FINAL REPORT

APPLICATION OF A STOCHASTIC SNOWMELT MODEL FOR PROBABILISTIC DECISIONMAKING

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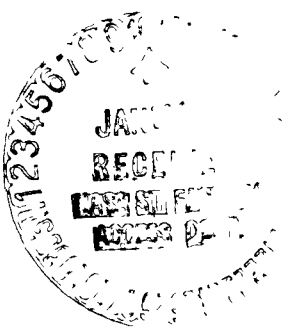

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SECTION I

INTRODUCTION

Existing snowmelt runoff models are used to make single-valued estimates for a specified decision period. Such models are based on the expected value theorem for statistical decisionmaking. A single valued estimate fails to provide a measure of the accuracy of an estimate. While a computed standard error may be used to indicate the accuracy, existing decisionmaking techniques that are based on the expected value theorem do not provide a mechanism for altering the decision based on the computed standard error. That is, existing decisionmaking techniques are not sensitive to the accuracy of the estimated value of snowmelt runoff. Models based on the expected value theorem also fail to provide for inaccuracies in the input data, especially the empirical coefficients that are specific to a particular watershed. Because these coefficients cannot be estimated with certainty for the specified decision period and may vary with time, error in the estimated values of the coefficients will cause an error in the output function, i.e., the runoff forecast. Incorrect decisions may result from inaccuracies in the output function.

The snowmelt-runoff model is a means of forecasting snowmelt runoff volumes using a small number of variables. The model first estimates the volume of snowmelt for each day of the forecast period and then estimates the amount of melt water that will leave the watershed on each day by using a recession curve. This model has been calibrated on a number of mountainous watersheds.

The depth of snowmelt for any day is estimated in the model by multiplying the number of degrees above freezing by a degree-day factor to obtain a depth measurement. This depth is converted into volume by multiplying the depth estimate by the area of the snowpack. The volume of precipitation for the day is then added in to get an estimate of the total amount of water available for runoff. This water is assumed to leave the watershed at a decaying rate. The model for predicting the snowmelt runoff on day n+1 is given by:

$$\hat{Q}_{n+1} = C_n\left[a_n(T_n+\Delta T_n)S_n+P_n\right]\left(\frac{0.01A}{86400}\right)(1-k_{n+1})+Q_n k_{n+1} \qquad (I-1)$$

in which Q is the average daily discharge (m^3/S); C is the runoff coefficient; a is the degree-day factor ($cm\cdot{}^\circ C^{-1}d^{-1}$) indicating the snowmelt depth resulting from 1 degree-day; T is the number of degree-days ($^\circ C\cdot d$); ΔT = the adjustment by temperature lapse rate necessary because of the altitude difference between the temperature station and the average hypsometric elevation of the basin or zone; S is the ratio of the snow-covered area to the total area; P is the precipitation contributing to runoff (cm); A is the area of the basin or zone (m^2); 0.01/86400 is a conversion from $CM\cdot m^2\cdot d^{-1}$ to m^3/S; k is the recession coefficient indicating the decline of discharge in a period without snowmelt or rainfall; and n is the notation indicating the day for which the value is given. The important parameters of the model that must be calibrated for any specific basin are the degree-day factor, the recession constant, and a runoff coefficient that is included to account for losses due to infiltration and evapotranspiration. Additionally, the value of ΔT must be considered as a variable because its value is not known exactly; it is based on an extrapolation and a mean zonal hypsometric elevation.

Project Objective

The goal of this project is to develop a stochastic form of the snowmelt runoff model that can be used for probabilistic decision-making. The use of probabilistic streamflow predictions instead of single-valued deterministic predictions should lead to greater accuracy in decisions. While the accuracy of the output function is important in decisionmaking, it is also important to understand the relative importance of the coefficients. Therefore, a sensitivity analysis will be made for each of the coefficients.

SECTION II

LITERATURE REVIEW

Many different models have been developed and used for predicting snowmelt runoff (Leaf, 1977; Baker and Carder, 1977; Zuzel and Cox, 1978). These models vary considerably in complexity; the simplest models are based solely on statistical techniques, while the most complex methods attempt to model the individual processes involved in the melting of a snowpack. Some models are designed to predict streamflow for any given day or series of days, (Leaf, 1977; Martinec, 1975; Tangborn, 1977) while other models give only seasonal predictions (Zuzel and Cox, 1978). Generally, snowmelt models may be categorized on the basis of complexity and length of forecast period.

Empirical models are based on statistical correlations between predictor variables and the criterion variable, volume of snowmelt runoff. This type of model is most often used for seasonal predictions. Snow water equivalent measurements, previous runoff volumes, and precipitation totals are the most common predictor variables (SCS, 1970; USACE, 1956). Theory is not very important in formulating empirical models; the objective is to explain as much of the variation in the criterion values as possible using whatever data are available. It is quite common for these models to include two predictors expressed in different units, such as snow water equivalent (in inches)

and previous winter runoff (in volumetric units).

Water balance models are more conceptual than the simple empirical models. The water balance is an accounting of all the water entering and leaving the basin. The volume of water stored in the snowpack is estimated from precipitation or water equivalent data; allowances are made for losses due to evaporation, groundwater storage, and transpiration; the remaining volume is the seasonal snowmelt runoff prediction (Zuzel and Cox, 1978). Loss rates may be estimated either empirically or conceptually, as may the snowpack storage. Most water balance models are somewhat empirical.

Short-term runoff predictions usually require models of greater complexity than the models used for seasonal runoff. Not only must the total volume of water stored in the snowpack be estimated, but also the proportion of that volume that will melt and leave the watershed as streamflow in a given time period must be estimated. The amount of water generated by melting snow is a function of the energy available for this purpose. Therefore, the most complex snowmelt models are generally based on an energy balance (Zuzel and Cox, 1978).

Energy balance procedures attempt to model the physical processes involved in snowmelt runoff. The amount of available energy is commonly estimated by the air temperature, although some models include such factors as incoming solar radiation, cloud cover, albedo, and net long-wave radiation (Anderson, 1976). These models often require that the watershed be subdivided into small,

homogeneous areas so that the available energy for each location can be estimated more accurately (Leaf, 1977). Since snowmelt models are generally used in mountainous areas, slope and aspect can result in large differences in incident energy from one area to another. Evaporation, transpiration and groundwater losses are also estimated conceptually in some energy budget models (Leaf, 1977).

Model Selection

To test the study objectives, models having significant differences in important characteristics had to be selected. Criteria for model selection include the frequency of current usage, input data requirements and whether or not these data are typically available, the degree of model complexity, and the length of forecast period. Additionally, because snow covered area (SCA) is more readily available than in previous decades, models that either included SCA or were capable of being modified to include it were given more consideration.

Three models were selected for comparison, with several methods of evaluation for each model. The model types studied were the regression model, the Tangborn model, and the Martinec model.

