and covariances $P(1)$ and thus a linearization was performed. In [14] a scalar formulation was presented and a first order linearization was performed about the nominal parameter estimate squared $(\hat{\theta}(0))^2$ and nominal covariance $\bar{P}(1)$. Also in [13,14] the vector case was presented and linearization to first order performed. To more accurately account for the dual effect a second order Taylor Series expansion is presented about $\hat{\theta}(0)$ and a first order expansion about the nominal covariance $\bar{P}(1)$. In addition (as will be presented subsequently) the covariance $P(1)$ will include a linearization to second order in $u(0)$. In [13,14], $P(1)$ was linearized to first order. It is believed that linearizations to second order are necessary to better account for the nonlinearity in $P(1)$ and $\hat{\theta}(1)$ of Eq. (A3.3) and in $u(0)$ of Eq. (A3.7) and (A3.8). In addition a nonlinear Newton algorithm is used in the second order approximation.

Linearization of Eq. (A3.3) about the nominal $\bar{\theta}(1) = \hat{\theta}(0)$ and $\bar{P}(1)$ using the nominal $\bar{u}(0)$ results in

$$J^*(1) = J^*[1, \hat{\theta}(0), \bar{P}(1)] + \frac{\partial J^*(1)}{\partial\hat{\theta}(1)} [\hat{\theta}(1) - \hat{\theta}(0)] + \frac{1}{2}[\hat{\theta}(1)-\hat{\theta}(0)]' \frac{\partial^2 J^*(1)}{\partial\hat{\theta}^2(1)}[\hat{\theta}(1)-\hat{\theta}(0)]$$

$$+ \sum_{\ell=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial J^*(1)}{\partial P^{\ell}_{i,j}(1)} [P^{\ell}_{i,j}(1) - \bar{P}^{\ell}_{i,j}(1)] \qquad \text{(A3.13)}$$

where the superscript ℓ represents the covariance matrix associated with the ℓ^{th} row of parameters and $P_{i,j}(1)$ is the i-j th element of the covariance matrix $P(1)$.

Eq. (A3.6) is rewritten as

$$\hat{\theta}^{\ell}(1) - \hat{\theta}^{\ell}(0) = K_{\ell}(1)\ \nu_{\ell}(1) \quad , \qquad \ell = 1, 2, \ldots n \qquad \text{(A3.14)}$$

Using (A3.14) the expected value of (A3.13) is

$$E[J^*(1)|Y^o] = J^*[1, \hat{\theta}(0), \bar{P}(1)] + \frac{1}{2}\, tr[\frac{\partial^2 J^*(1)}{\partial\hat{\theta}^2(1)} K(1)\ E\{\nu(1)\nu'(1)|Y^o\}\ K'(1)]$$

$$+ \sum_{\ell=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial J^*(1)}{\partial P^{\ell}_{i,j}(1)} [P^{\ell}_{i,j}(1) - \bar{P}^{\ell}_{i,j}(1)] \qquad \text{(A3.15)}$$

Using the innovation covariance

$$E\{\nu_{\ell}(1)\ \nu'_{\ell}(1)|Y^o\} = H(1)\ P^{\ell}(0)H'(1) + W_{\ell} \qquad \text{(A3.16)}$$

and (A3.7) and (A3.8), Eq. (A3.15) can be written as

$$E[J^*(1)|Y^o] = J^*[1, \hat{\theta}(0), \bar{P}(1)] + \sum_{\ell=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \Bigg\{ -\frac{1}{2} \frac{\partial}{\partial\hat{\theta}_i(1)} \frac{\partial J^*(1)}{\partial\hat{\theta}_j(1)} \left[P^{\ell}_{i,j}(1) - P_{i,j}(0) - V^{\ell}_{i,j}\right]$$

$$+ \frac{\partial J^*(1)}{\partial P^{\ell}_{i,j}(1)} [P^{\ell}_{i,j}(1) - \bar{P}^{\ell}_{i,j}(1)] \Bigg\} \qquad \text{(A3.17)}$$

The expected future cost is shown to be a function of the predicted covariance $P^{\ell}_{i,j}(1)$ with a multiplier given by the sensitivity $\frac{\partial J^*(1)}{\partial P^{\ell}_{i,j}(1)}$ and $\frac{\partial\ \partial J^*(1)}{\partial\hat{\theta}_i(1)\partial\hat{\theta}_j(1)}$. Since the covariance $P^{\ell}_{i,j}(1)$ depends on the control $u(0)$ the control has the dual effect. It should be noted that the importance of the dual effect depends upon the sensitivity of the expected future cost with respect to both the covariance and parameter estimate.

The optimal control $u(0)$ can be computed by minimization of (A3.2) using (A3.17). Since $P^{\ell}_{i,j}(1)$ is nonlinear in $u(0)$ a numerical search procedure is required. This is accomplished using a second order linearization in $u(0)$.

Thus Eq. (A3.8) is linearized to second order about the control $u^I(0)$, which is in the vicinity of the optimal control.

$$P^{\ell}_{i,j}(1) \simeq \bar{P}^{\ell}_{i,j}(1) + \left.\frac{\partial P^{\ell}_{i,j}(1)}{\partial u(0)}\right|_{u^I(0)} [u(0)-u^I(0)] + \frac{1}{2}[u(0)-u^I(0)]' \left.\frac{\partial^2 P^{\ell}_{i,j}(1)}{\partial u^2(0)}\right|_{u^I(0)} [u(0)-u^I(0)] \tag{A3.18}$$

The expected future cost as given by (A3.17) and (A3.18) is quadratic in $u(0)$ and thus a closed form solution $u^*(0)$ is obtained by minimization of (A3.2).

The optimal dual control $u^*(0)$ can now be computed from (A3.2) using (A3.17) and (A3.18). It is obtained by solving

$$\frac{\partial}{\partial u(0)} E\{x'(1)Q\,x(1) + u'(0)Ru(0) + J^*(1)\,|Y^o\} = 0 \tag{A3.19}$$

The optimal $u^*(0)$ is thus

$$u^*(0) = -[\hat{B}'(0)Q\hat{B}(0) + \sum_{\ell=1}^{n} (q_{\ell}P^{\ell}_{B}(0) + F_{\ell}) + R]^{-1}[\hat{B}'(0)Q\hat{c}(0) + \sum_{\ell=1}^{n} (q_{\ell}P^{\ell}_{Bc}(0) + f_{\ell})] \tag{A3.20}$$

where the matrix F_{ℓ} and the vector f_{ℓ} are

$$F_{\ell} = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{2}\left(\frac{\partial J^*(1)}{\partial P^{\ell}_{i,j}(1)} - \frac{1}{2}\frac{\partial}{\partial \hat{\theta}^{\ell}_{i}(1)}\frac{\partial J^*(1)}{\partial \hat{\theta}^{\ell}_{j}(1)}\right) \left.\frac{\partial}{\partial u(0)}\frac{\partial P^{\ell}_{ij}(1)}{\partial u(0)}\right|_{u^I(0),\hat{\theta}(0),\bar{P}(1)} \tag{A3.21}$$

$$f_{\ell} = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{2}\left(\frac{\partial J^*(1)}{\partial P^{\ell}_{i,j}(1)} - \frac{1}{2}\frac{\partial}{\partial \hat{\theta}^{\ell}_{i}(1)}\frac{\partial J^*(1)}{\partial \hat{\theta}^{\ell}_{j}(1)}\right)\left.\left(\frac{\partial P^{\ell}_{i,j}(1)}{\partial u(0)} - \frac{\partial}{\partial u(0)}\frac{\partial P^{\ell}_{i,j}(1)}{\partial u(0)}\,u^I(0)\right)\right|_{u^I(0),\hat{\theta}(0),\bar{P}(1)} \tag{A3.22}$$

Initially the nominal value of $\bar{u}(0)$ is computed from (A3.20) with F_{ℓ} and f_{ℓ} equal to zero. Then a gradient search is performed until in the vicinity of the optimal $u^*(0)$. Then (A3.20) - (A3.22) are used until convergnece is achieved. This iteration procedure is essentially Newton's method for minimization of a nonlinear function. The gradient search is used because the stochastic cost in (A3.2) being minimized is a high order nonlinear equation and the gradient procedure is used until $u^I(0)$ is in the vicinity of the minimum before switching to the Newton method. The nominal covariance $\bar{P}^{\ell}(1)$ is computed from (A3.7) - (A3.11) with $u(0) = \bar{u}(0)$. The sensitivity (partials) in (A3.21) and

(A3.22) of the cost $J^*(1)$ are computed from partial derivatives of $J^*(1)$ (Eq. (A3.3)) and $P^{\ell}(1)$ (Eq. (A3.7) through (A3.9)) evaluated at the nominal. The partials of the covariance are evaluated at $u^I(0)$ which is evaluated at the previous iteration I.

The approximate two-step ahead dual control of Eqs. (A3.20), (A3.21) and (A3.22) can be interpreted as a modification to the cautious controller by the terms F_{ℓ} and f_{ℓ}. These terms depend upon the sensitivity of the future nominal cost $J^*(1)$ with respect to the parameters $\hat{\theta}_i^{\ell}(1)\hat{\theta}_j^{\ell}(1)$ for all i, j and their covariance $P_{i,j}^{\ell}(1)$ for each row ℓ of parameters. Whenever these sensitivities are large the terms F_{ℓ} and f_{ℓ} will be significant (that is the dual effect will be important). Thus the sensitivities take into account in the control solution the sensitivity of the nominal future cost due to parameter variation and uncertainty. The larger this sensitivity the more important will be the dual effect.

The resulting dual controller (A3.20) exhibits a robustness property with respect to parameter variations and uncertainty of the future cost by including a term which appears in the denominator of the dual controller. In addition, a probing term also appears in the numerator.

A4. SCALAR EXAMPLE WITH ONE UNKNOWN PARAMETER

To further understand the dual control solution a scalar example with one unknown parameter b is presented (a multi-variable simulation is currently under development). The approximate dual control solution for the scalar case from Eqs. (A3.20), (A3.21) and (A3.22) using Q = 1, R = 0, is

$$u^*(0) = -\frac{\hat{b}(0)c + f_{\ell}}{\hat{b}^2(0)+P_b(0)+F_{\ell}} \tag{A4.1}$$

$$F_{\ell} = \frac{1}{2}\left(\frac{\partial J^*(1)}{\partial P_b(1)} - \frac{1}{2}\frac{\partial^2 J^*(1)}{\partial\hat{b}(1)\,\partial\hat{b}(1)}\right)\frac{\partial^2 P_b(1)}{\partial u(0)\,\partial u(0)} \tag{A4.2}$$

$$f_{\ell} = \frac{1}{2}\left(\frac{\partial^* J(1)}{\partial P_b(1)} - \frac{1}{2}\frac{\partial^2 J^*(1)}{\partial\hat{b}(1)\,\partial\hat{b}(1)}\right)\left(\frac{P_b(1)}{\partial u(0)} - \frac{\partial^2 P_b(1)}{\partial u(0)\,\partial u(0)}u^I(0)\right) \tag{A4.3}$$

$$\left.\frac{\partial J^*(1)}{\partial P_b(1)}\right|_{\hat{b}(0),\bar{P}_b(1)} = \frac{c^2\hat{b}^2(0)}{(\hat{b}^2(0)+\bar{P}_b(1))^2} \tag{A4.4}$$

$$\left.\frac{\partial^2 J^*(1)}{\partial\hat{b}(1)\partial\hat{b}(1)}\right|_{\hat{b}(0),\bar{P}_b(1)} = -2c^2\bar{P}_b(1)\left[\frac{\bar{P}_b(1)-3\hat{b}^2(0)}{(\hat{b}^2(0)+\bar{P}_b(1))^3}\right] \tag{A4.5}$$

$$\left.\frac{\partial P_b(1)}{\partial u(0)}\right|_{u^I(0)} = -\frac{2P_b^2(0)W\,u^I(0)a^2}{(P_b(0)u^{I^2}(0)+W)^2} \tag{A4.6}$$

$$\left.\frac{\partial^2 P_b(1)}{\partial u(0)\,\partial u(0)}\right|_{u^I(0)} = -2P_b^2(0)W\left[\frac{W-3P_b(0)u^{I^2}(0)}{(P_b(0)u^{I^2}(0)+W)^3}\right]a^2 \tag{A4.7}$$

where the nominal $\bar{u}(0)$ and $\bar{P}_b(1)$ are

$$\bar{u}(0) = -\frac{\hat{b}(0)c}{\hat{b}^2(0)+P_b(0)} \tag{A4.8}$$

$$\bar{P}_b(1) = a^2\left\{P_b(0) - \frac{P_b^2(0)\bar{u}^2(0)}{P_b(0)\bar{u}^2(0)+W}\right\}+V = \frac{a^2P_b(0)W}{P_b(0)\bar{u}^2(0)+W} + V \tag{A4.9}$$

The parameter estimate $\hat{b}(0)$ and $P_b(0)$ are computed using data up to $k = 0$ (i.e. $y(0)$).

The future cost evaluated at the nominal is

$$J^*(1, \hat{b}(0), \bar{P}(1)) = c^2 - c^2\frac{\hat{b}^2(0)}{\hat{b}^2(0)+\bar{P}_b(1)} \tag{A4.10}$$

and the expected future cost based upon the linearization of Eq. (A3.17)

$$E\{J^*(1)|Y^o\} = J^*(1, \hat{b}(0), \bar{P}(1)) - \frac{1}{2}\frac{\partial^2 J^*(1)}{\partial\hat{b}^2(1)}(P_b(1)-P_b(0)-V) + \frac{\partial J^*(1)}{\partial P_b(1)}(P_b(1)-\bar{P}_b(1))$$

(A4.11)

A4.1 Evaluation of the Cautious Controller

The performance of the cautious controller can be evaluated using (A3.2) with u(0) evaluated at the nominal

$$J(0) = [E\{x^2(1)\,|Y^o\} + E\{J^*(1)\,|Y^o\}]_{u(0)=\bar{u}(0)} \qquad (A4.12)$$

The first term in Eq. (A4.12) represents the expected cost at k = 1 and the second term in Eq. (A.12) represents the expected future cost at k = 2 using the cautious control at k = 2 (i.e. u(1)) and using the cautious control at k = 1 (i.e. $u(0) = \bar{u}(0)$). Eq. (A4.12) is evaluated using data Y^o.

Eq. (A4.12) is evaluated for the scalar example using (A4.10), (A4.11) and (A4.7) - (A4.9), which results in

$$J(0) = c^2 - \frac{\hat{b}^2(0)c^2}{\hat{b}^2(0)+P_b(0)} + c^2 - c^2\frac{\hat{b}^2(0)}{\hat{b}^2(0)+P_b(1)} + \frac{1}{2}\frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)}\cdot\frac{P_b^2(0)\bar{u}^2(0)}{P_b(0)\bar{u}^2(0)+W} \qquad (A4.13)$$

where $\frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)}$ is computed from (A4.5).

The last term in (A4.11) is zero since $P_b(1)$ evaluated at the nominal control (i.e. cautious control) equals $\bar{P}_b(1)$.

The first two terms in (A4.13) represent the average cost at step k = 1 and the last three terms represent the expected future cost at k = 2 using the cautious control.

Eq. (A4.13) can be used with a simple example to demonstrate when the cautious control is expected to behave poorly.

Assume a scalar example with one unknown b parameter and let

$$\hat{b}(0) = .05 \quad , \quad P(0) = .5 \quad , \quad a = 1.0 \qquad (A4.14)$$

$$V = .1 \quad , \quad W = .1 \quad , \quad c = 1$$

The expected cost at k = 1 and k = 2 is computed from the nominal, $\bar{u}(0)$, $\bar{P}_b(1)$ and $\frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)}$ which yields

$$\bar{u}(0) = -.1 \quad , \quad \bar{P}_b(1) = .575 \quad , \quad \frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)} = -3.47 \qquad (A4.15)$$

and

$$J(0) \simeq c^2 + c^2 \quad , \quad c = 1 \qquad (A4.16)$$

Thus the cautious control applied at $k = 0$ results in no reduction in the cost at $k = 1$ due to large uncertainty $P(1)$ and also no reduction in the future expected cost since $\bar{u}(0)$ is small and no improvement in parameter accuracy occurs at step $k = 1$.

A4.2 Evaluation of the Dual Controller

The dual controller given by (A4.1) - (A4.9) can be evaluated by computing the average cost of (A4.12) using (A4.7) - (A4.11) and the covariance

$$P_b(1) = \frac{P_b(0)W}{P_b(0)u^2(0)+W} + V \tag{A4.17}$$

The expected future cost (A4.11) reduces to

$$E\{J^*(1)|Y^o\}\Big|_{u^*(0)} = c^2 - c^2 \frac{\hat{b}^2(0)}{\hat{b}^2(0)+\bar{P}_b(1)} + \frac{1}{2}\frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)} \frac{P_b^2(0)u^{*2}(0)}{P_b(0)u^{*2}(0)+W}$$

$$- \frac{\partial J^*(1)}{\partial P_b(1)} \left(\frac{P_b^2(0)u^{*2}(0)}{P_b(0)u^{*2}(0)+W} - \frac{P_b^2(0)\bar{u}^2(0)}{P_b(0)\bar{u}^2(0)+W} \right) \tag{A4.18}$$

and the total expected cost at $k = 1$ and $k = 2$ using (A4.12) is

$$J^*(0) = E\{x^2(1)|Y^o\}\Big|_{u^*(0)} + E\{J^*(1)|Y^o\}\Big|_{u^*(0)} \tag{A4.19}$$

where

$$E\{x^2(1)|Y^o\}\Big|_{u^*(0)} = c^2 + 2\hat{b}(0)u^*(0)c + (\hat{b}^2(0) + P_b(0))u^{*2}(0) \tag{A4.20}$$

Examination of (A4.18) shows that the dual control can reduce the expected future cost over the cautious control since the last two expressions in (A4.18) can be negative if $u^{*2}(0) > \bar{u}^2(0)$. Thus the dual property can have a desirable effect on the future cost.

The cost $J^*(0)$ is computed using the scalar example previously discussed for the cautious controller. A search procedure is used on (A4.19) using

(A4.18) and (A4.20) with the parameter values from (A4.14), and $u^*(0)$ is iterated until in the vicinity of the minimum yielding

$$\left.\frac{\partial J^*(1)}{\partial P_b(1)}\right|_{\bar{u}(0)=-.1} = .0075 \quad , \quad \left.\frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)}\right|_{\bar{u}(0)=-.1} = -3.47 \quad , \quad \left.\frac{\partial P_b(1)}{\partial u(0)}\right|_{u^I(0)=-.6} = .382 \quad ,$$

$$\left.\frac{\partial^2 P_b(1)}{\partial u^2(0)}\right|_{u^I(0)=-.6} = +1.0 \qquad F_\ell = .87 \quad , \quad f_\ell = .85 \qquad (A4.21)$$

Eq. (A4.21) was evaluated in the vicinity of the optimal $u^I(0) = -.6$ and $P_b(1) = .278$. The optimal control $u^*(0)$ is from Eq. (A4.1) using $u^I(0) = -.6$

$$u^*(0) = -\frac{\hat{b}(0)\,c + .85}{\hat{b}^2(0) + P_b(0) + .87} \quad , \quad c = 1 \qquad (A4.22)$$

$$= -.62$$

Eq. (A4.22) shows that $u^*(0)$ is considerably different than the cautious control $\bar{u}(0) = -.1$ and is a result of large values of F_ℓ and f_ℓ which in turn are due to large values of $\frac{\partial^2 P_b(1)}{\partial u^2(0)}$ and $\frac{\partial^2 J(1)}{\partial \hat{b}^2(1)}$

The corresponding future expected cost using Eq. (A4.19) is

$$E\{J^*(1)|Y^o\}\Big|_{u^*(0)} \simeq c^2 + \frac{1}{2}\,\frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)}\,\frac{P_b^2(0)u^{*2}(0)}{P_b(0)u^{*2}(0)+W}$$

$$\simeq c^2 - \frac{1}{2}\,(3.47)(.321)\;c^2$$

$$\simeq c^2 - .557\;c^2$$

$$\simeq .442\;c^2 \quad , \quad c = 1 \qquad (A4.23)$$

The result of this example shows that the dual control of Eq. (A4.22) reduces the expected future cost to 44% of the original c^2 with no control. The cautious control resulted in no reduction of the future cost. The terms responsible for the improvement with dual control are the second order sensitivities $\frac{\partial^2 P(1)}{\partial u^2(0)}$ and $\frac{\partial^2 J^*(1)}{\partial \hat{b}^2(1)}$.

The dual control of Eq. (A4.22) differs from the cautious control by the terms $F_\ell = .87$ in the denominator and $f_\ell = .85$ in the numerator. The denominator term in effect provides more "caution" whereas the numerator term is an additive probing effect. The term F_ℓ provides a "robustness" property in that the sensitivity of the future cost to parameter uncertainties as they appear in the controller (i.e. $\hat{b}^2(0)$) are minimized. Thus a new interpretation of the dual control is that it contains robustness and learning (via probing). These concepts are applicable to the multivariable dual controller in Eq. (A3.20) through Eq. (A3.22).

A5. SIMULATION RESULTS

Performance was evaluated from 100 Monte Carlo runs for the following controllers where $\hat{b}(0)$ was set equal to b(0) with covariance $P_b(0)$:

1. Cautious Controller
2. A two step dual based on the first order Taylor Series expansion [13,14]
3. The new dual controller based on the second order Taylor Series expansion.

The above algorithms were tested for two cases:

a) Time varying case, $b(0) = .05$, $P_b(0) = 1.0$, $V = .1$, $c = 1.0$

$W = .01$ and $W = .1$, $a = 0.9$

b) Constant case, with $b(0) = .05$, $P_b(0) = 1.0$, $V = 0$, $c = 1.0$

$W = .01$ and $W = .1$, $a = 1.0$

Example a

Table 6 summarizes the results of the simulation runs. All three algorithms were tested on this example for two different levels of measurement noise covariance, W = .01 and W = .1, 100 Monte Carlo runs were performed, each of 40 time steps. For each run, an average cost was computed over 40 time steps and then the averages over 100 runs are tabulated in Table 6 and Table 7. The tables clearly indicate that the dual controller based on the second order sensitivity functions shows the lower cost. The dual effect shows a larger improvement for larger measurement noise (i.e. W = .1). Run numbers 7 and 14 of the 100 Monte Carlo runs were selected for plotting. The cautious control is shown by the circle symbol, the first order dual by the triangle, and the second order dual by the plus symbols. The cost and parameter value are plotted in Figures 138 - 141. It is evident that the second order dual improves upon the other two on the average.

Example b

In this case the true parameter was close to zero (i.e., b(0) = .05) but constant. Table 6 summarizes the result. The average cost obtained by the second order dual is much lower than the other two. The second order dual was always found to exhibit excellent convergence whereas the other controllers performed poorly. In addition the new controller consistently avoided turn off and burst [28]. This was an important common feature in all the Monte Carlo runs. Runs 26 and 80 are plotted in Figures 142 and 143 respectively, as typical examples.

The simulation study has shown that the new dual controller improves upon the cost on the average. The magnitude of the improvement on the average appears to be relatively small for the noise levels used. However, the real advantage of the new dual controller is the improvement in those instances where the cautious controller and the dual controller of [13,14] yields unacceptable results. Although the dual controller of [13,14] shows improvement over the cautious controller, it has been found to be unacceptable at many time points.

A6. CONCLUSION

A new adaptive dual control solution based upon the sensitivity functions of the expected future cost has been presented. This controller takes into account the dual effect better by performing the second order Taylor series expansion of the expected future cost. The form of this controller is a modification of the one step cautious controller. The approximate dual control of [13,14] did not have the denominator correction term like the present one. This adds stability to the new control design. Simulation results of a scalar model have shown the improvement obtained using the new dual algorithm.

APPENDIX B

DERIVATION OF THE SENSITIVITY FUNCTIONS FOR THE SECOND ORDER DUAL CONTROLLER FOR A TWO STATE VECTOR MODEL

The concept of a dual controller is introduced in Section 2.2 and a dual controller based upon a second-order Taylor's series expansion is derived in Appendix A. This controller is given by

$$u^*(0) = -[\hat{B}'(0)Q\,\hat{B}(0) + \sum_{\ell=1}^{n} (q\,P_B^{\ell}(0) + F_\ell) + R]^{-1}[\hat{B}'(0)Q\hat{c}(0) + \sum_{\ell=1}^{n} (q_\ell P_{Bc}^{\ell}(0) + f_\ell)] \tag{B.1}$$

where the matrix F_ℓ and the vector f_ℓ are

$$F_\ell = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} \left(\frac{\partial J^*(1)}{\partial P_{i,j}^{\ell}(1)} - \frac{1}{2} \frac{\partial}{\partial \hat{\theta}_i^{\ell}(1)} \frac{\partial J^*(1)}{\partial \hat{\theta}_j^{\ell}(1)} \right) \frac{\partial}{\partial u(0)} \frac{\partial P_{i,j}^{\ell}(1)}{\partial u(0)} \Bigg|_{u^I(0),\hat{\theta}(0),\bar{P}(1)} \tag{B.2}$$

$$f_\ell = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} \left(\frac{\partial J^*(1)}{\partial P_{i,j}^{\ell}(1)} - \frac{1}{2} \frac{\partial}{\partial \hat{\theta}_i^{\ell}(1)} \frac{\partial J^*(1)}{\partial \hat{\theta}_j^{\ell}(1)} \right) \left(\frac{\partial P_{i,j}^{\ell}(1)}{\partial u(0)} - \frac{\partial}{\partial u(0)} \frac{\partial P_{i,j}^{\ell}(1)}{\partial u(0)}\, u^I(0) \right) \Bigg|_{u^I(0),\hat{\theta}(0),\bar{P}(1)} \tag{B.3}$$

This approximate two-step ahead dual control is a modification of the cautious controller by the terms F_ℓ and f_ℓ. These terms depend upon the sensitivity of the future expected cost $J^*(1)$ with respect to the parameters $\hat{\theta}_i^{\ell}(1)$ $\hat{\theta}_j^{\ell}(1)$, for all i,j and their covariance $P_{i,j}^{\ell}(1)$ for each row ℓ of parameters. The sensitivity evaluations of the future covariance $P_{i,j}(1)$ with respect to the current control $u(0)$ are also necessary. These functions are derived in detail in the following sections of this appendix.

Variable Definitions

For the sake of completeness, we may rewrite Eq. (3.3) of Appendix A here, assuming $Q = \text{diag}\ (q_\ell)$, as

$$J^*(1) = c'(1)\ Qc(1) + \sum_{\ell=1}^{n} q_\ell\ P_c^\ell(1)$$

$$- [\hat{c}'(1)\ Q\ \hat{B}(1) + \sum_{\ell=1}^{n} q_\ell\ P_{cB}^\ell(1)][\hat{B}'(1)Q\hat{B}(1) +$$

$$\sum_{\ell=1}^{n} q_\ell\ P_B^\ell(1) + R]^{-1} \cdot [\hat{B}'(1)Q\hat{c}(1) + \sum_{\ell=1}^{n} q_\ell P_{Bc}^\ell(1)] \tag{B.4}$$

The plant equations are described by

$$x_1(k+1) = \theta_1(k) + \theta_2(k)\ u_1(k) + \theta_3(k)\ u_2(k) \tag{B.5}$$

$$x_2(k+1) = \theta_4(k) + \theta_5(k)\ u_1(k) + \theta_6(k)\ u_2(k) \tag{B.6}$$

whose measurements are according to

$$y_1(k) = x_1(k) + w_1(k) \tag{B.7}$$

$$y_2(k) = x_2(k) + w_2(k) \tag{B.8}$$

with

$$\left.\begin{array}{l} E(w(k)) = 0 \\ E\{w(k)w'(j)\} = W\delta_{kj} \quad ; \quad w(k) \triangleq (w_1(k) \quad w_2(k))' \end{array}\right. \tag{B.9}$$

and the parameter of Eqn. (B.5) and (B.6) varying as

$$\theta(k+1) = \theta(k) + v(k) \tag{B.10}$$

where

$$\left.\begin{array}{l} E(v(k)) = 0 \quad ; \quad E\{v(k)\ v'(j)\} = V\delta_{kj} \\ E\{w(k)v'(j)\} = 0 \quad ; \quad v(k) \triangleq (v_1(k) \quad v_2(k))' \end{array}\right. \tag{B.11}$$

Following the notations of [13] we have,

$$\hat{c}(1) = \begin{bmatrix} \hat{\theta}_1(1) \\ \hat{\theta}_4(1) \end{bmatrix}, \quad \hat{B}(1) = \begin{bmatrix} \hat{\theta}_2(1) & \hat{\theta}_3(1) \\ \hat{\theta}_5(1) & \hat{\theta}_6(1) \end{bmatrix} \tag{B.12}$$

and the future expected cost,

$$J^*(1) = q_1(\hat{\theta}_1^2(1) + P_{1,1}(1)) + q_2(\hat{\theta}_4^2(1) + P_{4,4}(1))$$

$$- \frac{1}{CD-E^2} \; (F^2D - 2FGE + G^2C) \qquad (B.13)$$

where $P_{m,n}(1)$ is the m-n element of the covariance matrix P(1), and

$$C = q_1(\hat{\theta}_2^2(1) + P_{2,2}(1)) + q_2(\hat{\theta}_5^2(1) + P_{5,5}(1)) + r_1$$

$$D = q_1(\hat{\theta}_3^2(1) + P_{3,3}(1)) + q_2(\hat{\theta}_6^2(1) + P_{6,6}(1)) + r_2$$

$$E = q_1(\hat{\theta}_2(1)\;\hat{\theta}_3(1) + P_{2,3}(1)) + q_2(\hat{\theta}_5(1)\;\hat{\theta}_6(1) + P_{5,6}(1)) \qquad (B.14)$$

$$F = q_1(\hat{\theta}_1(1)\;\hat{\theta}_2(1) + P_{1,2}(1)) + q_2(\hat{\theta}_4(1)\;\hat{\theta}_5(1) + P_{4,5}(1))$$

$$G = q_1(\hat{\theta}_1(1)\;\hat{\theta}_3(1) + P_{1,3}(1)) + q_2(\hat{\theta}_4(1)\;\hat{\theta}_6(1) + P_{4,6}(1))$$

First Order Sensitivity $\dfrac{\partial J^*(1)}{\partial P_{i,j}(1)}$

$$\frac{\partial J^*(1)}{\partial P_{1,1}(1)} = q_1 \qquad (B.15)$$

$$\frac{\partial J^*(1)}{\partial P_{1,2}(1)} = -\frac{2q_1(FD-GE)}{(CD-E^2)} \qquad (B.16)$$

$$\frac{\partial J^*(1)}{\partial P_{1,3}(1)} = -\frac{2q_1(GC-FE)}{(CD-E^2)} \qquad (B.17)$$

$$\frac{\partial J^*(1)}{\partial P_{2,2}(1)} = -\frac{q_1\{G^2(CD-E^2) - (F^2D-2FGE+G^2C)D\}}{(CD-E^2)^2} \qquad (B.18)$$

$$\frac{\partial J^*(1)}{\partial P_{2,3}(1)} = -\frac{q_1\{(-2FG)(CD-E^2)-(F^2D-2FGE+G^2C)(-2E)\}}{(CD-E^2)^2} \qquad (B.19)$$

$$\frac{\partial J^*(1)}{\partial P_{3,3}(1)} = -\frac{q_1\{F^2(CD-E^2)-(F^2D-2FGE+G^2C)C\}}{(CD-E^2)^2} \qquad (B.20)$$

The derivatives of $J^*(1)$ with respect to the variances of the parameters involved in the second state (B.6) are the same as above with q_1 replaced by q_2.

First Order Sensitivity $\dfrac{\partial J^*(1)}{\partial \hat{\theta}_i(1)}$

Using equations (B.13) and (B.14) we derive below $\dfrac{\partial J^*(1)}{\partial \hat{\theta}(1)}$

$$\frac{\partial J^*(1)}{\partial \hat{\theta}_1(1)} = 2q_1 \ \hat{\theta}_1(1) - \frac{2q_1(\ FD\hat{\theta}_2(1) - EG\hat{\theta}_2(1) - FE\hat{\theta}_3(1) + GC\hat{\theta}_3(1))}{(CD-E^2)} \qquad (B.21)$$

$$\frac{\partial J^*(1)}{\partial \hat{\theta}_2(1)} = -\ 2q_1\{(FD\hat{\theta}_1(1) - GE\hat{\theta}_1(1) - FG\hat{\theta}_3(1) + G^2\hat{\theta}_2(1))(CD-E^2)$$
$$- (F^2D-2FGE+G^2C)(\hat{\theta}_2(1)D - \hat{\theta}_3(1)E)\} \ / \ (CD-E^2)^2 \qquad (B.22)$$

$$\frac{\partial J^*(1)}{\partial \hat{\theta}_3(1)} = -\ 2q_1\{(F^2\hat{\theta}_3(1) - FE\hat{\theta}_1(1) - FG\hat{\theta}_2(1) + GC\hat{\theta}_1(1)) \ (CD-E^2)$$
$$- (F^2D-2FGE+G^2C)(C\hat{\theta}_3(1) - E\hat{\theta}_2(1)\} \ / \ (CD-E^2)^2 \qquad (B.23)$$

The partials of $J^*(1)$ with respect to the parameters of the second state (B.6) are similar to the above with q_1 replaced by q_2, and the parameters $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$ by($\hat{\theta}_4$, $\hat{\theta}_5$, $\hat{\theta}_6$) respectively ones.

Second Order Sensitivity $\dfrac{\partial^2 J^*(1)}{\partial\hat{\theta}_i(1)\,\partial\hat{\theta}_j(1)}$

$$\frac{\partial^2 J^*(1)}{\partial\hat{\theta}_1^2(1)} = 2q_1 - \frac{1}{(CD-E^2)} \cdot (2q_1^2\hat{\theta}_2^2(1)D - 4q_1^2\hat{\theta}_2(1)\hat{\theta}_3(1)E + 2q_1^2\hat{\theta}_3^2(1)C) \tag{B.24}$$

$$\begin{aligned}\frac{\partial^2 J^*(1)}{\partial\hat{\theta}_1(1)\,\partial\hat{\theta}_2(1)} = -\frac{1}{(CD-E^2)^2} \cdot \{&(2q_1^2\hat{\theta}_1(1)\hat{\theta}_2(1)D - 2q_1^2\hat{\theta}_1(1)\hat{\theta}_3(1)E - 2q_1^2\hat{\theta}_3^2(1)F \\ &+ 2q_1^2\hat{\theta}_2(1)\hat{\theta}_3(1)G - 2GEq_1 + 2FDq_1)(CD-E^2) \\ &- (2q_1\hat{\theta}_2(1)FD - 2q_1\hat{\theta}_2(1)GE - 2q_1\hat{\theta}_3(1)FE \\ &+ 2q_1\hat{\theta}_3(1)GC)\ (2q_1\hat{\theta}_2(1)D - 2q_1\hat{\theta}_3(1)E)\}\end{aligned} \tag{B.25}$$

$$\begin{aligned}\frac{\partial^2 J^*(1)}{\partial\hat{\theta}_1(1)\,\partial\hat{\theta}_3(1)} = -\frac{1}{(CD-E^2)^2} \cdot \{&(2q_1^2\hat{\theta}_2(1)\hat{\theta}_3(1)F - 2q_1^2\hat{\theta}_1(1)\hat{\theta}_2(1)E - 2q_1^2\hat{\theta}_2^2(1)G \\ &- 2FEq_1 + 2GCq_1 + 2q_1^2\hat{\theta}_1(1)\hat{\theta}_3(1)C)\ (CD-E^2) \\ &- (2q_1\hat{\theta}_2(1)FD - 2q_1\hat{\theta}_2(1)GE - 2q_1\hat{\theta}_3(1)FE + 2q_1\hat{\theta}_3(1)GC) \cdot \\ &(C\cdot 2q_1\hat{\theta}_3(1) - 2Eq_1\hat{\theta}_2(1))\}\end{aligned} \tag{B.26}$$

$$\begin{aligned}\frac{\partial^2 J^*(1)}{\partial\hat{\theta}_2^2(1)} = -\frac{1}{(CD-E^2)^4}\ [&\{(2q_1^2\hat{\theta}_1^2(1)D - 4q_1^2\hat{\theta}_1(1)\hat{\theta}_3(1)G + 2q_1G^2)(CD-E^2) \\ &- (F^2D - 2FGE + G^2C)(2q_1D - 2q_1^2\hat{\theta}_3^2(1))\}\ (CD-E^2)^2 \\ &- \{(2q_1\hat{\theta}_1(1)FD - 2q_1\hat{\theta}_1(1)GE - 2q_1\hat{\theta}_3(1)FG + 2q_1\hat{\theta}_2(1)G^2) \cdot \\ &(CD-E^2) - (F^2D - 2FGE + G^2C)(2q_1\hat{\theta}_2(1)D - 2q_1\hat{\theta}_3(1)E)\} \\ &\cdot 2\{(CD-E^2)(2q_1\hat{\theta}_2(1)D - 2q_1\hat{\theta}_3(1)E)\}]\end{aligned} \tag{B.27}$$

$$\frac{\partial^2 J^*(1)}{\partial\hat{\theta}_2(1)\,\partial\hat{\theta}_3(1)} = -\frac{1}{(CD-E^2)^4}\Big[\{(2q_1^2\hat{\theta}_1(1)\hat{\theta}_3(1)F + 2q_1^2\hat{\theta}_1(1)\hat{\theta}_2(1)G - 2q_1^2\hat{\theta}_1^2(1)E$$

$$- 2q_1FG)(CD-E^2) + (2q_1\hat{\theta}_3(1)F^2 - 2q_1\hat{\theta}_1(1)FE$$

$$- 2q_1\hat{\theta}_2(1)FG + 2q_1\hat{\theta}_1(1)GC)\ (2q_1\hat{\theta}_2(1)D - 2q_1\hat{\theta}_3(1)E)$$

$$- (2q_1\hat{\theta}_1(1)FD - 2q_1\hat{\theta}_1(1)GE - 2q_1\hat{\theta}_3(1)FG + 2q_1\hat{\theta}_2(1)G^2)\ \cdot$$

$$(2q_1\hat{\theta}_3(1)C - 2q_1\hat{\theta}_2(1)E) - (F^2D - 2FGE + G^2C)(2q_1^2\hat{\theta}_2(1)\hat{\theta}_3(1)$$

$$- 2q_1E)\}\ (CD-E^2)^2 - \{(2q_1\hat{\theta}_3(1)F^2 - 2q_1\hat{\theta}_1(1)FE \tag{B.28}$$

$$-2q_1\hat{\theta}_2(1)FG + 2q_1\hat{\theta}_1(1)GC)(CD-E^2) - (F^2D - 2FGE + G^2C)\ \cdot$$

$$(2q_1\hat{\theta}_3(1)C - 2q_1\hat{\theta}_2(1)E)\}\ \{2(CD-E^2)(2q_1\hat{\theta}_2(1)D - 2q_1\hat{\theta}_3(1)E\}\Big]$$

$$\frac{\partial^2 J^*(1)}{\partial\hat{\theta}_3^2(1)} = -\frac{1}{(CD-E^2)^4}\Big[\{(2q_1F^2 - 4q_1^2\hat{\theta}_1(1)\hat{\theta}_2(1)F + 2q_1^2\hat{\theta}_1^2(1)C)(CD-E^2)$$

$$- (F^2D - 2FGE + G^2C)(2q_1C - 2q_1^2\hat{\theta}_2^2(1))\}\ (CD-E^2)^2 \tag{B.29}$$

$$- \{(2q_1\hat{\theta}_3(1)F^2 - 2q_1\hat{\theta}_1(1)FE - 2q_1\hat{\theta}_2(1)FG + 2q_1\hat{\theta}_1(1)GC)(CD-E^2)$$

$$- (F^2D - 2FGE + G^2C)(2q_1\hat{\theta}_3(1)C - 2q_1\hat{\theta}_2(1)E)\}\ \{2(CD-E^2)(2q_1\hat{\theta}_3(1)C$$

$$-2q_1\hat{\theta}_2(1)E)\}\Big]$$

First and Second Order Sensitivities $\frac{\partial P(1)}{\partial u(0)}$ and $\frac{\partial^2 P(1)}{\partial u^2(0)}$

Next we shall compute the terms $\frac{\partial P(1)}{\partial u(0)}$ and $\frac{\partial^2 P(1)}{\partial u^2(0)}$. Referring to (B.5-B.11) we know that at any time the covariance is block diagonal. The parameters in (B.5) are uncorrelated with those in (B.6). The measurement matrix H(k) composed of the controls is also block diagonal. Thus we write the following.

$$P(k) \triangleq \begin{bmatrix} P^{(1)}(k) & 0 \\ 0 & P^{(2)}(k) \end{bmatrix} \tag{B.30}$$

$$\tilde{H}(k) = \begin{bmatrix} H(k) & 0 \\ 0 & H(k) \end{bmatrix} \tag{B.31}$$

Following any covariance update equation (3.8) of Appendix A we need $\tilde{H}(1)P(0)\tilde{H}'(1)$. Thus

$$\tilde{H}(1)P(0)\tilde{H}'(1) = \begin{bmatrix} H(1) & 0 \\ 0 & H(1) \end{bmatrix} \begin{bmatrix} P^{(1)}(0) & 0 \\ 0 & P^{(2)}(0) \end{bmatrix} \begin{bmatrix} H'(1) & 0 \\ 0 & H'(1) \end{bmatrix}$$

$$= \begin{bmatrix} H(1)P^{(1)}(0)H'(1) & 0 \\ 0 & H(1)P^{(2)}(0)H'(1) \end{bmatrix} \tag{B.32}$$

Let us look into $H(1)P^{(1)}(0)H'(1)$ further.

$$H(1)P^{(1)}(0)H'(1) = [\ 1 \quad u_1(0) \quad u_2(0)\ ] \begin{bmatrix} P_{1,1}(0) & P_{1,2}(0) & P_{1,3}(0) \\ P_{2,1}(0) & P_{2,2}(0) & P_{2,3}(0) \\ P_{3,1}(0) & P_{3,2}(0) & P_{3,3}(0) \end{bmatrix} \cdot \begin{bmatrix} 1 \\ u_1(0) \\ u_2(0) \end{bmatrix}$$

$$= P_{1,1}(0) + 2u_1(0)P_{1,2}(0) + 2u_2(0)P_{1,3}(0) + 2u_1(0)u_2(0)P_{2,3}(0) + u_1^2(0)P_{2,2}(0) + u_2^2(0)\ P_{3,3}(0) \tag{B.33}$$

Similarly,

$$H\ (1)P^{(2)}(0)H'(1) = P_{4,4}(0) + 2u_1(0)P_{4,5}(0) + 2u_2(0)\ P_{4,6}(0) + 2u_1(0)u_2(0)P_{5,6}(0) + u_1^2(0)P_{5,5}(0) + u_2^2(0)P_{6,6}(0) \tag{B.34}$$

Thus

$$[\tilde{H}(1)\ P(0)\ \tilde{H}'(1) + W]^{-1} = \frac{1}{A \cdot B} \begin{bmatrix} B & 0 \\ 0 & A \end{bmatrix} \tag{B.35}$$

where

$$A = H(1)P^{(1)}(0)\ H'(1) + W_1$$
$$B = H(1)P^{(2)}(0)\ H'(1) + W_2 \tag{B.36}$$

The innovation covariance in equation (3.8) of Appendix A is

$$= -\frac{1}{A\cdot B}\begin{bmatrix} P^{(1)}(0) & 0 \\ 0 & P^{(2)}(0)\end{bmatrix}\begin{bmatrix} H'(1) & 0 \\ 0 & H'(1)\end{bmatrix}\begin{bmatrix} B & 0 \\ 0 & A\end{bmatrix}\begin{bmatrix} H(1) & 0 \\ 0 & H(1)\end{bmatrix}\begin{bmatrix} P^{(1)}(0) & 0 \\ 0 & P^{(2)}(0)\end{bmatrix}$$

$$= -\frac{1}{A\cdot B}\begin{bmatrix} BP^{(1)}(0)H'(1)H(1)P^{(1)}(0) & 0 \\ 0 & AP^{(2)}(0)H'(1)H(1)P^{(2)}(0)\end{bmatrix} \tag{B.37}$$

With our standard notation $P^{(1)}(0)H'(1)H(1)P^{(1)}(0)$ may be rewritten as

$$= \begin{bmatrix} P_{1,1}(0) & P_{1,2}(0) & P_{1,3}(0) \\ P_{2,1}(0) & P_{2,2}(0) & P_{2,3}(0) \\ P_{3,1}(0) & P_{3,2}(0) & P_{3,3}(0)\end{bmatrix}\begin{bmatrix} 1 \\ u_1(0) \\ u_2(0)\end{bmatrix}[1 \;\; u_1(0) \;\; u_2(0)]\begin{bmatrix} P_{1,1}(0) & P_{1,2}(0) & P_{1,3}(0) \\ P_{2,1}(0) & P_{2,2}(0) & P_{2,3}(0) \\ P_{3,1}(0) & P_{3,2}(0) & P_{3,3}(0)\end{bmatrix}$$

$$= \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}[\alpha \;\; \beta \;\; \gamma] \tag{B.38}$$

$$= \begin{bmatrix} \alpha^2 & \alpha\beta & \alpha\gamma \\ \beta\alpha & \beta^2 & \beta\gamma \\ \gamma\alpha & \gamma\beta & \gamma^2 \end{bmatrix}$$

with

$$\begin{aligned} \alpha &= P_{1,1}(0) + P_{1,2}(0)u_1(0) + P_{1,3}(0)u_2(0) \\ \beta &= P_{1,2}(0) + P_{2,2}(0)u_1(0) + P_{2,3}(0)u_2(0) \\ \gamma &= P_{1,3}(0) + P_{2,3}(0)u_1(0) + P_{3,3}(0)u_2(0) \end{aligned} \tag{B.39}$$

Further introducing ,

$$\delta = P_{4,4}(0) + P_{4,5}(0)u_1(0) + P_{4,6}(0)u_2(0)$$

$$\psi = P_{4,5}(0) + P_{5,5}(0)u_1(0) + P_{5,6}(0)u_2(0) \qquad \text{(B.40)}$$

$$\phi = P_{4,6}(0) + P_{5,6}(0)u_1(0) + P_{6,6}(0)u_2(0)$$

we may write the innovation covariance as

$$= -\frac{1}{A\cdot B}\left[\begin{array}{ccc|ccc} B\alpha^2 & B\alpha\beta & B\alpha\gamma & & & \\ B\alpha\beta & B\beta^2 & B\beta\gamma & & 0 & \\ B\alpha\gamma & B\beta\gamma & B\gamma^2 & & & \\ \hline & & & A\delta^2 & A\delta\psi & A\delta\phi \\ & 0 & & A\delta\psi & A\psi^2 & A\psi\phi \\ & & & A\delta\phi & A\psi\phi & A\phi^2 \end{array}\right] \qquad \text{(B.41)}$$

Let us define

$$AU1 = \frac{\partial A}{\partial u_1(0)} = 2P_{1,2}(0) + 2u_2(0)P_{2,3}(0) + 2u_1(0)P_{2,2}(0)$$

$$\text{(B.42)}$$

$$AU2 = \frac{\partial A}{\partial u_2(0)} = 2P_{1,3}(0) + 2u_1(0)P_{2,3}(0) + 2u_2(0)P_{3,3}(0)$$

$$BU1 = \frac{\partial B}{\partial u_1(0)} = 2P_{4,5}(0) + 2u_2(0)P_{5,6}(0) + 2u_1(0)P_{5,5}(0)$$

$$\text{(B.43)}$$

$$BU2 = \frac{\partial B}{\partial u_2(0)} = 2P_{4,6}(0) + 2u_1(0)P_{5,6}(0) + 2u_2(0)P_{6,6}(0)$$

Thus we may next evaluate $\frac{\partial P(1)}{\partial u(0)}$ and $\frac{\partial^2 P(1)}{\partial u^2(0)}$.