The Regression Models

The most common form of empirical model is the linear regression. These models are widely used for snowmelt runoff predictions in the western U.S. (USACE, 1956; SCS, 1970). They are

easily calibrated and can use many different hydrologic variables as predictor variables. These models are used for making seasonal runoff forecasts, but due to the empirical nature of the method, they may also be used to give predictions for shorter time periods.

Linear regression models are based on the assumption that there is a linear relationship between the predictor variables and the criterion variable. This assumption implies that as the value of the predictor variable increases, the value of the criterion variable changes at a constant rate. The equation that relates the value of the criterion to the value of the predictor is of the form:

$$Y = a + bX \qquad (2\text{-}1)$$

in which Y is the criterion variable, X is the predictor variable, and a and b are the regression coefficients (Miller and Freund, 1977).

Many hydrologic variables have approximately linear relationships with the volume of snowmelt runoff. A few of these variables are snow water equivalent, winter precipitation, and snow covered area. The linearity of the relationships is due to the fact that these variables are indicators of the volume of water stored in the snowpack. Because the relationships between these predictor variables and the volume of runoff are only approximately linear, many different lines may be drawn which appear to fit the data. Some of the lines pass

through a number of the data points, but due to deviations from linearity, a straight line that will pass through all of the data points can not be drawn.

The method of selecting the best regression line for a set of data points is based on minimizing the sum of squares of the errors. For each observed value of the predictor, two values of the criterion variable appear; the first is the corresponding observed value and the second is the value predicted by the regression equation. The difference between these two values is termed the error of prediction. The regression line is defined as the line that results in the minimum value of the sum of the squares of the errors. The coefficients of the regression line can be derived using the equations:

$$b = \frac{\Sigma XY - (\Sigma X \Sigma Y)/n}{\Sigma X^2 - (\Sigma X)^2/2} \qquad (2\text{-}2)$$

and

$$a = (\Sigma Y)/n - b(\Sigma X)/n \qquad (2\text{-}3)$$

in which X and Y are the predictor and criterion variables, respectively, and n is the number of observations (Hays, 1965). By using these equations, the line of best fit can be determined.

In natural systems the value of the criterion variable is often a function of more than one predictor. The relationships between the criterion variable and the predictors may be assumed to be linear, resulting in a prediction equation of the form:

$$Y = a + \sum_{i=1}^{p} b_i X_i \qquad (2\text{-}4)$$

in which Y is the criterion variable, X_i is the i^{th} predictor variable, and a and b_i are the regression coefficients. Models of this type are called multiple linear regressions. The regression coefficients are unique and may be calculated from equations similar to Eqs. 2-2 and 2-3. In many cases, the inclusion of more than one predictor variable results in a more accurate model (Davis, 1973).

The Tangborn Model

The Tangborn equation is a water balance model (Tangborn and Rasmussen, 1976). The structure of the model was established conceptually, but calibration is accomplished using regression methods. The model may be used for any length of forecast period from one day to the entire snowmelt season. The only data required are daily precipitation and runoff values, although daily temperature may be included for short forecast periods.

The basic form of the model is:

$$R_s^* = a\ P_w + b - R_w \qquad (2\text{-}5)$$

in which R_s^* is the predicted runoff volume, P_w is the total depth of precipitation observed during the preceding winter, R_w is the winter runoff, and a and b are regression coefficients. The structure of the model is based on the assumption that the volume of water stored on the watershed is equal to the amount of winter precipitation minus the winter runoff. The regression

coefficients represent losses and modifications such as transpiration, groundwater storage, and evaporation.

An important feature of the Tangborn model is the test season modification. In using this method, a short test season prediction model with the structure of Eq. 2-5 is developed. At the end of the test season, the error of the test season prediction is evaluated and used to modify the prediction for the forecast season. The form of the forecast model becomes:

$$R_s^{**} = R_s^* - ce_t = a(P_w+P_t) + b - (R_w+R_t) - ce_t \qquad (2\text{-}6)$$

in which R_s^{**} is the revised runoff prediction; R_s^* is the original prediction; P_w and P_t are the winter and test season precipitation, respectively; R_w and R_t are the winter and test season runoff volumes, respectively; a, b, and c are coefficients; and e_t is the error of the test season prediction. The reasoning behind this modification is that the test season error is a result of the inaccuracy of estimating basin storage by subtracting winter runoff from winter precipitation. Because the forecast season prediction is based on the same estimate, the test season error should be related to the prediction season error.

Figure 1 shows the relationship of the various seasons. In order to use the test season approach, data from the present and a number of previous years are compiled. For each year, precipitation and runoff totals are computed for the winter and test seasons; runoff totals are also computed for the prediction season of each year, except for the current year (the value for