These are given below.

$$\frac{\partial P_{1,1}(1)}{\partial u_1(0)} = \frac{\partial}{\partial u_1(0)} \left(- \frac{\alpha^2}{A}\right) = - \frac{2\alpha P_{1,2}(0)A - \alpha^2 \cdot AU1}{A^2} \tag{B.44}$$

$$\frac{\partial P_{1,1}(1)}{\partial u_2(0)} = \frac{\partial}{\partial u_2(0)} \left(- \frac{\alpha^2}{A}\right) = - \frac{2\alpha P_{1,3}(0)A - \alpha^2 \cdot AU2}{A^2} \tag{B.45}$$

$$\frac{\partial P_{1,2}(1)}{\partial u_1(0)} = \frac{\partial}{\partial u_1(0)} \left(- \frac{\alpha\beta}{A}\right) = - \frac{P_{1,2}(0)\beta A + \alpha P_{2,2}(0)A - \alpha\beta AU1}{A^2} \tag{B.46}$$

$$\frac{\partial P_{1,2}(1)}{\partial u_2(0)} = \frac{\partial}{\partial u_2(0)} \left(- \frac{\alpha\beta}{A}\right) = - \frac{P_{1,3}(0)\beta A + \alpha P_{2,3}(0)A - \alpha\beta \cdot AU2}{A^2} \tag{B.47}$$

$$\frac{\partial P_{1,3}(1)}{\partial u_1(0)} = \frac{\partial}{\partial u_1(0)} \left(- \frac{\alpha\gamma}{A}\right) = - \frac{P_{1,2}(0)\gamma A + \alpha P_{2,3}(0)A - \alpha\gamma \cdot AU1}{A^2} \tag{B.48}$$

$$\frac{\partial P_{1,3}(1)}{\partial u_2(0)} = \frac{\partial}{\partial u_2(0)} \left(- \frac{\alpha\gamma}{A}\right) = - \frac{P_{1,3}(0)\gamma A + \alpha P_{3,3}(0)A - \alpha\gamma \cdot AU2}{A^2} \tag{B.49}$$

$$\frac{\partial P_{2,2}(1)}{\partial u_1(0)} = \frac{\partial}{\partial u_1(0)} \left(- \frac{\beta^2}{A}\right) = - \frac{2\beta P_{2,2}(0)A - \beta^2 \cdot AU1}{A^2} \tag{B.50}$$

$$\frac{\partial P_{2,2}(1)}{\partial u_2(0)} = \frac{\partial}{\partial u_2(0)} \left(- \frac{\beta^2}{A}\right) = - \frac{2\beta P_{2,3}(0)A - \beta^2 \cdot AU2}{A^2} \tag{B.51}$$

$$\frac{\partial P_{2,3}(1)}{\partial u_1(0)} = \frac{\partial}{\partial u_1(0)} \left(-\frac{\beta\gamma}{A}\right) = -\frac{P_{2,2}(0)\gamma A + \beta P_{2,3}(0)A - \beta\gamma\cdot AU1}{A^2} \tag{B.52}$$

$$\frac{\partial P_{2,3}(1)}{\partial u_2(0)} = \frac{\partial}{\partial u_2(0)} \left(-\frac{\beta\gamma}{A}\right) = -\frac{P_{2,3}(0)\gamma A + \beta P_{3,3}(0)A - \beta\gamma\cdot AU2}{A^2} \tag{B.53}$$

$$\frac{\partial P_{3,3}(1)}{\partial u_1(0)} = \frac{\partial}{\partial u_1(0)} \left(-\frac{\gamma^2}{A}\right) = -\frac{2\gamma P_{2,3}(0)A - \gamma^2\cdot AU1}{A^2} \tag{B.54}$$

$$\frac{\partial P_{3,3}(1)}{\partial u_2(0)} = \frac{\partial}{\partial u_2(0)} \left(-\frac{\gamma^2}{A}\right) = -\frac{2\gamma P_{3,3}(0)A - \gamma^2\cdot AU2}{A^2} \tag{B.55}$$

$$\frac{\partial^2 P_{1,1}(1)}{\partial u_1^2(0)} = -\frac{1}{A^4}\,[2(P_{1,2}^2(0)A - \alpha^2 P_{2,2}(0))\;A^2 - (2\alpha\; P_{1,2}(0)A - \alpha^2 \cdot AU1)(2A\cdot AU1)] \tag{B.56}$$

$$\frac{\partial^2 P_{1,1}(1)}{\partial u_1(0)\,\partial u_2(0)} = -\frac{1}{A^4}\,[2(P_{1,2}(0)P_{1,3}(0)A + \alpha P_{1,3}(0)AU1 - \alpha AU2P_{1,2}(0)$$

$$- \alpha^2 P_{2,3}(0))A^2 - (2\alpha P_{1,3}(0)A - \alpha^2 AU2)(2A\cdot AU1)] \tag{B.57}$$

$$\frac{\partial^2 P_{1,1}(1)}{\partial u_2^2(0)} = -\frac{1}{A^4}\;[2(P_{1,3}^2(0)A - \alpha^2 P_{3,3}(0))A^2 - (2\alpha P_{1,3}(0)A - \alpha^2 \cdot AU2)(2A\cdot AU2)] \tag{B.58}$$

$$\frac{\partial^2 P_{1,2}(1)}{\partial u_1^2(0)} = -\frac{1}{A^4}\,[2(P_{1,2}(0)P_{2,2}(0)A - \alpha\beta P_{2,2}(0))A^2 - (P_{1,2}(0)\beta A + \alpha P_{2,2}(0)A$$

$$- \alpha\beta AU1)\;(2A\cdot AU1)] \tag{B.59}$$

$$\frac{\partial^2 P_{1,2}(1)}{\partial u_1(0)\,\partial u_2(0)} = -\frac{1}{A^4}\,[(P_{1,3}(0)P_{2,2}(0)A + P_{1,3}(0)\,\beta AU1 + P_{1,2}(0)P_{2,3}(0)A + \alpha P_{2,3}(0)AU1$$

$$- P_{1,2}(0)\,\beta AU2 - \alpha P_{2,2}(0)AU2 - 2\alpha\beta P_{2,3}(0))A^2 \tag{B.60}$$

$$- (P_{1,3}(0)\,\beta A + \alpha P_{2,3}(0)A - \alpha\beta AU2)(2A\cdot AU1)]$$

$$\frac{\partial^2 P_{1,2}(1)}{\partial u_2^2(0)} = -\frac{1}{A^4}\,[2(P_{1,3}(0)P_{2,3}(0)A - \alpha\beta P_{3,3}(0))A^2 - (P_{1,3}(0)\,\beta A + \alpha P_{2,3}(0)A$$

$$- \alpha\beta AU2)\;(2A\cdot AU2)] \tag{B.61}$$

$$\frac{\partial^2 P_{1,3}(1)}{\partial u_1^2(0)} = -\frac{1}{A^4}\,[2(P_{1,2}(0)P_{2,3}(0)A - \alpha\gamma P_{2,2}(0))A^2 -$$

$$- (P_{1,2}(0)\gamma A + \alpha P_{2,3}(0)A - \alpha\gamma AU1)\;(2A\cdot AU1)] \tag{B.62}$$

$$\frac{\partial^2 P_{1,3}(1)}{\partial u_1(0)\,\partial u_2(0)} = -\frac{1}{A^4}\,[(P_{1,3}(0)P_{2,3}(0)A + P_{1,3}(0)\gamma AU1 + P_{1,2}(0)P_{3,3}(0)A$$

$$+ \alpha P_{3,3}(0)AU1 - P_{1,2}(0)\gamma AU2 - \alpha P_{2,3}(0)AU2 \qquad (B.63)$$

$$- 2\alpha\gamma P_{2,3}(0))A^2 - (P_{1,3}(0)\gamma A + \alpha P_{3,3}(0)A - \alpha\gamma AU2)\cdot(2A\cdot AU1)]$$

$$\frac{\partial^2 P_{1,3}(1)}{\partial u_2^2(0)} = -\frac{1}{A^4}\,[2(P_{1,3}(0)P_{3,3}(0)A - \alpha\gamma P_{3,3}(0))A^2 - (P_{1,3}(0)\gamma A + \alpha P_{3,3}(0)A$$

$$- \alpha\gamma AU2)\ (2A\cdot AU2)] \qquad (B.64)$$

$$\frac{\partial^2 P_{2,2}(1)}{\partial u_1^2(0)} = -\frac{1}{A^4}\,[2(P_{2,2}^2(0)A - \beta^2\cdot P_{2,2}(0))A^2 - (2\beta P_{2,2}(0)A - \beta^2 AU1)(2A\cdot AU1)] \qquad (B.65)$$

$$\frac{\partial^2 P_{2,2}(1)}{\partial u_1(0)\,\partial u_2(0)} = -\frac{1}{A^4}\,[2(P_{2,3}(0)P_{2,2}(0)A + \beta P_{2,3}(0)AU1 - \beta AU2P_{2,2}(0)$$

$$- \beta^2 P_{2,3}(0))A^2 - (2\beta P_{2,3}(0)A - \beta^2 AU2)\ (2A\cdot AU1)] \qquad (B.66)$$

$$\frac{\partial^2 P_{2,2}(1)}{\partial u_2^2(0)} = -\frac{1}{A^4}\,[2(P_{2,3}^2(0)A - \beta^2 P_{3,3}(0))A^2 - (2\beta P_{2,3}(0)A - \beta^2 AU2)(2A\cdot AU2)] \qquad (B.67)$$

$$\frac{\partial^2 P_{2,3}(1)}{\partial u_1^2(0)} = -\frac{1}{A^4}\,[2(P_{2,2}(0)P_{2,3}(0)A - \beta\gamma P_{2,2}(0))A^2 - (P_{2,2}(0)\gamma A + \beta P_{2,3}(0)A$$

$$- \beta\gamma AU1)\ (2A\cdot AU1)) \qquad (B.68)$$

$$\frac{\partial^2 P_{2,3}(1)}{\partial u_1(0)\,\partial u_2(0)} = -\frac{1}{A^4}\,[P_{2,3}^2(0)A + P_{2,3}(0)\gamma AU1 + P_{2,2}(0)P_{3,3}(0)A + \beta P_{3,3}(0)AU1$$

$$- P_{2,2}(0)\gamma AU2 - \beta P_{2,3}(0)AU2 - \beta\gamma\ 2P_{2,3}(0))A^2 \qquad (B.69)$$

$$- (P_{2,3}(0)\gamma A + \beta P_{3,3}(0)A - \beta\gamma AU2)(2A\cdot AU1)]$$

$$\frac{\partial^2 P_{2,3}(1)}{\partial u_2^2(0)} = -\frac{1}{A^4}[2(P_{2,3}(0)P_{3,3}(0)A - \beta\gamma P_{3,3}(0))A^2 - (P_{2,3}(0)\gamma A + \beta P_{3,3}(0)A - \beta\gamma AU2)\ (2A\cdot AU2)] \tag{B.70}$$

$$\frac{\partial^2 P_{3,3}(1)}{\partial u_1^2(0)} = -\frac{1}{A^4}[2(P_{2,3}^2(0)A - \gamma^2 P_{2,2}(0))A^2 - (2\gamma P_{2,3}(0)A - \gamma^2 AU1)(2A\cdot AU1)] \tag{B.71}$$

$$\frac{\partial^2 P_{3,3}(1)}{\partial u_1(0)\,\partial u_2(0)} = -\frac{1}{A^4}[2(P_{2,3}(0)P_{3,3}(0)A + \gamma P_{3,3}(0)AU1 - \gamma P_{2,3}(0)AU2 - \gamma^2 P_{2,3}(0))A^2 - (2\gamma P_{3,3}(0)A - \gamma^2 AU2)(2A\cdot AU1)] \tag{B.72}$$

$$\frac{\partial^2 P_{3,3}(1)}{\partial u_2^2(0)} = -\frac{1}{A^4}[2(P_{3,3}^2(0)A - \gamma^2 P_{3,3}(0))A^2 - (2\gamma P_{3,3}(0)A - \gamma^2 AU2)\cdot(2A\cdot AU2)] \tag{B.73}$$

The partials of the covariances associated with the parameters of (B.6) are similar to the above with α, β, γ, A, AU1, AU2 replaced respectively by δ, ψ, ϕ, B, BU1, BU2 and the appropriate covariances of the parameters of (B.6).

REFERENCES

1. Molusis, J. A.; Hammond, C. E.; and Cline, J. H.: A Unified Approach to the Optimal Design of Adaptive and Gain Scheduled Controllers to Achieve Minimum Helicopter Rotor Vibration. 37th Annual Forum of the AHS, May 1981.

2. Taylor, R. B.; Zwicke, P. E.; Gold, P.; and Miao, W.: Analytical Design and Evaluation of an Active Control System for Helicopter Vibration Reduction and Gust Response Alleviation. NASA CR 152377, July 1980.

3. Johnson, W.: Self-Tuning Regulators for Multicyclic Control of Helicopter Vibration. NASA Technical Paper 1996, March 1982.

4. Chopra, I.; and McCloud, J.L.: Considerations of Open-Loop, Closed-Loop, and Adaptive Multicyclic Control Systems, Preprint No. 13. Presented at the American Helicopter Society Northeast Region Specialists' Meeting on Helicopter Vibration, Hartford, Connecticut, Nov. 1981.

5. Weisbrich, R.; Perley, R.; and Howes H.: Design Study of a Feedback Control System for the Multicyclic Flap System Rotor (MFS). NASA CR 151060, Jan. 1977.

6. Davis, M. W.: Refinement and Evaluation of Helicopter Real-Time Self-Adaptive Active Vibration Control Algorithms. Contract NAS2-11260, Nov. 1983.

7. Hammond, C. E.: Wind Tunnel Results Showing Rotor Vibratory Loads Reduction Using Higher Harmonic Blade Pitch. 36th Annual Forum of the AHS, May 1980.

8. Shaw, J.; and Albion, N.: Active Control of the Helicopter Rotor for Vibration Reduction. Paper No. 80-68, 36th Annual Forum of the American Helicopter Society, Washington, D. C., May 1980.

9. Wood, E. R.; Powers, R. W.; Cline, J. H.; and Hammond, C. E.: On Developing and Flight Testing a Higher Harmonic Control System. Presented at the 39th Annual Forum of the American Helicopter Society, May 1983.

10. Higher Harmonic Control Government/Industry Briefing on Second Flight Test Results of the OH-6A Helicopter, Mesa, Arizona, May 10, 1984.

11. Feldbaum, A. A.: Optimal Control Systems. Academic Press, New York, 1965.

12. Bar-Shalom, Y.; and Tse, E.: Dual Effect, Certainty Equivalence and Separation in Stochastic Control. IEEE Trans. Auto. Control, vol AC-19, Oct. 1974, 494-500.

13. Mookerjee, P.: The Control of Helicopter Vibration. M.S. Thesis, Department of Electrical Engineering and Computer Science, The University of Connecticut, Storrs, March 1983.

14. Bar-Shalom, Y.; Mookerjee, P.; and Molusis, J. A.: A Linear Feedback Dual Controller for a Class of Stochastic Systems. Outils et Modeles Mathematiques Pour L'Automatique, L'Analyse De Systemes et Le Traitement Du Signal, Volume 3, I. D. Landau (Coordinator), Centre National de la Recherche Scientifique, Paris, 1983.

15. DeKeyser, R.; and Van Cauwenberghe, A.: Self-Adjusting Control of Multivariate Nonstationary Processes. MECO symposium, Athens, Greece 1978.

16. Sternby, J.: A Regulator for Time-Varying Stochastic Systems. Proc. 7th IFAC World Congress, Helsinki, Finland, June 1978, 2025-2032.

17. Molusis, J. A.: The Importance of Nonlinearity on the Higher Harmonic Control of Helicopter Vibration, presented at the 39th Annual Forum of the American Helicopter Society, St. Louis, Missouri, May 1983.

18. Molusis, J.A.; Mookerjee, P.; and Bar-Shalom, Y.: Dual Adaptive Control Based Upon Sensitivity Functions, submitted for presentation at the 23rd CDC, Las Vegas, 1984.

19. Bellman, R.: Dynamic Programming. Princeton University Press, Princeton, New Jersey, 1957.

20. Bertsekas, D. P.: Dynamic Programming and Stochastic Control. Academic Press, New York, 1976.

21. Bar-Shalom, Y.; and Tse, E.: Concepts and Methods in Stochastic Control. Control and Dynamic Systems, Advances in Theory and Applications, C. T. Leondes, (ed.), Academic Press, New York, 1976.

22. Molusis, J. A.; Mookerjee, P.; and Bar-Shalom, Y.: Evaluation of the Effect of Vibration Nonlinearity on Convergence Behavior of Adaptive Higher Harmonic Controllers. NASA CR-166424, January 1983.

23. Molusis, J. A.: Investigation of Unexplained Behavior and Nonlinearity Observed in Wind Tunnel Tests of Higher Harmonic Control. USAAVRADCOM-TR-83-D-8, August 1983.

24. Himmelblau, D. M.: Applied Nonlinear Programming, McGraw Hill,Inc., 1972.

25. Vygodsky, M.: Mathematical Handbook-Higher Mathematics, translated from Russian by G. Yankovsky, English translation, Mir Publishing, Moscow, 1975.

26. Eykhoff, P.: System Identification - Parameter and State Estimation, John Wiley and Sons, 1974.

27. Bar-Shalom, Y., "Stochastic Dynamic Programming, Caution and Probing", IEEE Trans. on Auto. Control, .C-26, 1184-1195, 1981.

28. Wittenmark, B., "An Active Suboptimal Dual Controller for Systems with Stochastic Parameters", Automatic Control Theory and Applications (Canada), Vol. 3, pp. 13-19, 1975.

29. Goodwin, G., and Payne, R., Dynamic Systems Identification: Experiment Design and Data Analysis, Mathematics in Science and Eng., Vol. 136, Academic Press, bY, 1977.

30. Milito, R., Padilla, C.S., Padilla, R.A. and Cadorin, D., "Dual Control Through Innovations", Proceedings of the 19th IEEE Conference on Decision and Control, Albuquerque, N. Mex., Dec. 1980.

31. Mosca, E., Rocchi, S., and Rappa, G., "A New Dual Control Algorithm", Proceedings of the 1978 IEEE Conference on Decision and Control, San Diego, Jan. 1979.

32. Sternby, J., "A Regulator for Time Varying Stochastic Systems", Proceedings of the 7th International Federation of Automatic Control World Congress, Helinski, Finland, June 1978.

33. Tse, E., Bar-Shalom, Y. and Meier, L., "Wide-Sense Adaptive Dual Control for Nonlinear Stochastic Systems", IEEE Trans. Automatic Control AC-18, 98-108, April 1973.

34. Padilla, C.S. and Cruz, J.B., "Sensitivity Adaptive Feedback with Estimation Redistribution", IEEE Trans. Automatic Control, vol. AC-23, No. 3, June 1978.

LONGITUDINAL HUB SHEAR STATE X1

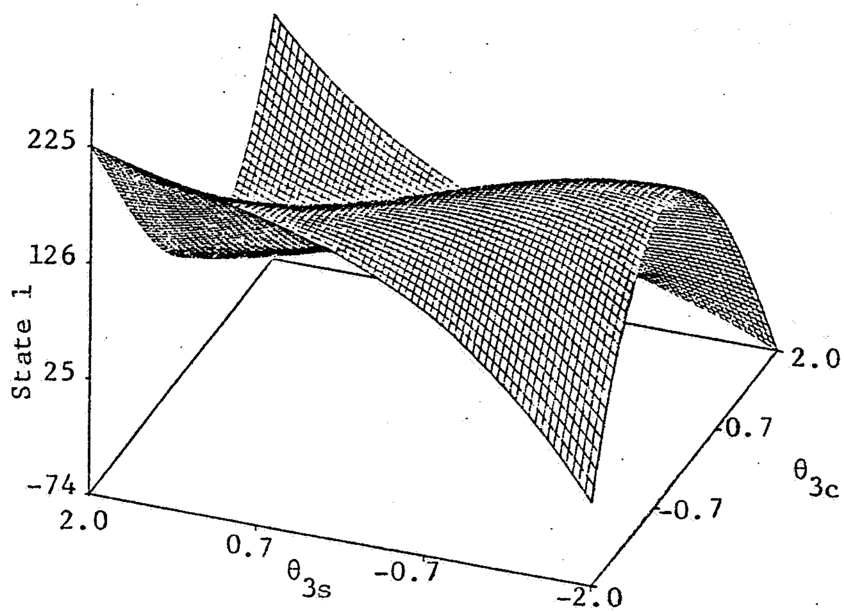

Fig. 1 Longitudinal hub cosine state x_1, vs the control inputs θ_{3c} and θ_{3s} from the identified third-order nonlinear model obtained from G400 simulation data (120 knots)

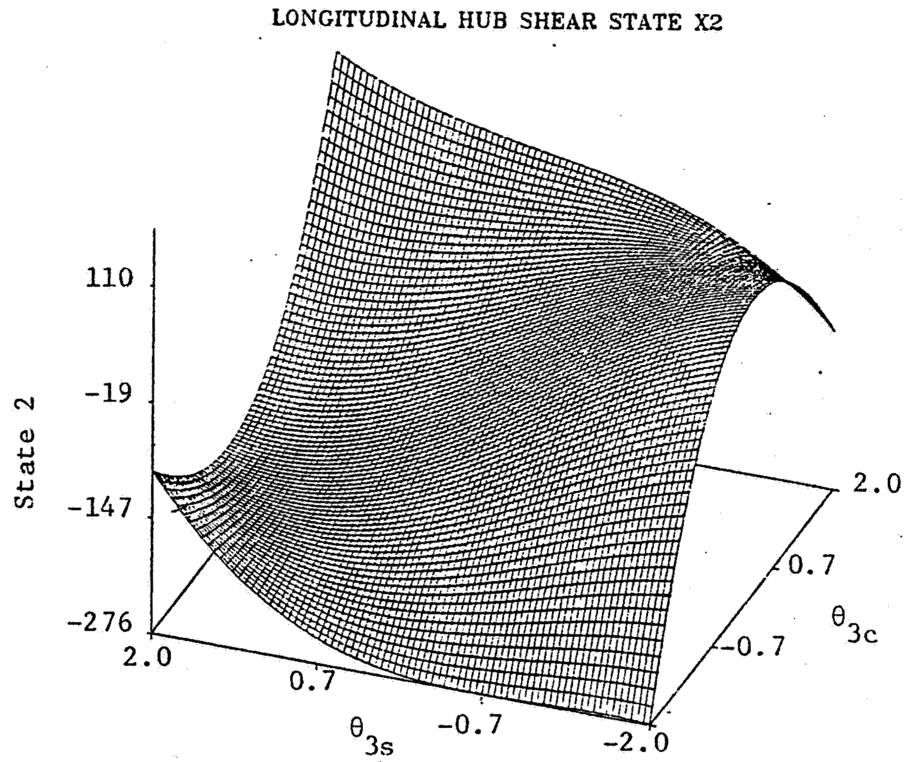

Fig. 2 - Longitudinal hub sine state x_2 vs the control inputs θ_{3c} and θ_{3s} from the identified third-order nonlinear model obtained from G400 simulation data (120 knots)

LONGITUDINAL COST

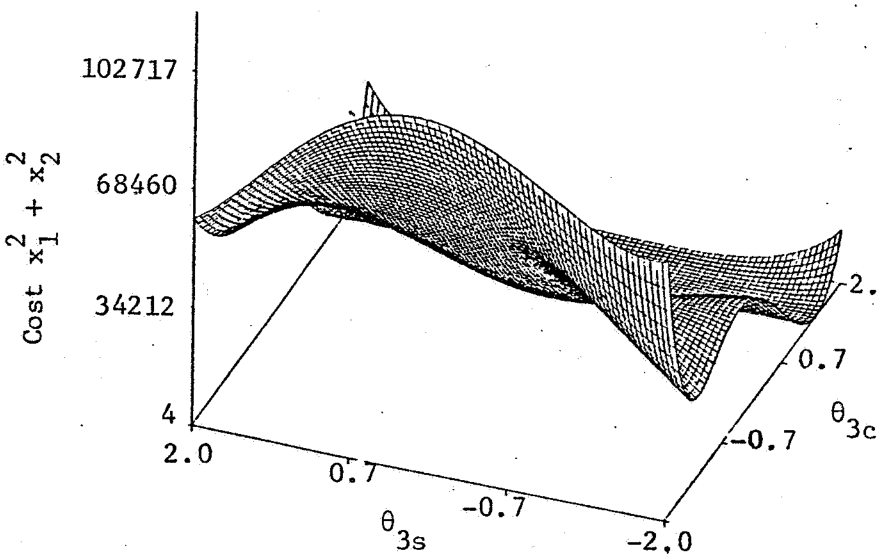

Fig. 3 Total longitudinal hub cost $x_1^2 + x_2^2$ vs the control inputs θ_{3c} and θ_{3s} from the identified third-order nonlinear model obtained from G400 simulation data (120 knots)

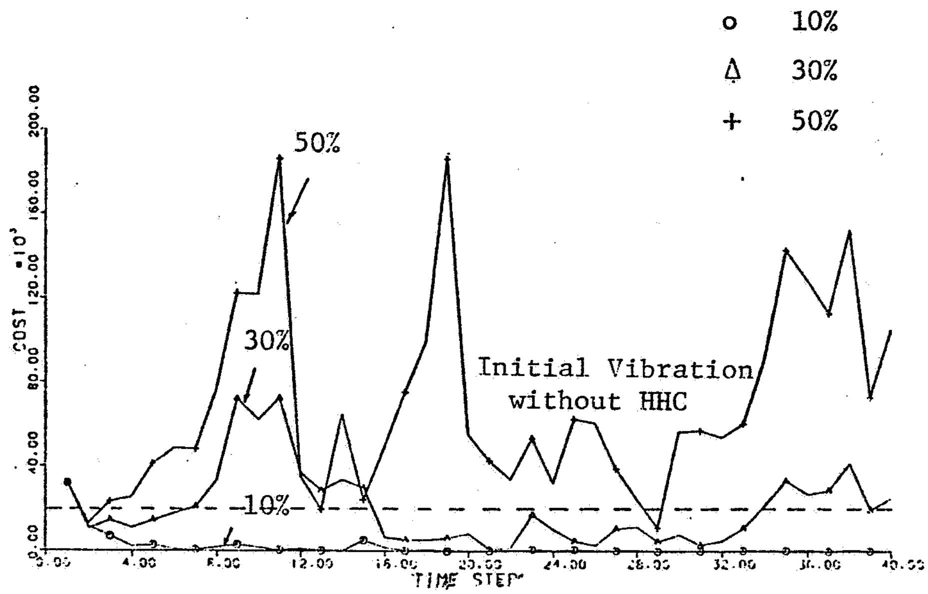

Fig. 4 Cautious control performance on the linear plant for different noise variance levels. (first method initialization, 10%, 30% 50% noise levels)

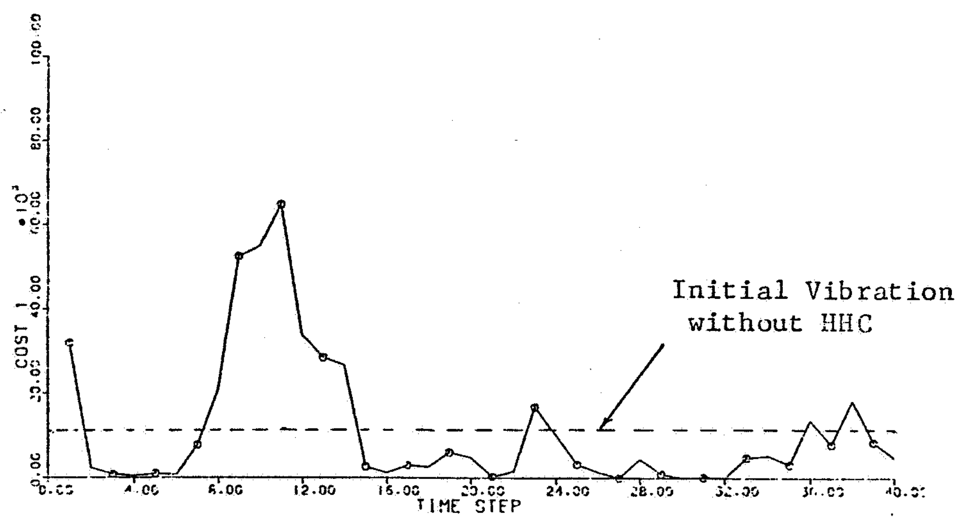

Fig. 5 Vibration contribution from the cosine component, using the cautious controller (30% noise)

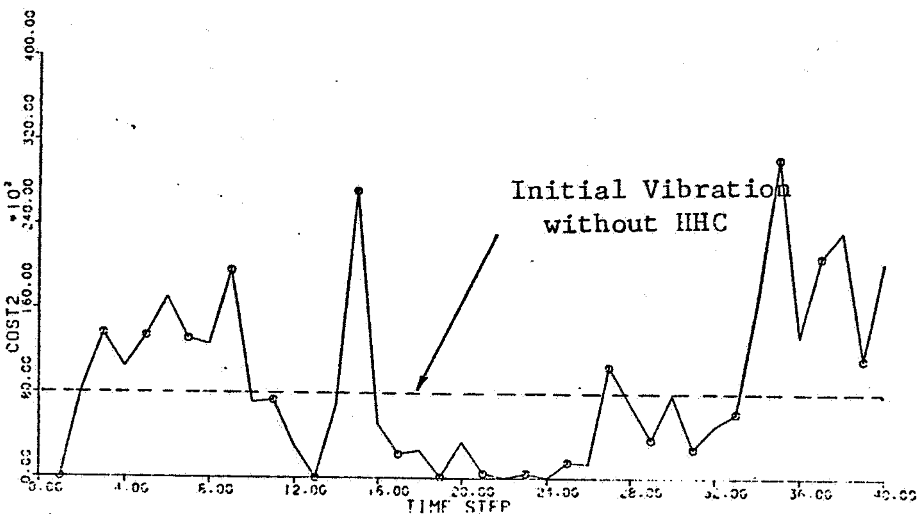

Fig. 6 Vibration contribution from the sine component, using the cautious controller

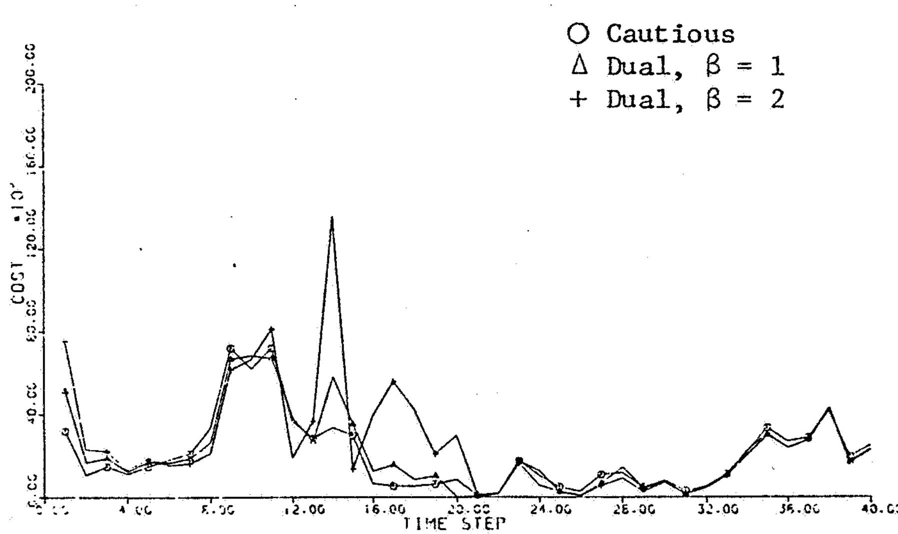

Fig. 7 Comparison of the cautious and dual controller's performances on the 30% varying parameter plant

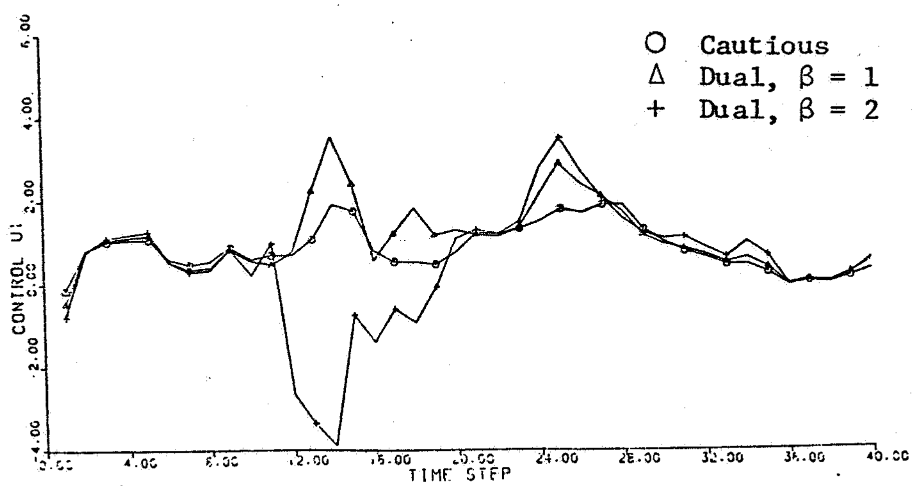

Fig. 8 Comparison of the cautious and dual controllers' performances on the 30% varying parameter plant

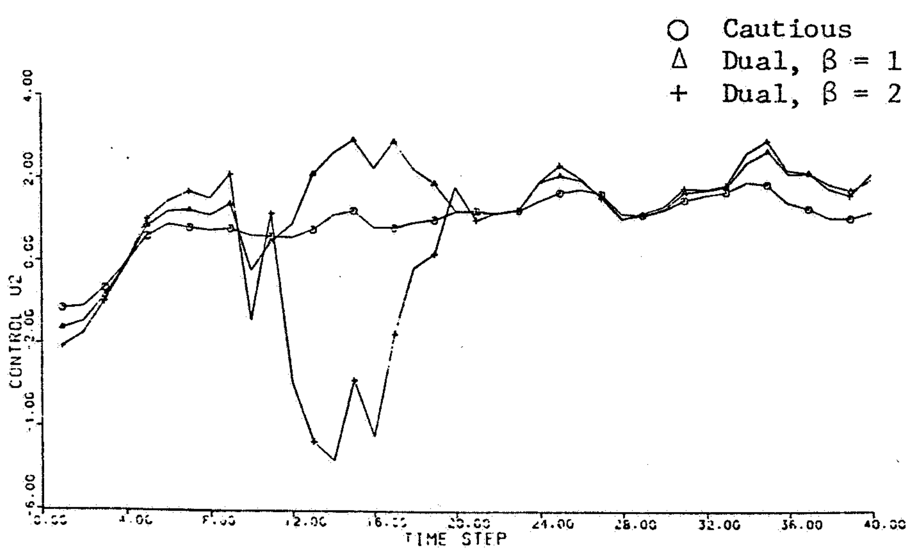

Fig. 9 Comparison of the cautious and dual controllers' performances on the 30% varying parameter plant

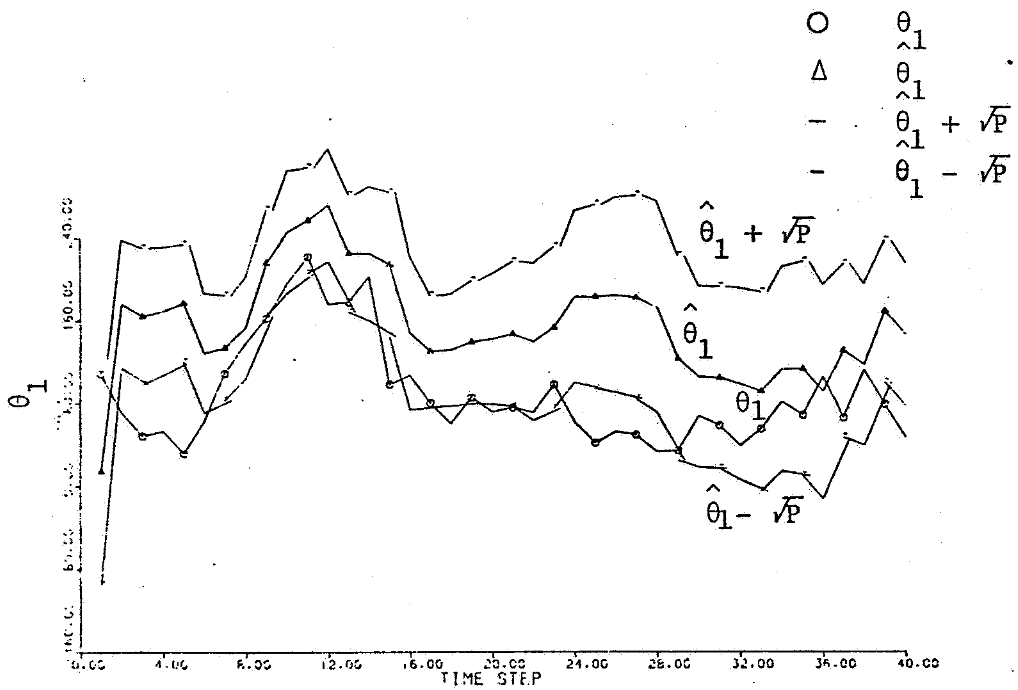

Fig. 10 Time history of θ_1, $\hat{\theta}_1$, $\hat{\theta}_1 \pm \sqrt{P}$, using the cautious controller

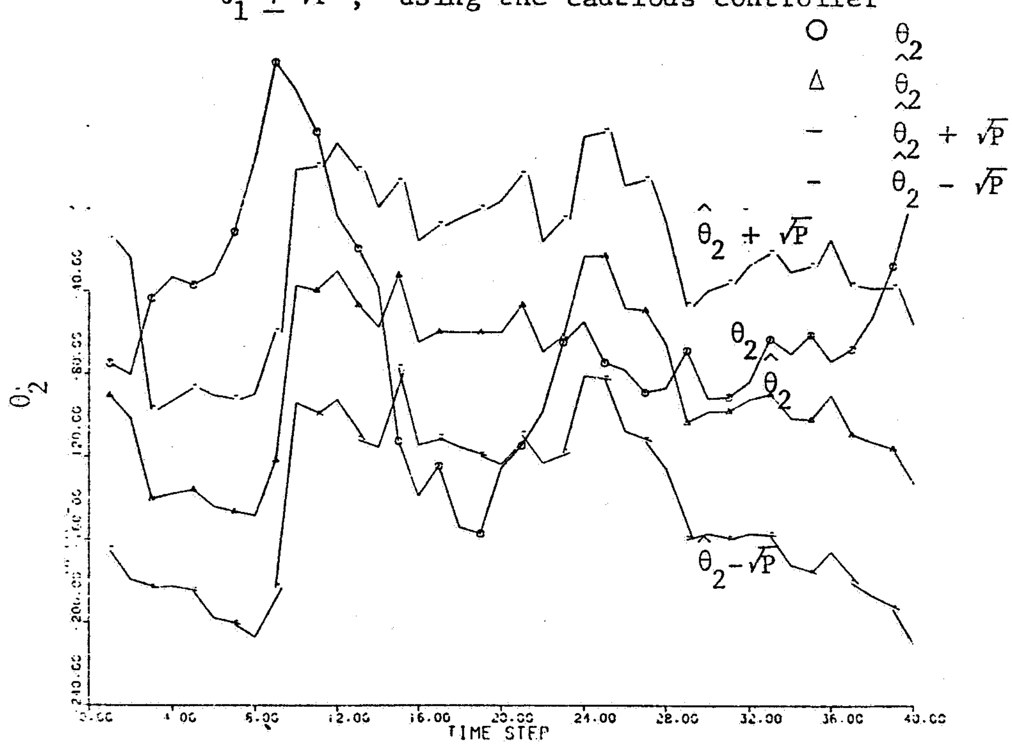

Fig. 11 Time history of θ_2, $\hat{\theta}_2$ $\hat{\theta}_2 \pm \sqrt{P}$, using the cautious controller

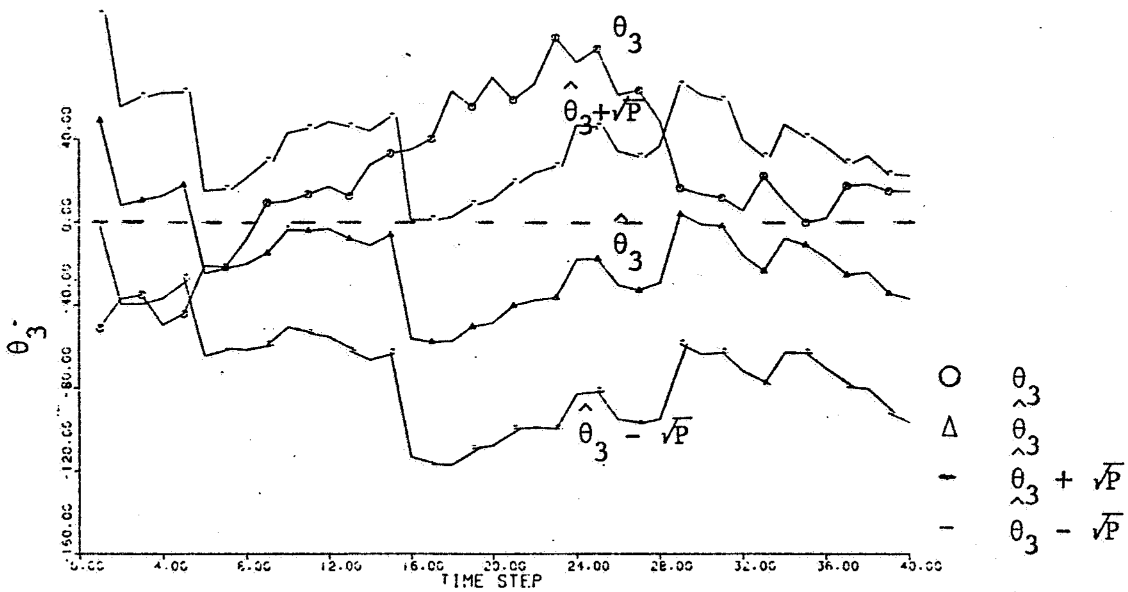

Fig. 12 Time history of θ_3, $\hat{\theta}_3$, $\hat{\theta}_3 \pm \sqrt{P}$, using the cautious controller

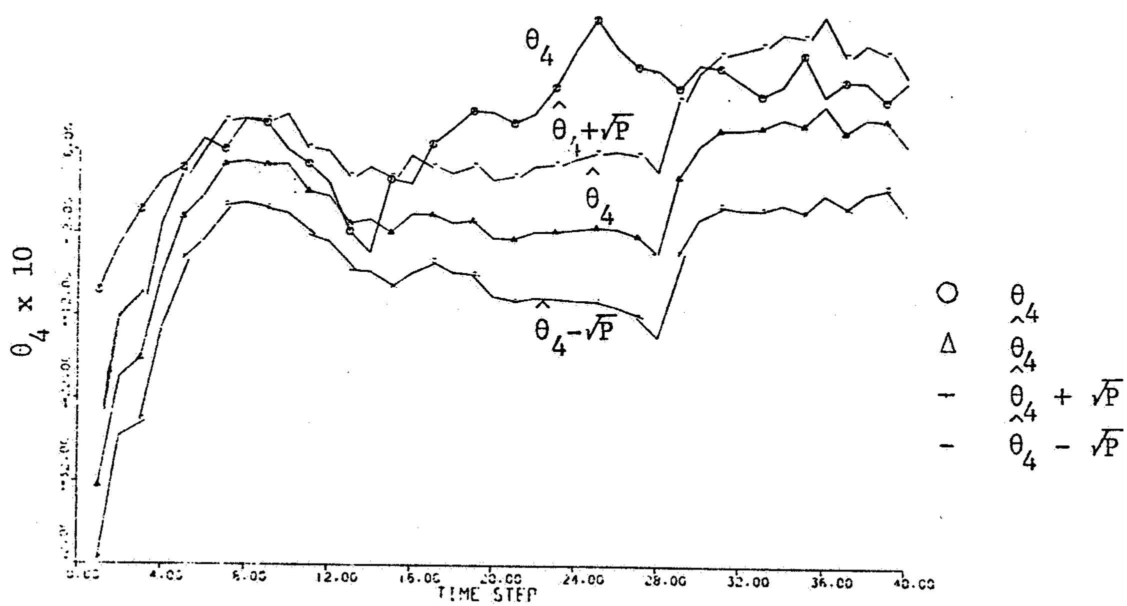

Fig. 13 Time history of θ_4, $\hat{\theta}_4$, $\hat{\theta}_4 \pm \sqrt{P}$, using the cautious controller

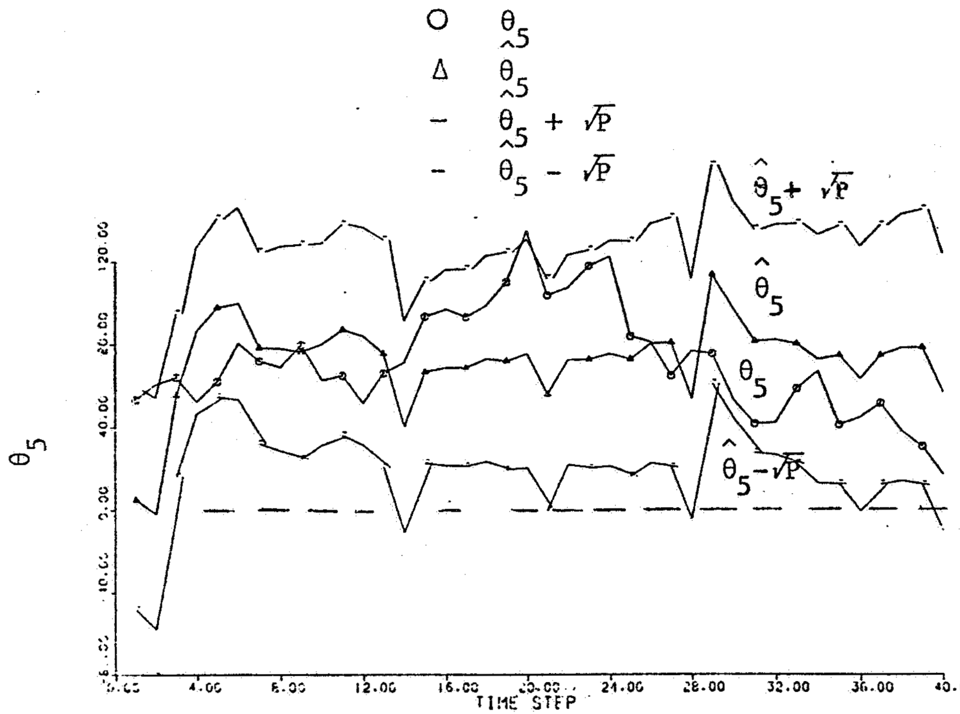

Fig. 14 Time history of θ_5, $\hat{\theta}_5$, $\hat{\theta}_5 \pm \sqrt{P}$, using the cautious controller

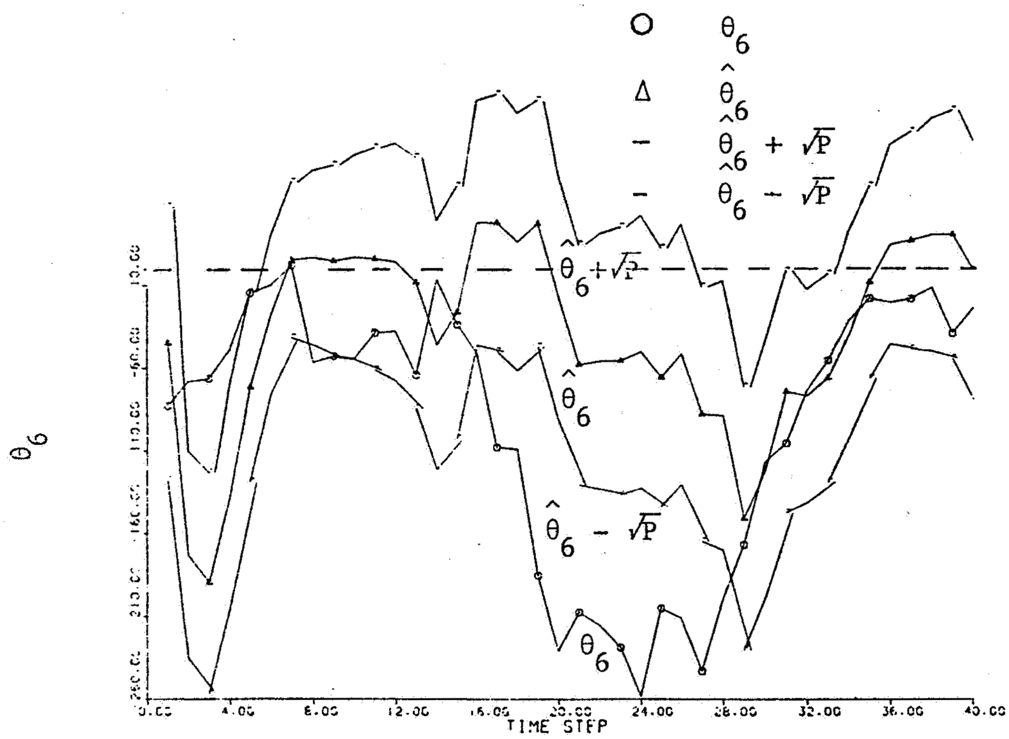

Fig. 15 Time history of θ_6, $\hat{\theta}_6$, $\hat{\theta}_6 \pm \sqrt{P}$, using the cautious controller

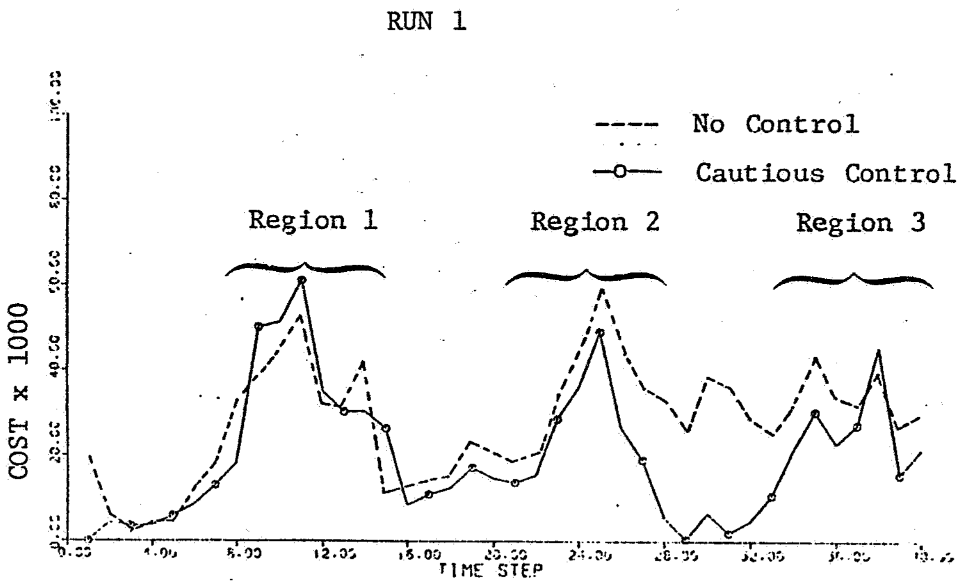

Fig. 16 Comparison of the performance of the cautious controller with that of no control (second method of initialization) (Run 1)

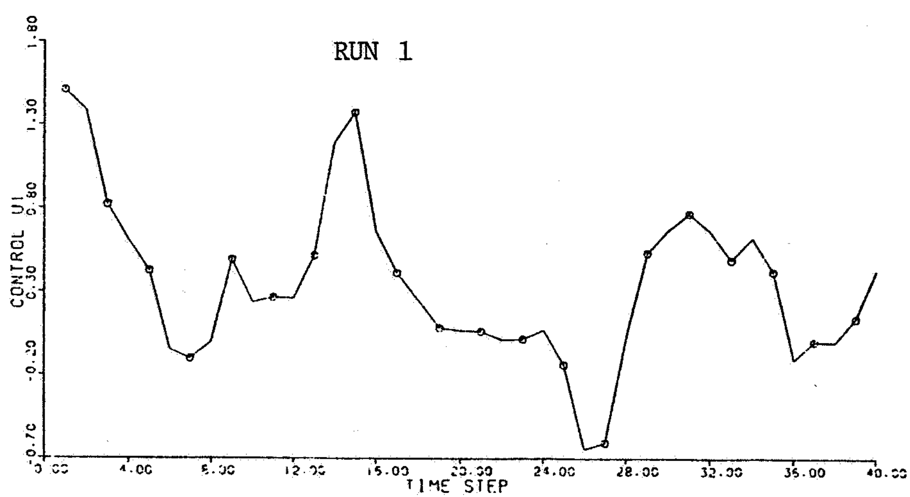

Fig. 17 Control u_1 with the cautious controller (second method of initialization) (Run 1)

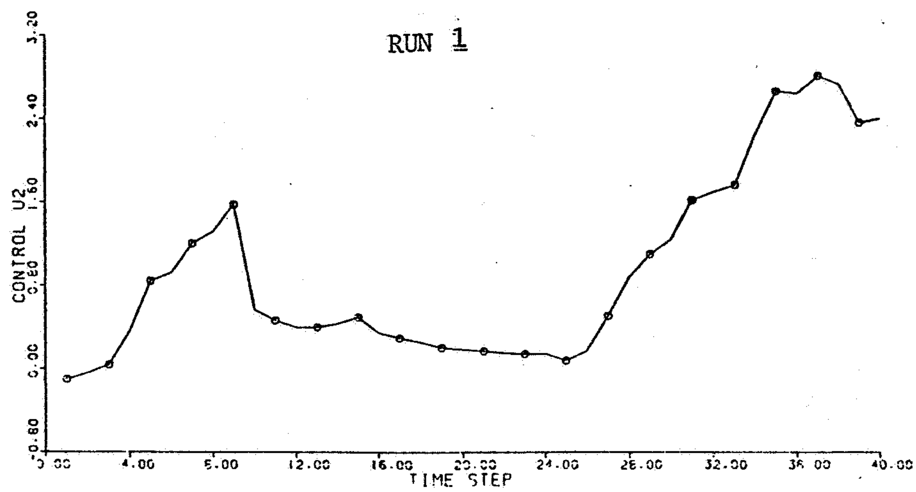

Fig. 18 Control u_2 with the cautious controller (second method of initialization) (Run 1)

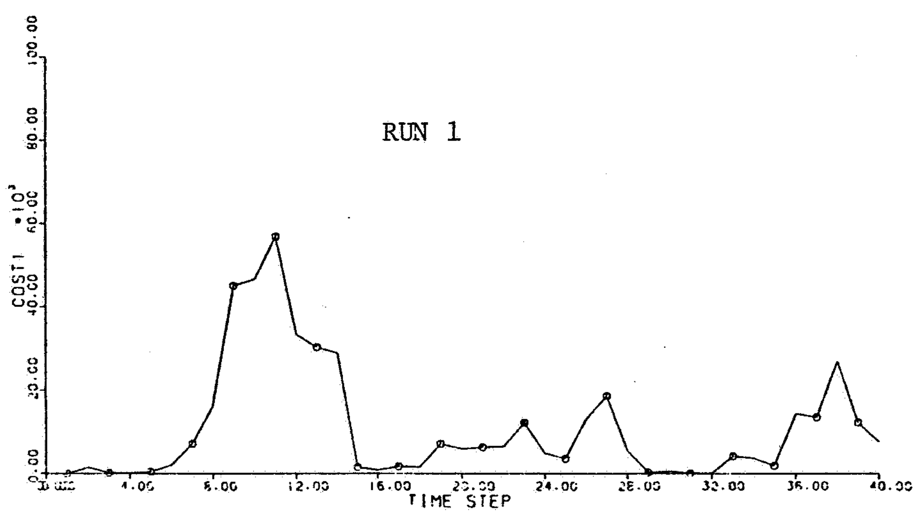

Fig. 19 Vibration contribution from the cosine component, using the cautious controller

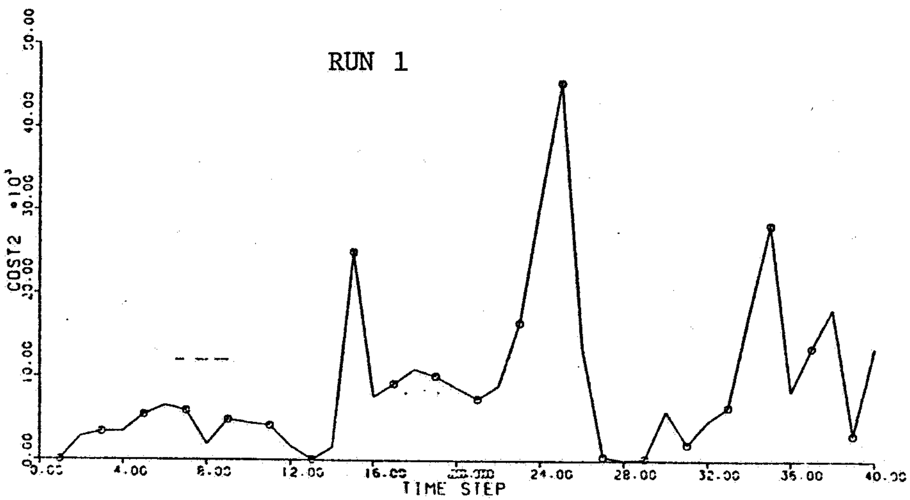

Fig. 20 Vibration contribution from the sine component using the cautious controller

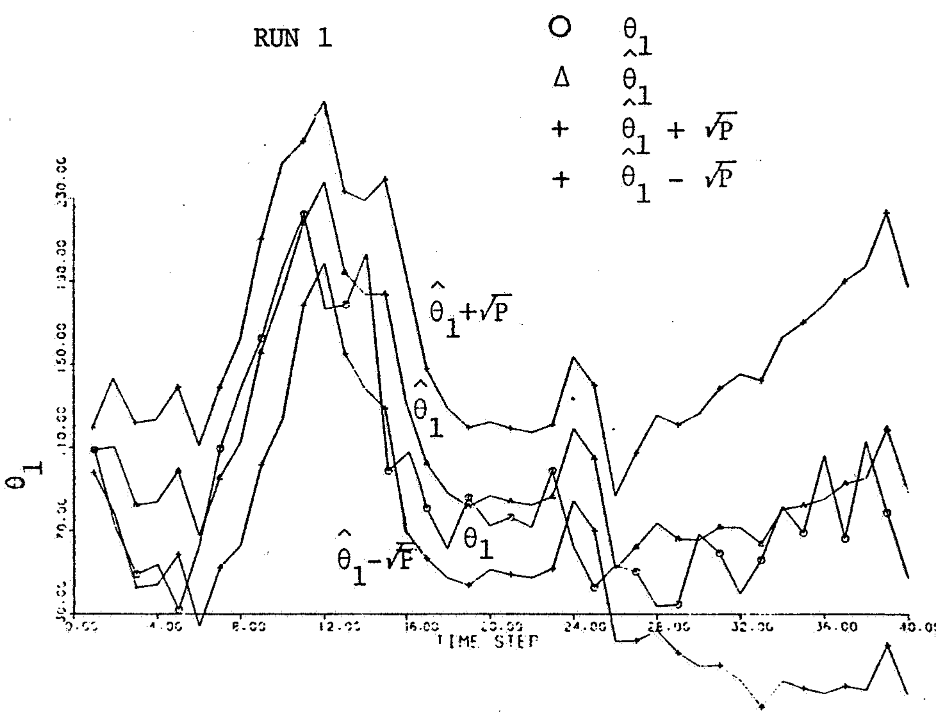

Fig. 21 Time history of θ_1, $\hat{\theta}_1$, $\hat{\theta}_1 \pm \sqrt{P}$, using the cautious controller

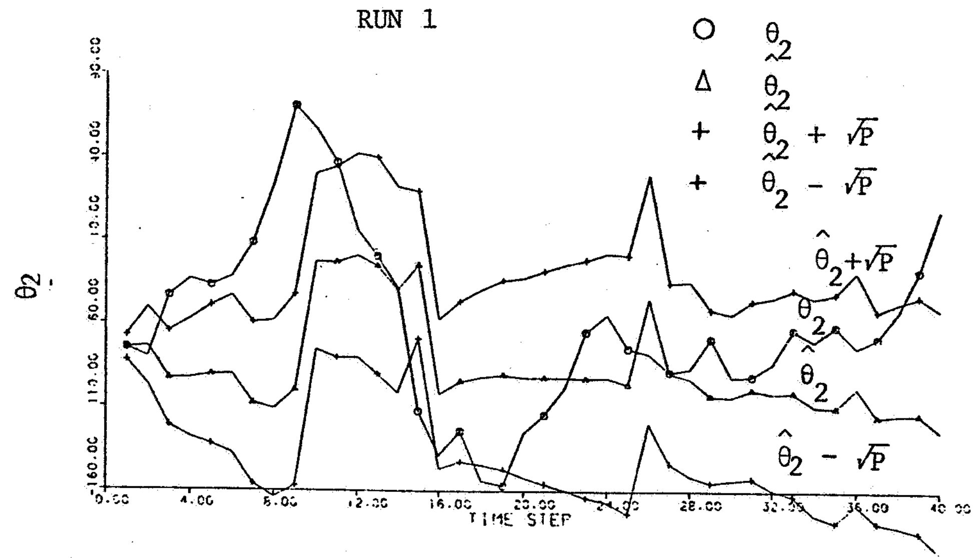

Fig. 22 Time history of θ_2, $\hat{\theta}_2$, $\hat{\theta}_2 \pm \sqrt{P}$, using the cautious controller

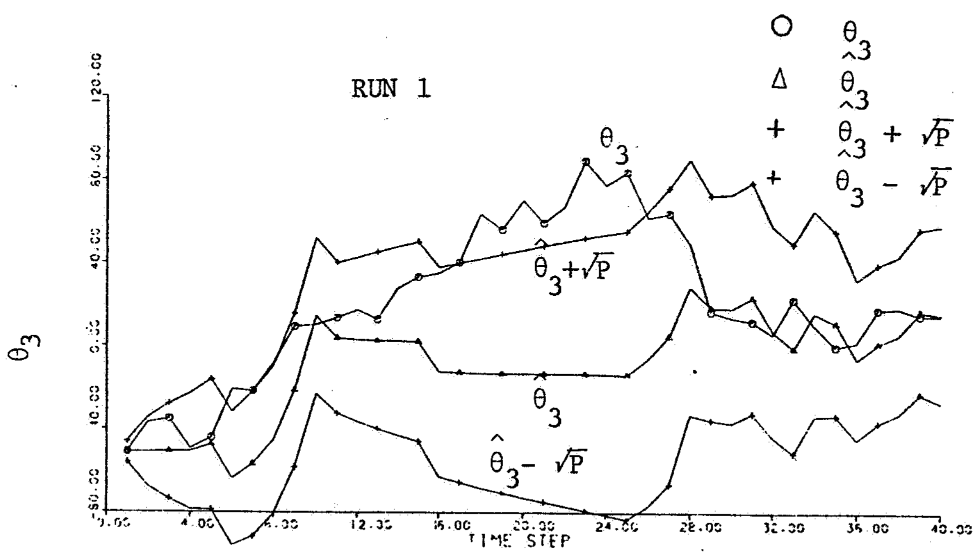

Fig. 23 Time history of θ_3, $\hat{\theta}_3$, $\hat{\theta}_3 \pm \sqrt{P}$, using the cautious controller

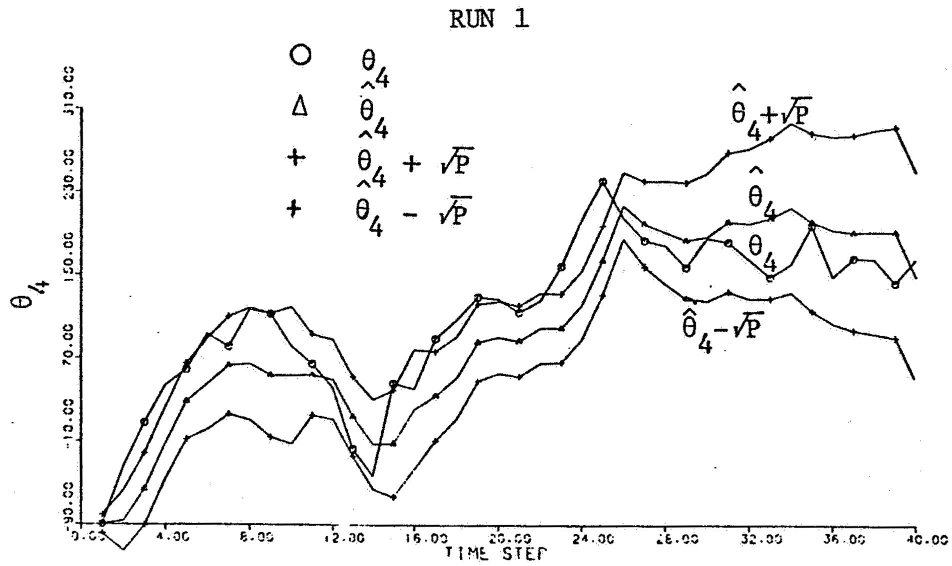

Fig. 24 Time history of θ_4, $\hat{\theta}4$,
$\hat{\theta}_4 \pm \sqrt{P}$, using the cautious controller

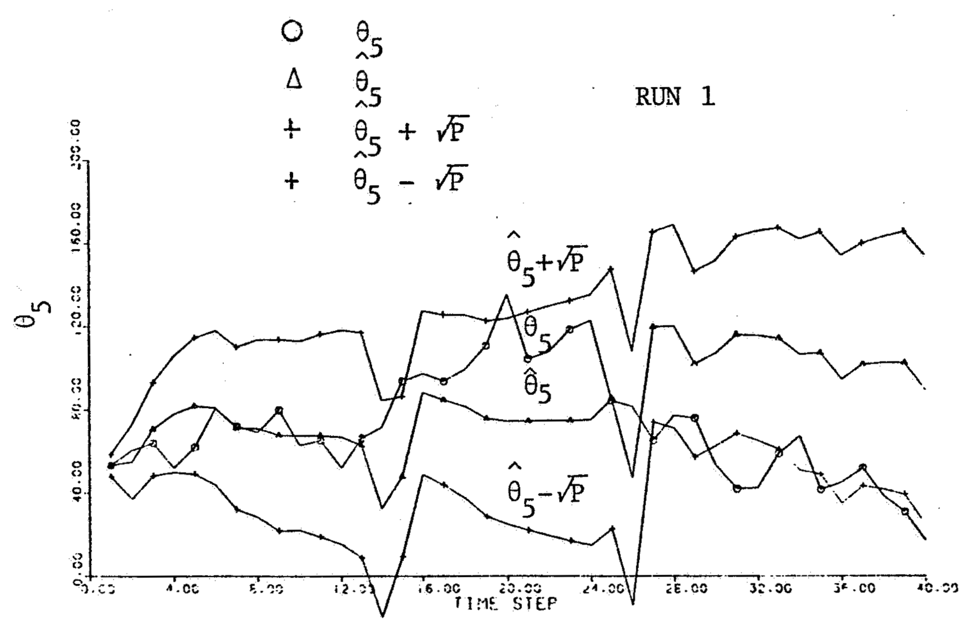

Fig. 25 Time history of θ_5, $\hat{\theta}_5$,
$\hat{\theta}_5 \pm \sqrt{P}$, using the cautious controller

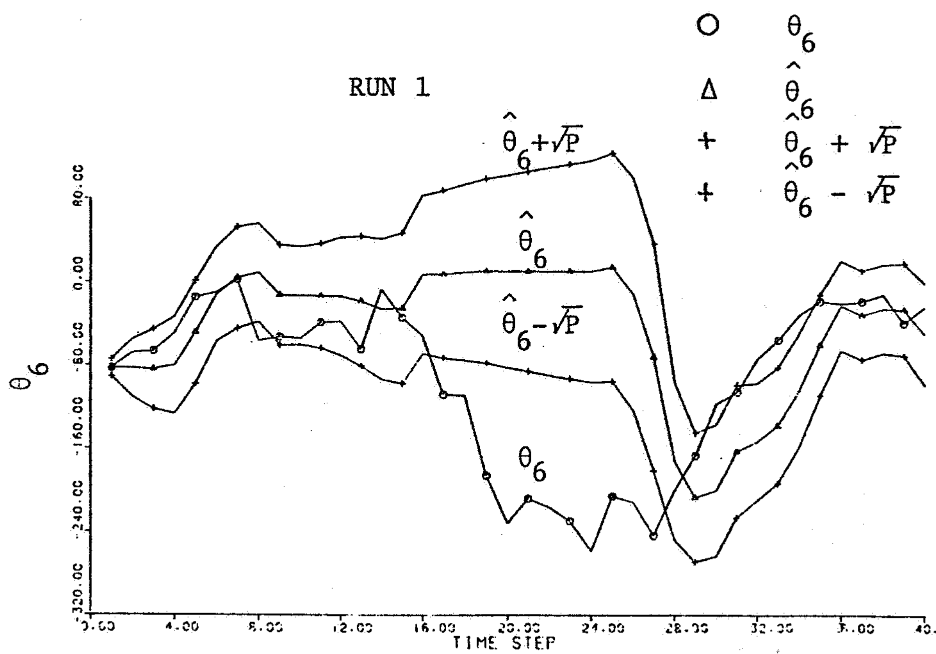

Fig. 26 Time history of θ_6, $\hat{\theta}_6$, $\hat{\theta}_6 \pm \sqrt{P}$, using the cautious controller

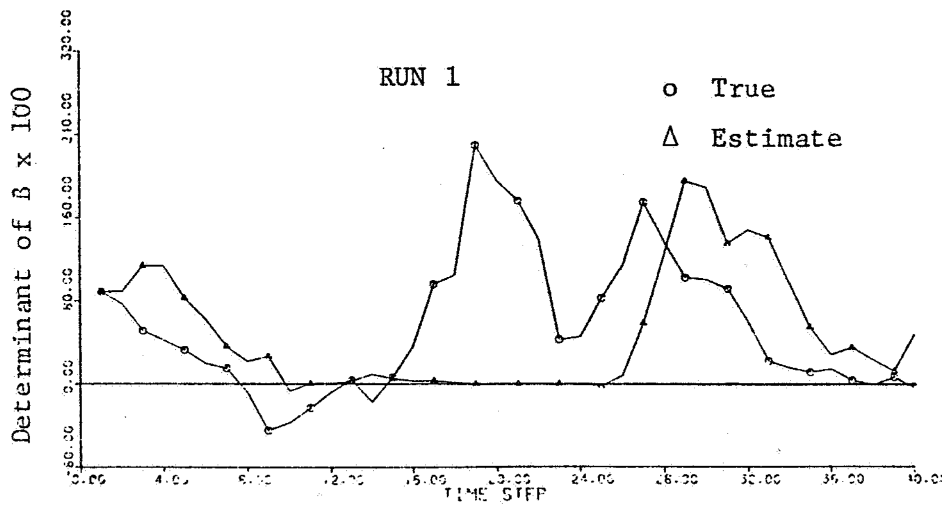

Fig. 27 Comparison of the determinants of the true and the estimated parameter transfer matrices for the cautious controller

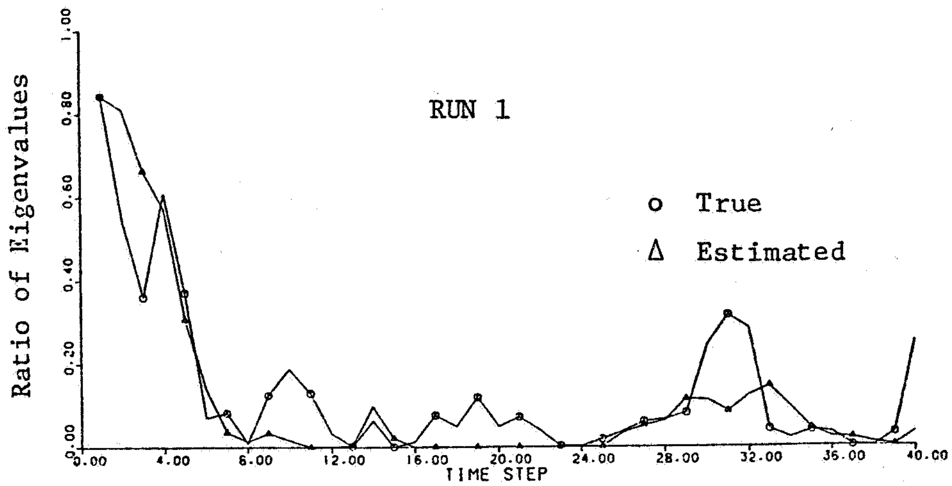

Fig. 28 Comparison of the ratio of the eigenvalues of the true and estimated parameter transfer matrices for the cautious controller