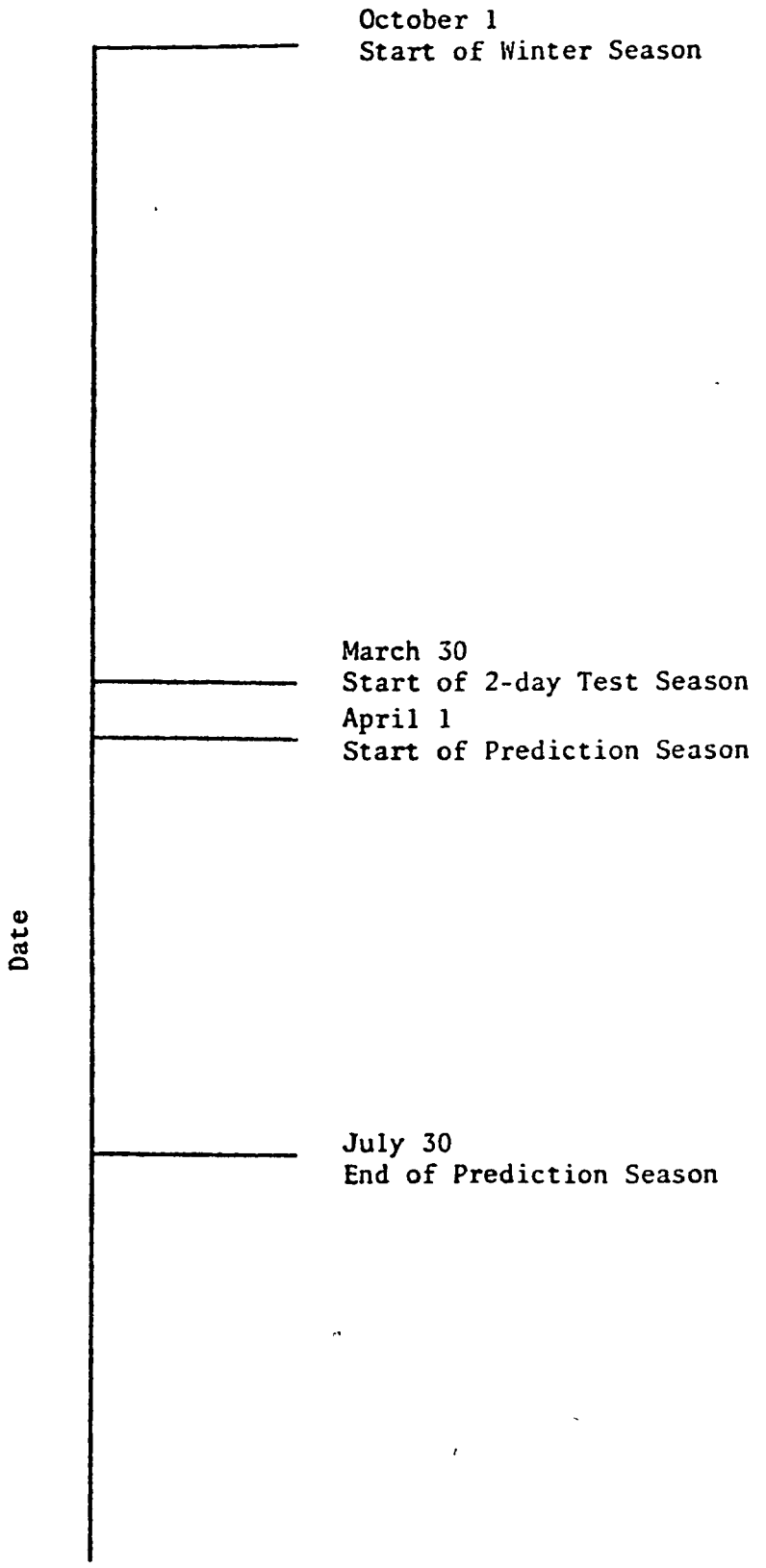

FIGURE 1. Relationship of the Winter, Test, and Prediction Seasons for the Tangborn Model

the current year is not yet known). Note that the prediction date, April 1, is at the end of the test season; therefore, observed values of runoff and precipitation during the test season are available for the current year. Once all the data has been obtained, the observed test season runoff volumes are regressed onto the winter precipitation values, resulting in a calibrated equation of the form:

$$R_t^* = a\ P_w + b - R_w \qquad (2\text{-}7)$$

in which R_t^* is the predicted test season runoff. The test season error in each year is then computed by the equation:

$$e_t = R_t^* - R_t \qquad (2\text{-}8)$$

Next, a model for estimating the prediction season runoff is formed by regressing the prediction season runoff on the sum of the winter and test season precipitation depths for each of the previous years:

$$R_s^* = a(P_w+P_t) + b - (R_w+R_t) \qquad (2\text{-}9)$$

The errors are then calculated in a manner similar to that used for the test season:

$$e_s = R_s^* - R_s \qquad (2\text{-}10)$$

in which e_s is the prediction season error. The coefficient of the test season error, c in Eq. (2-6), can then be determined.

The coefficient is computed using the test season and prediction season errors from previous years, according to the equation:

$$c = \frac{\Sigma(e_t * e_s)}{\Sigma(e_t)^2} \qquad (2\text{-}11)$$

The original runoff season prediction, R_s^*, which was calculated for the current year in Eq. (2-9), is adjusted by the product of c and the current year test season error; the final prediction is:

$$R_s^{**} = R_s^* - ce_t = a(P_w+P_t) + b - (R_w+R_t) - ce_t \qquad (2\text{-}12)$$

in which R_s^{**} is the final prediction.

When using the Tangborn model for prediction periods of a few days, accuracy may be increased by including temperature in the model (Tangborn, 1978). Tangborn suggested the following composite temperature variable, A_t:

$$A_t = \alpha\overline{T} + (1-\alpha)\Delta T \qquad (2\text{-}13)$$

in which $\overline{T}$ is the daily mean temperature, ΔT is the daily range of temperature, and α is a coefficient. The daily mean temperature is computed from the observed maximum and minimum temperatures for the day; the range of temperature is the difference between the maximum and minimum observed values. The reasoning behind this equation is that the average daily mean temperature is an

estimator of the amount of convective energy available for melting snow, and that the difference between maximum and minimum temperatures can be used to estimate the amount of radiant energy available for this purpose. Large differences between the daily maximum and minimum are indicative of clear skies, while a small daily range of temperature indicates cloud cover and, therefore, less radiant energy. The relative importance of the two components (radiative and convective) is controlled by the coefficient α. When the temperature term is included in the Tangborn model, the equation becomes:

$$R_s^{***} = a(P_w+P_t) + b - (R_w+R_t) - ce_t - e_s^* \qquad (2\text{-}14)$$

in which e_s^* is the prediction season error estimated from the temperature function, and R_s^{***} is the revised runoff prediction. The value of the prediction season error is estimated from the temperature function A_t using the equation:

$$e_s^* = dA_t + e \qquad (2\text{-}15)$$

in which d and e are coefficients determined by regression. Tangborn reports a minimum reduction in standard error of estimate of nine percent due to inclusion of this temperature term (Tangborn, 1978).

The Martinec Model

The Martinec model is conceptually derived and may be used for prediction periods of one day or longer (Martinec,1975).

The amount of energy available for anowmelt runoff is estimated by a daily temperature index. Data requirements include daily temperature, precipitation, and snow covered area. The form of the model is:

$$Q_n^* = c(dTSCA+P)\ A(1-K) + KQ_{n-1} \qquad (2\text{-}16)$$

in which Q_n^* is the predicted volume of runoff for day n, c is a dimensionless runoff coefficient, d is a degree-day factor, T is the value of the daily temperature index on day n, A is the total area of the watershed, SCA is the percentage of the area that is covered by snow on day n, K is a dimensionless recession coefficient, and Q_{n-1} is the volume of runoff observed on the previous day. The value of the daily temperature index is computed using hourly data if available; otherwise, the daily maximum and minimum temperatures are used. The daily index is a measure of the average number of degrees above freezing for the temperature on that day. The values are expressed in degree-days celsius.

The first term of Eq. (2-16) represents the amount of water that is generated by precipitation and melting snow on day n and that is expected to leave the watershed on that day. The value of the degree-day factor, d, is expressed in inches of water per degree Celsium; therefore, when the temperature index is multiplied by this factor, an estimate of the depth of water generated by snowmelt is obtained. This depth is multiplied by the total area of the watershed, A, and by the percentage of the total area that is covered by snow (SCA) to get an estimate of the volume of water produced by melting snow on day n. The precipitation, P, is assumed to be a constant depth over the entire watershed; therefore, the product

of P and A is an estimate of the volume of rainfall on day n. The sum of the volume of melted snow and the volume of precipitation is referred to as the generated runoff.

Not all of the generated runoff leaves the watershed on the day of generation. Some is lost to groundwater storage and evapotranspiration; this proportion is represented by c, the runoff coefficient. Furthermore, on large watersheds the outlet of the basin is quite a distance from the source of much of the generated melt; therefore, much of the water is in transit to the outlet for several days. The proportion of water that does not reach the outlet on the day that it is generated is represented by K, the recession coefficient. Thus, only the proportion (1-K) of the runoff generated on day n actually reaches the outlet on day n.