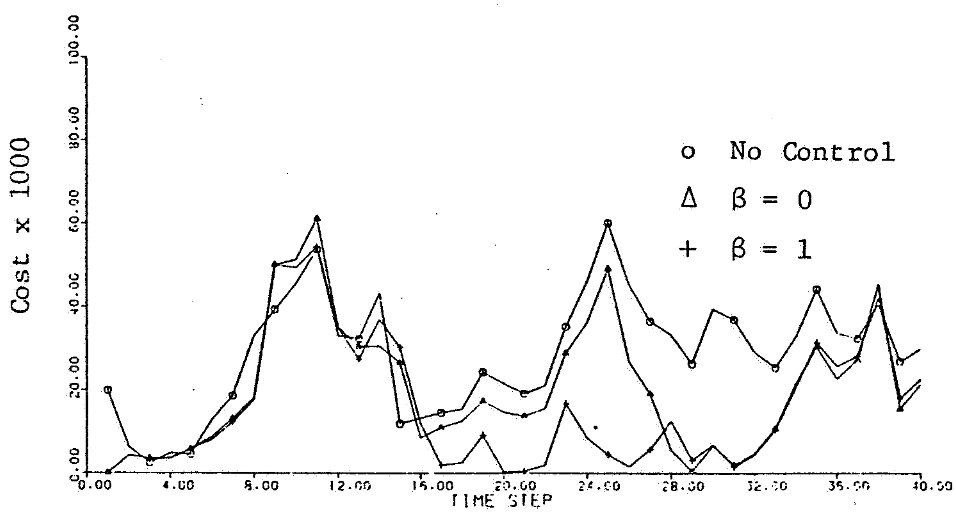

Fig. 29 Comparison of no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 1

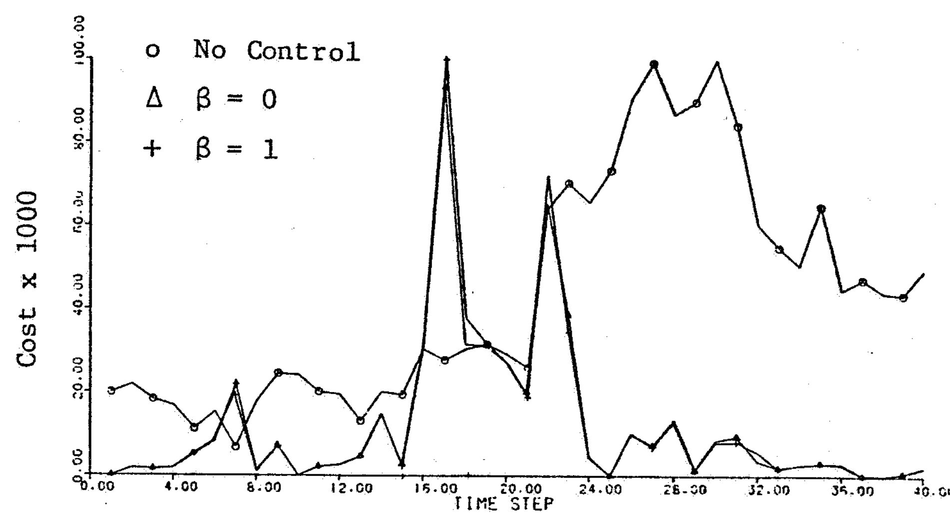

Fig. 30 Comparison of no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 2

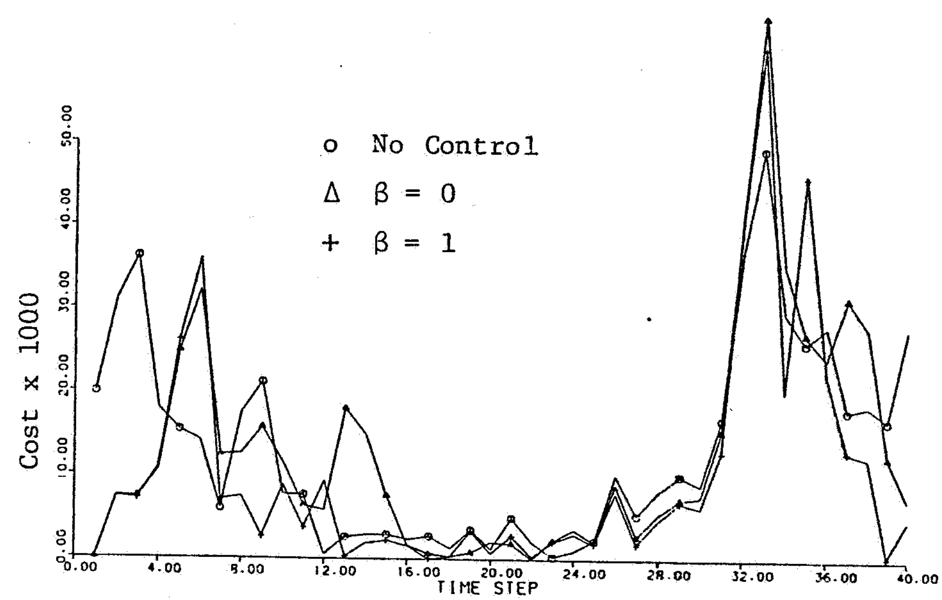

Fig. 31 Comparison of no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 3

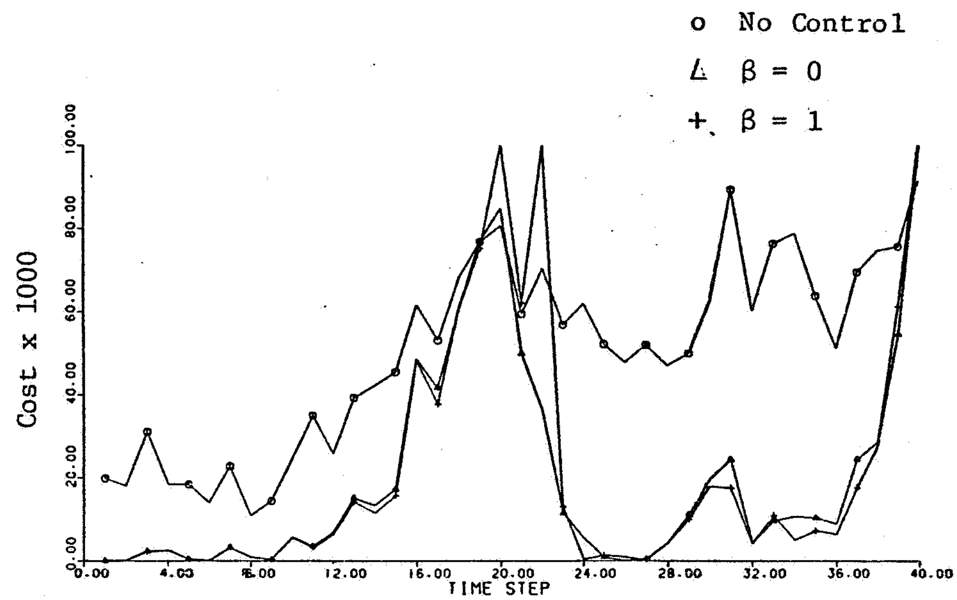

Fig. 32 Comparison of no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 4

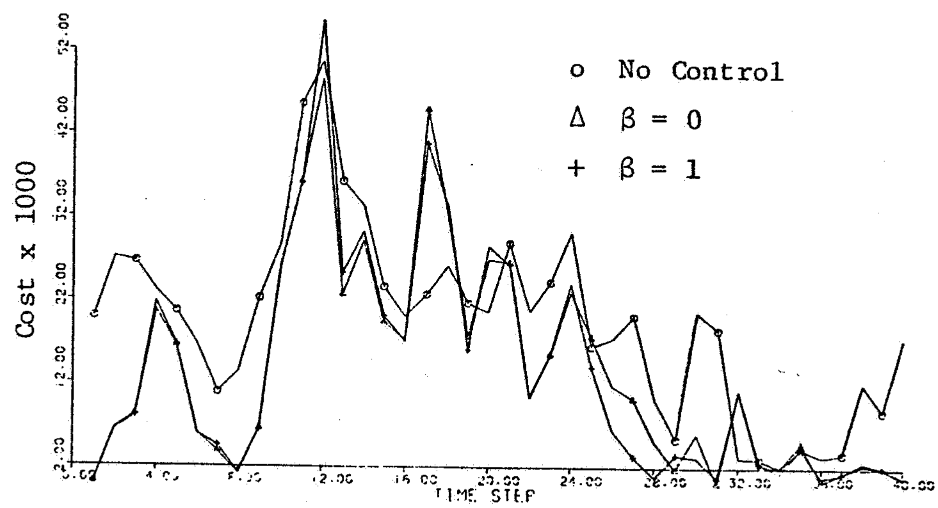

Fig. 33 Comparison of no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 5

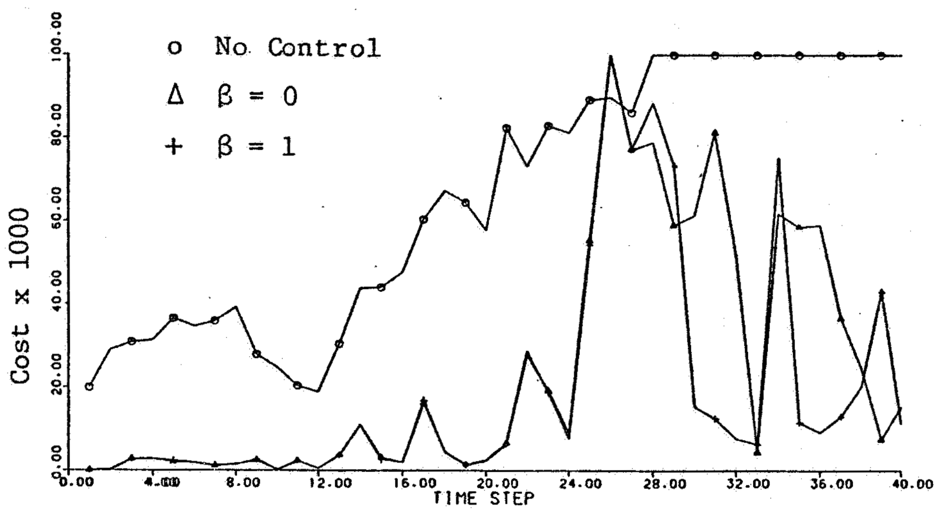

Fig. 34 Comparison of no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 6

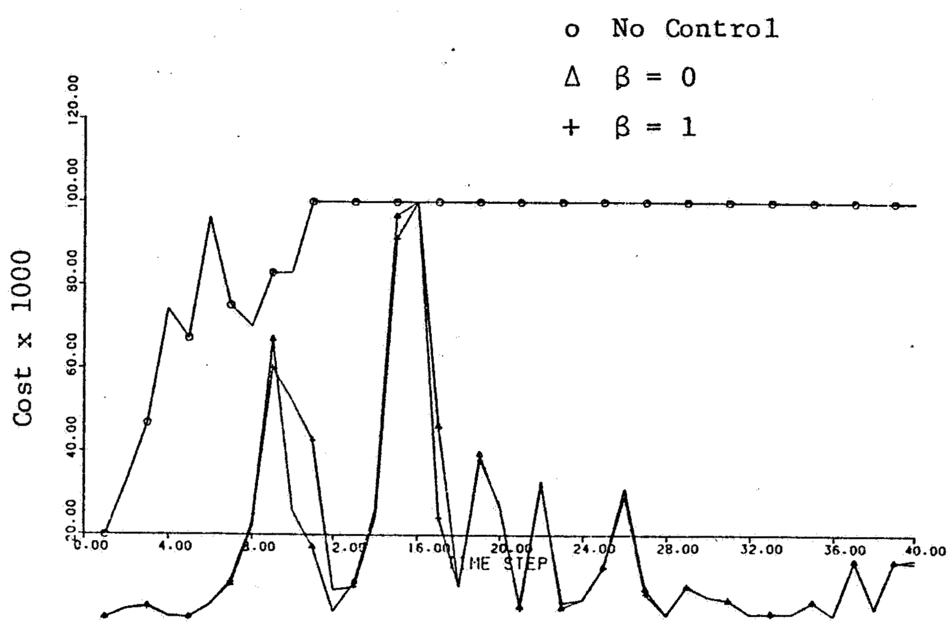

Fig. 35 Comparison of no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 7

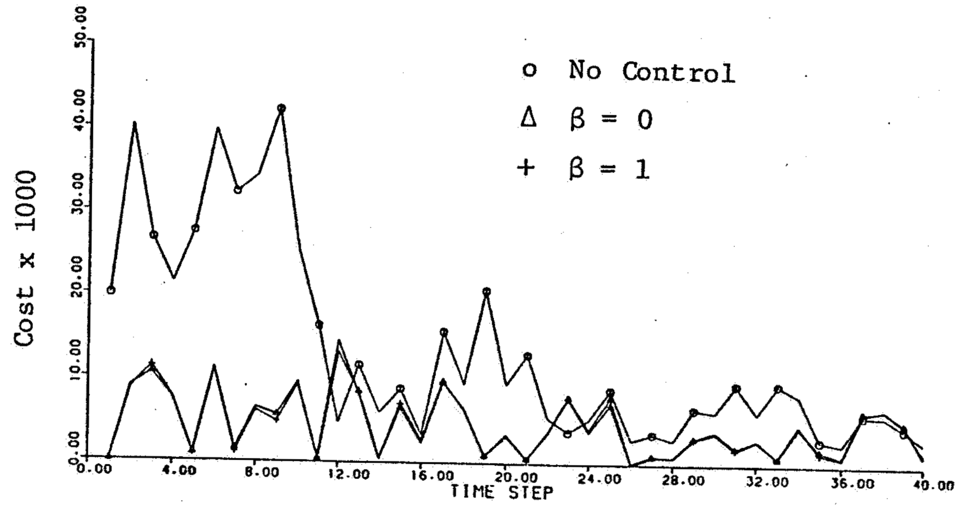

Fig. 36 Comparison of no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 8

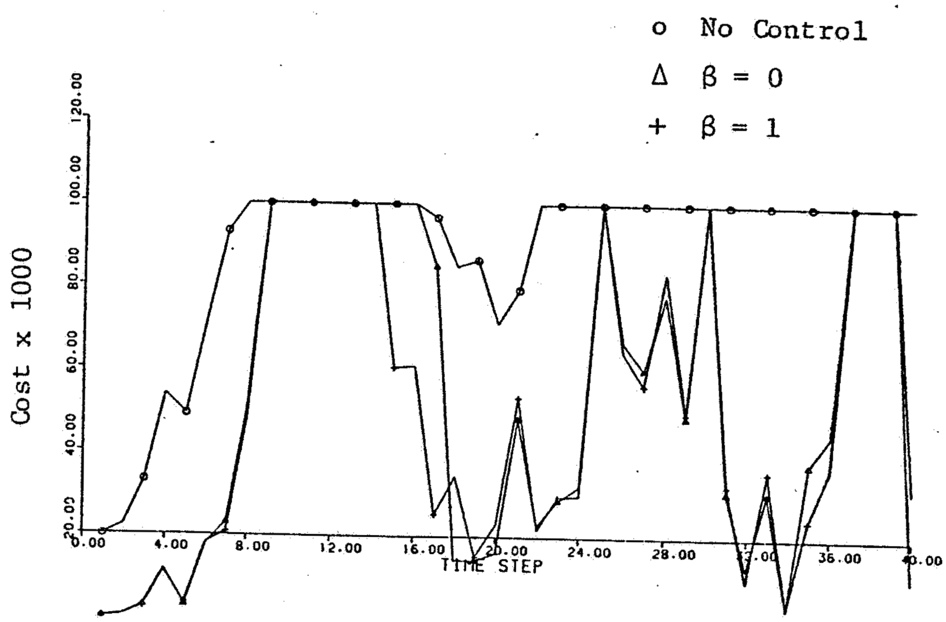

Fig. 37 Comparison of no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 9

C-2

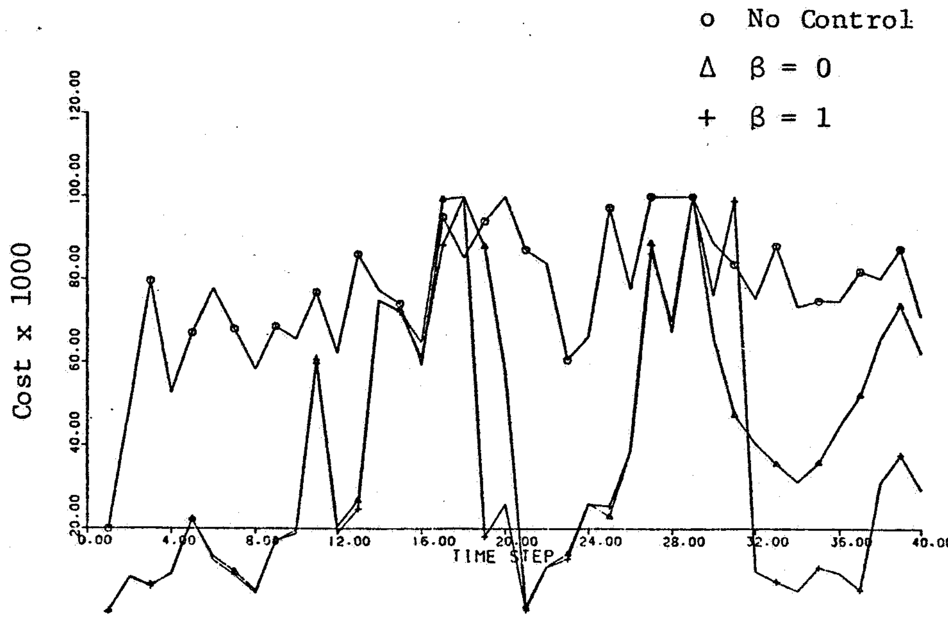

Fig. 38 Comparison of no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 10

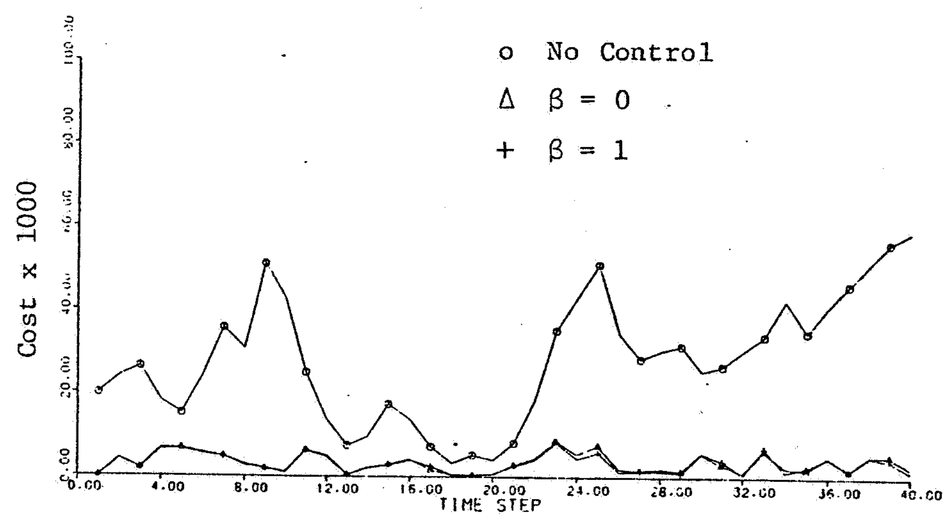

Fig. 39 Comparison of no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 11

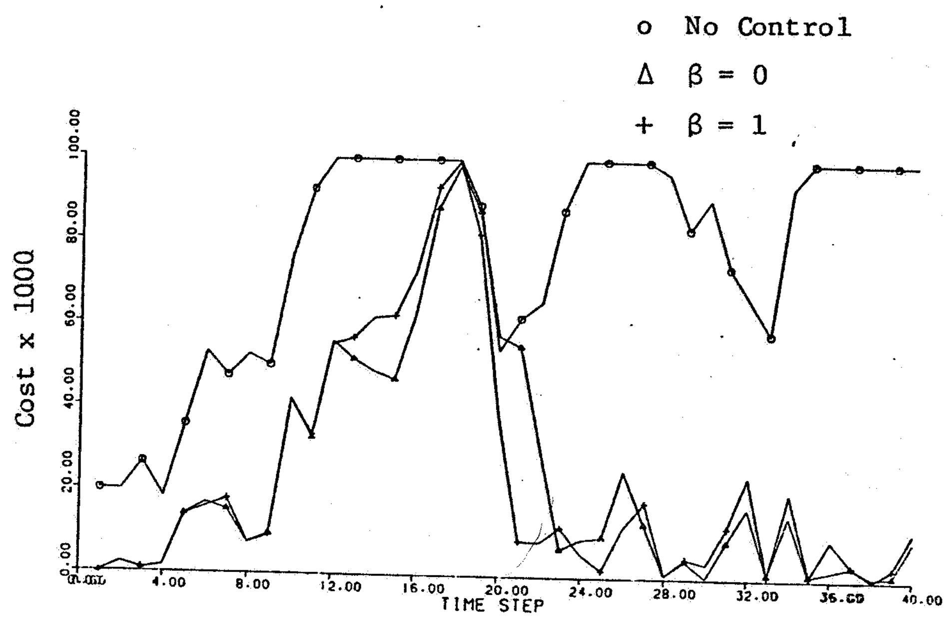

Fig. 40 Comparison of no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 12

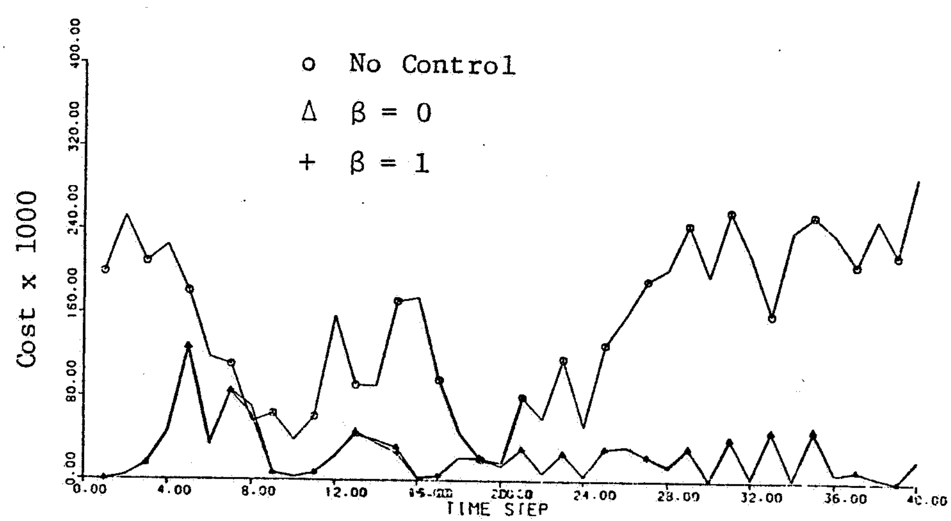

Fig. 41 Comparison of no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 13

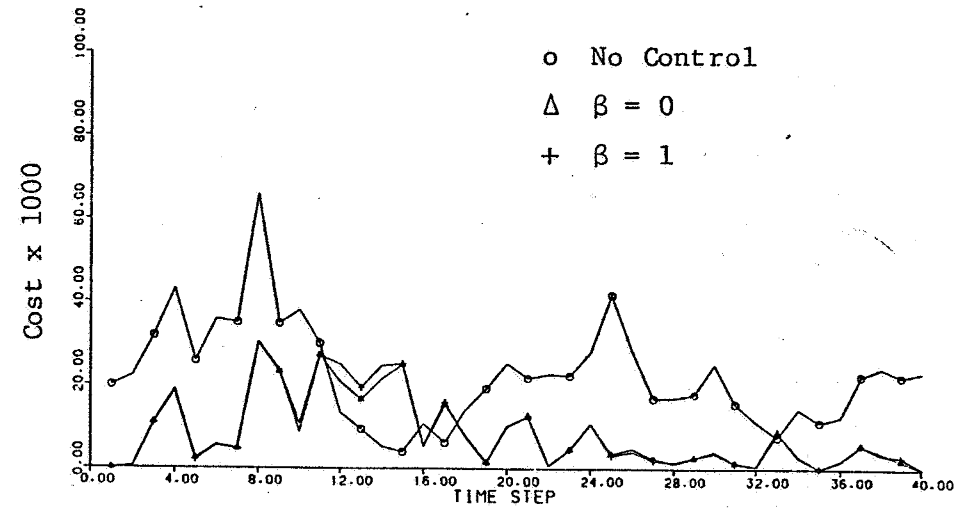

Fig. 42 Comparison of the no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 14

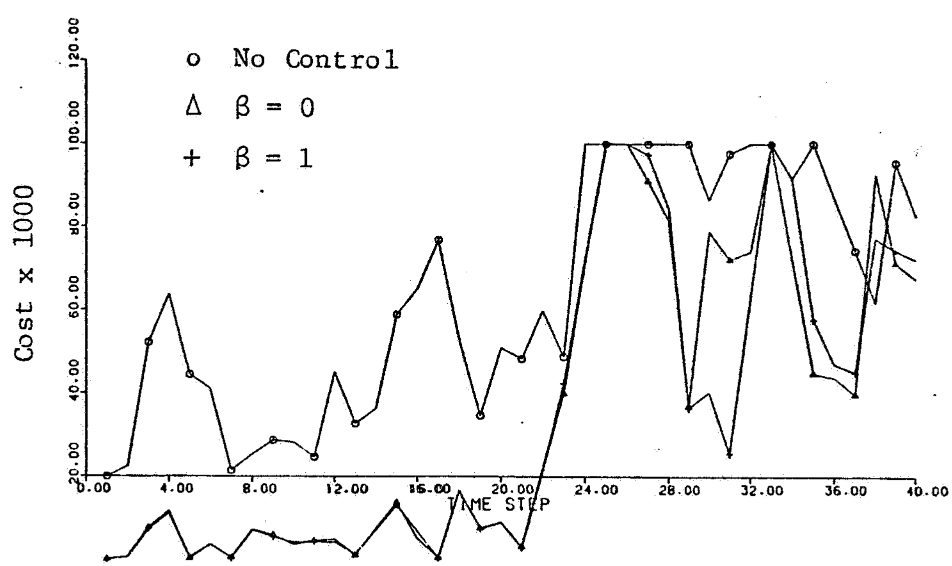

Fig. 43 Comparison of the no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 15

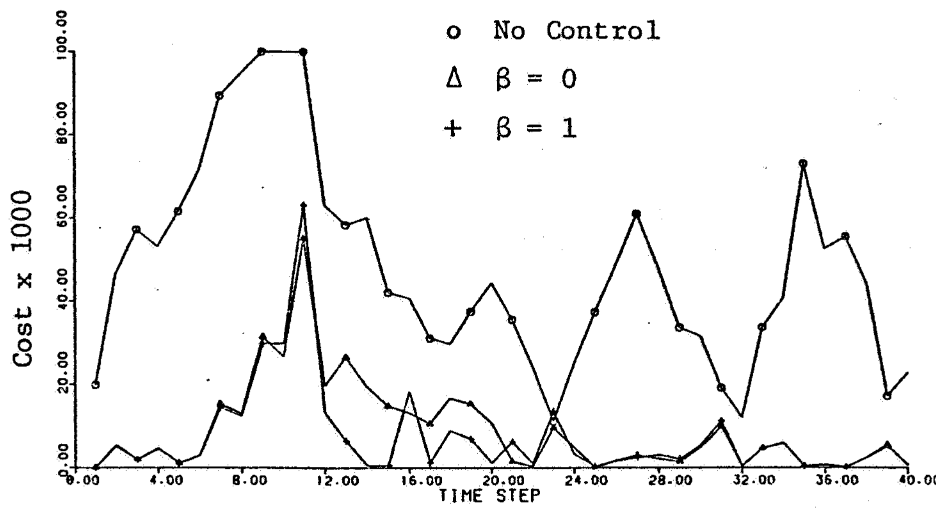

Fig. 44 Comparison of the no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 16

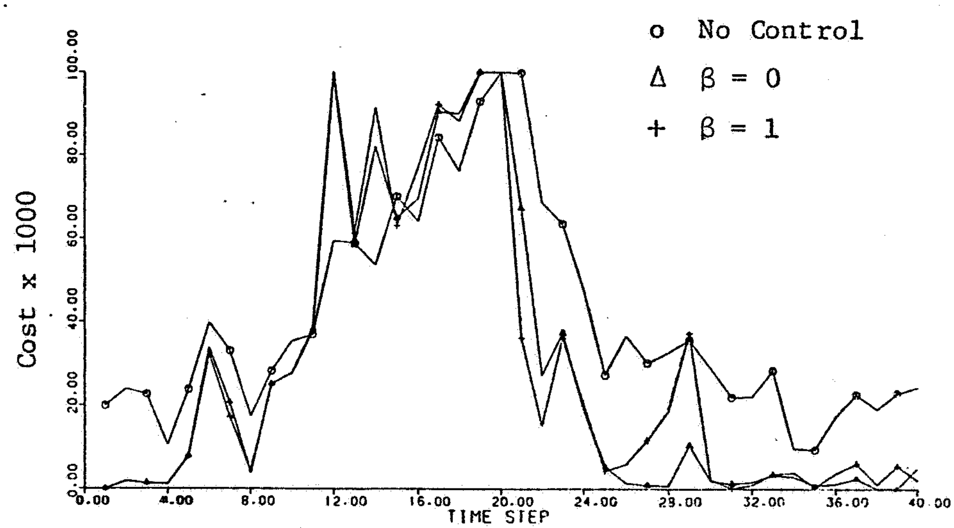

Fig. 45 Comparison of the no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 17

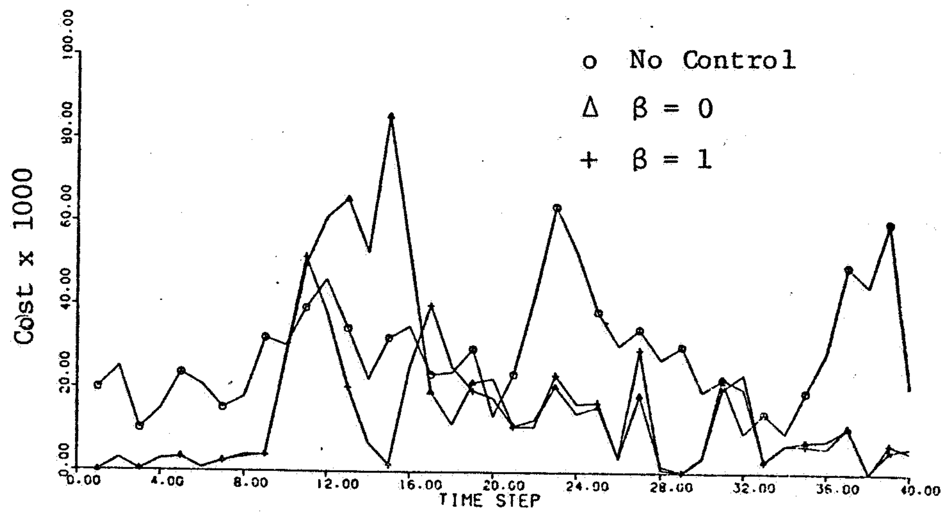

Fig. 46 Comparison of the no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 18

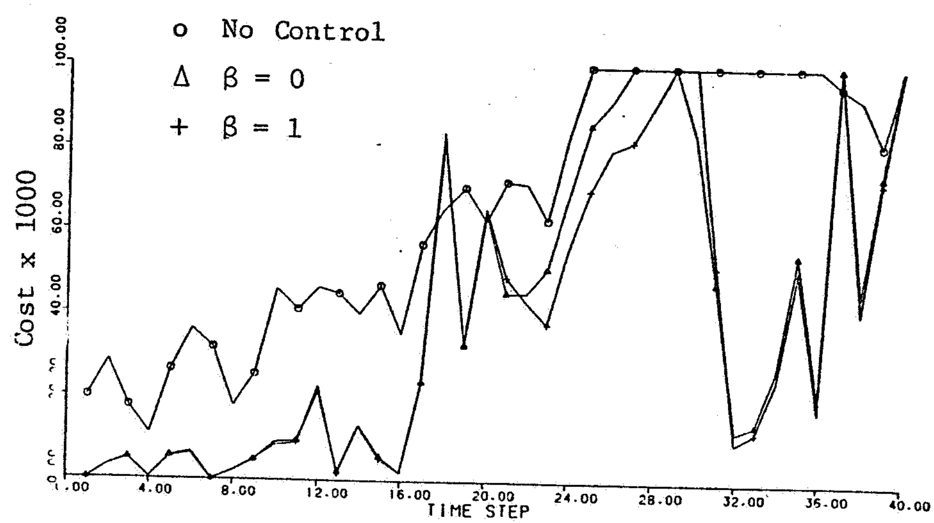

Fig. 47 Comparison of the no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 19

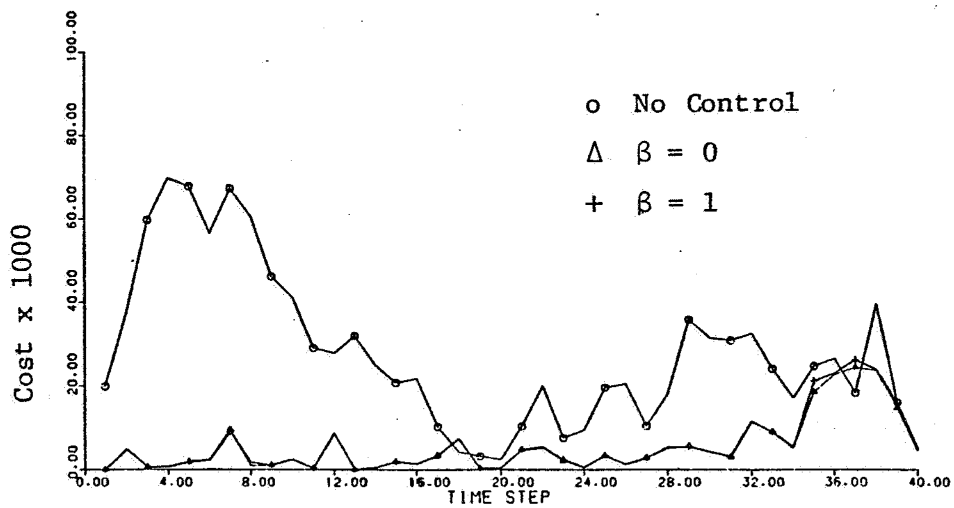

Fig. 48 Comparison of the no control, cautious and the first order dual controller's performances on the time varying parameter case (30%) Run 20

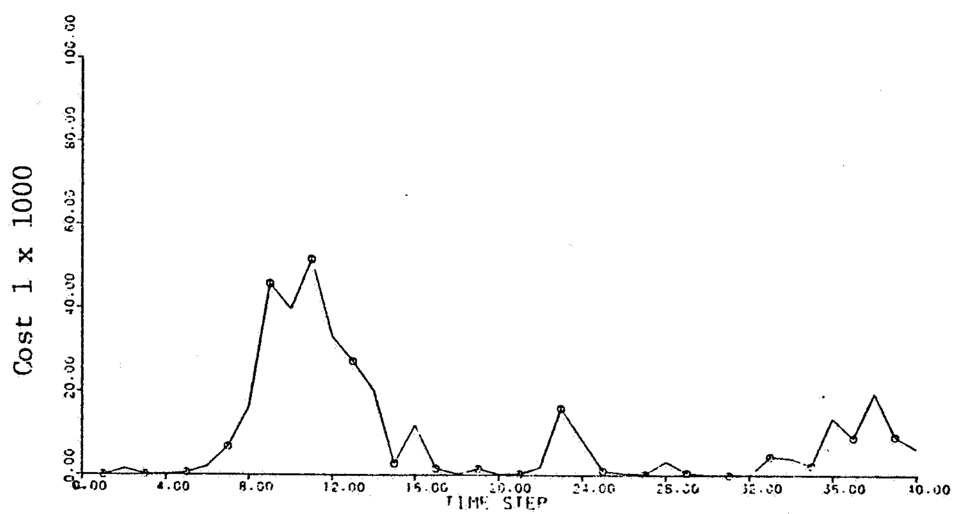

Fig. 49 Vibration contribution from the cosine component, using the dual controller (second method of initialization) (Run 1)

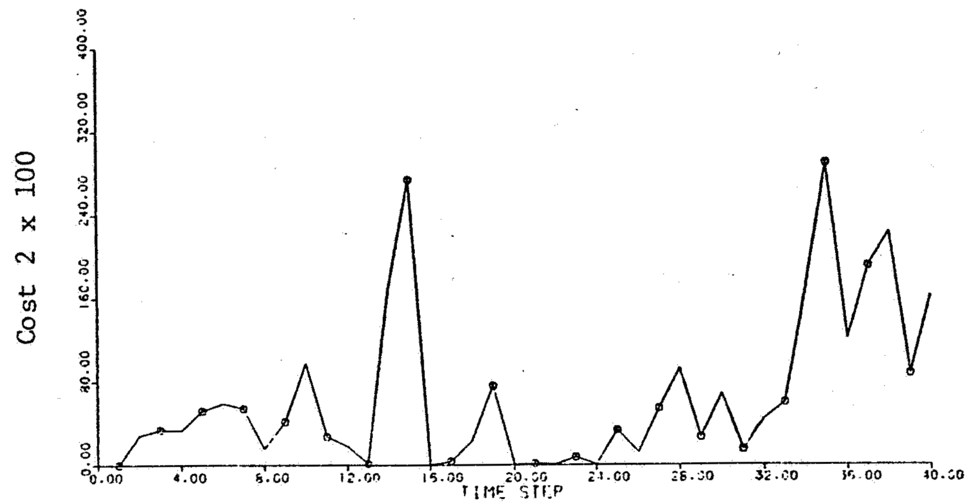

Fig. 50 Vibration contribution from the sine component, using the dual controller (second method of initialization) (Run 1)

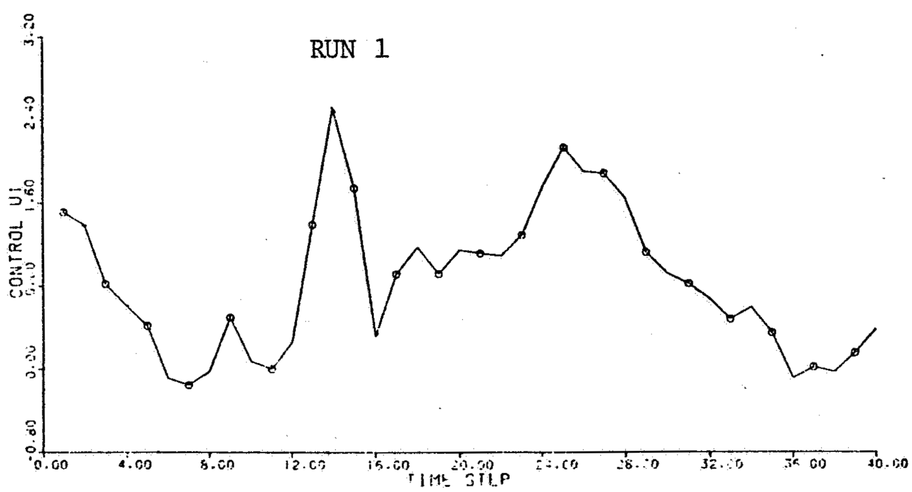

Fig. 51 Time history of control 1 used by the dual controller

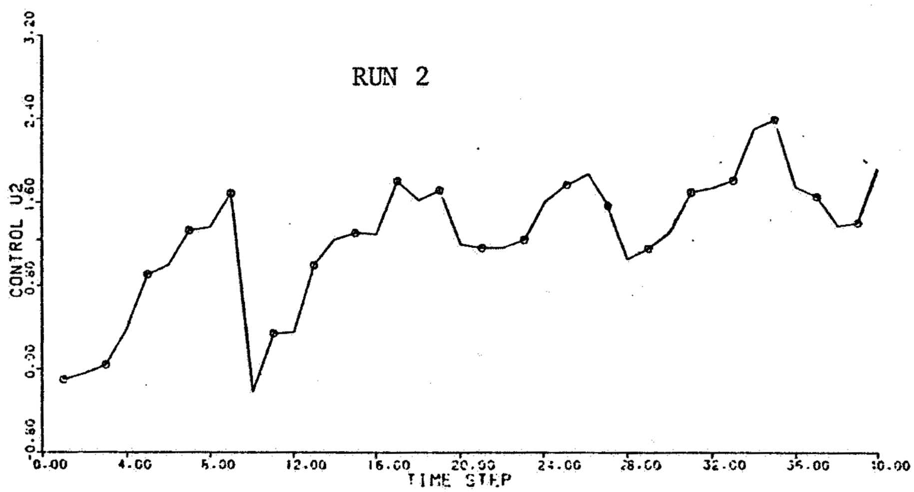

Fig. 52 Time history of control 2 used by the dual controller

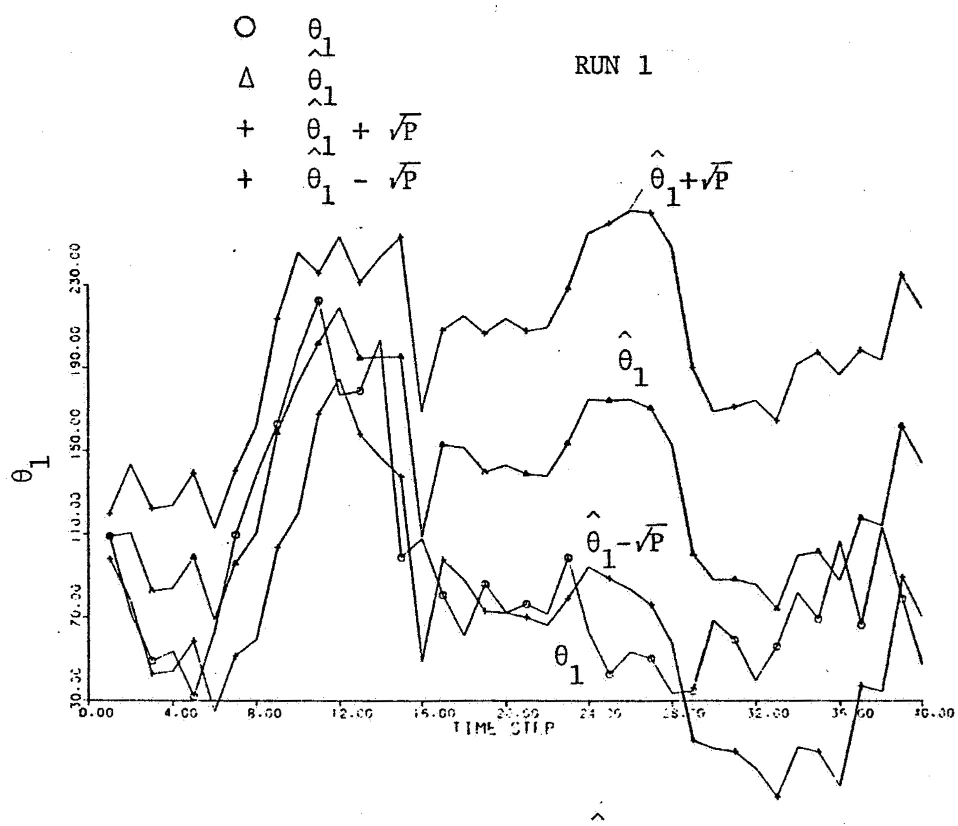

Fig. 53 Time history of θ_1, $\hat{\theta}_1$, $\hat{\theta}_1 \pm \sqrt{P}$, using the dual controller

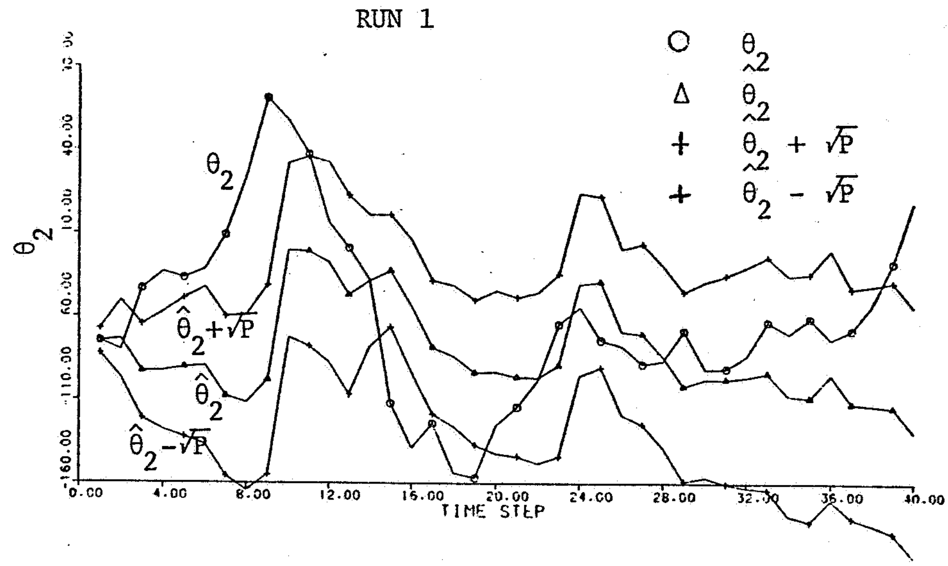

Fig. 54 Time history of θ_2, $\hat{\theta}_2$, $\hat{\theta}_2 \pm \sqrt{P}$, using the dual controller

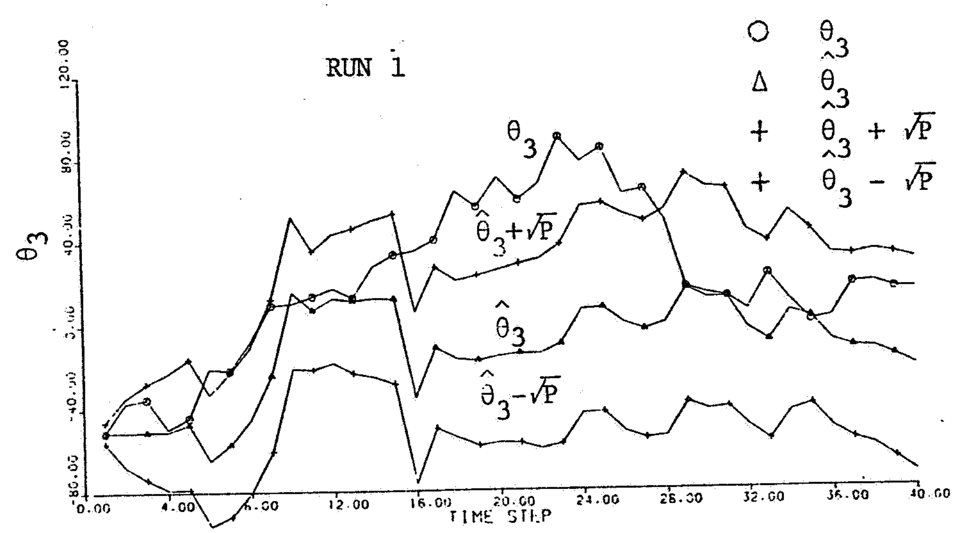

Fig. 55 Time history of θ_3, $\hat{\theta}_3$, $\hat{\theta}_3 \pm \sqrt{P}$, using the dual controller

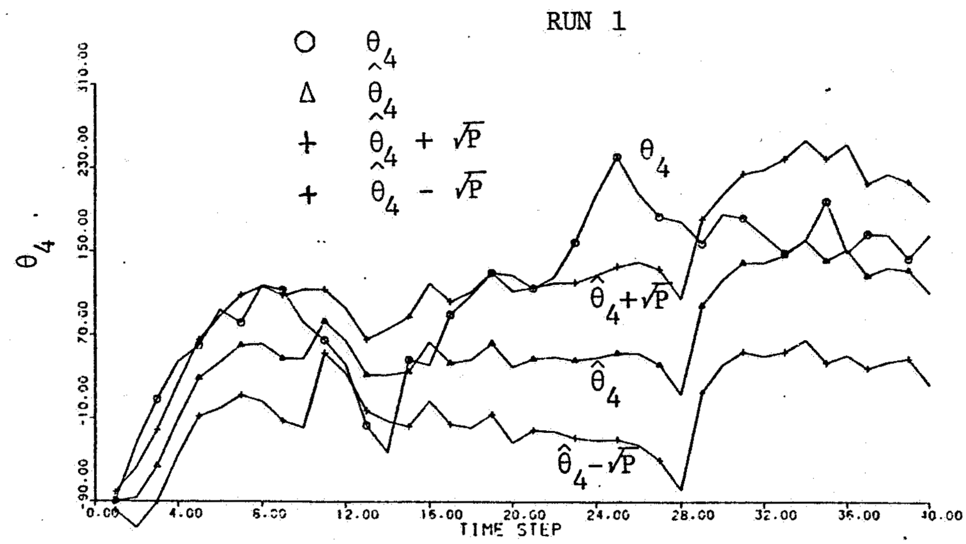

Fig. 56 Time history of θ_4, $\hat{\theta}_4$, $\hat{\theta}_4 \pm \sqrt{P}$, using the dual controller