The second term in the equation, $K \cdot Q_{n-1}$, is called the recession term. It represents the amount of water generated on previous days that is expected to appear as runoff on day n. Because K is nearly equal to 1 on large watersheds, this recession term is often considerably larger than the generated runoff term.

The Snowmelt Runoff Model

The snowmelt-runoff model is an advanced form of the Martinec model. Both models can be used to forecast daily streamflow in basins where snowmelt is a primary contribution to the total runoff. The snowmelt-runoff model is designed to be used with remotely sensed estimates of snow covered area, as well as temperature and precipitation data. The User's Manual (Martinec, et al., 1983) provides information on the computer program, the required input, and the application of the model.

SECTION III

SENSITIVITY ANALYSIS OF THE SNOWMELT-RUNOFF MODEL

Hydrologic modeling is a procedure in which one or more of the phases of the hydrologic cycle are represented by a simplified system. Although the physical principles underlying the specific hydrologic processes should be considered in formulating a model structure, the final design invariably is only an approximation of the processes being modeled. In spite of the approximations involved, models can often provide insight into those parts of the physical process in which knowledge of the underlying principles is deficient.

In addition to being familiar with the physical principles being modeled, the designer must also be familiar with modeling tools that aid in formulating, calibrating, and verifying conceptual representations of the unknown parts of the physical processes. Sensitivity analysis is a modeling tool that, if properly used, can provide a model designer with a better understanding of the correspondence between the model and the physical processes being modeled. Sensitivity of model components and parameters is potentially useful in the formation, calibration, and verification of a hydrologic model. However, in the past the use of sensitivity has been limited to the determination of an optimal set of model parameters and identifying the effect of variability in a parameter on the optimal solution. Parameter optimization techniques categorized as gradient climbing procedures use estimates of parameter sensitivity to derive an optimal set of model parameters. In post-optimization analysis, sensitivity has frequently been used as a means of ranking the variables in order of relative importance.

MATHEMATICAL FOUNDATIONS OF SENSITIVITY

Definition

Sensitivity is the rate of change in one factor with respect to change in another factor. Although such a definition is vague in terms of the factors involved it nevertheless implies a quotient of two differentials. Stressing the nebulosity of the definition is important because, in practice, the sensitivity of model parameters is rarely recognized as a special case of the concept of sensitivity. The failure to recognize the generality of sensitivity has been partially responsible for the limited use of sensitivity as a tool for the design and analysis of hydrologic models.

The Sensitivity Equation

The general definition of sensitivity can be expressed in mathematical form by considering a Taylor series expansion of the explicit function:

$$0 = f(F_1, F_2, \ldots, F_n) \qquad \text{(III-1)}$$

The change in factor 0 resulting from change in a factor F_i is given by:

$$f(F_i+\Delta F_i, F_{j|j\neq i}) = O_0 + \frac{\partial O_0}{\partial F_i}\Delta F_i + \frac{1}{2!}\frac{\partial^2 O_0}{\partial F_i^2}\Delta F_i^2 + \ldots \qquad \text{(III-2)}$$

in which O_0 is the value of O at some specified level of each F_i. If the nonlinear terms are small in comparison with the linear terms, Eq. III-2 reduces to:

$$f(F_i+\Delta F_i, F_{j|j\neq i}) = O_0 + \frac{\partial O_0}{\partial F_i}\Delta F_i \qquad \text{(III-3)}$$

Thus:

$$\Delta O_o = f(F_i+\Delta F_i,\ F_{j|j\neq i})-O_0=(\partial O_0/\partial F_i)\Delta F_i \qquad \text{(III-4)}$$

Equation III-4 is referred to herein as the linearized sensitivity equation. It measures thechange in factor O that results from change in factor F_i. The linearized sensitivity equation can be extended to the case where more than one parameter is changed simultaneously. The general definition of sensitivity is derived from Eq. III-1 and III-4:

$$S=\partial O_0/\partial F_i=[f(F_i+\Delta F_i,F_{j|j\neq i})-f(F_1,F_2,\ldots,F_n)]/\Delta F_i \qquad \text{(III-5)}$$

Computational Methods

The general definition of sensitivity which is expressed in mathematical form by Eq. III-5 suggests two methods of computation. The left-hand side of Eq. III-5 suggests that the sensitivity of O to changes in factor F_i can be estimated by differentiating the explicit relationship of Eq. 1 with respect to factor F_i:

$$S =\partial O_o/\partial F_i \qquad \text{(III-6)}$$

Analytical differentiation has not been used extensively for analyzing hydrologic models because the mathematical framework of sensitivity has not been sufficiently developed. It will be used even less frequently as hydrologic models become more complex.

The method of factor perturbation, which is the second computational method suggested by Eq. III-5, is the more commonly used method in hydrologic analysis. The right-hand side of Eq. III-5 indicates that the sensitivity of O to change in F_1 can be derived by incrementing F_i and computing the resulting change in the solution O. The sensitivity is the ratio of the two differentials and can be expressed in finite difference form:

$$S=\Delta O_0/\Delta F_i=[f(F_i+\Delta F_i, F_{j|j\neq i})-f(F_1, F_2, \ldots, F_n)]/\Delta F_i \qquad \text{(III-7)}$$

However, use of the method of parameter perturbation is often impractical for a complete sensitivity analysis of multiparameter systems because of the extensive computational effort required for complex models. It is used when analytical analysis of the sensitivity is not possible.

Parametric and Component Sensitivity

A simplified system or a component of a more complex system is described by three functions: the input function, the output function and the system response function. The response function is the function that transforms the input function into the output function and is often defined by a distribution function which depends on one or more parameters. In thepast, sensitivity analyses of models have been limited to measuring the effect of parametric variations on the output. Such analyses focus on the output and response functions. Using the form of Eq. III-5 parametric sensitivity can be mathematically expressed as:

$$S_{p_i} = \frac{\partial O}{\partial p_i} = \frac{f(P_i+\Delta P_i, P_{j|j\neq i})-f(P_1, P_2, \ldots, P_n)}{\Delta P_i} \qquad \text{(III-8)}$$

where O represents the output function and P_i is the parameter under consideration.