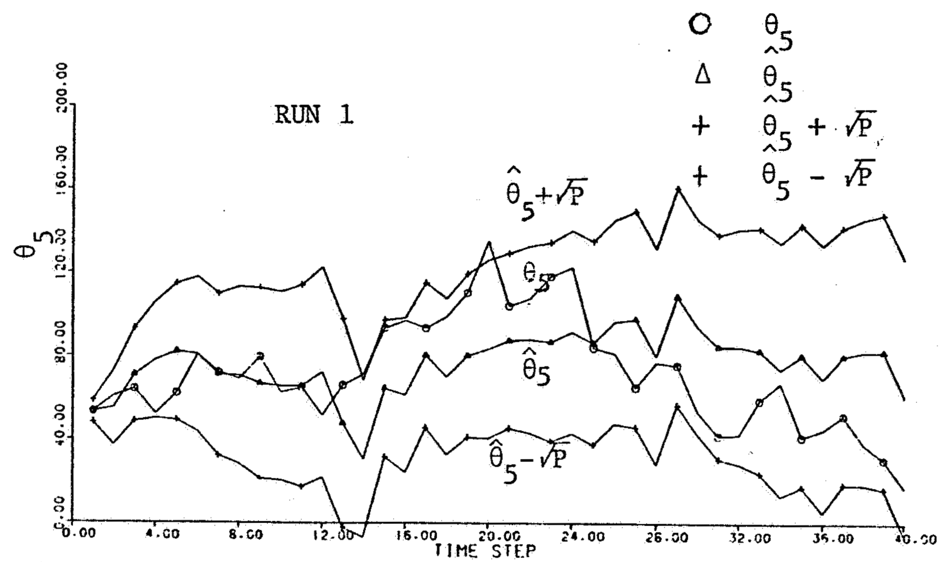

Fig. 57 Time history of θ_5, $\hat{\theta}_5$, $\hat{\theta}_5 \pm \sqrt{P}$, using the dual controller

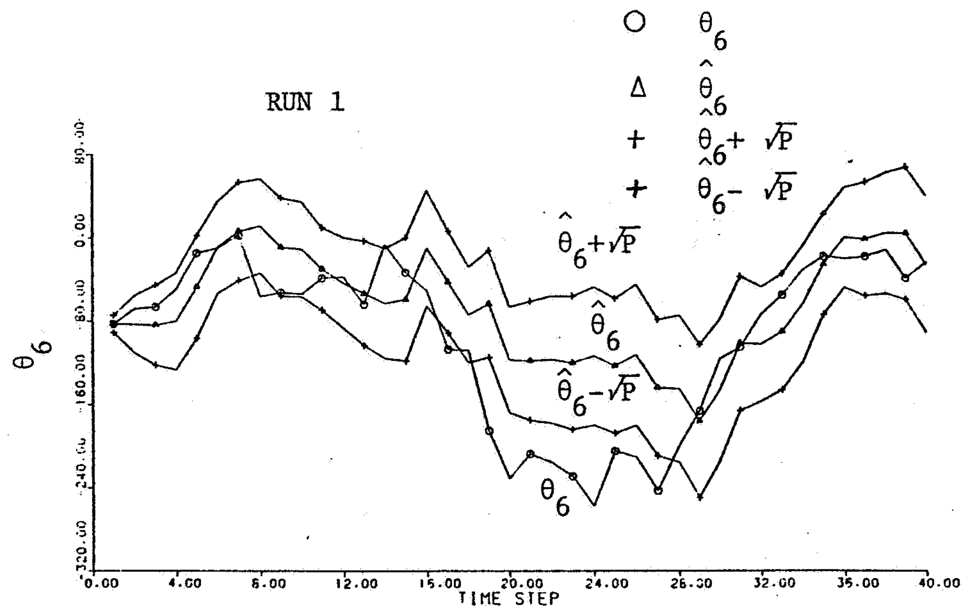

Fig. 58 Time history of θ_6, $\hat{\theta}_6$, $\hat{\theta}_6 \pm \sqrt{P}$, using the dual controllers

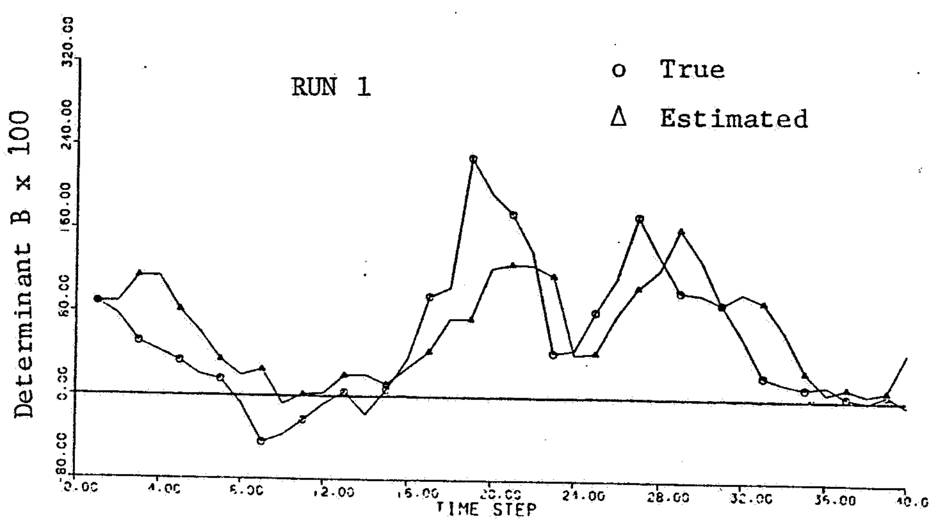

Fig. 59 Comparison of the determinants of the true and the estimated parameter transfer matrix for the dual controller

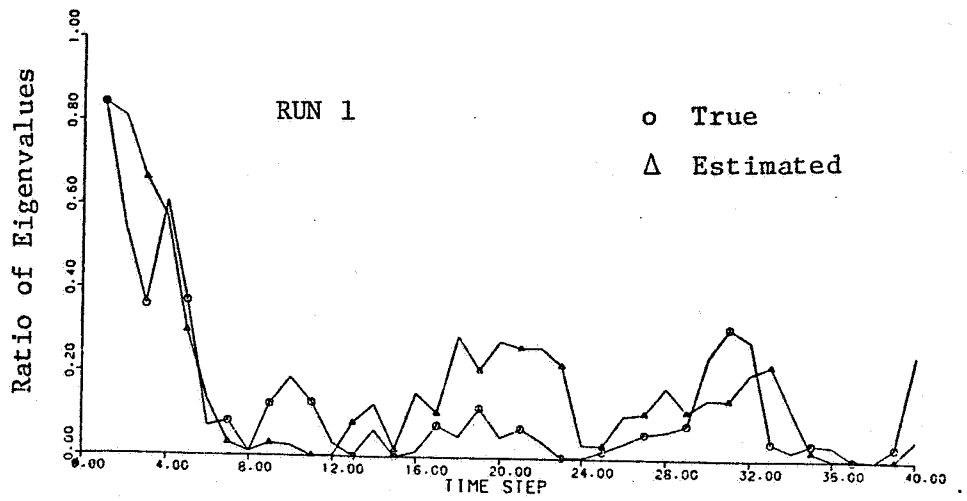

Fig. 60 Comparison of the ratio of the eigenvalues of the true and estimated parameter transfer matrices for the dual controller

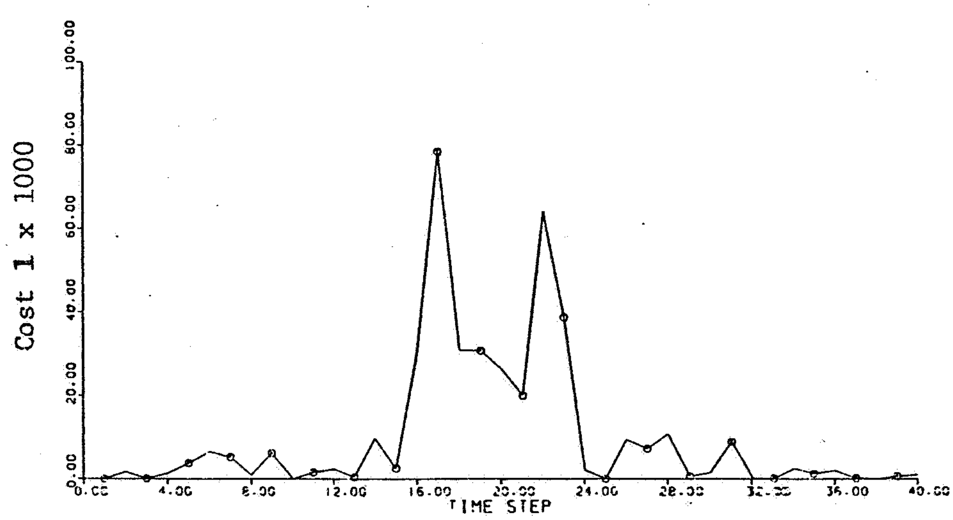

Fig. 61 Vibration contribution from the cosine component using the cautious controller (second method of initialization, Run 2)

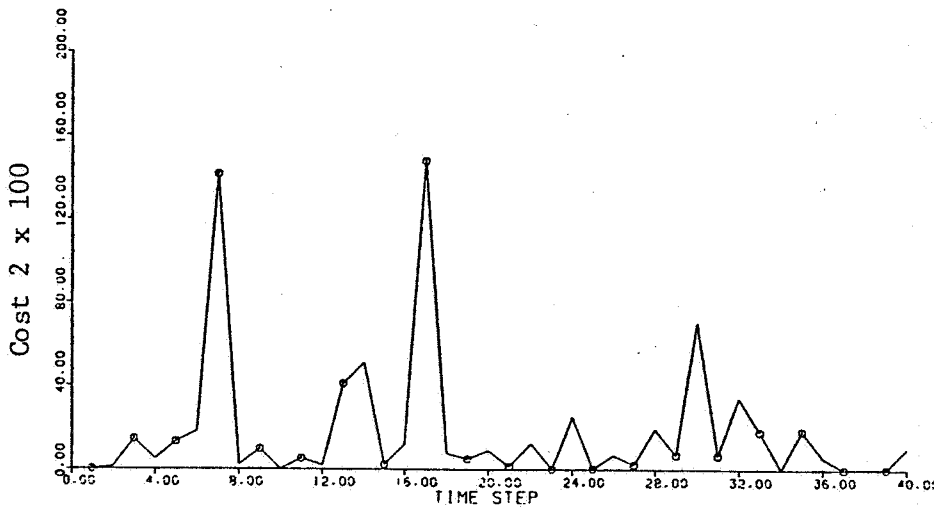

Fig. 62 Vibration contribution from the sine component using the cautious controller (second method of initialization, Run 2)

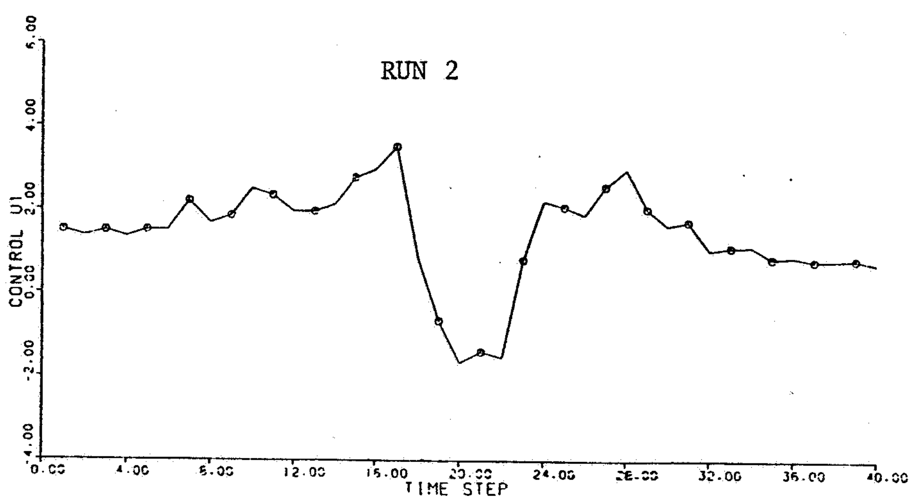

Fig. 63 Time history of control 1 used by the cautious controller

RUN2

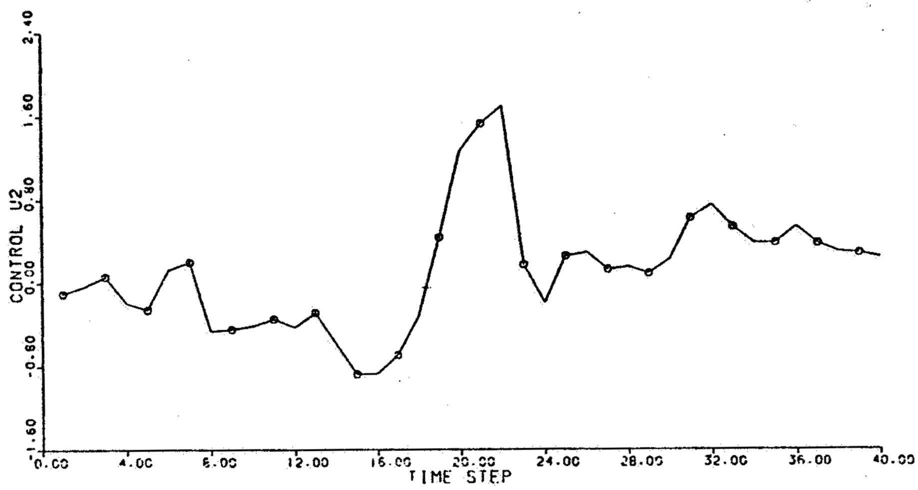

Fig. 64 Time history of control 2 used by the cautious controller

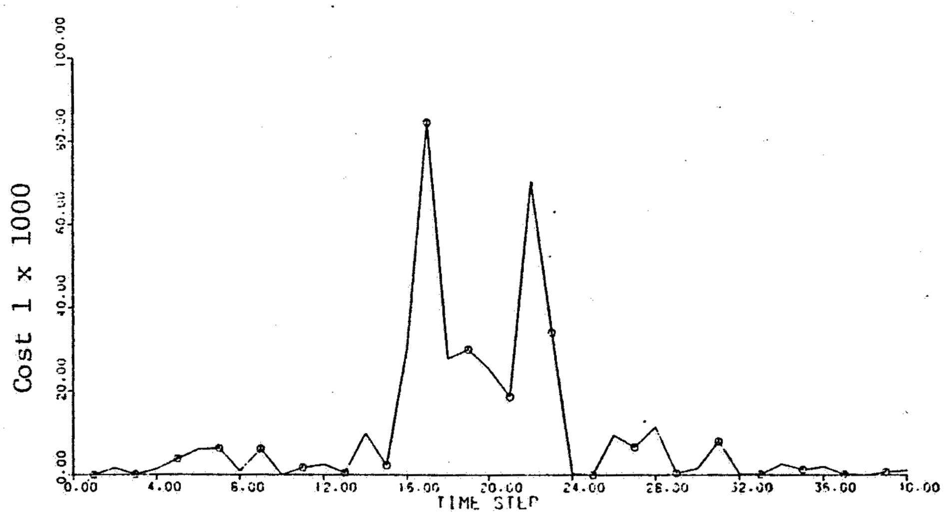

Fig. 65 Vibration contribution from the cosine component using the dual controller (Run 2)

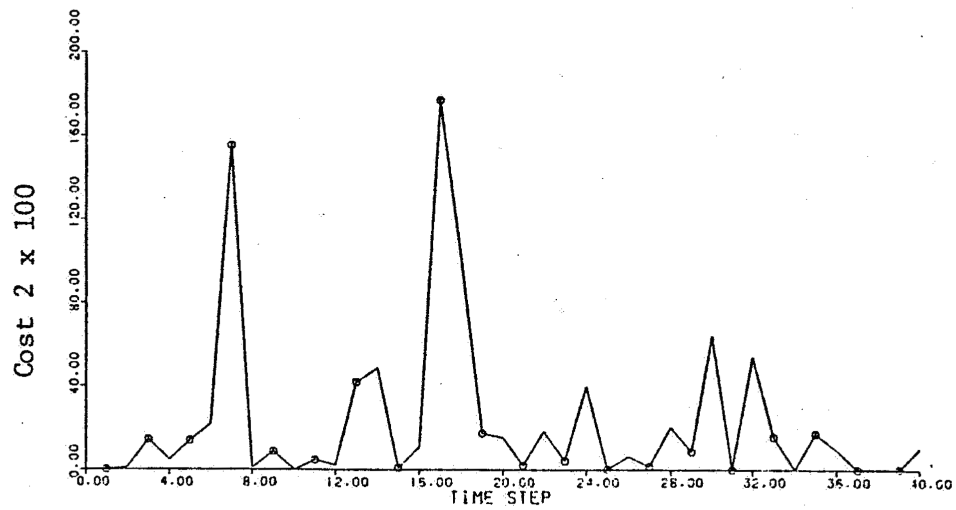

Fig. 66 Vibration contribution from the sine component using the dual controller (Run 2)

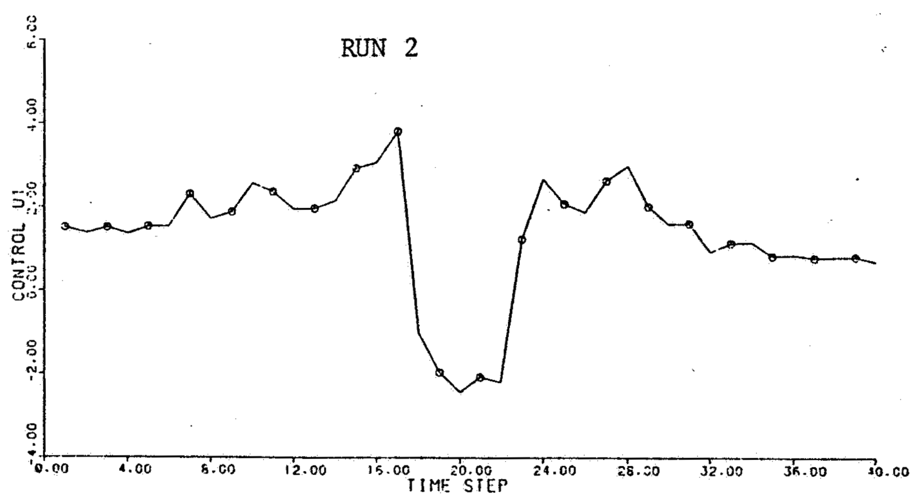

Fig. 67 Time history of control 1 used by the dual controller

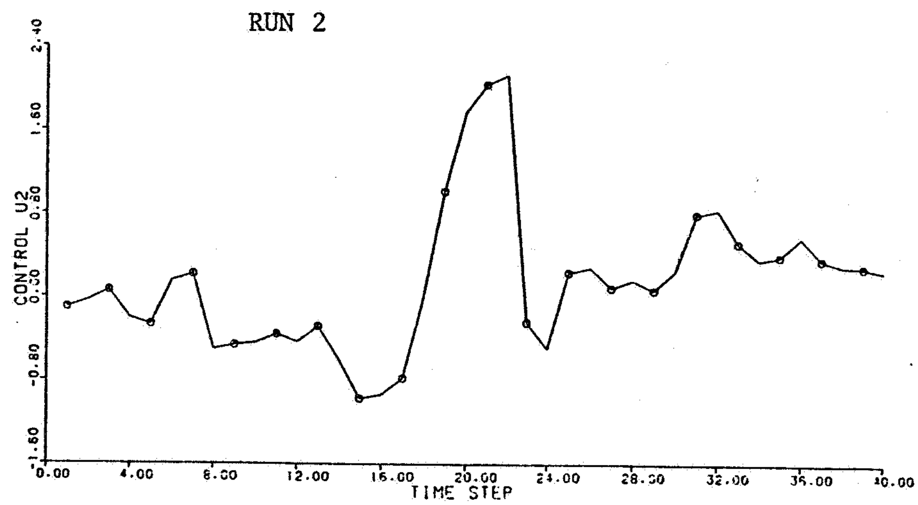

Fig. 68 Time history of control 2 used by the dual controller

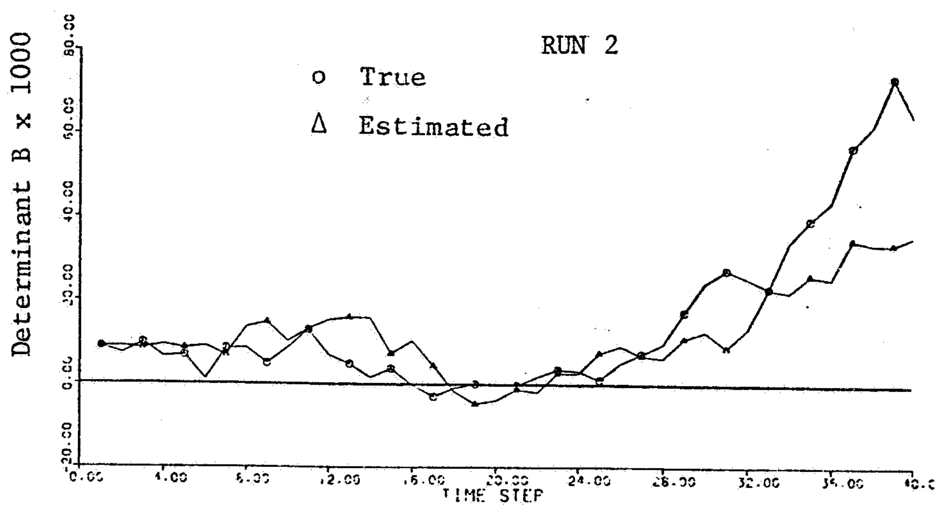

Fig. 69 Comparison of the determinants of the true and the estimated parameter transfer matrices for the cautious controller

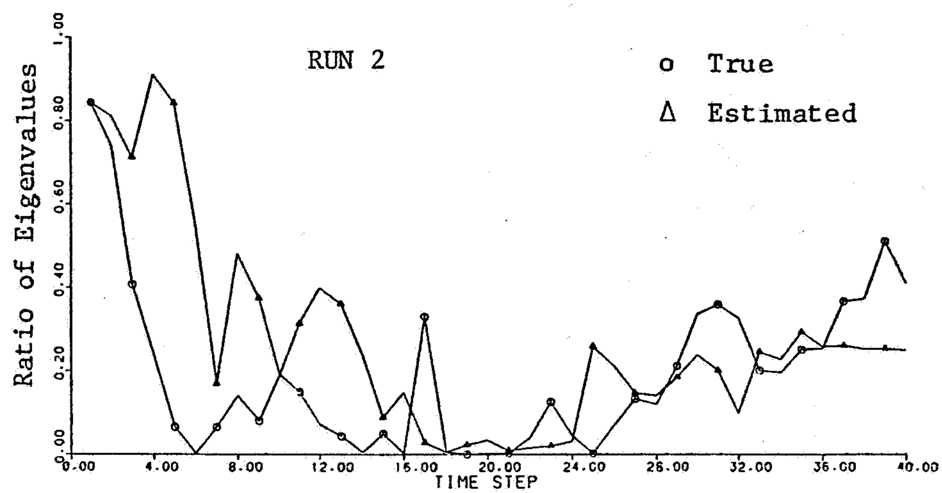

Fig. 70 Comparison of the ratio of the eigenvalues of the true and the estimated parameter transfer matrices for the cautious controller

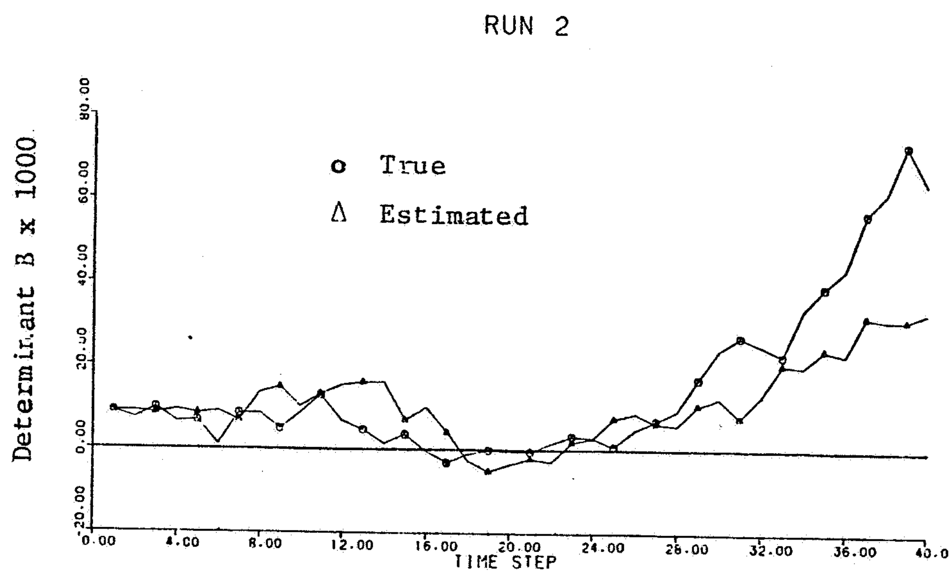

Fig. 71 Comparison of the determinants of the true and the estimated parameter transfer matrices for the dual controller

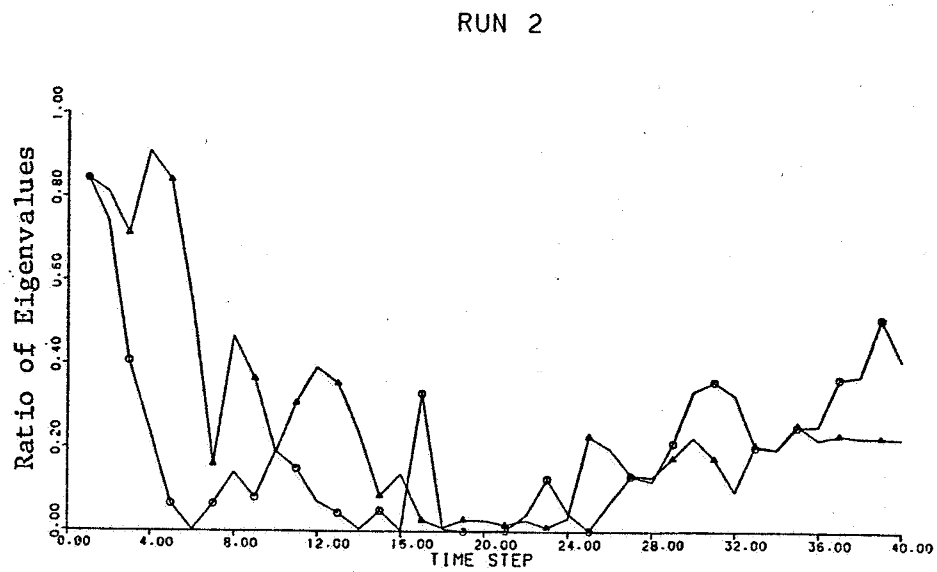

Fig. 72 Comparison of the ratio of the eigenvalues of the true and the estimated parameter transfer matrices for the dual controller

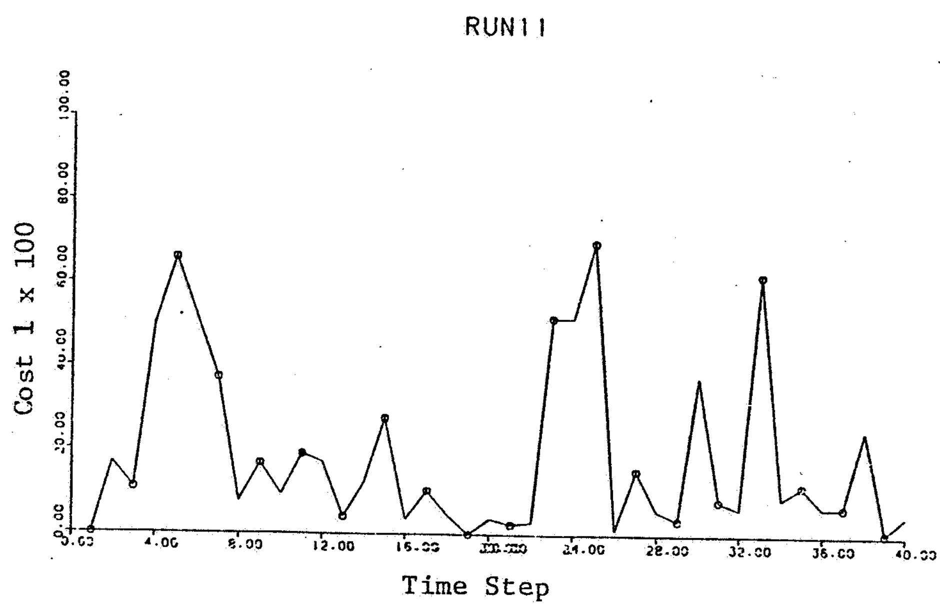

Fig. 73 Comparison of the vibration contribution from the cosine component using the cautious controller (second method of initialization, Run 11)

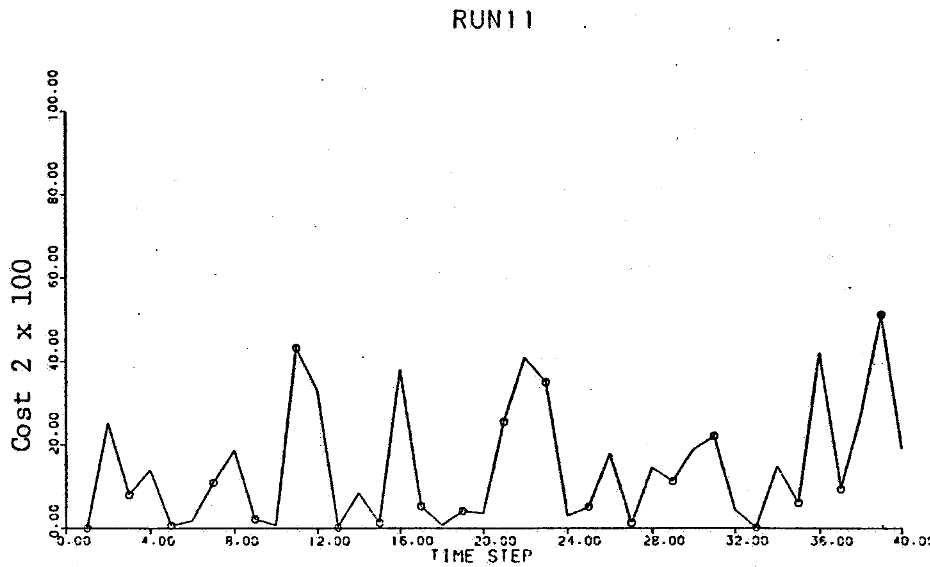

Fig. 74 Vibration contribution from the sine component using the cautious controller (second method of initialization, Run 11)

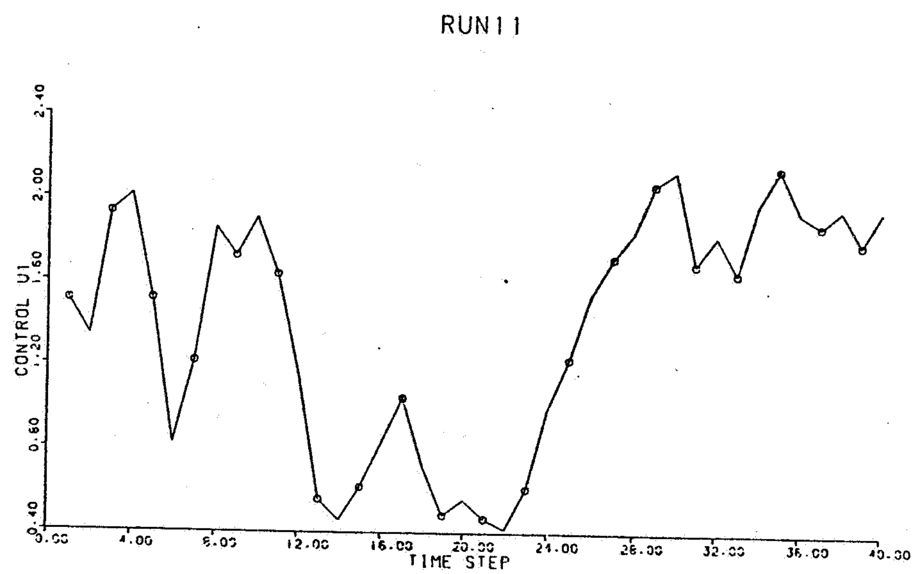

Fig. 75 Time history of control 1 used by the cautious controller

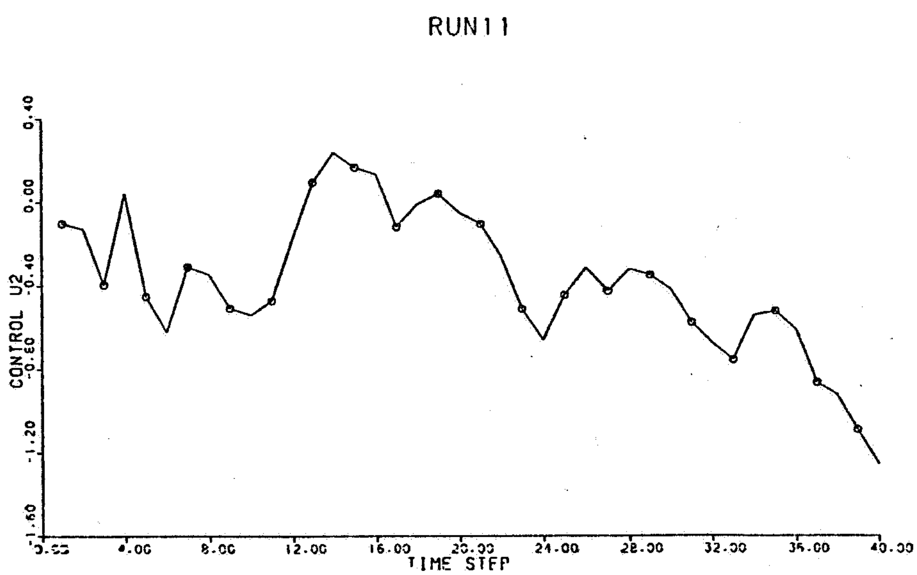

Fig. 76 Time history of control 2 used by the cautious controller

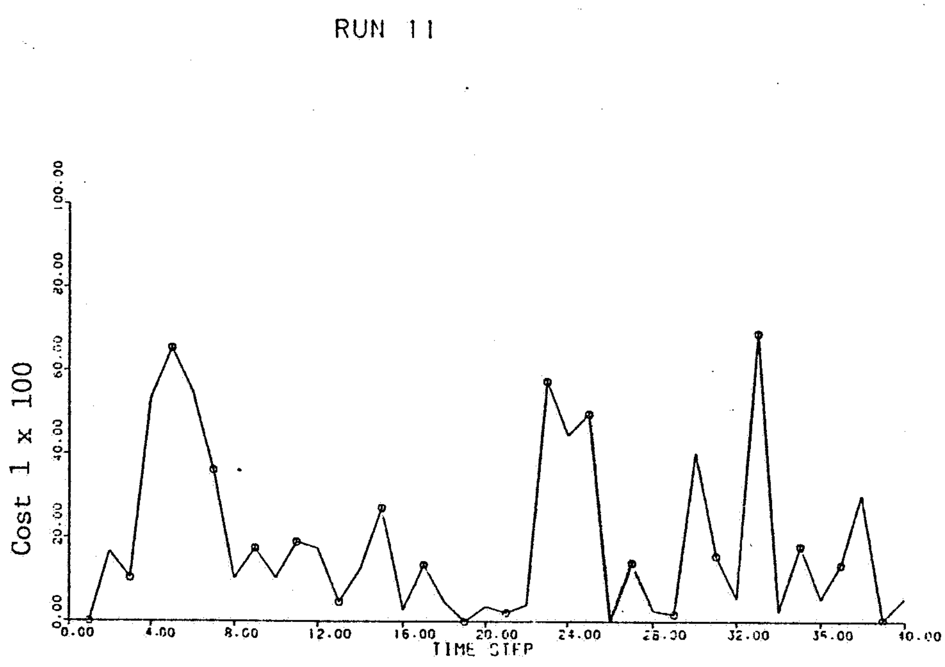

Fig. 77 Vibration contribution from the cosine component using the dual controller (Run 11)

RUN 11

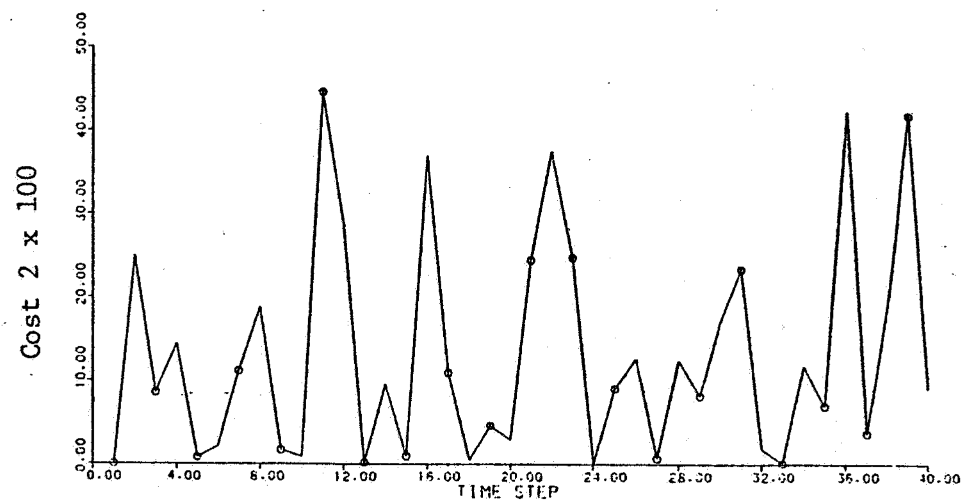

Fig. 78 Vibration contribution from the sine component using the dual controller (Run 11)

RUN11

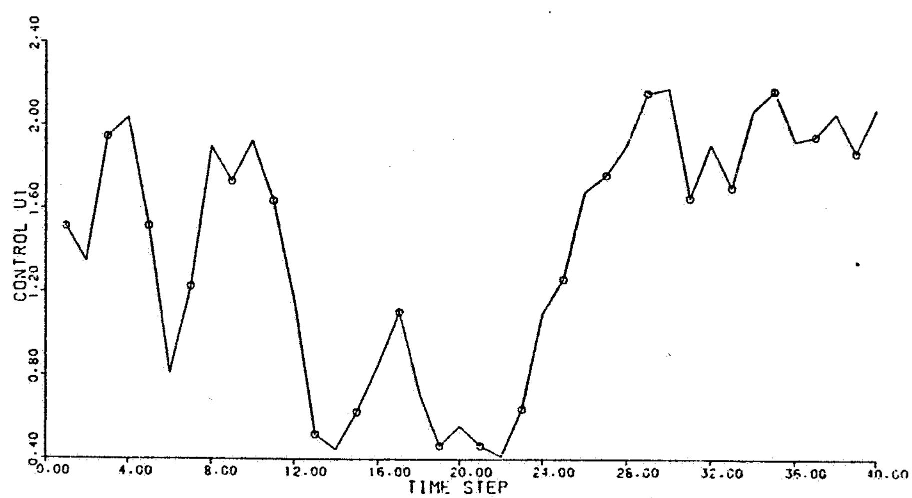

Fig. 79 Time history of control 1 used by the dual controller

RUN 11

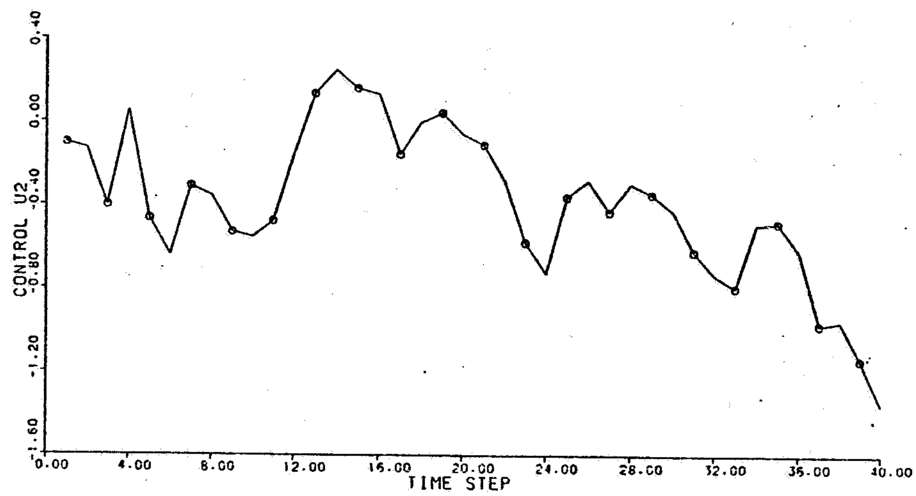

Fig. 80 Time history of control 2 used by the dual controller

RUN 11

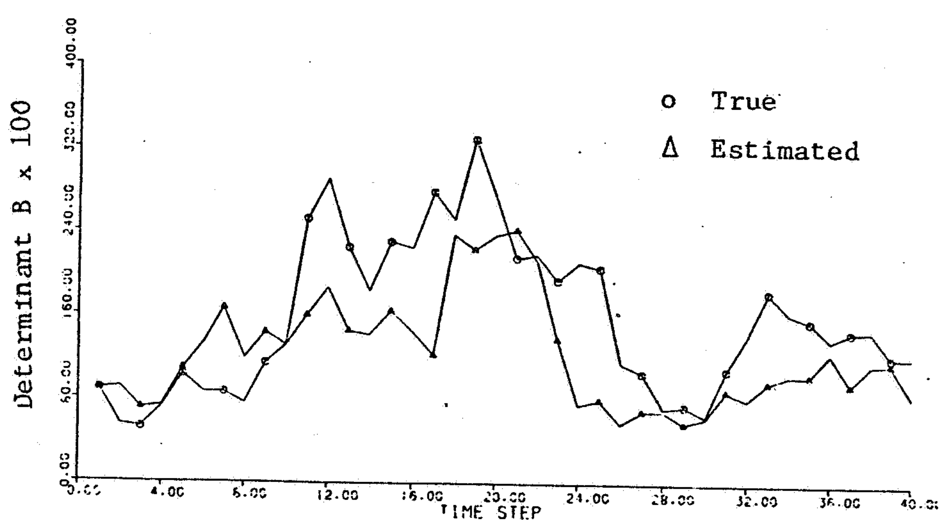

Fig. 81 Comparison of the determinants of the true and the estimated parameter transfer matrices for the cautious controller

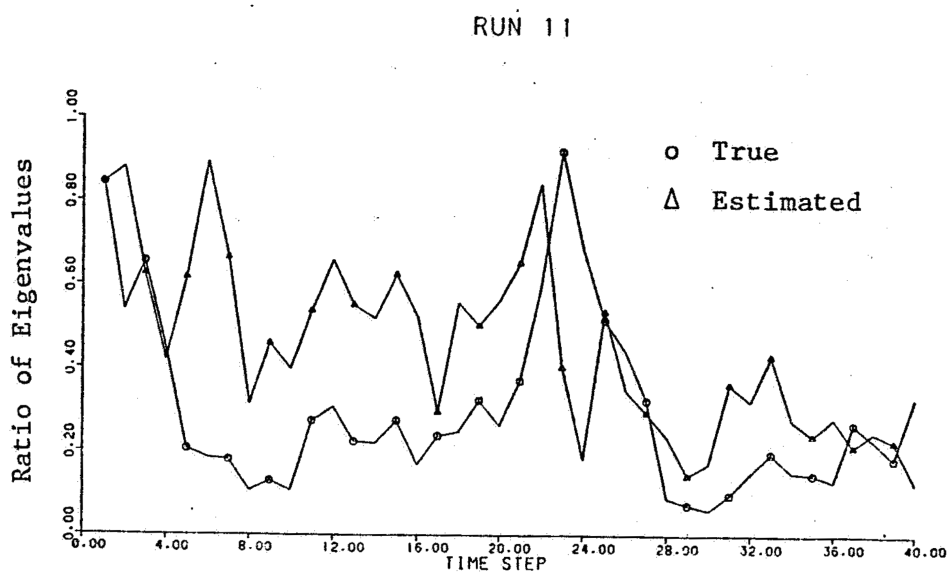

Fig. 82 Comparison of the ratio of the eigenvalues of the true and the estimated parameter transfer matrices for the cautious controller