Unfortunately, the general concept of sensitivity has been overshadowed by parametric sensitivity. As models have become more complex the derivation of parametric sensitivity estimates have become increasingly more difficult and, most often, impossible, to compute. However, by considering the input and output functions the general

definition of sensitivity, Eq. III-5, can be used to derive another form of sensitivity. Component sensitivity measures the effect of variation in the input function I on the output function:

$$S_c = \frac{\partial O}{\partial I} = \frac{\Delta O}{\Delta I} \qquad \text{(III-9)}$$

Combining component and parameter sensitivity functions makes it feasible to estimate the sensitivity of parameters of complex models. For example, in the simplified two component model of Fig. III-1, the sensitivity of Y to variation in P_1 and the sensitivity of Z to variation in P_2 are readily computed using sensitivity as defined by Eq. III-6:

$$S_1 = \partial Y/\partial P_1 \qquad \text{and} \qquad S_2 = \partial Z/\partial P_2 \qquad \text{(III-10)}$$

However, the sensitivity of the output from component 2 to change in the parameter of component 1 cannot always be estimated directly from the differential $\partial Z/\partial P_1$. In such cases, the component sensitivity function of component 2 can be used with the parametric sensitivity function S_1 to estimate the sensitivity of Z to change in P_1. Specifically, the sensitivity of $\partial Z/\partial P_1$ equals the product of the component sensitivity function $\partial Z/\partial Y$ and the parametric sensitivity function $\partial Y/\partial P_1$:

$$\partial Z/\partial P_1 = (\partial Z/\partial Y) \cdot (\partial Y/\partial P_1) \qquad \text{(III-11)}$$

Whereas the differentials $\partial Z/\partial Y$ and $\partial Y/\partial P_1$ are often easily derived, an explicit sensitivity function $\partial Z/\partial P_1$ can be computed only for very simple models. When a solution cannot be obtained analytically, then the numerical method of Eq. III-7 must be used.

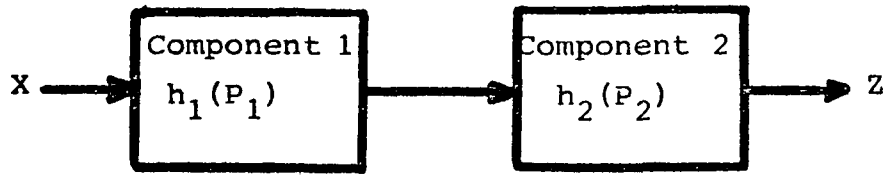

FIGURE III-1. Sensitivity Analysis of Two-Component Model

Absolute and Relative Sensitivity

Sensitivity can be expressed in two forms: absolute and relative. The form in which sensitivity values are presented depends on the intended use. Sensitivity values computed with the definition of Eq. III-5 are in absolute form. Such a definition is inappropriate for the comparison of sensitivity values because values computed using Eq. III-5 are not invariant to the magnitude of either factor O or F_i. Dividing the numerator of Eq. III-5 by O_0 and the denominator by F_i provides an estimate of the relative change in O with respect to a relative change in F_i:

$$R_S = \frac{\partial O/O_0}{\partial F_i/F_i} = \frac{\partial O}{\partial F_i} \cdot \frac{F_i}{O_o} \qquad \text{(III-12)}$$

Parametric Sensitivity in the Optimization Process

Parametric sensitivity is a vital part of all calibration strategies. It is used in analytical optimization when the derivatives of the objective function with respect to the unknowns are taken. For example, the principle of least squares computes the derivative of the sum of the squares of the errors with respect to each of the regression coefficients. In Lagrangian optimization, it is also necessary to take derivatives with respect to coefficients in the constraint(s).

In addition to analytical optimization, sensitivity analysis plays a central role in numerical optimization. A set of parametric sensitivity coefficients defined by Eq. III-5 represents the gradient of the output function. The parametric response surface defined by the output function of multiparameter hydrologic models invariably

contains a considerable number of stationary points (points at which the gradient is zero). Thus, the gradient techniques which are useful for locating a stationary point in a localized region are mathematically insufficient for locating a global optimum. Selecting the global optimum from among the many stationary points requires a more complete optimization strategy. However, parametric sensitivity will be a vital component of any strategy selected for numerical optimization.

Systematic optimization techniques based on the gradient of the output function in the parametric hyperspace are rarely used with complex simulation models. Use of such techniques are limited by the inability to efficiently estimate the sensitivity of many parameters. For models involving as few as five parameters that require evaluation, the method of parameter perturbation requires a considerable amount of computer time to evaluate the sensitivity of each parameter. Component sensitivity may, in the future, demonstrate the efficiency of the direct method of differentiation (Eq. III-6) for estimating the optimal values of unknown parameters.

Stability of the Optimum Solution

A sensitivity plot is a graphical comparison of the percent change in output and the percent change in a parameter value. The change in the value of an objective function is often used to represent the change in output. The sensitivity plot can be used to examine the stability of a parameter of the optimum solution. Derivation of the sensitivity plot has traditionally involved an iterative procedure in which the percentage change in the value of the objective function or output is computed for different percentage changes in a parameter value. For multi-parameter models the required computer time is often

excessive. The direct method of differentiation (Eq. III-6) is an alternate means of examining optimum solution stability having the advantage of requiring significantly less computer time.

The sensitivity function approach is strictly valid only when the nonlinear terms of Eq. III-2 are insignificant. However, even for large changes in a parameter value the sensitivity function approach provides a reliable qualitative indication of the stability of the optimum solution. Furthermore, the sensitivity function approach requires considerably less computational effort than the derivation of a sensitivity plot.

Sensitivity of Initial Value Estimate

An estimate of the initial state of the system is often necessary. For example, simulation models involving storage of water in snow pack or soil zones require estimates of the initial water content of each storage component. Also, models for estimating watershed retention rates often require an estimate of the initial rate. If such models are to be used for design purposes, then it is desirable to estimate the effect of error in initial value estimates on the design variables. A sensitivity analysis of a proposed initial estimate can be used to estimate the effect of error in initial estimates on the computed output.

The method of parameter perturbation is usually used to measure the effect of error in initial value estimates. However, the differential approach to estimating sensitivity requires considerably less computational effort than the perturbation technique, especially for complex models.

Sensitivity and Data Error Analysis

Data used for model verification or for estimating values of model parameters invariably contain error. The magnitude of distribution of error often cannot be evaluated. For example, air speed is often included in evaporation models as a measure of air instability. But whether or not air speed measurements quantitatively measure the effect of air instability on evaporation rates is difficult to assess. However, the effects of data error from other sources can be quantitatively evaluated. For example, instrument specifications supplied by manufacturers can be used to estimate the potential error in an observed measurement due to the inaccuracy of the recording instrument. Reproducibility is a measure of how well different testers can estimate the value of a property with the same input information and procedure. The lack of reproducibility is a measure of random error. Sensitivity and error analyses are a means of examining the effect of the lack of reproducibility on a hydrologic design variable.

An error analysis can be made using the linearized sensitivity equation of Eq. III-4. In this case, the derivative $\partial O_0/\partial F_i$ is the sensitivity of O with respect to F_i, ΔF_i is the error in F_i, and ΔO_0 is the error in O_0 that results from ΔF_i. One convenient method is to use the standard error of a factor as the error variation ΔF_i. Thus, Eq. III-4 becomes:

$$\Delta O_0 = (\partial O_0/\partial F_i) S_{ei} \qquad \text{(III-13)}$$

in which S_{ei} is the standard error of F_i. If Eq. III-13 is computed for each factor, then the values of ΔO_0 for each factor can be computed

and compared. For example, if S_{ei} were the standard errors of the measurement error, then the computed value of ΔO_0 for each F_i would indicate the relative importance of the measurement error in the prediction of the output O.