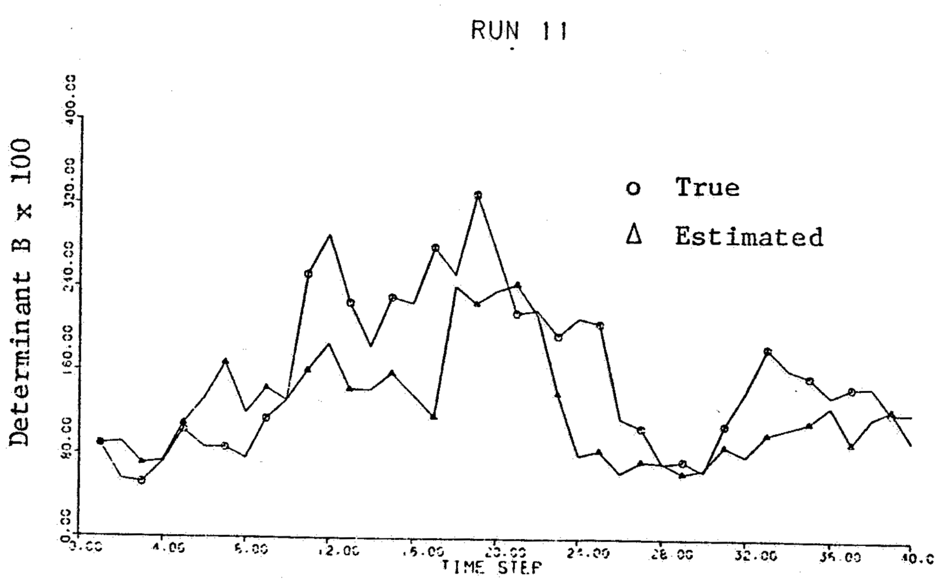

Fig. 83 Comparison of the determinants of the true and the estimated parameter transfer matrices for the dual controller

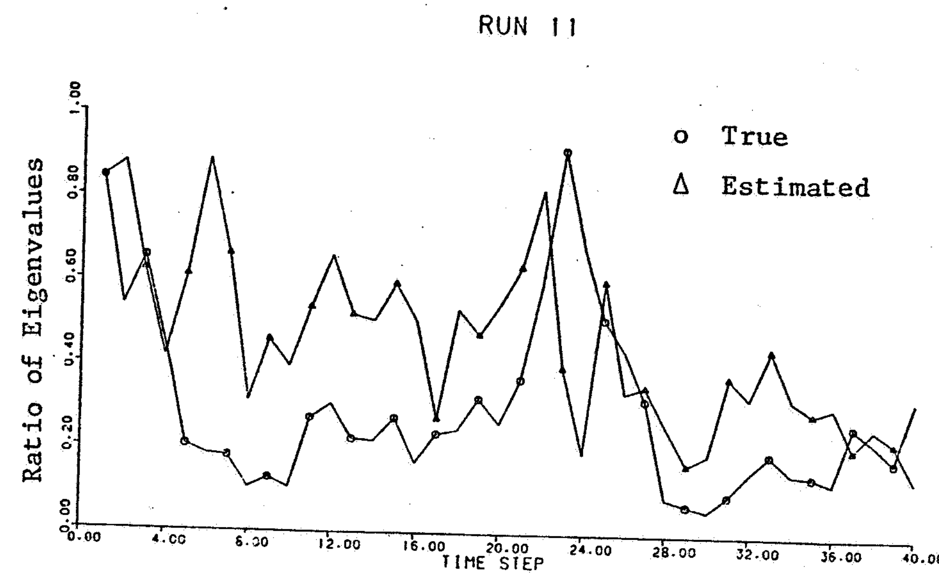

Fig. 84 Comparison of the ratio of the eigenvalues of the true and the estimated parameter transfer matrices for the dual controller

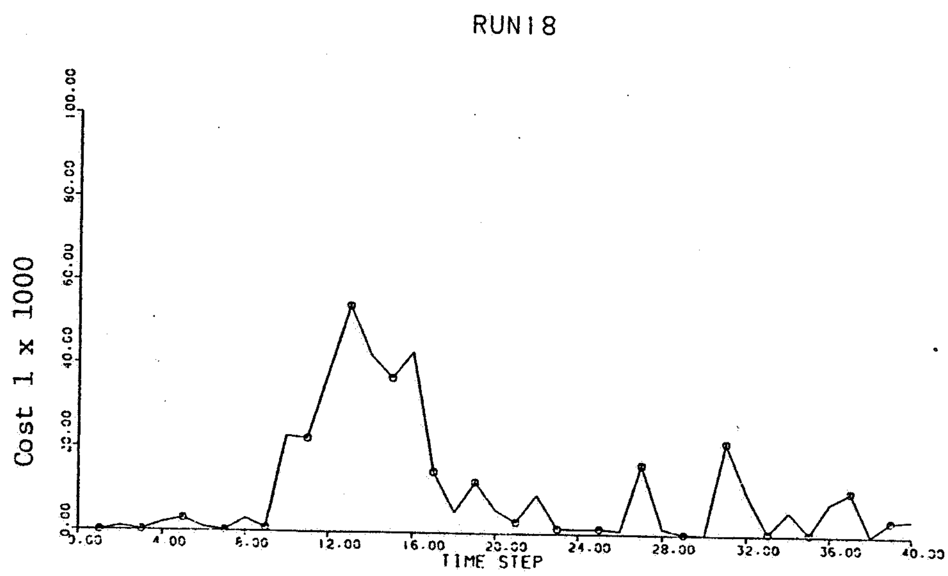

Fig. 85 Vibration contribution from the cosine component using the cautious controller (second method of initialization, Run 18)

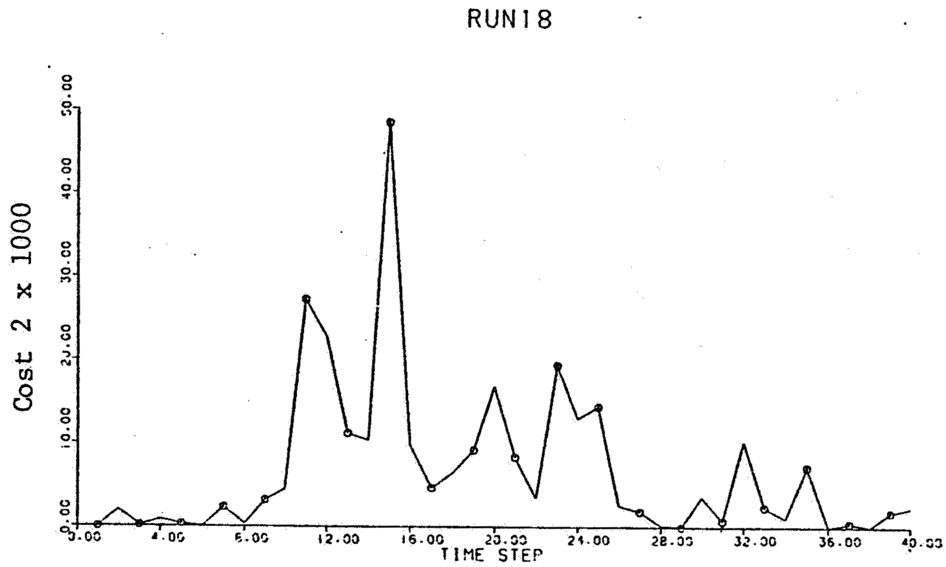

Fig. 86 Vibration contribution from the sine component using the cautious controller (second method of initialization, Run 18)

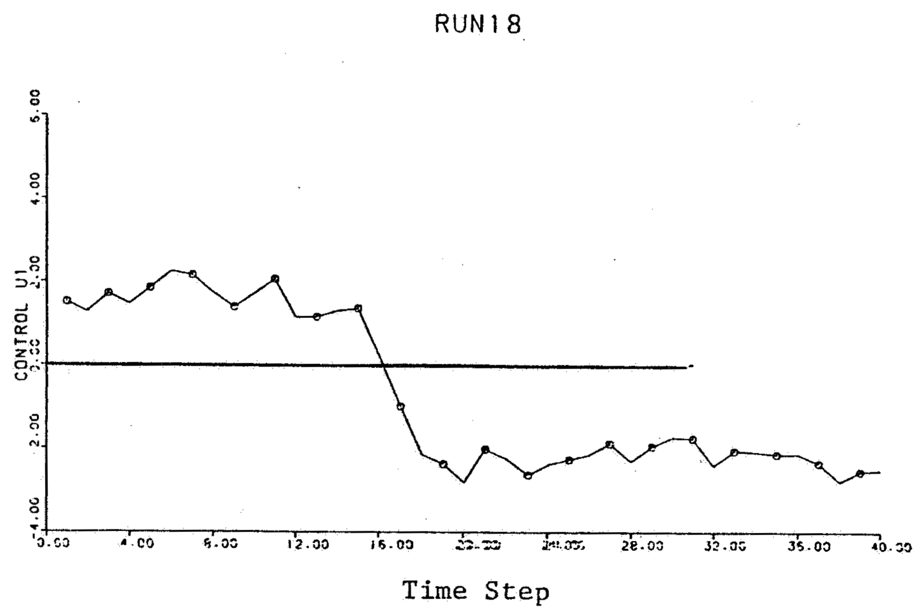

Fig. 87 Time history of control 1 used by the cautious controller

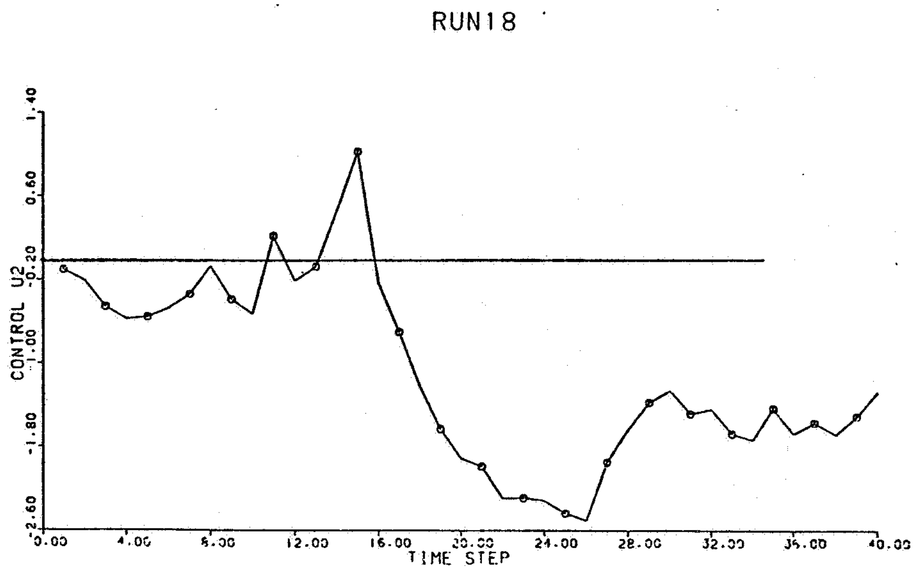

Fig. 88 Time history of control 2 used by the cautious controller

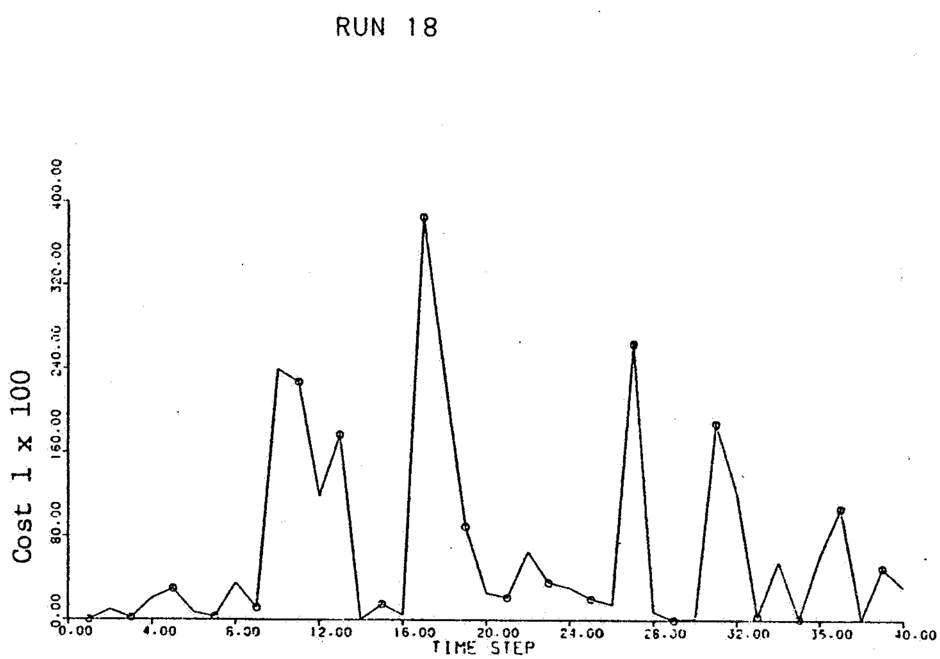

Fig. 89 Vibration contribution from cosine component using the dual controller (Run 18)

RUN 18

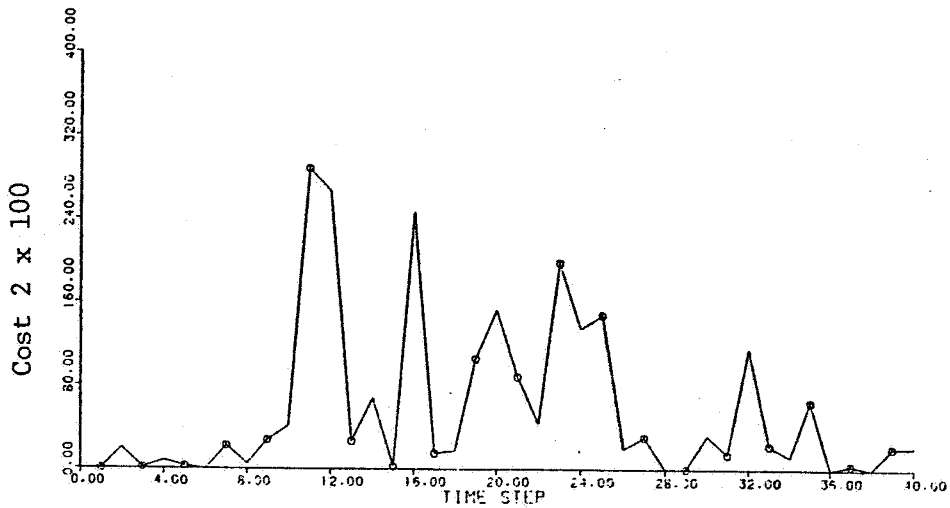

Fig. 90 Vibration contribution from the sine component using the dual controller (Run 18)

RUN18

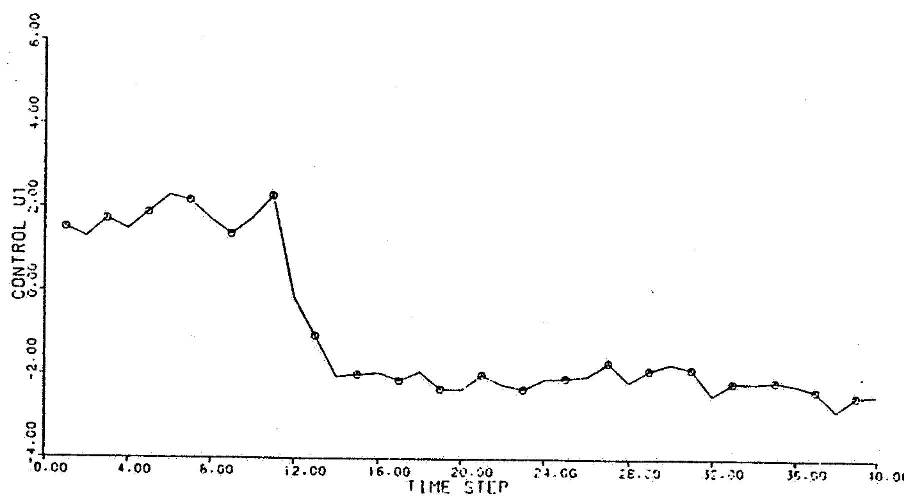

Fig. 91 Time history of control 1 used by the dual controller

RUN 18

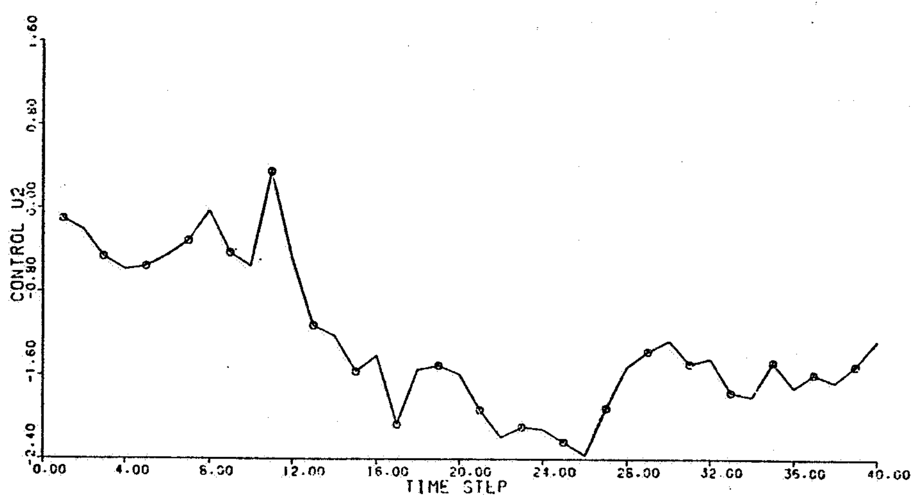

Fig. 92 **Time** history of control 2 used by the dual controller

RUN 18

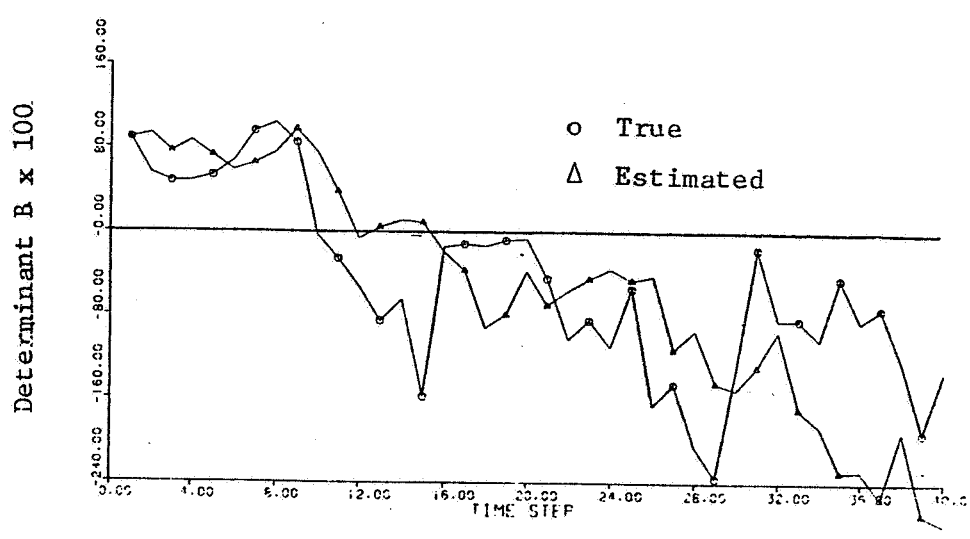

Fig. 93 Comparison of the determinants of the true and the estimated parameter transfer matrices for the cautious controller

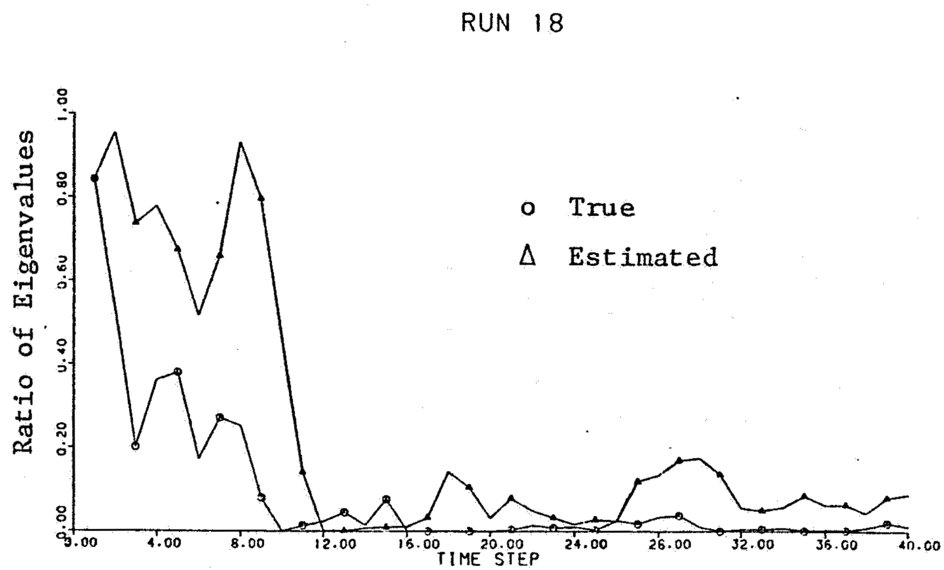

Fig. 94 Comparison of the ratio of the eigenvalues of the true and the estimated parameter transfer matrices for the cautious controller

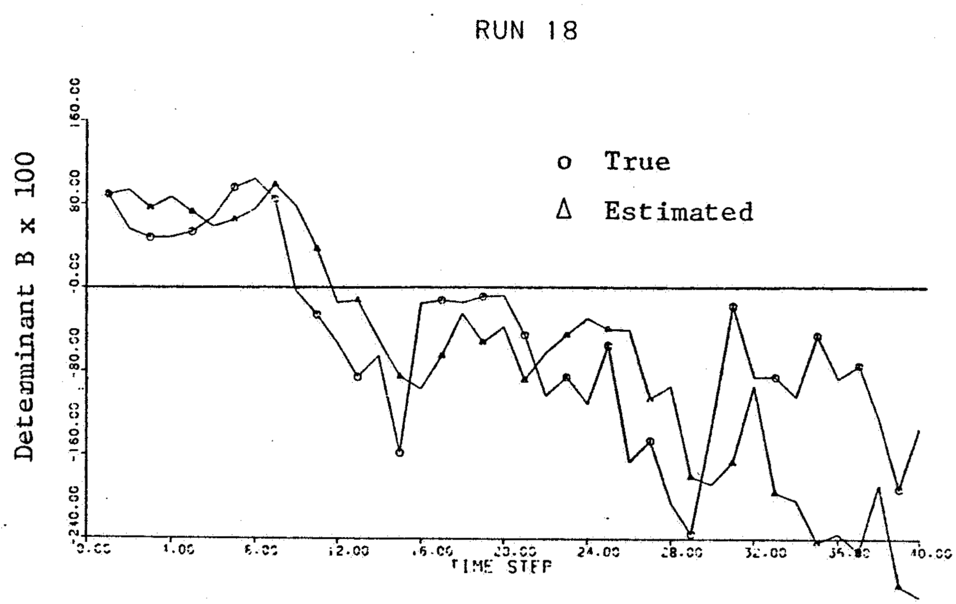

Fig. 95 Comparison of the determinants of the true and the estimated parameter transfer matrices for the dual controller

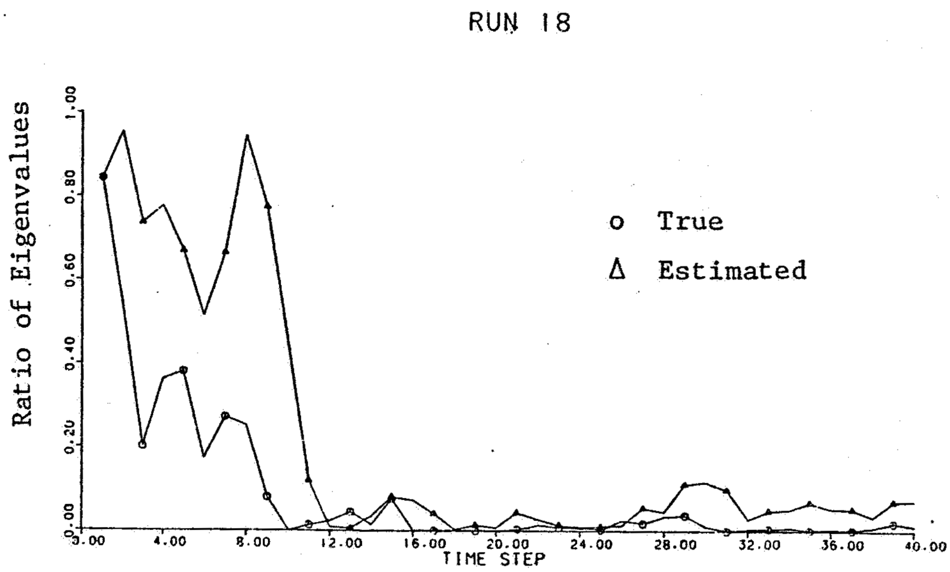

Fig. 96 Comparison of the ratio of the eigenvalues of the true and the estimated parameter transfer matrices for the dual controller

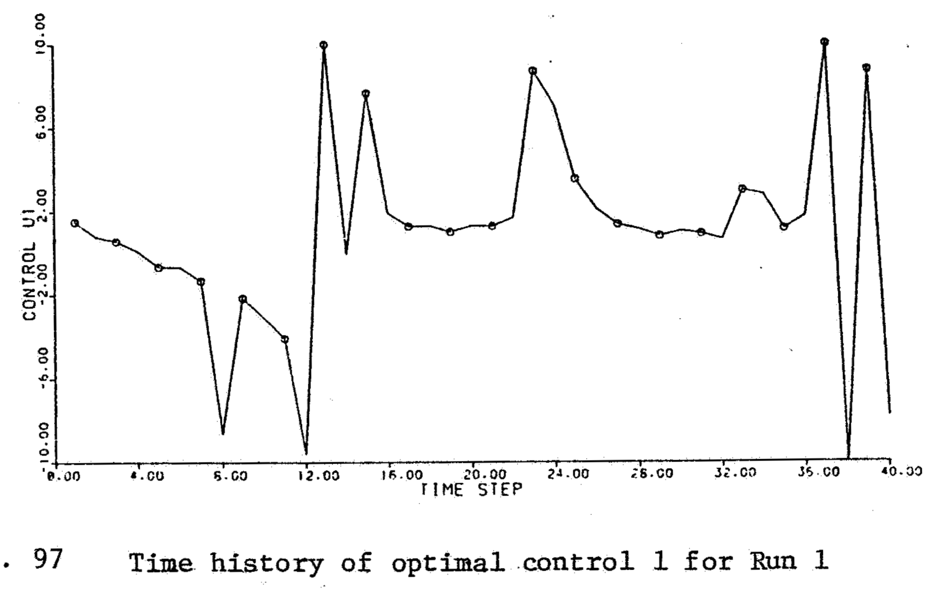

Fig. 97 Time history of optimal control 1 for Run 1

RUN 1

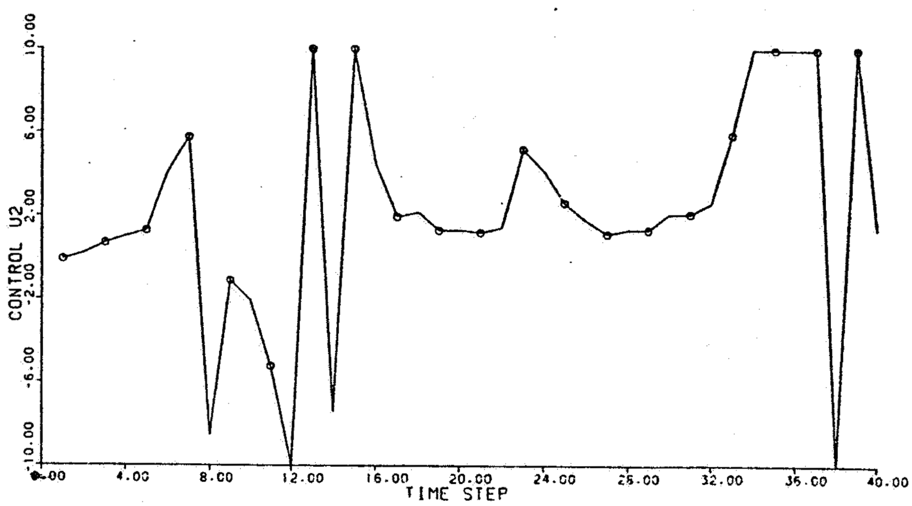

Fig. 98 Time history of optimal control 2 for Run 1

RUN 2

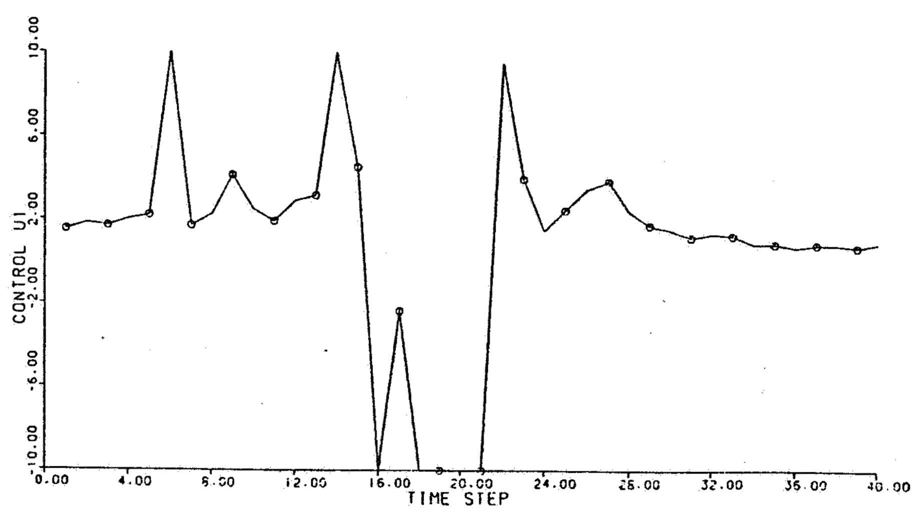

Fig. 99 Time history of optimal control 1 for Run 2

RUN 2

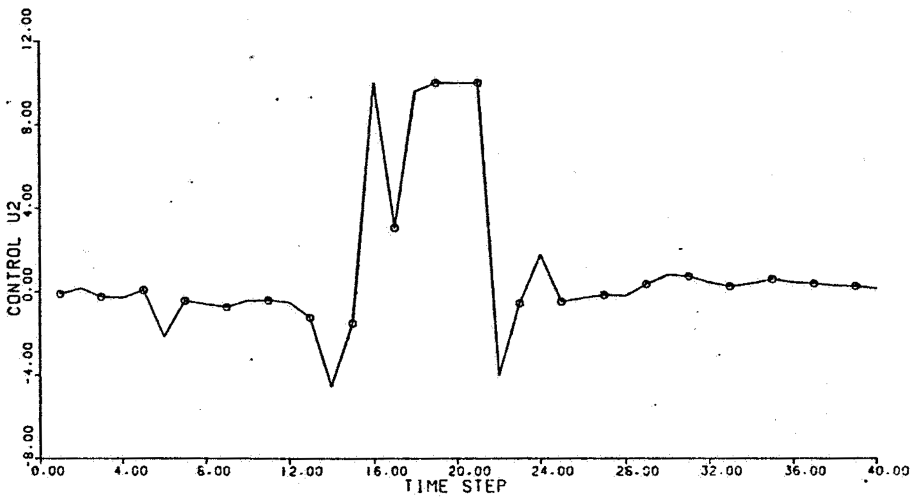

Fig. 100 Time history of optimal control 2 for Run 2

RUN 11

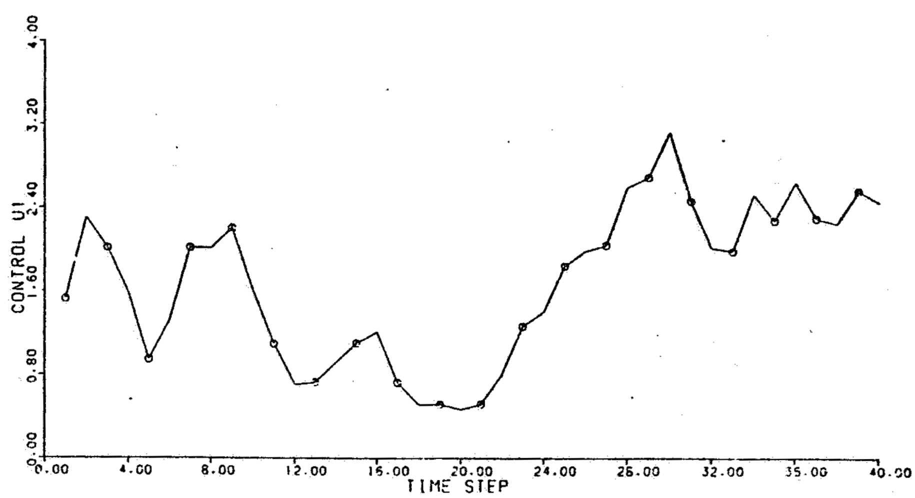

Fig. 101 Time history of optimal control 1 for Run 11

RUN 11

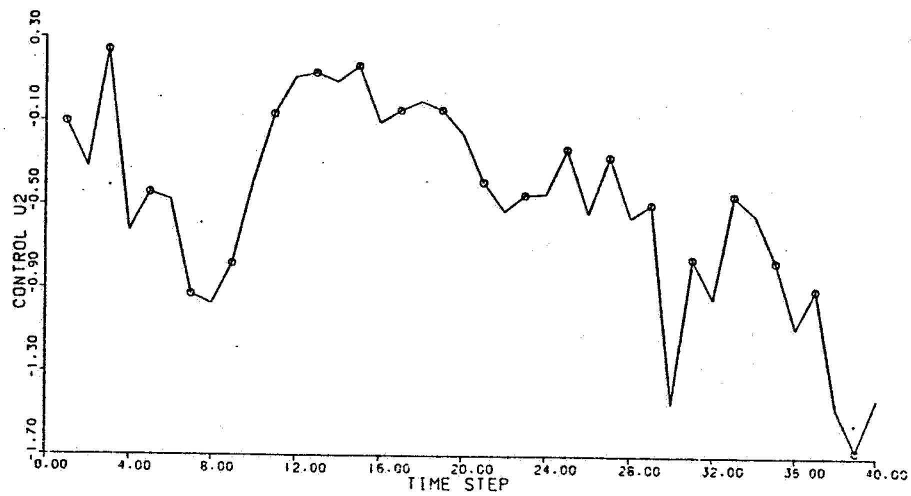

Fig. 102 Time history of optimal control 2 for Run 11

RUN 18

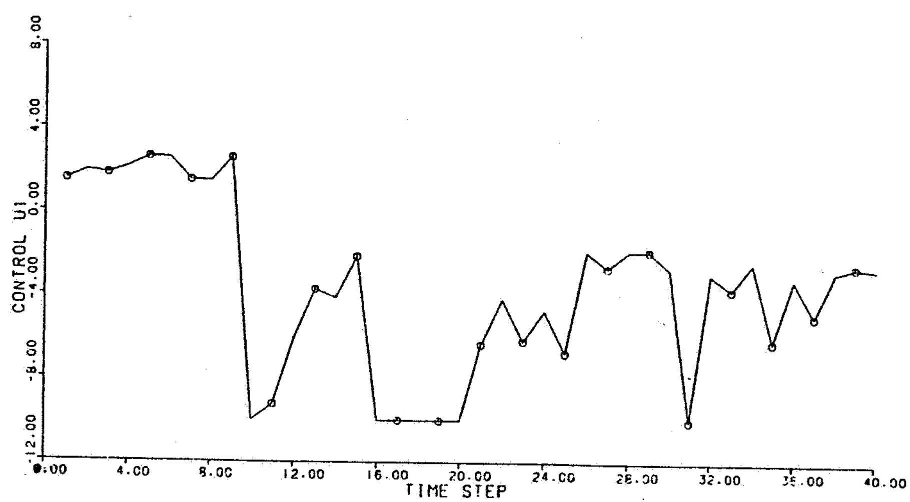

Fig. 103 Time history of optimal control 1 for Run 18

RUN 18

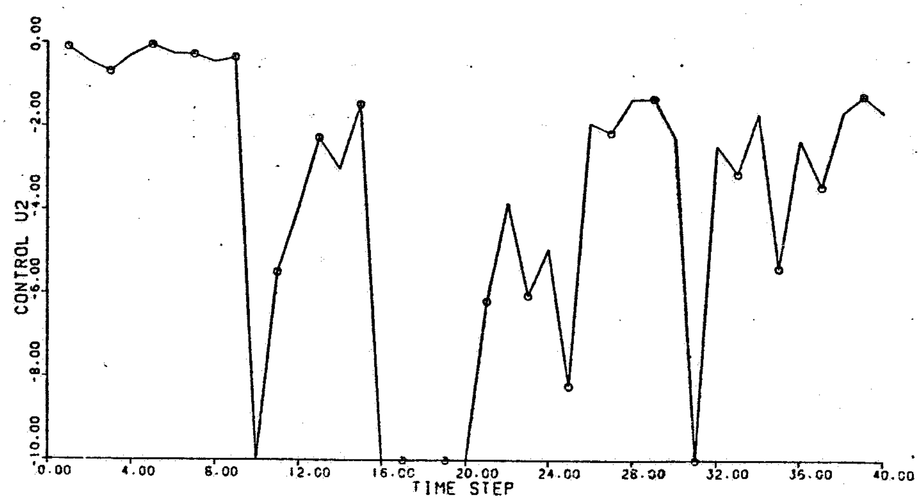

Fig.104 Time history of optimal control 2 for Run 18

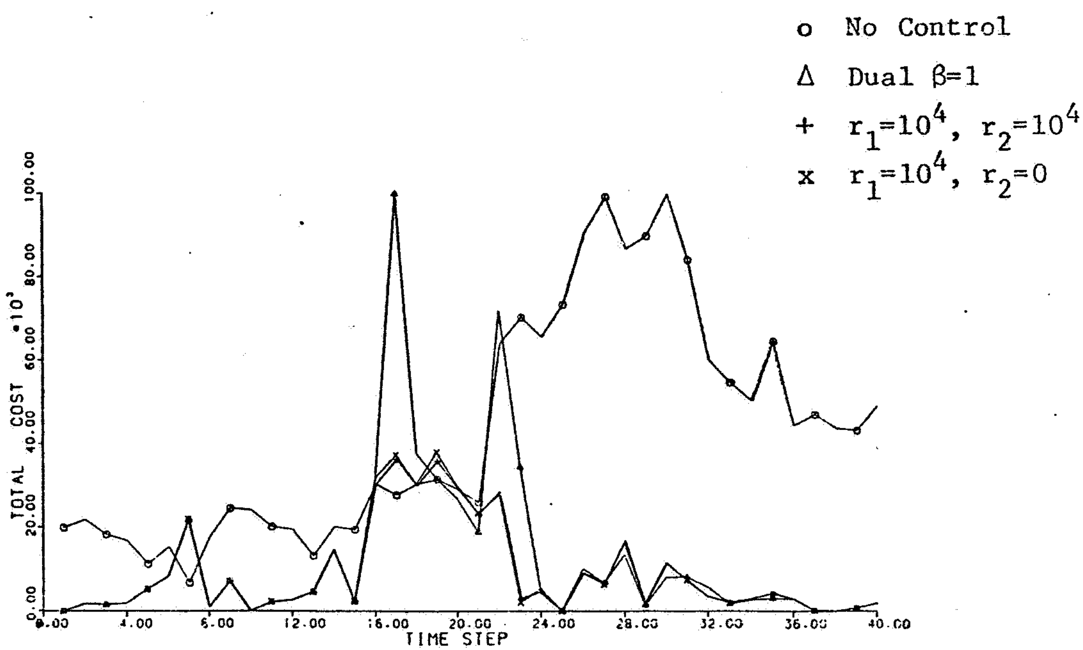

Fig. 105 Time history variation of the total cost with different switches on the control weights 'R' for Run 2. (Q=diag(1,1))

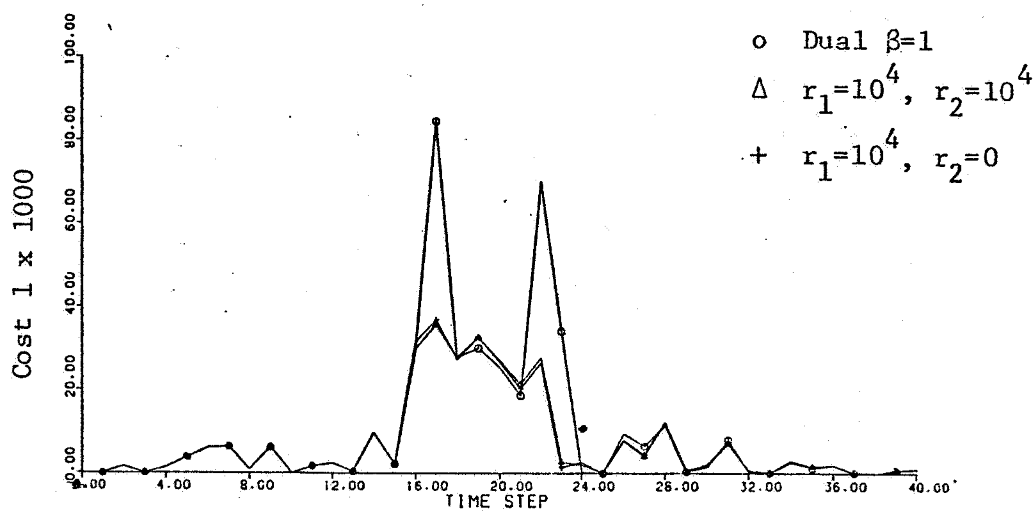

Fig. 106 Time history the variation of the cosine contribution with different switches on the control weight 'R' for Run 2. (Q=diag(1,1))

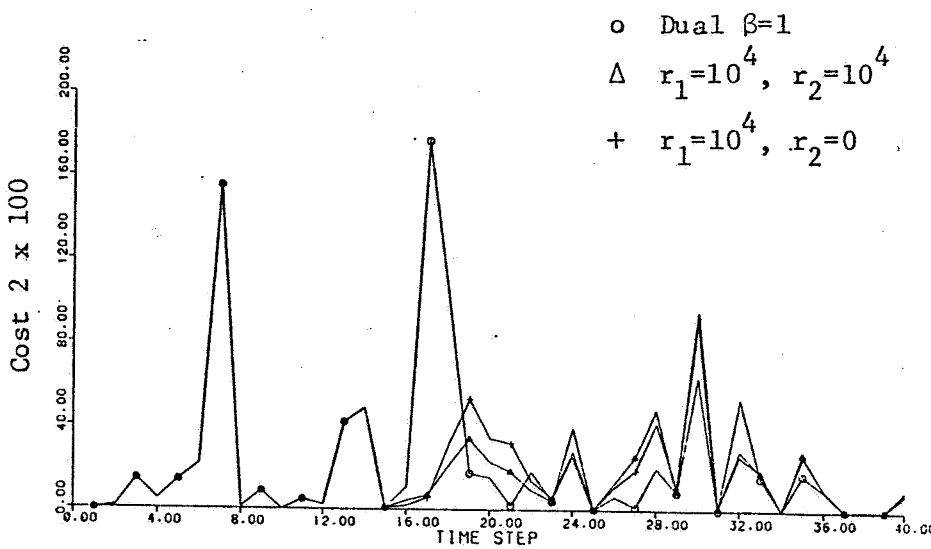

Fig. 107 Time history the variation of the sine contribution with different switches on the control weights 'R' for Run 2. (Q=diag(1,1))

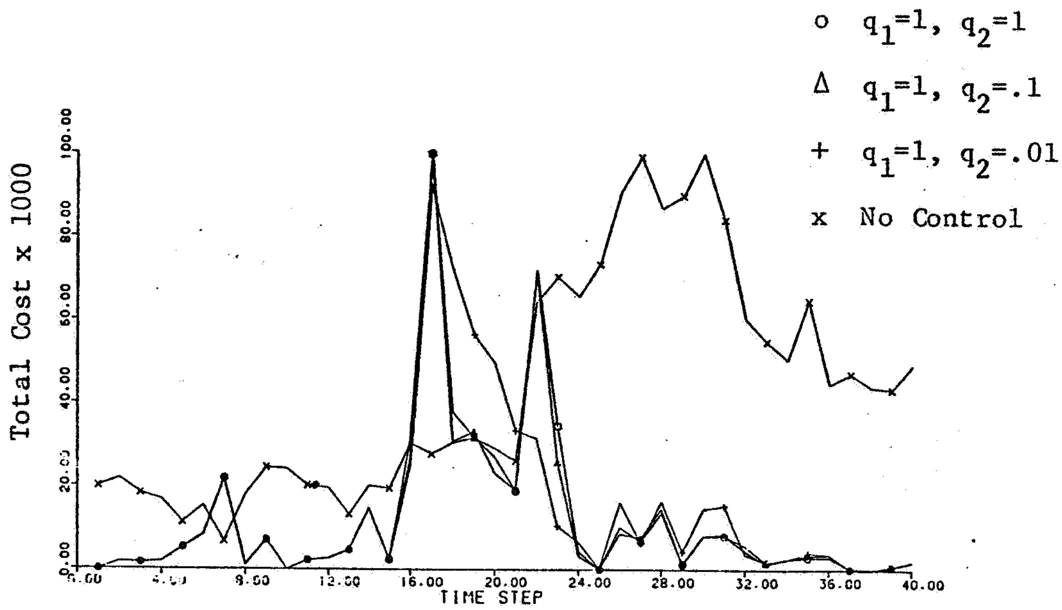

Fig. 108 Time history the variation of the total cost with different switches on the state weights 'Q' for Run 2. (R=diag(0,0))

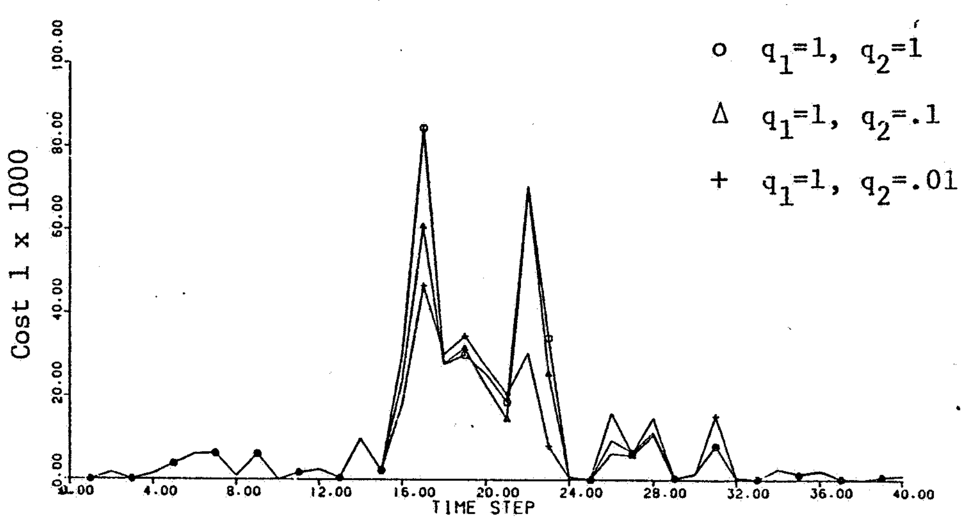

Fig. 109 Time history the variation of the cosine contribution with different switches on the state weights 'Q' for Run 2. (R=diag(0,0))

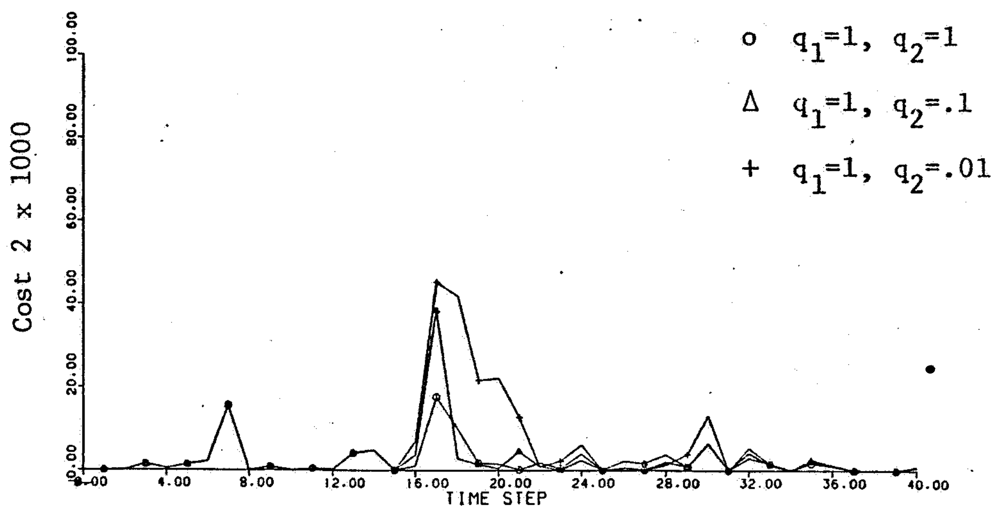

Fig. 110 Time history variation of the sine contribution with different switches on the state weights 'Q' for Run 2. (R=diag(.01,.01))

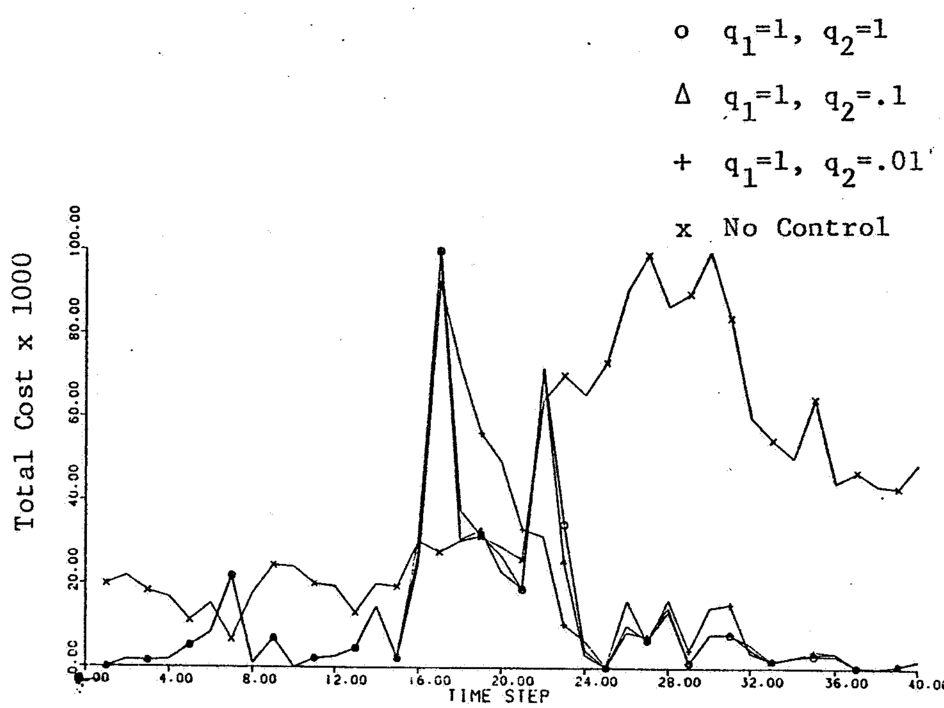

Fig. 111 Time history the variation of the total cost with different switches on the state weights 'Q' for Run 2. (R=diag(.01,.01))

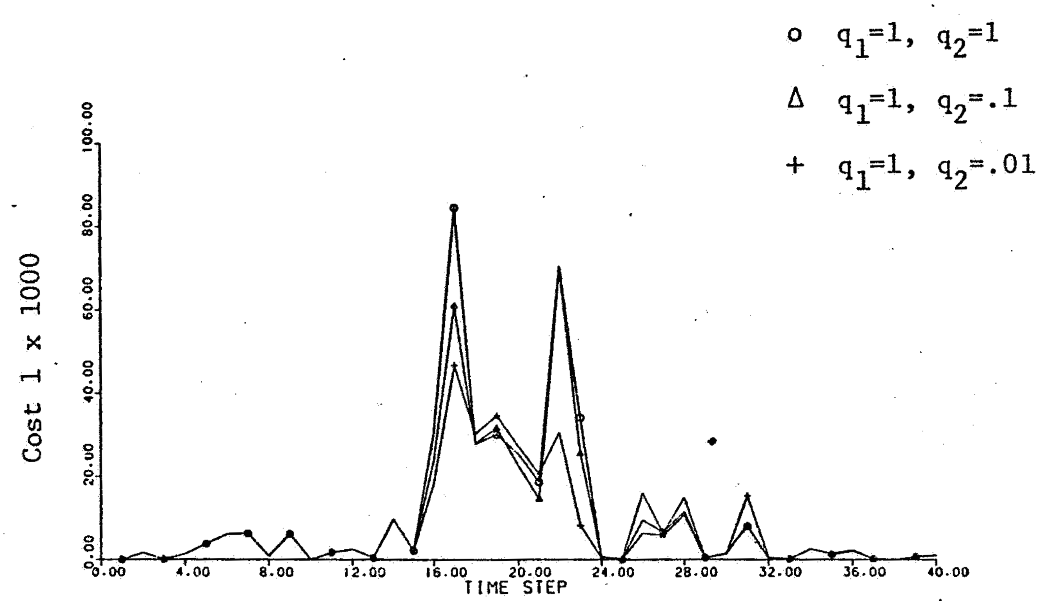

Fig. 112 Time history the variation of the cosine contribution with different switches on the state weights 'Q' for Run 2. (R=diag(.01,.01))

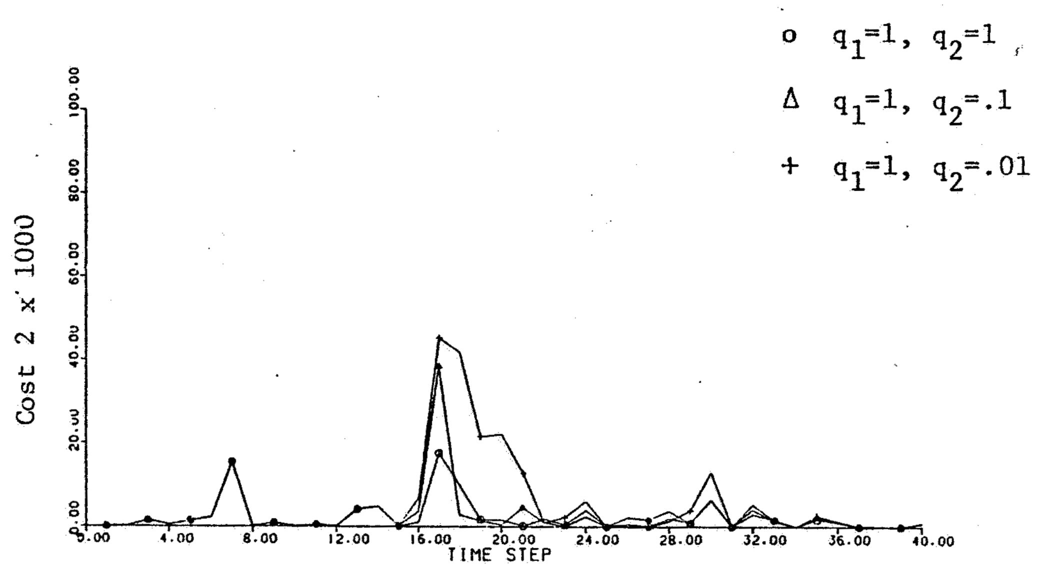

Fig. 113 Time history variation of the sine contribution with different switches on the state weights 'Q' for Run 2. (R=diag(.01,.01))

NORMAL INITIAL VARIANCE

BETA=0.0, 1.0; 10% NOISE

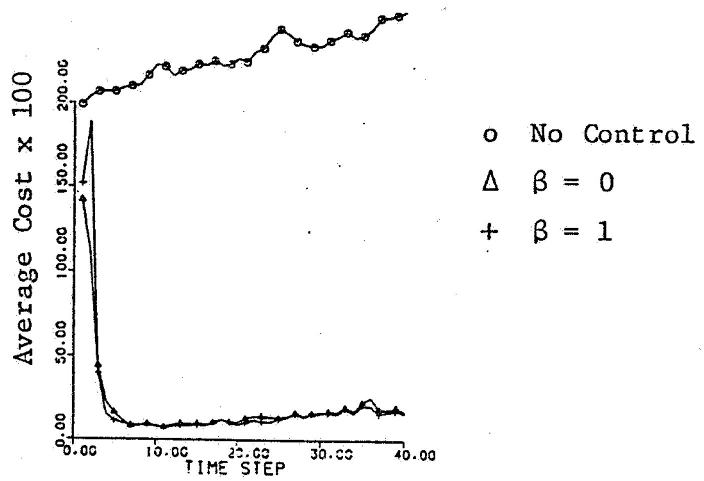

Fig. 114 Effect of the initial covariance (first method of initialization) on the cautious and the dual controllers

NORMAL INITIAL VARIANCE/4

BETA=0.0, 1.0, 10% NOISE

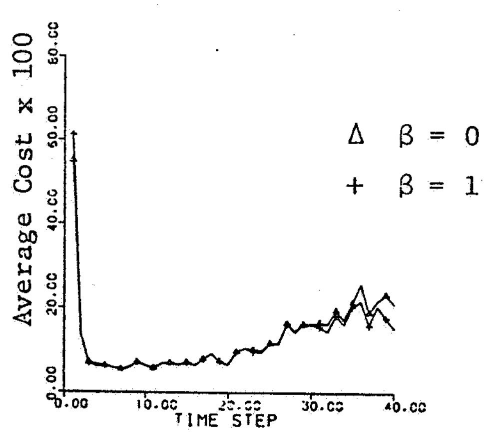

Fig. 115 Effect of the initial covariance (first method of initialization) on the cautious and the dual controllers

NORMAL INITIAL VARIANCE*4

BETA=0.0, 1.0, 10% NOISE

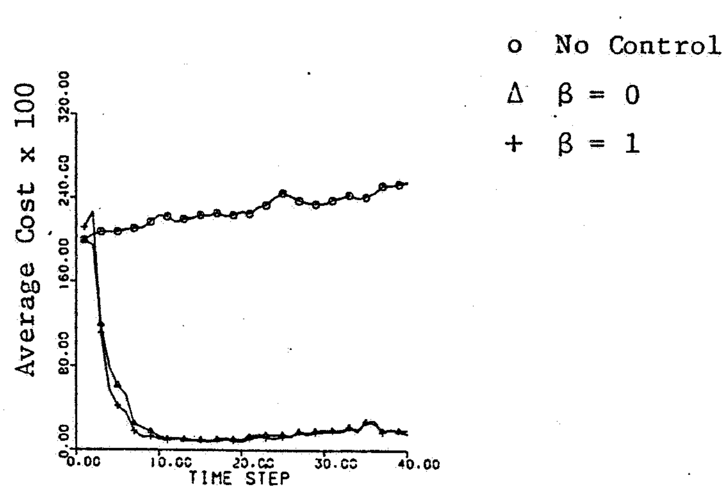

Fig. 116 Effect of the initial covariance (first method of initialization) on the cautious and the dual controllers

NORMAL INITIAL VARIANCE

BETA=0.0, 1.0, 10% NOISE

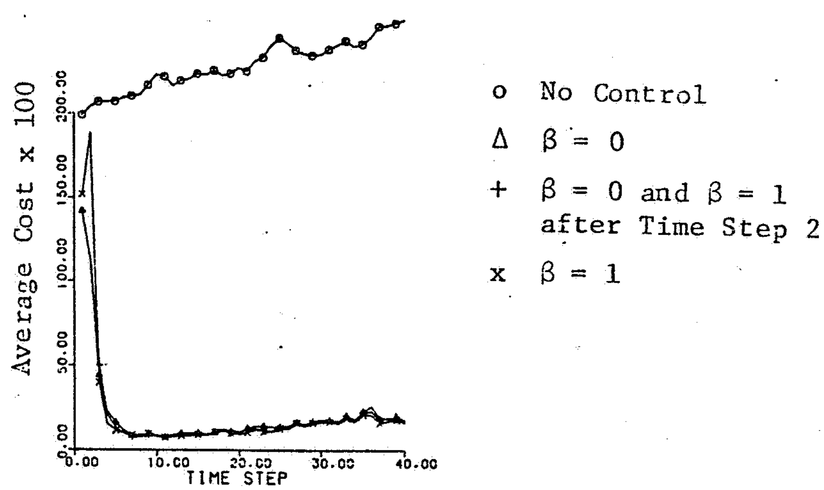

Fig. 117 Effect of the initial covariance (first method of initialization) on the cautious and the dual controllers

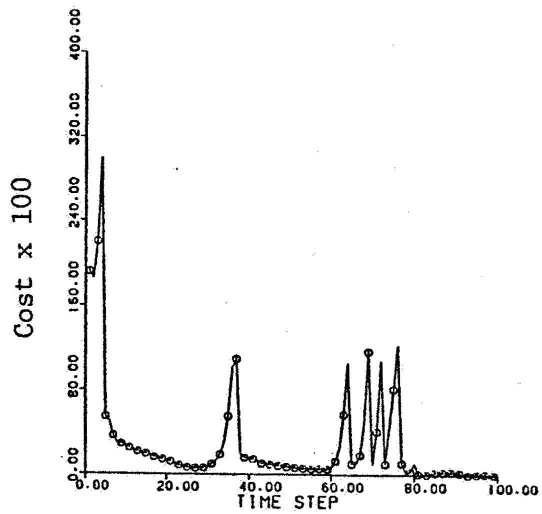

Fig. 118 Time history convergence of cost for the global linear adaptive cautious controller showing the divergence points when the control values are in a very nonlinear region. Q=diag(10^{-5}, 5×10^{-8}) ; R=diag(10^{-4}, 10^{-4})

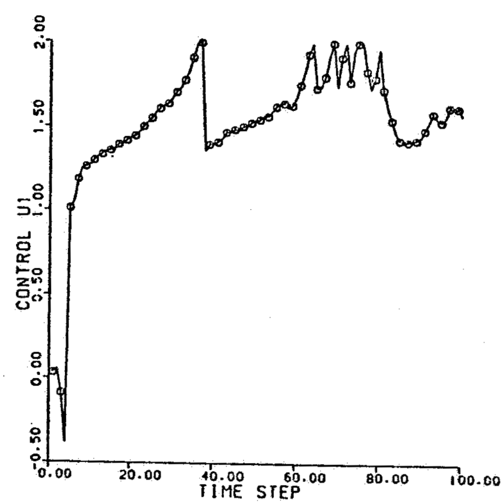

Fig. 119 Time history convergence of Control 1 for the global linear adaptive cautious controller showing the divergence points when the control values are in a very nonlinear region. Q=diag(10^{-5}, 5×10^{-8}) ; R=diag(10^{-4}, 10^{-4})

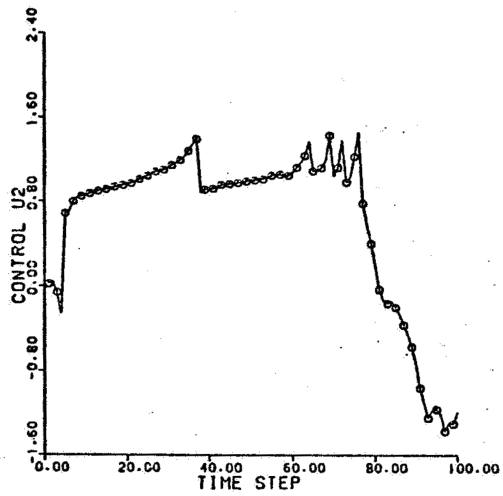

Fig. 120 Time history convergence of Control 2 for the global linear adaptive cautious controller showing the divergence points when the control values are in a very nonlinear region. $Q=diag(10^{-5}, 5x10^{-8})$; $R=diag(10^{-4}, 10^{-4})$

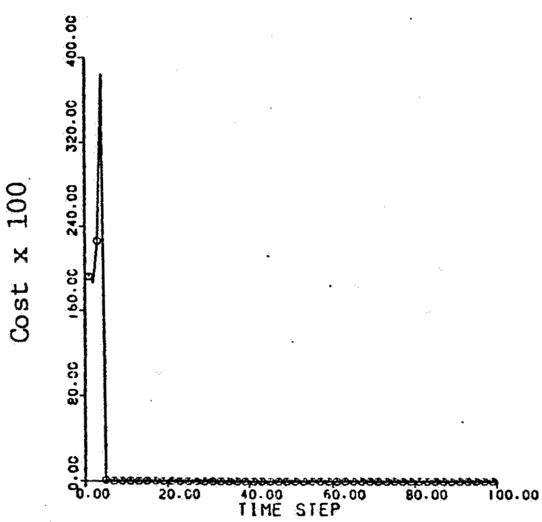

Fig. 121 Time history convergence of cost for the global linear adaptive cautious controller showing the divergence points when the control values are in a very nonlinear region. Q=diag(1,1) ; R=diag(0,0)

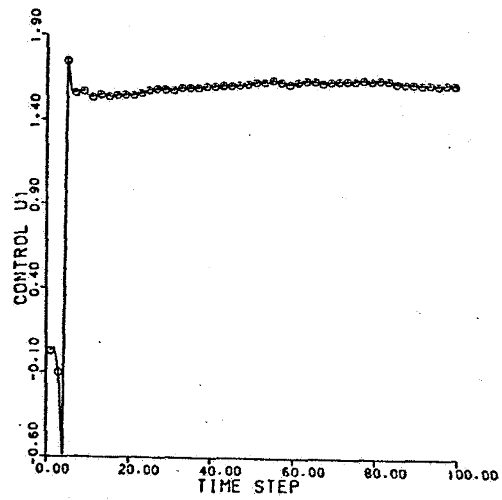

Fig. 122 Time history convergence of Control 1 for the global linear adaptive cautious controller showing the divergence points when the control values are in a very nonlinear region. Q=diag(1,1) ; R=diag(0,0)

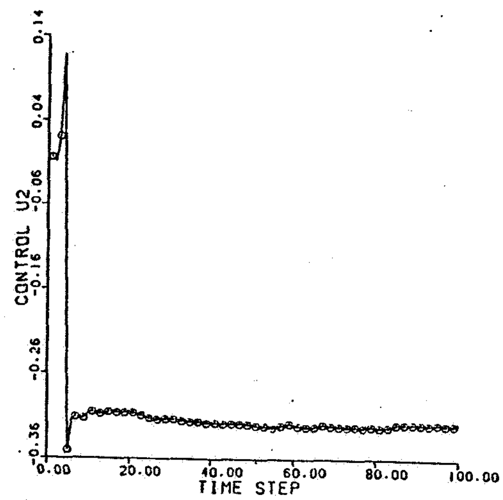

Fig. 123 Time history convergence of Control 2 for the global linear adpative cautious controller showing the divergence points when the control values are in a very nonlinear region. Q=diag(1,1) ; R=diag(0,0)

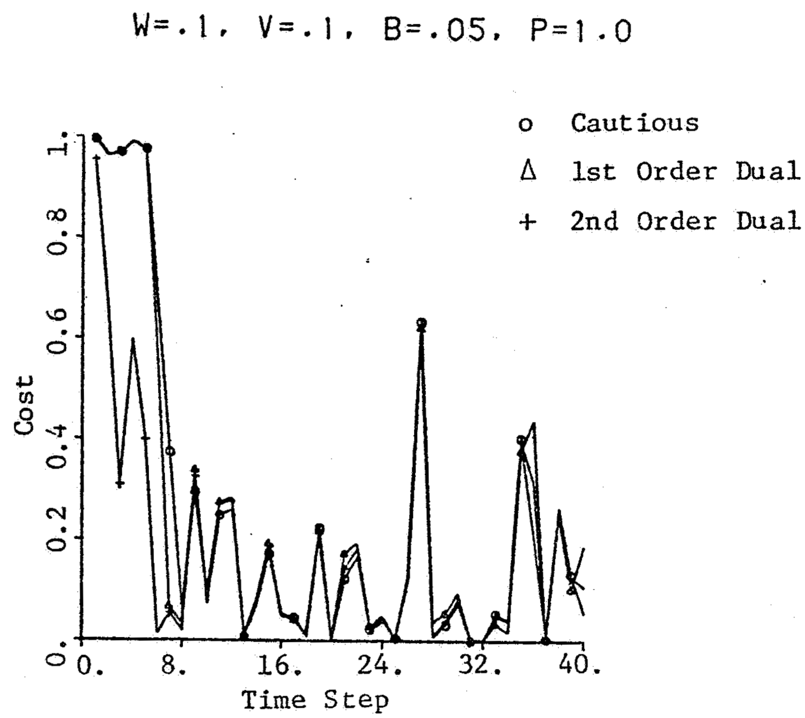

Fig. 124 Comparison of the costs using the cautious, the first order dual and the new dual (Time varying parameter case. Run 2 from 100 Monte Carlo Runs)

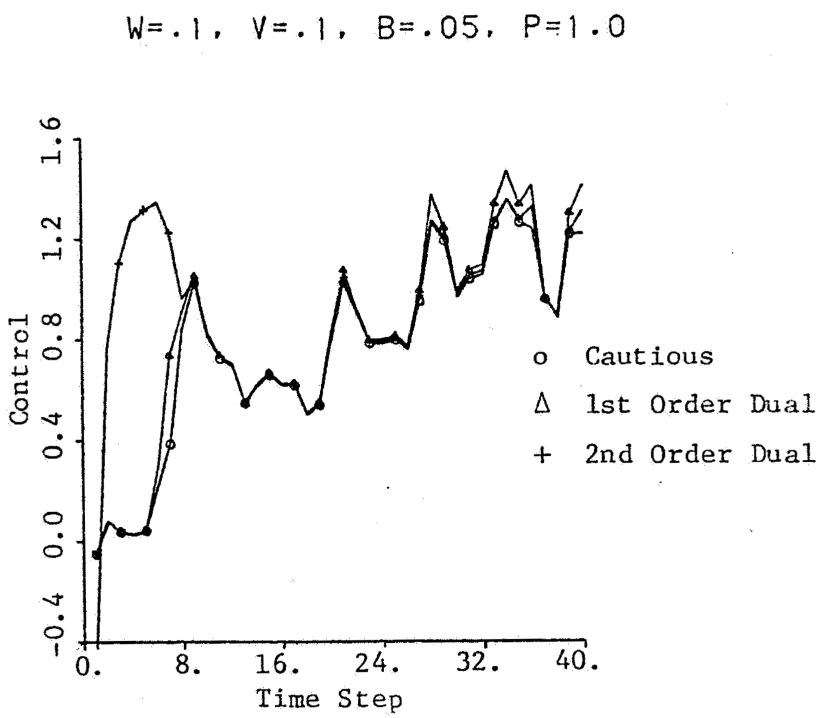

Fig. 125 Comparison of the controls using the cautious, the first order dual and the new dual (Time varying parameter case. Run 2 from 100 Monte Carlo Runs)

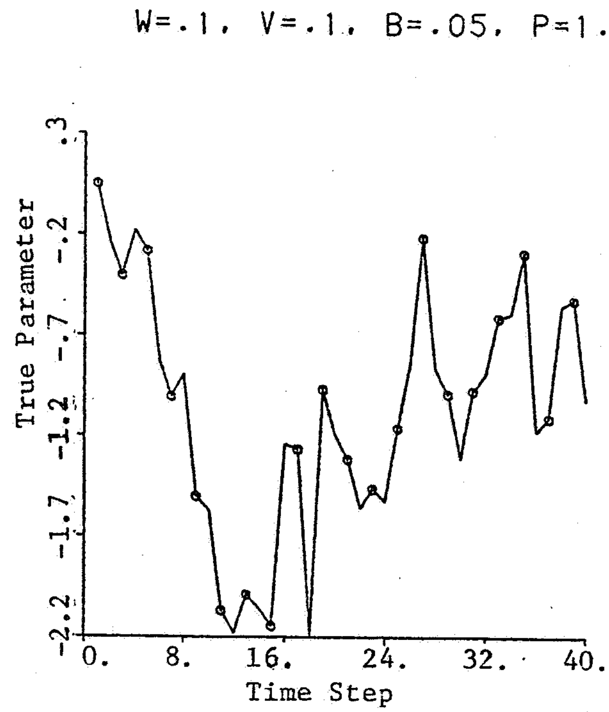

Fig. 126 Time history of the true parameter for Run 2 from 100 Monte Carlo Runs (Time Varying Case)

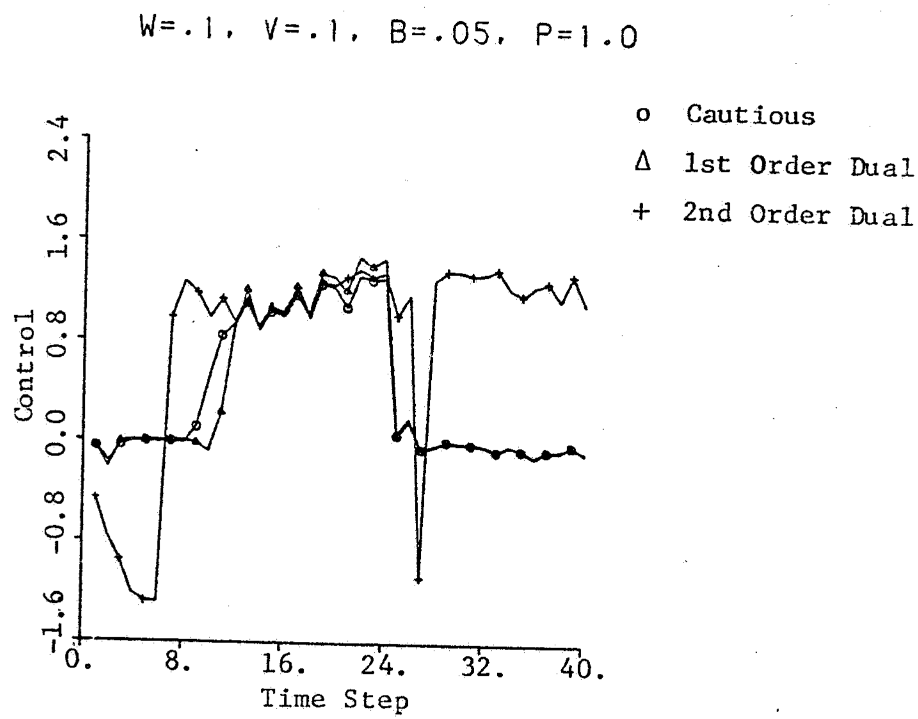

Fig. 127 Comparison of the controls using the cautious, the first order dual and the new dual (Time varying parameter case: Run 7 from 100 Monte Carlo Runs)

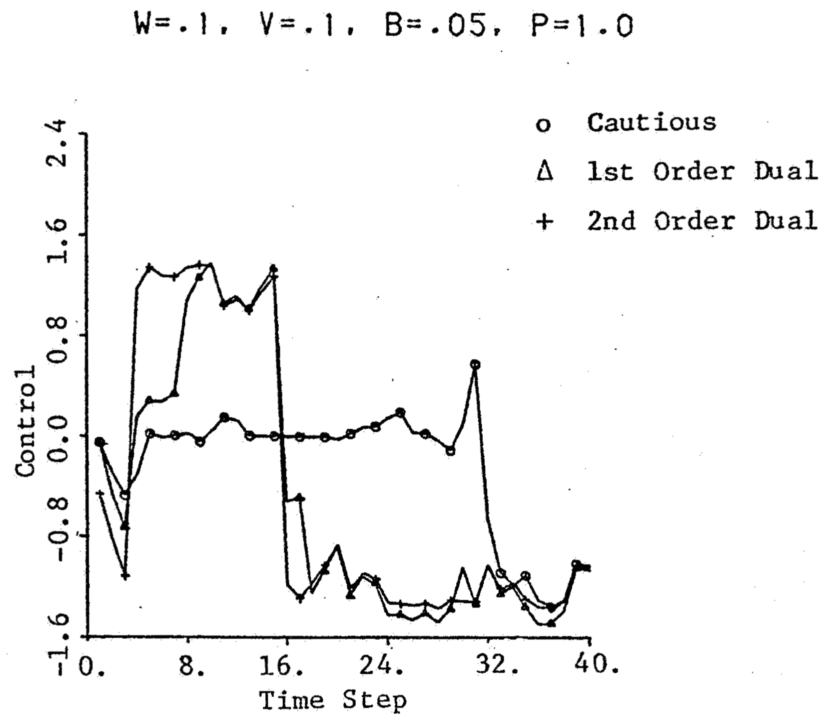

Fig. 128 Comparison of the controls using the cautious, the first order dual and the new dual (Time varying parameter case: Run 14 from 100 Monte Carlo Runs)

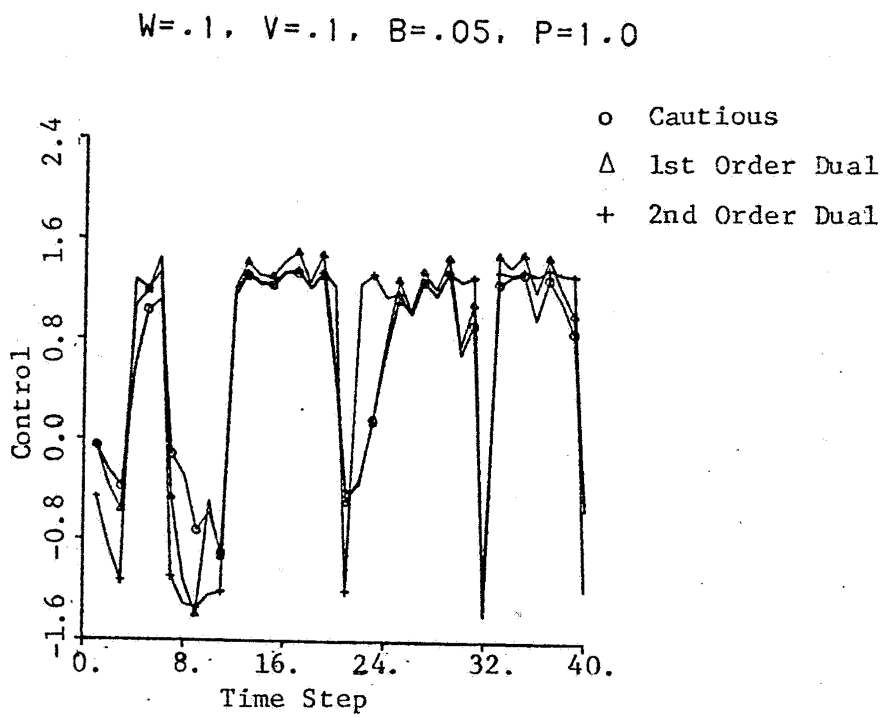

Fig. 129 Comparison of the controls using the cautious, the first order dual and the new dual (Time varying parameter case: Run 21 from 100 Monte Carlo Runs)

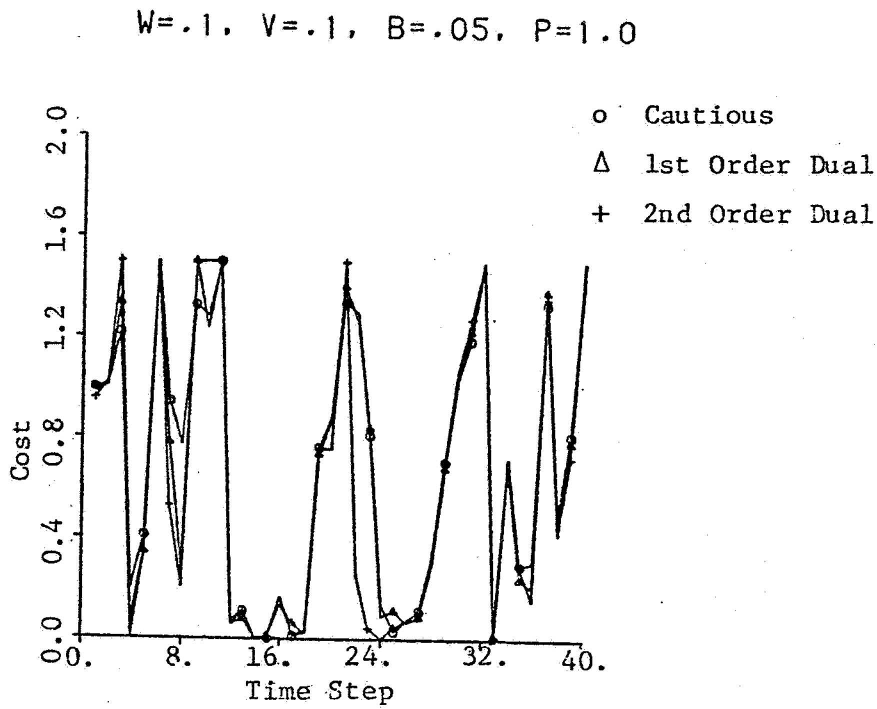

Fig. 130 Comparison of the cost using the cautious, the first order dual and the new dual (Time varying parameter case: Run 21 from 100 Monte Carlo Runs)

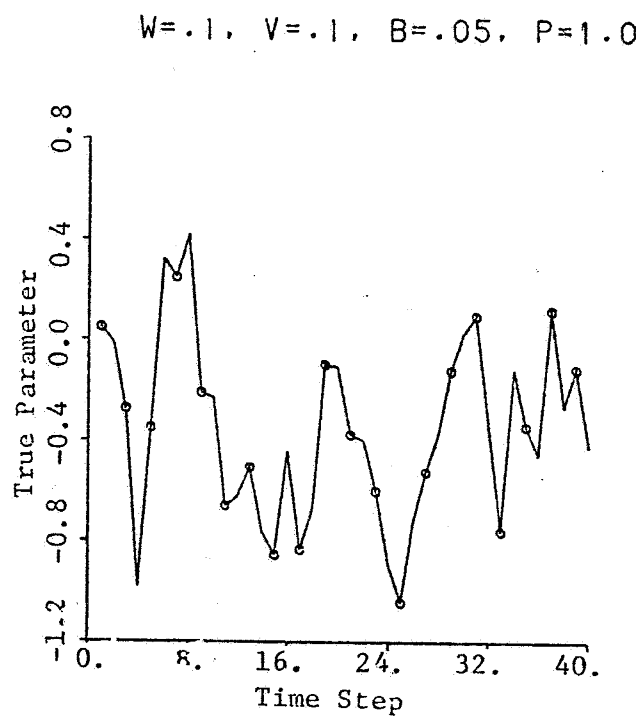

Fig. 131 Time history of the true parameter for Run 21 from 100 Monte Carlo Runs (Time varying case)

W=.1, V=.0, B=.05, P=1.0

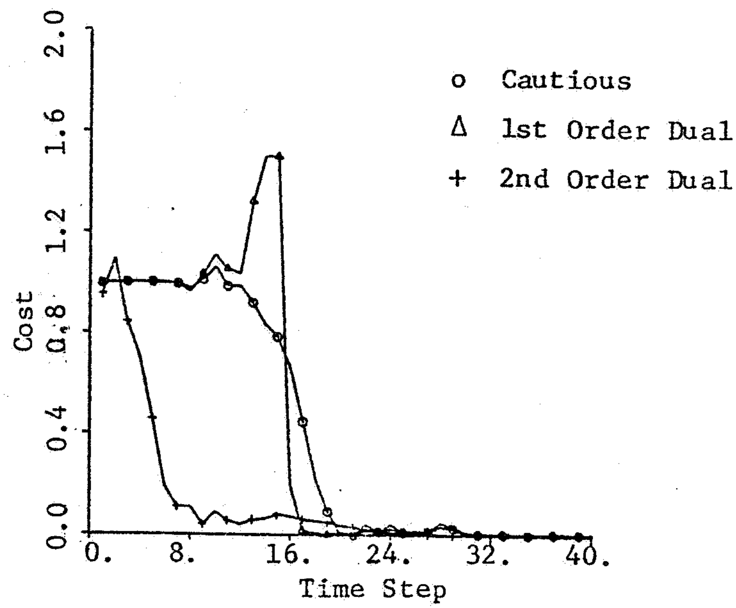

Fig. 132 Time history of the cost using the cautious, dual and the new dual solutions (Constant parameter case: Run 18 from 100 Monte Carlo Runs)