Limitations of Sensitivity Analysis

Sensitivity analysis was shown to be a useful tool for all phases of modeling (formulation, calibration and verification), as well as part of an error analysis for decisionmaking. However, it has some limitations. First, sensitivity analysis is usually applied using the linear sensitivity equation (Eq. III-4); however, the linear form is valid only over a limited range of the variable in the denominator. Since most hydrologic models are nonlinear, including the snowmelt-runoff model, the sensitivity changes as the value of each parameter changes. In most applications, sensitivity coefficients are usually computed using the means of the variables as a base point. However, the sensitivity at the extreme values of the physical conditions may also be of primary interest.

The univariate nature of sensitivity is a second limitation. In general, sensitivity functions are derived while hodling the values of the other variables constant at some base point value. Similarly, for the error analysis the reproducibility errors represent the "independent" effects of the input variables. That is, the reproducibility errors assume error only in the single variable of interest. An underlying assumption is that there is no interaction between the variables. This is usually not true. Also, we cannot assume that any user on one design will make an error on only one variable.

However, because of the interaction between variables one cannot just simply add the reproducibility errors. For example, even if a user made an error of one standard error in each of the variables, the net affect would not be the sum of the reproducibility errors. Thus, the univariate nature of sensitivity is a limiting factor for large errors or deviations in the variables.

The third, and probably the most important, limitation of sensitivity analysis is that it provides only a single-valued indication of the effect on the criterion or dependent variable. Ideally, one would like to have some idea of the distribution of the design variable. One could approximate this by using the linearized sensitivity equation with the distribution of the error or variation of the indpendent or input variable, but this approach is limited in usefulness because of the first two limitations, i.e., linearity and univariate. A method that circumvents the limitations of sensitivity analysis would be an improvement for many analyses of design methods. This does not imply that senitivity is not of value, only that advanced forms of sensitivity analysis are needed. The linear, univariate form of sensitivity analysis will still be of value for analyses described previously especially for analytical and numerical optimization.

APPLICATION OF SENSITIVITY ANALYSIS TO THE SNOWMELT-RUNOFF MODEL

The general structure of the snowmelt-runoff model is given by:

$$Q_{n+1}=C_n[a_n(T_n+\Delta T_n)S_n+P_n]\frac{A(0.01)}{86400}(1-k_{n+1})+Q_n k_{n+1} \qquad \text{(III-14)}$$

in which Q is the average daily discharge (m^3/S); C is the runoff

coefficient expressing the losses as a ratio (runoff/precipitation); a is the degree-day factor (CM/°Cd) indicating the snowmelt depth resulting from 1-degree-day; T is the number of degree-days (°C·d); ΔT is the adjustment by temperature lapse rate necessary because of the altitude difference between the temperature station and the average hypsometric elevation of the basin or zone; S is the ratio of the snow covered area to the total area; P is the precipitation contributing to runoff (CM); A is the area of the basin or zone (M^2); k is the recession coefficient indicating the decline of discharge in a period without snowmelt or rainfall (i.e., $k = Q_{n+1}/Q_n$); n is the sequence of days during the discharge computation period; and 0.01/86400 is a conversion from $CM \cdot M^2/d$ to M^3/S. Eq. III-14 assumes that there is a single watershed unit; for watersheds with nonhomogeneity of runoff, rainfall, or temperature characteristics, the watershed should be subdivided into homogeneous subareas. When the watershed is subdivided, Eq. III-14 must be modified to reflect the subdivision. This requires repeating the first term of the right-hand side of Eq. III-14 for each subarea.

The model of Eq. III-14 contains eight quantities. The drainage area is constant for a watershed or subwatershed. Time-varying input variables include T, S, and P; these will vary daily. The remaining four quantities (a, C, ΔT, and k) may vary on a daily basis; however, they are not directly measureable, and so without additional input, they would probably be considered as constants for a given snowmelt season. The assumption that these quantities (i.e., c, a, k, and ΔT) are constant will introduce an error, which can be assessed using a sensitivity analysis. Rewriting Eq. III-14 yields:

$$Q_{n+1} = [C\ a\ S_n(T_n+\Delta T)+CP_n]\left(\frac{0.01A}{86400}\right)(1-k)+Q_n k \qquad \text{(III-15)}$$

The subscripts for C, a, ΔT, and k were dropped to indicate that they are assumed to be constant. The derivatives of Eq. III-15 with respect to the four constants are:

$$\frac{\partial Q_{n+1}}{\partial C} = \left(\frac{0.01A}{86400}\right)(1-k)[a\ S_n(T+\Delta T)+P_n] \qquad \text{(III-16)}$$

$$\frac{\partial Q_{n+1}}{\partial a} = \left(\frac{0.01A}{86400}\right)(1-k)[C\ S_n(T_n+\Delta T)] \qquad \text{(III-17)}$$

$$\frac{\partial Q_{n+1}}{\partial \Delta T} = \left(\frac{0.01A}{86400}\right)(1-k)\ C\ a\ S_n \qquad \text{(III-18)}$$

$$\frac{\partial Q_{n+1}}{\partial k} = Q_n - \left(\frac{0.01A}{86400}\right)[C\ a\ S_n(T+\Delta T)+CP_n] \qquad \text{(III-19)}$$

An error analysis can be performed by multiplying the derivatives of Eqs. III-16 to III-19 by the error of the corresponding variable (see Eq. III-4). The error in Q_{n+1}, which can be denoted as ΔQ_{n+1}, is a function of the daily values of S, T, and P, and for computing the error with respect to k it would also be a function of Q_n.

To illustrate the use of Eqs. III-16 to III-19, consider the following values for an 8.9 sq. km. subwatershed: C=0.95, a=0.45, k=0.87, S=0.8, T=1.15, ΔT=0.65, P=0.21, and Q_n=0.453. Eq. III-14 yields a predicted runoff rate of 0.503 M^3/S. Eqs. III-16 to III-19 yield the following derivatives: $\partial Q_{n+1}/\partial C$=0.297, $\partial Q_{n+1}/\partial a$=0.183, $\partial Q_{n+1}/\partial \Delta T$=0.0458, and $\partial Q_{n+1}/\partial k$=-0.3866. These values cannot be compared to measure the relative importance of the four quantities (C, a, ΔT, and k).

In order to use the derivatives as part of an error analysis, values of the change in C, a, ΔT, and k must be estimated. Based on the data for the Dischma watershed given in the Snowmelt-Runoff Model User's Manual the following errors were estimated: $\Delta C=0.025$, $\Delta a=0.05$, $\Delta(\Delta T)=0.01$, and $\Delta k=0.15$. The errors in Q_{n+1} that would result from these errors can be estimated using Eq. III-4:

$$\Delta Q_{n+1} = \frac{\partial Q_{n+1}}{\partial C} \cdot \Delta C = 0.297(0.025) = 0.007 \qquad \text{(III-20)}$$

$$\Delta Q_{n+1} = \frac{\partial Q_{n+1}}{\partial a} \cdot \Delta a = 0.183(0.05) = 0.009 \qquad \text{(III-21)}$$

$$\Delta Q_{n+1} = \frac{\partial Q_{n+1}}{\partial \Delta T} \cdot \Delta(\Delta T) = 0.0458(0.01) = 0.0005 \qquad \text{(III-22)}$$

$$\Delta Q_{n+1} = \frac{\partial Q_{n+1}}{\partial k} \cdot \Delta k = -0.3866(0.15) = -0.058 \qquad \text{(III-23)}$$

A comparison of the values of Eqs. III-20 to III-23 indicate that the largest error for this data and watershed would occur because of imprecision in k, with the error due to ΔT being insignificant. Obviously, the effect of error in the variables would be different for other values of any of the inputs (i.e., C, a, ΔT, k, T, S, P).

While Eqs. III-20 to III-23 provide a measure of the effect of errors, the derivatives can be used to compute the relative sensitivity (Eq. III-12). For the data of the error analysis, the relative sensitivities of C, a, ΔT, and k are:

$$R_C = \frac{\partial Q_{n+1}}{\partial C} \cdot \frac{C}{Q_{n+1}} = 0.297\left(\frac{0.95}{0.503}\right) = 0.561 \qquad \text{(III-24)}$$

$$R_a = \frac{\partial Q_{n+1}}{\partial a} \cdot \frac{a}{Q_{n+1}} = 0.183\left(\frac{0.45}{0.503}\right) = 0.164 \qquad \text{(III-25)}$$

$$R_{\Delta T} = \frac{\partial Q_{n+1}}{\partial \Delta T} \quad \frac{\Delta T}{Q_{n+1}} = 0.0453\left(\frac{0.65}{0.503}\right) = 0.059 \qquad \text{(III-26)}$$

$$R_k = \frac{\partial Q_{n+1}}{\partial k} \quad \frac{k}{Q_{n+1}} = -0.3866\left(\frac{0.87}{0.503}\right) = -0.669 \qquad \text{(III-27)}$$

The values of Eqs. III-24 to III-27 are dimensionless and indicate that k and C are the most important variables, with a being only moderately important and ΔT have very low importance.

SECTION IV

PROBABILISTIC MODELING

Probabilistic modeling is an extension of sensitivity analysis with the following important distinctions: 1) it is not limited to the univariate form, 2) it allows for the interaction of the input variables, and 3) it provides the distribution of the design variable (i.e., dependent or criterion variable) and not just a single-valued measure of the dispersion of the design variable. Of course, there is a price to pay for the additional information. First, we must know the distributions of the input variables, and second, the computational requirement for a probabilistic analysis is much greater than that required for a linear, univariate sensitivity analysis.

A probabilistic analysis is based on an iterative analysis of the design model for a sufficient number of conditions that defines the distribution of the design variable. While a numerical analysis of sensitivity requires two iterations of the design model in order to define the sensitivity by Eq. III-7, the probabilistic approach involves numerous solutions of the design model. While the sensitivity analysis of Eq. III-7 is univariate, with each variable being perturbed independently of the others, probabilistic analysis uses a simultaneous perturbation of all of the design variables. While the sensitivity analysis requires only a measure of the dispersion or error variation of the input variables, a probabilistic analysis requires knowledge of the entire distribution function of the input variables.

Actually, probabilistic analysis is quite simple. Given the distribution functions of the input variables, values of the input variables are generated using values of random variables having the

distribution functions and used to compute a single value of the design variable. This process is repeated until a sufficient number of values of the design variables has been generated to define the distribution function of the design variable.

A simple computational example can be used to illustrate the methodology. Let's assume a design process consists of two input variables, X_1 and X_2, which are both normally distributed. The variable X_1 has a mean and standard deviation of 5 and 1, respectively. Given that both have a normal distribution, values of X_1 and X_2 can be generated using a random normal number generator with the appropriate statistics. A sample of 25 was generated for each of the input variables (see Table IV-1). The value of a design variable Y is related to X_1 and X_2 by

$$Y = 7.0 + 0.6X_1 + 1.6X_2 \qquad \text{(IV-1)}$$

The 25 values of Y generated with Eq. IV-1 are also given in Table IV-1. It is known from theory that the sum of m independent random variables, each normally distributed, is also normally distributed for a linear combination. Thus, since Eq. IV-1 is a linear equation and X_1 and X_2 are normally distributed, then Y must also be normally distributed. It can also be shown from theory that the mean and variance of Y are sums of the linear combinations of the input variables X_1 and X_2. However, the important point here is that we used randomly generated values of the input variables to generate the values of the design variable, from which an estimate of the underlying distribution of Y was determined. In the example used, the true

TABLE IV-1. Generated Values of a Design Variable (Y) as a Function of Two Input Variables (X_1 and X_2)

i	X_1	X_2	Y
1	5.080	4.889	17.870
2	1.874	4.884	15.939
3	2.574	5.948	18.061
4	4.796	4.309	16.772
5	2.726	5.952	18.159
6	0.830	3.422	12.973
7	-0.736	5.677	15.642
8	3.122	3.740	11.857
9	1.932	3.270	13.391
10	3.508	5.421	17.778
11	2.640	3.702	11.507
12	2.618	5.705	17.699
13	3.322	4.956	16.923
14	2.754	4.740	16.236
15	2.570	4.842	16.289
16	5.506	5.321	18.817
17	2.708	5.285	17.081
18	0.414	4.444	14.359
19	2.468	5.920	17.953
20	-0.638	5.245	15.009
21	4.338	4.239	16.385
22	4.620	5.461	18.510
23	0.608	3.746	13.358
24	4.422	6.241	19.639
25	2.264	2.892	12.986

distribution could have been derived from theory only because the input variables had a normal distribution and Eq. IV-1 defines Y to be a linear combination of X_1 and X_2. In practice, the input variables are frequently non-normal and the relationships between Y and the input variables are not linear. In that case, it would be impossible to derive the distribution of the design variable Y from theory.

PROBABILISTIC ANALYSIS OF SNOWMELT-RUNOFF MODEL

The sensitivity analysis indicated that errors in the coefficients C, a, ΔT, and k can introduce significant errors into the computed runoff, withthe value of a, C, and k being the most important for the example provided. For a value of Q_{n+1} of 0.503, the individual errors in absolute value of 0.058, 0.009, and 0.007 for k, a, and C, respectively, are 11.5, 1.8, and 1.4 percent. These errors assume that each of the other coefficients are known with certainty, which is certainly not the case. When the errors in the coefficients occur simultaneously, then the accuracy of the computed snowmelt runoff will be considerably less than the accuracy when only one coefficient is in error.

Distributions of the Coefficients

A necessary input to a probabilistic analysis is the distribution functions of the variables, which would be the coefficients for the snowmelt runoff model. The data base was very insufficient to determine the distribution functions of the coefficients; therefore, the normal distribution was assumed. This assumption will probably not be critical to the conclusions, although one could make a good argument that for values of the recession coefficient near either zero or one, the error distribution of the recession coefficient would be highly skewed.