W=.1, V=.0, B=.05, P=1.0

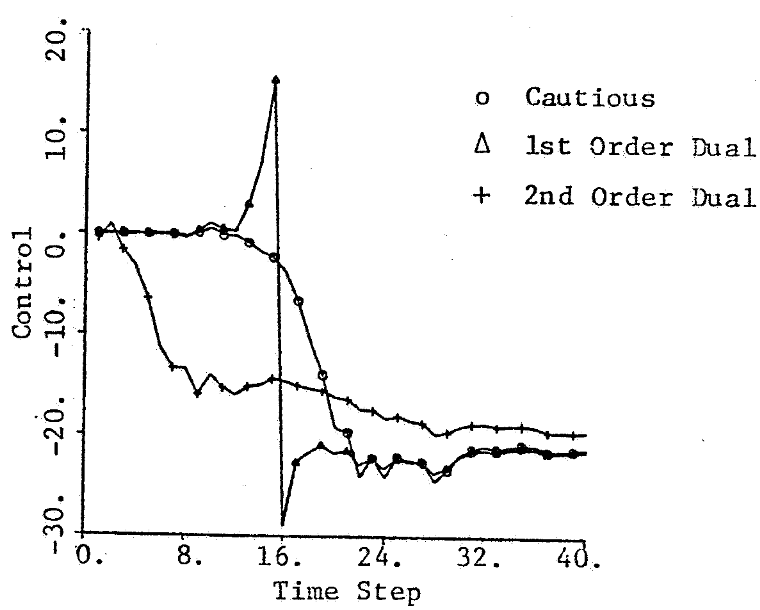

Fig. 133 Time history of the control using the cautious, dual and the new dual solutions (Constant parameter case: Run 18 from 100 Monte Carlo Runs)

W=.1, V=.0, B=.05, P=1.0

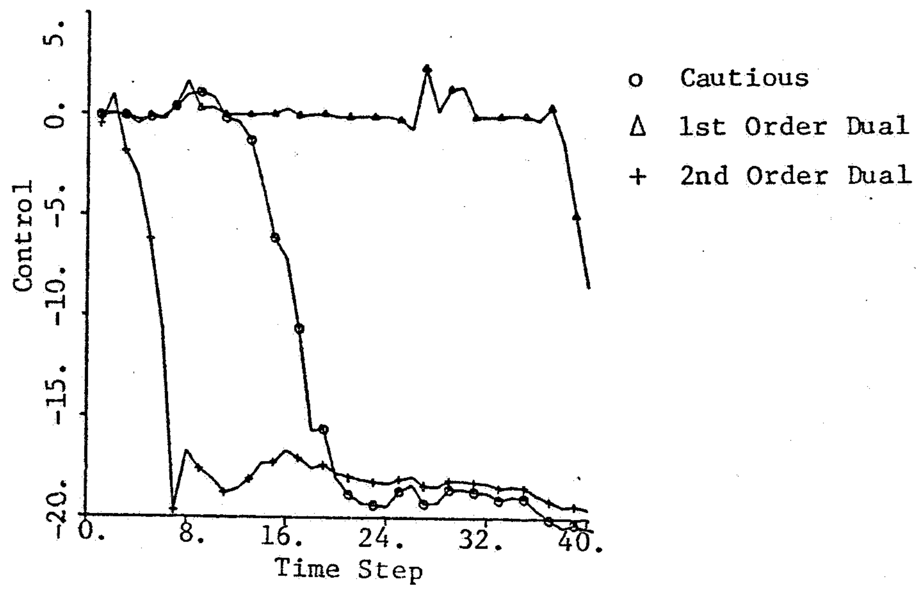

Fig. 134 Time history of the control using the cautious, dual and the new dual solutions (Constant parameter case: Run 26 from 100 Monte Carlo Runs)

W=.1, V=.0, B=.05, P=1.0

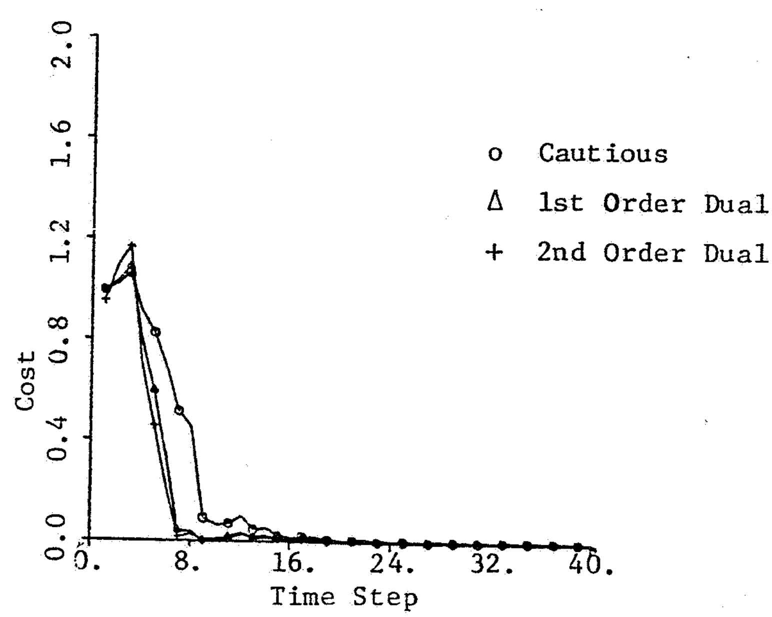

Fig. 135 Time history of the cost using the cautious, dual and the new dual solutions (Constant parameter case: Run 44 from 100 Monte Carlo Runs)

W=.1, V=.0, B=.05, P=1.0

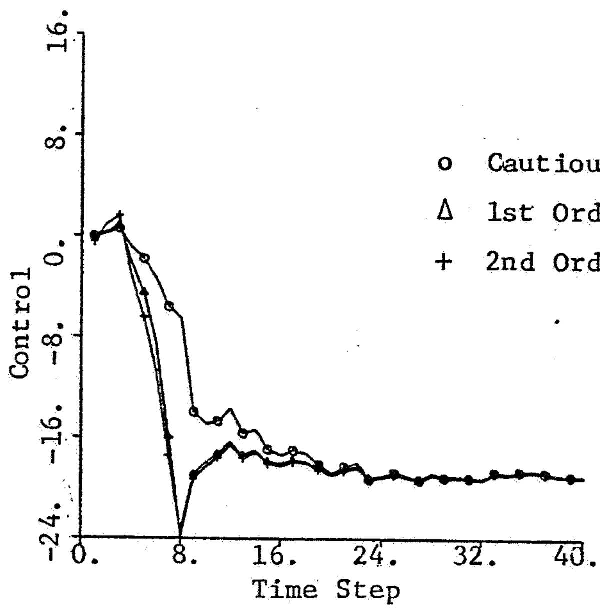

Fig. 136 Time history of the control using the cautious, dual and the new dual solutions (Constant parameter case: Run 44 from 100 Monte Carlo Runs)

W=.1, V=.0, B=.05, P=1.0

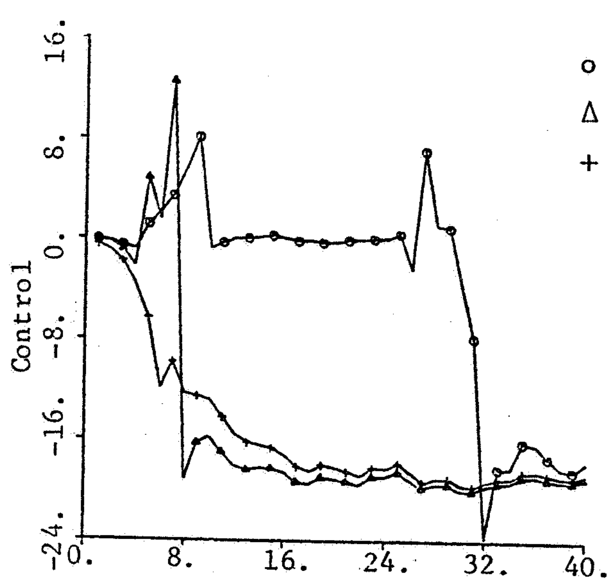

Fig. 137 Time history of the control using the cautious, dual and the new dual solutions (Constant parameter case: Run 80 from 100 Monte Carlo Runs)

W=.1, V=.1, B=.05, P=1.0

RUN NUMBER 7

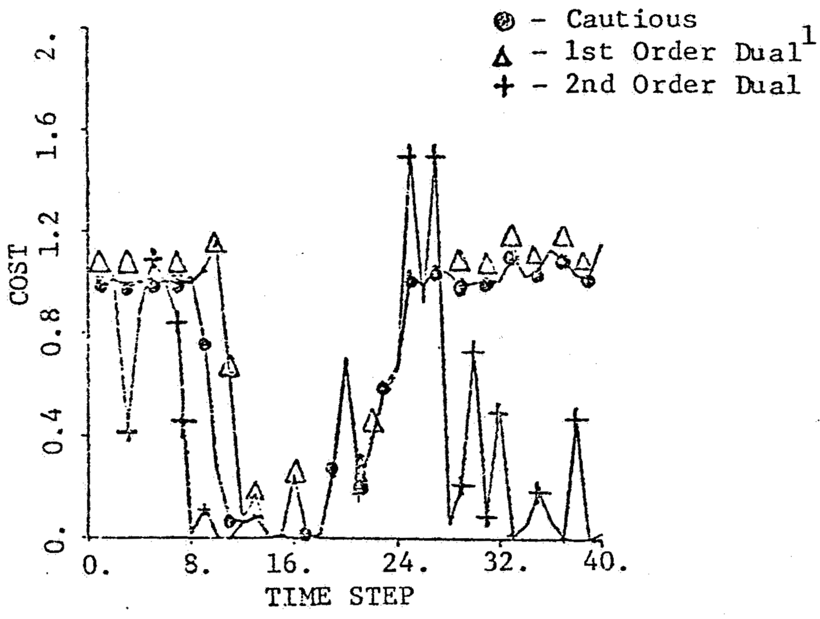

Figure 138. Time history of cost comparing the new dual, dual of Reference 1 and 2, and the cautious controller (Time varying parameter case: Run No. 7 form 100 Monte Carlo Runs)

W=.1, V=.1, B=.05, P=1.0

RUN NUMBER 14

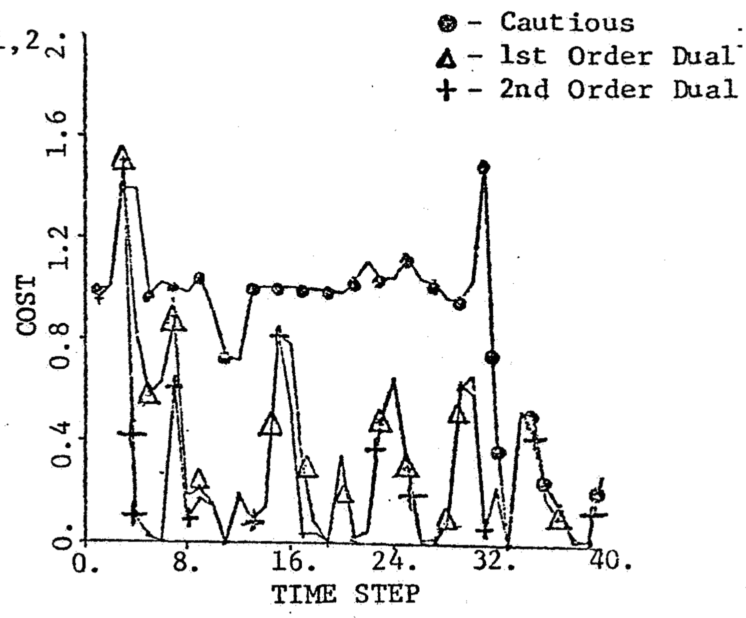

Figure 140. Time history of cost comparing the new dual, dual of Reference 1 and 2, and the cautious controller (Time varying parameter case: Run No. 14 from 100 Monte Carlo Runs)

W=.1, V=.1, B=.05, P=1.0

RUN NUMBER 7

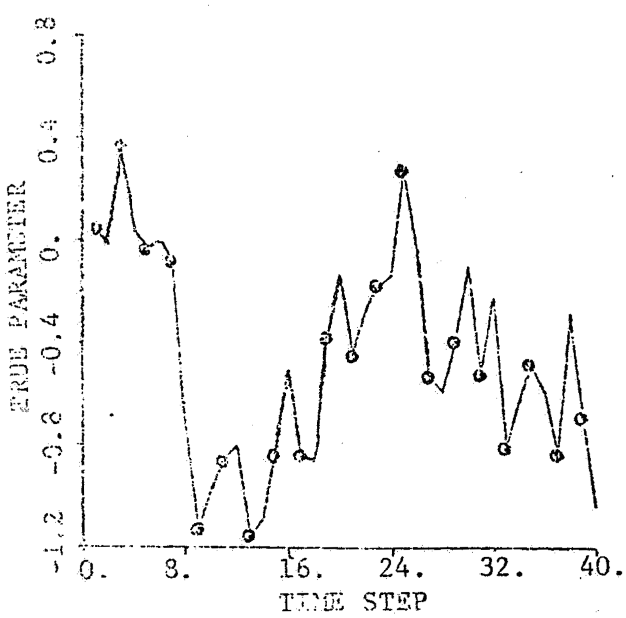

Figure 139. Time history of parameter for Run No. 7 from the 100 Monte Carlo Runs (Time Varying Case)

W=.1, V=.1, B=.05, P=1.0

RUN NUMBER 14

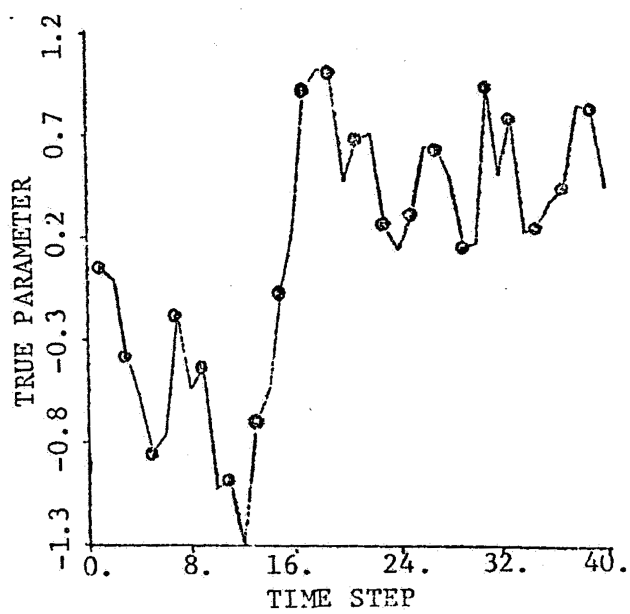

Figure 141. Time history of parameter for Run No. 14 from the 100 Monte Carlo Runs (Time Varying Case)

W = .1, V = .0, B = .05, P = 1.0

RUN NUMBER 26

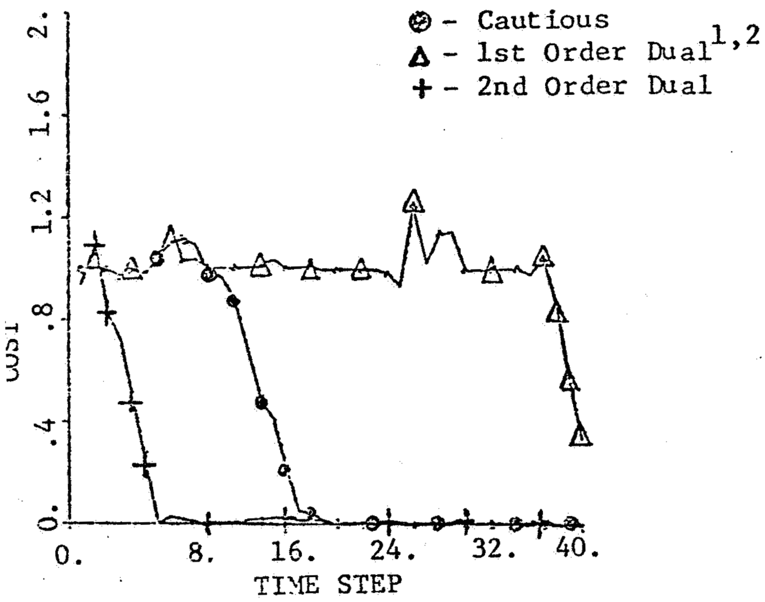

Figure 142. Time history of cost comparing the new dual, dual of Reference 1 and 2, and the cautious controller (Constant parameter case: Run No. 26 from 100 Monte Carlo Runs)

W = .1, V = .0, B = .05, P = 1.0

RUN NUMBER 80

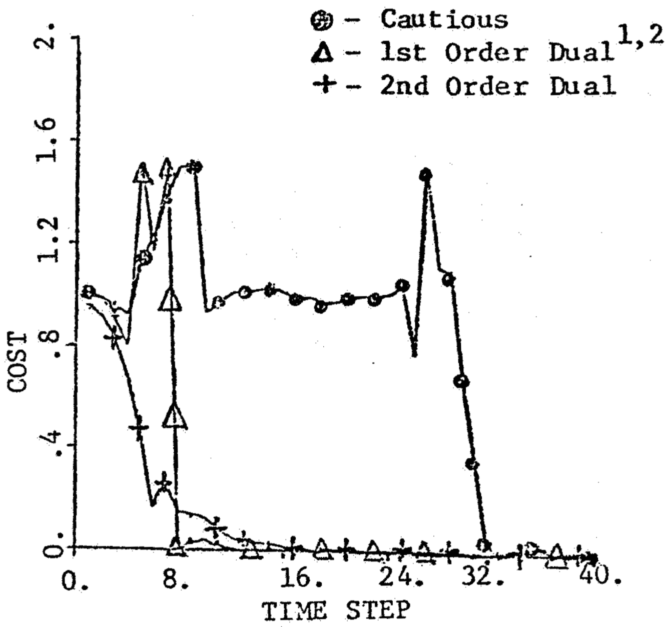

Figure 143. Time history of cost comparing the new dual, dual of Reference 1 and 2, and the cautious controller (Constant parameter case: Run No. 80 from 100 Monte Carlo Runs)

	NO CONTROL	CAUTIOUS CONTROL	FIRST ORDER DUAL CONTROL
COLUMN	C1	C2	C3
COUNT	100	100	100
ROW			
1	29583.	20215.7	15764.2
2	43381.	12057.8	12571.0
3	13404.	13034.1	9945.3
4	50353.	20402.7	24502.0
5	19526.	13650.5	12529.1
6	98981.	24216.8	18934.9
7	145625.	17844.4	19558.6
8	13333.	4515.5	4440.5
9	128733.	68898.3	58591.4
10	73020.	43092.1	34656.7
11	28091.	3360.7	3230.2
12	88536.	23937.9	23491.6
13	15391.	2454.7	2392.5
14	22420.	8319.9	8380.3
15	68898.	36004.2	36422.2
16	49979.	9125.8	9718.3
17	49882.	29413.5	31975.5
18	28850.	17064.0	12511.5
19	71219.	42689.5	36412.3
20	28107.	5566.4	6045.1
21	93340.	54353.8	56635.1
22	18255.	4186.4	4224.0
23	136934.	46734.9	37071.6
24	63349.	47029.4	47367.5
25	26768.	2731.2	2816.2
26	10338.	12941.2	12575.2
27	49111.	28825.9	31378.5
28	15025.	16803.9	17198.7
29	39763.	10345.1	10571.0
30	24362.	3471.2	3441.4
31	78134.	5411.6	5368.2
32	99055.	26284.7	21871.2
33	181477.	34191.8	40623.4
34	19294.	16688.0	14996.0
35	31517.	8046.2	5585.5
36	22659.	3155.8	3104.3
37	40703.	7587.5	6238.2
38	34430.	7443.4	6118.6
39	12744.	11197.6	7963.5
40	51355.	10266.9	10391.1
41	29767.	8306.3	6929.6
42	59979.	17791.0	18006.5
43	35569.	5255.3	5336.4
44	17551.	2103.3	2152.5
45	7542.	3194.2	3312.1
46	99625.	41247.9	46671.4
47	56021.	11742.9	16181.9
48	164647.	39493.9	40825.1

Table 1. - Cost Values for the 100 Monte-Carlo Runs For 30% Process Noise (No Control, Cautious Control, First Order Dual Control)

	NO CONTROL	CAUTIOUS CONTROL	FIRST ORDER DUAL CONTROL
49	30097.	9734.6	7329.8
50	10307.	9931.9	10811.2
51	53184.	13510.8	13252.4
52	50729.	4210.2	4203.9
53	58082.	37536.4	33789.5
54	128404.	5531.1	5453.6
55	24251.	4851.5	4915.2
56	28350.	5755.2	6763.1
57	92454.	29068.3	29249.0
58	71128.	27227.1	29648.5
59	12923.	2838.6	2737.8
60	50150.	19194.0	17414.1
61	61581.	5055.7	5100.6
62	39167.	22284.4	14690.5
63	128726.	59607.8	63676.0
64	49263.	17388.0	16184.6
65	89602.	18362.7	11733.3
66	74954.	54910.7	50597.4
67	32930.	8820.3	7394.6
68	46825.	10346.8	9781.0
69	42513.	4605.5	4581.0
70	57483.	12977.0	6851.2
71	49134.	27630.5	29720.5
72	94879.	47743.1	43412.0
73	30835.	21408.3	22353.1
74	105892.	5279.1	5417.2
75	42173.	43911.5	22868.5
76	10164.	3052.9	3067.1
77	14932.	4910.6	4917.6
78	64611.	34968.2	37813.5
79	9705.	3012.9	3228.5
80	15149.	11303.1	10163.5
81	16436.	2971.0	2827.1
82	38890.	21377.7	31838.4
83	10240.	4177.8	4126.1
84	26971.	6805.4	8825.1
85	30797.	5289.7	4788.5
86	6279.	6468.3	4024.6
87	22690.	11025.6	11231.4
88	52721.	12340.1	9472.4
89	21636.	9925.7	8608.5
90	62764.	32778.4	31479.5
91	10244.	6283.4	6445.6
92	23055.	6020.5	5298.4
93	98388.	72592.9	68177.6
94	73330.	22093.3	17316.1
95	99183.	15872.6	17329.8
96	10049.	6202.7	7427.8
97	43503.	23657.6	24194.7
98	81788.	25497.7	25847.4
99	43204.	3935.7	3984.1
100	46252.	18141.4	15749.8

Aver No Control = 50531.
Aver Cautious Control = 18051.
Aver Dual Control = 17141.

Table 1. (Cont.)

	CAUTIOUS	FIRST ORDER DUAL	SECOND ORDER DUAL
COLUMN	C1	C2	C3
COUNT	100	100	100
ROW			
1	0.293600	0.294100	0.283000
2	0.183000	0.175800	0.170000
3	0.400200	0.347600	0.297300
4	0.573600	0.577100	0.562700
5	0.339700	0.348500	0.376900
6	0.511700	0.491000	0.453300
7	0.427500	0.410800	0.414300
8	0.583200	0.618600	0.561700
9	0.422200	0.425900	0.415900
10	0.536900	0.505600	0.522900
11	0.710600	0.708100	0.752400
12	0.358900	0.347400	0.340200
13	0.742500	0.764100	0.701800
14	0.315500	0.312300	0.298500
15	0.371400	0.403100	0.406100
16	0.369600	0.367100	0.373000
17	0.770200	0.770700	0.759900
18	0.442300	0.431200	0.418400
19	0.741700	0.722300	0.756000
20	0.523500	0.520800	0.492200
21	0.624300	0.607500	0.561500
22	0.237300	0.237300	0.227100
23	0.536700	0.553000	0.583800
24	0.134800	0.129200	0.125700
25	0.402500	0.393000	0.423700
26	0.160400	0.135000	0.111100
27	0.578600	0.569300	0.553800
28	0.175400	0.175100	0.189200
29	0.724000	0.740900	0.740900
30	0.286600	0.292800	0.298400
31	0.605300	0.610400	0.524800
32	0.565300	0.557200	0.523900
33	0.430300	0.439700	0.451900
34	0.720400	0.700200	0.678100
35	0.601300	0.592400	0.566900
36	0.279300	0.283200	0.208100
37	0.327100	0.329900	0.330100
38	0.690000	0.688700	0.674500
39	0.489300	0.468000	0.436900
40	0.373900	0.367500	0.341100
41	0.162800	0.168800	0.184400
42	0.616100	0.622300	0.586700
43	0.616200	0.629600	0.657400
44	0.431000	0.432100	0.458800
45	0.724200	0.800000	0.651900
46	0.583200	0.588200	0.567500
47	0.210400	0.209400	0.215000
48	0.737300	0.681200	0.627300
49	0.517800	0.473000	0.470000
50	0.480200	0.510800	0.453900
51	0.503100	0.506100	0.456200
52	0.640000	0.634600	0.622600
53	0.589200	0.573400	0.568800
54	0.594300	0.615900	0.620000
55	0.303000	0.305300	0.301800

Table 2. - Cost Values for the 100 Monte-Carlo Runs for the time varying scalar model, using the cautious, first order dual and second order dual controllers ($b(0) = .05$, $P_b(0) = 1$, $V = .1$, $c = 1$, $W = .01$, No Control Cost = 1)

	CAUTIOUS	FIRST ORDER DUAL	SECOND ORDER DUAL
56	0.516900	0.470100	0.462500
57	0.579000	0.577400	0.588400
58	0.350200	0.375600	0.377000
59	0.347500	0.345200	0.348000
60	0.533700	0.546800	0.562600
61	0.293900	0.297300	0.314700
62	0.440000	0.445700	0.427700
63	0.460100	0.451800	0.436700
64	0.598000	0.571900	0.548400
65	0.273900	0.382200	0.238500
66	0.619700	0.646600	0.641100
67	0.424700	0.423500	0.440100
68	0.478600	0.489100	0.496800
69	0.197900	0.193200	0.194300
70	0.459600	0.437600	0.396200
71	0.271500	0.239000	0.230200
72	0.512100	0.505500	0.493200
73	0.408200	0.416000	0.385600
74	0.565600	0.560300	0.555700
75	0.798100	0.788900	0.749900
76	0.700800	0.709900	0.741500
77	0.589000	0.509000	0.471400
78	0.294700	0.295000	0.318700
79	0.326400	0.319200	0.306900
80	0.737700	0.668900	0.673200
81	0.423400	0.427000	0.416100
82	0.314700	0.319700	0.326100
83	0.915400	0.763900	0.751200
84	0.451600	0.451100	0.537700
85	0.481900	0.503000	0.469500
86	0.196800	0.197100	0.192700
87	0.449600	0.412600	0.402200
88	0.383100	0.328100	0.348700
89	0.493900	0.488100	0.468900
90	0.708500	0.695700	0.694700
91	0.483300	0.505400	0.543300
92	0.423000	0.431400	0.426400
93	0.653900	0.664800	0.648400
94	0.526100	0.484900	0.474500
95	0.538800	0.528000	0.516100
96	0.493100	0.475900	0.452100
97	0.525900	0.518600	0.494600
98	0.319200	0.317400	0.382000
99	0.253900	0.259200	0.210600
100	0.427100	0.379800	0.328900

Average Cautious = .47520
Average First Order Dual = .46988
Average Second Order Dual = .45814

Table 2. (Cont.)

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	CAUTIOUS	FIRST ORDER DUAL	SECOND ORDER DUAL
COLUMN	C1	C2	C3
COUNT	100	100	100
ROW			
1	0.37730	0.37950	0.314000
2	0.24700	0.24900	0.179600
3	0.38090	0.37370	0.458400
4	0.59300	0.76590	0.680600
5	0.51770	0.53640	0.494600
6	0.63830	0.87830	0.511600
7	0.73070	0.77080	0.425400
8	0.58150	0.57850	0.530000
9	0.49980	0.49640	0.434800
10	0.91900	0.89960	0.620300
11	0.92020	0.88570	0.613200
12	0.40770	0.34300	0.346700
13	0.79250	0.75520	0.805900
14	0.87560	0.36210	0.303500
15	0.43750	0.50390	0.425000
16	0.39720	0.35780	0.331500
17	0.92480	0.94370	0.849800
18	0.57040	0.55480	0.429900
19	0.98940	0.90890	0.788800
20	0.67070	0.63930	0.646300
21	0.72580	0.72450	0.732700
22	0.42910	0.32980	0.195600
23	0.76760	0.70550	0.670000
24	0.22040	0.21400	0.138700
25	0.42100	0.45810	0.476600
26	0.41910	0.21630	0.200200
27	0.65590	0.74280	0.585800
28	0.17620	0.17340	0.170900
29	0.82550	0.82530	0.681600
30	0.29370	0.29590	0.296100
31	0.92650	0.93770	0.693500
32	0.49660	0.48500	0.514200
33	0.70690	0.74210	0.557700
34	0.89090	0.90810	0.711500
35	0.93930	0.72410	0.610500
36	0.37420	0.56320	0.270300
37	0.40930	0.42160	0.394600
38	0.74820	0.72170	0.688700
39	0.62200	0.53430	0.470900
40	0.60030	0.57520	0.397500
41	0.24620	0.24150	0.172600
42	0.78480	0.76770	0.794200
43	0.72420	0.70390	0.678100
44	0.49820	0.47430	0.484700
45	0.77450	0.74520	0.641700
46	0.63010	0.63000	0.628300
47	0.34190	0.30030	0.232000
48	0.76070	0.79820	0.719700
49	0.77530	0.63850	0.615000
50	0.78100	0.75330	0.552600
51	0.79230	0.55620	0.498500
52	0.75490	0.72380	0.711600
53	0.97680	0.91870	0.657300
54	0.85630	0.66350	0.599100

Table 3. - Cost Values for the 100 Monte-Carlo Runs for the time varying scalar model, using the cautious, first order dual and second order dual controllers ($b(0) = .05$, $P_b(0) = 1$, $V = .1$, $c = 1$, $W = .1$, No Control Cost = 1)

	CAUTIOUS	FIRST ORDER DUAL	SECOND ORDER DUAL
55	0.47710	0.44530	0.478000
56	0.55660	0.54210	0.486000
57	0.87470	0.88870	0.706500
58	0.43060	0.43240	0.385200
59	0.44260	0.32380	0.312300
60	0.55830	0.63130	0.650800
61	0.46280	0.34690	0.371100
62	0.58940	0.58580	0.426100
63	0.55230	0.53890	0.485700
64	0.66680	0.61320	0.724500
65	0.40090	0.40910	0.393200
66	0.82600	0.79980	0.821000
67	0.54170	0.48200	0.412600
68	0.50030	0.50240	0.480000
69	0.35060	0.50270	0.204200
70	0.83090	0.84170	0.420200
71	0.28470	0.29180	0.306700
72	0.77530	0.75140	0.552200
73	0.45820	0.52040	0.433500
74	0.74170	0.70080	0.687000
75	0.87310	0.89910	0.702400
76	0.75730	0.74240	0.359100
77	0.84690	0.93530	0.451900
78	0.32950	0.33070	0.328900
79	0.64820	0.45360	0.456500
80	0.94980	0.90930	0.829100
81	0.46220	0.57210	0.485300
82	0.31220	0.31460	0.306600
83	1.04020	1.02040	0.798700
84	0.66560	0.63370	0.629800
85	0.61360	0.63510	0.663300
86	0.25040	0.98580	0.212500
87	0.46860	0.43310	0.450300
88	0.44730	0.52820	0.465900
89	0.74990	0.75400	0.485100
90	0.83940	0.75590	0.731200
91	0.65150	0.78070	0.588800
92	0.73680	0.88110	0.538000
93	0.38380	0.71730	0.680900
94	0.77620	0.49630	0.511200
95	0.82680	0.71890	0.599700
96	1.04330	0.55810	0.541800
97	0.70900	0.70760	0.635200
98	0.60010	0.62640	0.366400
99	0.25800	0.23070	0.270300
100	0.47860	0.49490	0.430200

Average Cautious = .62293
Average First Order Dual = .60787
Average Second Order Dual = .51389

Table 3. (Cont.)

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	CAUTIOUS	FIRST ORDER DUAL	SECOND ORDER DUAL
COLUMN	C1	C2	C3
COUNT	100	100	100
ROW			
1	0.119300	0.084000	0.102200
2	0.092000	0.065200	0.061100
3	0.172300	0.153700	0.063600
4	0.095300	0.062600	0.055200
5	0.117700	0.064800	0.062100
6	0.143500	0.109300	0.059300
7	0.161900	0.057000	0.051400
8	0.086800	0.085800	0.099700
9	0.093300	0.085300	0.060600
10	0.102400	0.080700	0.066000
11	0.086100	0.059200	0.052400
12	0.174400	0.166700	0.065800
13	0.080600	0.073700	0.064600
14	0.072500	0.068500	0.056900
15	0.169000	0.096600	0.104200
16	0.114900	0.089000	0.058500
17	0.085000	0.085600	0.087900
18	0.200000	0.082900	0.065600
19	0.079800	0.072000	0.057300
20	0.080100	0.063400	0.061700
21	0.073400	0.065000	0.054500
22	0.166500	0.057500	0.051600
23	0.104300	0.089700	0.070800
24	0.100600	0.084800	0.089400
25	0.083300	0.068000	0.058500
26	0.089700	0.072600	0.069300
27	0.075700	0.069500	0.055900
28	0.094600	0.087200	0.061100
29	0.097500	0.091100	0.091000
30	0.144400	0.098700	0.059300
31	0.109300	0.065800	0.061800
32	0.082200	0.077600	0.064300
33	0.162200	0.063800	0.058800
34	0.158200	0.057200	0.051900
35	0.091600	0.082500	0.067100
36	0.128900	0.097500	0.176300
37	0.087800	0.073700	0.069900
38	0.098100	0.083800	0.068900
39	0.082600	0.061200	0.052400
40	0.117400	0.147400	0.084100
41	0.075600	0.069000	0.055600
42	0.109500	0.091600	0.089300
43	0.088800	0.090200	0.109800
44	0.090100	0.090600	0.107500
45	0.109900	0.086700	0.088700
46	0.137600	0.064000	0.060700
47	0.129700	0.060500	0.054000
48	0.076800	0.073500	0.065900
49	0.122100	0.208600	0.078300
50	0.090500	0.091100	0.098900
51	0.078800	0.074700	0.064000
52	0.108000	0.103000	0.069100
53	0.115200	0.093000	0.102700
54	0.107300	0.082600	0.056200
55	0.134900	0.165700	0.094500

Table 4. - Cost Values for the 100 Monte-Carlo Runs for the constant parameter scalar model, using the cautious, first order dual and second order dual controllers ($b(0)=.05$, $P_b(0)=1$, $V=0$, $c=1$, $W=.01$, No Control Cost=1)

	CAUTIOUS	FIRST ORDER DUAL	SECOND ORDER DUAL
56	0.093700	0.084300	0.090100
57	0.112800	0.067900	0.064100
58	0.162400	0.063600	0.058500
59	0.166700	0.139400	0.056800
60	0.085700	0.085700	0.060100
61	0.085900	0.091200	0.058500
62	0.084200	0.068700	0.065300
63	0.125000	0.071500	0.058500
64	0.119500	0.077700	0.060700
65	0.193400	0.063900	0.061700
66	0.075700	0.069400	0.061900
67	0.077200	0.063900	0.061100
68	0.125300	0.072600	0.070500
69	0.191100	0.060300	0.055500
70	0.093200	0.069500	0.065500
71	0.087900	0.060300	0.053400
72	0.112800	0.222300	0.115800
73	0.135100	0.109300	0.060700
74	0.121300	0.103500	0.112700
75	0.110300	0.091100	0.072200
76	0.139000	0.330400	0.063800
77	0.084200	0.065600	0.056800
78	0.075000	0.067400	0.059700
79	0.120400	0.092200	0.088400
80	0.078400	0.065700	0.055800
81	0.085200	0.086100	0.084800
82	0.076700	0.073100	0.064700
83	0.111600	0.087500	0.062800
84	0.103400	0.091900	0.070400
85	0.095400	0.090200	0.097900
86	0.100700	0.077800	0.058600
87	0.076400	0.070100	0.066100
88	0.087300	0.071000	0.067900
89	0.075300	0.067500	0.054600
90	0.185000	0.078200	0.053200
91	0.079400	0.065000	0.053400
92	0.118700	0.064300	0.061300
93	0.103700	0.124900	0.061900
94	0.096500	0.109100	0.063300
95	0.114600	0.067400	0.058700
96	0.081900	0.064100	0.056300
97	0.074400	0.066500	0.055900
98	0.133600	0.093200	0.062800
99	0.091500	0.097600	0.058400
100	0.098300	0.064000	0.057700

Average Cautious = .10907
Average First Order Dual = .08739
Average Second Order Dual = .06927

Table 4. (Cont.)

	CAUTIOUS	FIRST ORDER DUAL	SECOND ORDER DUAL
COLUMN	C1	C2	C3
COUNT	100	100	100
ROW			
1	0.224600	0.207400	0.131200
2	0.490300	0.429800	0.129100
3	0.451900	0.143300	0.166700
4	0.191100	0.153100	0.110700
5	0.321000	0.242800	0.146800
6	0.833900	0.282800	0.128300
7	0.503800	0.323500	0.106100
8	0.261900	0.185500	0.136700
9	0.237100	0.162100	0.108700
10	0.354000	0.327500	0.124100
11	0.284400	0.171900	0.145200
12	0.346700	0.308600	0.163200
13	0.239600	0.145400	0.119000
14	0.496600	0.102200	0.101600
15	0.645100	0.183700	0.140800
16	0.211100	0.245300	0.246600
17	0.487400	0.266900	0.119500
18	0.405700	0.473400	0.131800
19	0.156300	0.131800	0.116200
20	0.290400	0.238800	0.138800
21	0.324700	0.307300	0.106300
22	0.465700	0.364300	0.117900
23	0.240000	0.204400	0.129800
24	0.732900	0.313100	0.143400
25	0.250500	0.210200	0.141400
26	0.375800	0.988200	0.115200
27	0.155600	0.117400	0.105900
28	0.187600	0.153000	0.119600
29	0.194700	0.163200	0.131700
30	0.258700	0.257900	0.120300
31	0.350100	0.343700	0.130500
32	0.293200	0.184500	0.174100
33	0.552400	0.265800	0.108700
34	0.372900	0.293000	0.420400
35	0.770800	0.344400	0.117800
36	0.517600	0.331900	0.143000
37	0.330200	0.193000	0.132100
38	0.705300	0.565300	0.145100
39	0.310800	0.215800	0.109800
40	0.288500	0.197900	0.129100
41	0.151800	0.119200	0.105600
42	0.340600	0.215700	0.144300
43	0.329500	0.269400	0.140400
44	0.176600	0.123800	0.120700
45	0.289700	0.192600	0.110600
46	0.396500	0.345100	0.205000
47	0.923700	0.175000	0.132600
48	0.293500	0.193700	0.126300
49	0.358500	0.187300	0.219400
50	0.218200	0.242000	0.286000
51	0.437300	0.197100	0.148400
52	0.305000	0.205700	0.140000
53	0.450300	0.438800	0.138300
54	0.301800	0.243800	0.112200

Table 5. - Cost Values for the 100 Monte-Carlo Runs for the constant parameter scalar model, using the cautious, first order dual and second order dual controllers ($\hat{b}(0)=.05$, $P_b(0)=1$, V=0, c=1, W=.1, No Control Cost=1)

	CAUTIOUS	FIRST ORDER DUAL	SECOND ORDER DUAL
55	0.180100	0.167200	0.151300
56	0.567900	0.411600	0.114900
57	0.290100	0.197000	0.124100
58	0.442300	0.325200	0.154700
59	0.273500	0.255300	0.115100
60	0.481900	0.158800	0.160500
61	0.308500	0.144800	0.124200
62	0.368600	0.201100	0.120800
63	0.352700	0.327200	0.127500
64	0.383400	0.323100	0.206300
65	0.330700	0.223000	0.142000
66	0.347800	0.209600	0.205200
67	0.403700	0.225800	0.124200
68	0.287900	0.219700	0.213700
69	0.340500	0.269600	0.112800
70	0.429100	0.371200	0.132500
71	0.579600	0.175900	0.109100
72	0.320900	0.345200	0.227400
73	0.306100	0.299300	0.121700
74	0.267100	0.221500	0.176500
75	0.323300	0.209700	0.127600
76	0.260900	0.314500	0.145100
77	0.340200	0.145900	0.139600
78	0.174100	0.124700	0.124600
79	0.235900	0.346800	0.154600
80	0.829700	0.230600	0.126400
81	0.342700	0.221700	0.123300
82	0.311100	0.322400	0.256300
83	0.472600	0.451900	0.109200
84	0.426300	0.313100	0.163000
85	0.363100	0.358200	0.149500
86	0.585300	0.169500	0.112200
87	0.246400	0.173900	0.151700
88	0.434400	0.253400	0.160000
89	0.189900	0.134100	0.113200
90	0.292500	0.240900	0.124200
91	0.222200	0.134200	0.110000
92	0.353200	0.293700	0.154900
93	0.225800	0.235500	0.122100
94	0.434400	0.157800	0.152000
95	0.280300	0.269900	0.134500
96	0.170400	0.139000	0.123200
97	0.342900	0.175300	0.104300
98	0.379100	0.182600	0.136000
99	0.237400	0.161600	0.136600
100	0.283700	0.168500	0.114900

Average Cautious = .359
Average First Order Dual = .250
Average Second Order Dual = .142

Table 5. (Cont.)

Measurement Noise Covariance W	Average Cost		
	Cautious	First Order Dual	Second Order Dual
.01	.475	.469	.458
.1	.623	.608	.514

Table 6. Average Cost for the three controllers on the time varying parameter model ($b(0)=.05$, $P_b(0)=1$, $V=.1$, $c=1$)

Measurement Noise Covariance W	Average Cost		
	Cautious	First Order Dual	Second Order Dual
.01	.109	.087	.069
.1	.359	.250	.142

Table 7. Average Cost for three controllers on the Constant Parameter Model ($b(0)=.05$, $P_b(0)=1$, $V=0$, $c=1$)

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15. Supplementary Notes

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16. Abstract

An investigation of the properties important for the design of stochastic adaptive controllers for the higher harmonic control of helicopter vibration is presented. Three different model types are considered for the transfer relationship between the helicopter higher harmonic control input and the vibration output; 1) Nonlinear, 2) linear with slow time varying coefficients, and 3) linear with constant coefficients. The stochastic controller formulations and solutions are presented for a dual, cautious, and deterministic controller for both linear and nonlinear transfer models. Extensive simulations are performed with the various models and controllers. It is shown that the cautious adaptive controller can sometimes result in unacceptable vibration control.

A new second order dual controller is developed which is shown to modify the cautious adaptive controller by adding numerator and denominator correction terms to the cautious control algorithm. The new dual controller is simulated on a simple single-control vibration example and is found to achieve excellent vibration reduction and significantly improves upon the cautious controller.

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