Assuming that each of the four coefficients have a normal distribution, then it is only necessary to identify the location and scale parameters of the distribution. Using the expected value theorem, the mean value provided in the User's Manual (Martinec, et al., 1983) will be used to represent the location parameter. The standard errors derived for the sensitivity analysis will be used to represent the scale parameters of the distributions. These values of the location and scale parameters completely define the assumed normal populations of the four coefficients.

Distribution of the Error in the Snowmelt Runoff

The usual procedure for decisionmaking is to compute an average, or expected value, of the runoff. The predicted value represents the best estimate of the snowmelt runoff. While the expected value is important, it is also important in decisionmaking to know the likely variation in the predicted value due to error in the input. Such information can be used to compute either confidence intervals on the computed value or tolerance limits on the distribution of the output.

Using the data for the Dischma watershed for 1974 (Martinec, et al., 1983), the mean one-day snowmelt runoff (m^3/s) was computed for each day from April 2 to July 31 and an error function was computed. The error about the mean predicted value was computed; this error distribution indicates the error due solely to error in the coefficients. Bias in the predicted values will not affect this error distribution.

The probabilistic modeling approach was used to assess the error distribution. The snowmelt-runoff model of Eq. I-1 was used with the Dischma data for 1974 and the one day runoff was computed for the 121

day period. A sequence was computed using the mean values of the coefficients; this sequence would be the predicted values assuming no error in the coefficients. Then random numbers were generated using a random number generator, and the values of the coefficients were computed using the probability distributions of the coefficients. A total of 250 sequences were generated, and the mean daily error was computed for each sequence. For the 250 mean daily errors, the mean of the means was 0.0115 m^3/sec and the standard deviation of the mean errors was 0.0468 m^3/S. The distribution of the means is shown in Fig. IV-1. For a 1 percent level of significance, the mean of 0.0115 m^3/S is statistically different from zero; therefore, the model produces slightly biased estimates for the data used. This bias is probably due to the high recession coefficient in the early days of the sequence; during this period, the distribution of the recession coefficient had to be truncated in order to maintain rationality. The truncation has only a minor effect on the distribution of the errors, as shown in Fig. IV-1. From this figure it is evident that the distribution is approximately normally distributed.

Confidence Intervals on the Error of the Mean

The above statistic can be used to derive confidence intervals on the mean daily estimated runoff for the 121 day period from April 2 to July 31. The two-sided 95 percent confidence interval would be ±.0918 m^3/S. The two-sided 99 percent confidence interval is ±0.121 m^3/S. These confidence intervals are valid for the mean error for the 121 day period and not the mean error for an individual day during that period.

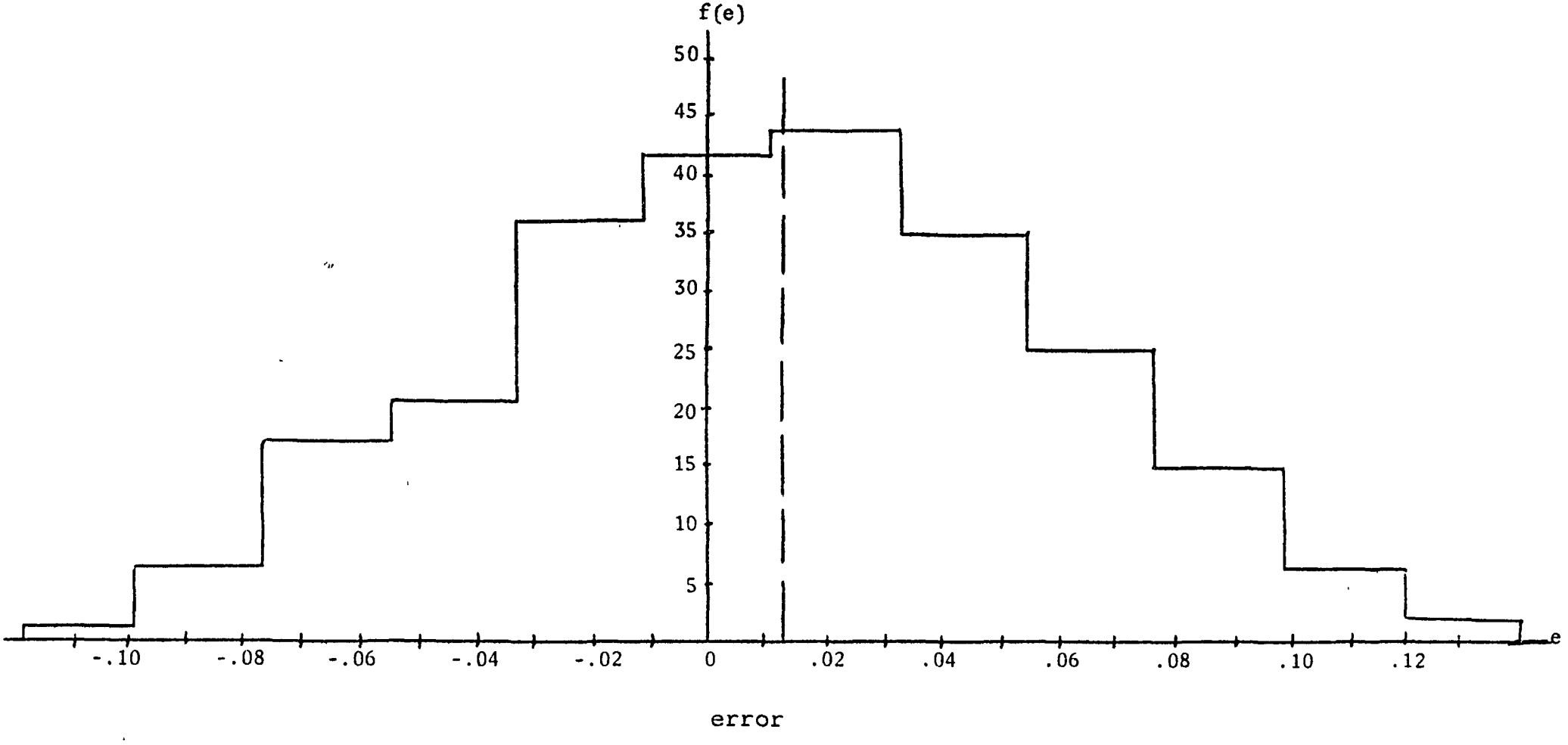

FIGURE IV-1. Sample Frequency Distribution of the Errors (e) for the Daily Mean Runoff

Also, these confidence intervals reflect only the error due to the inability to accurately predict the values of the four input coefficients; they do not reflect the error due to the inability of the model structure to represent the physical processes being modeled. Such errors would be substantially greater.

The computed confidence intervals indicate that the error in the mean daily discharge due to the inaccuracy of the coefficients is not very significant. Thus, for mean long-term discharge estimates, the accuracy of the coefficients should not be viewed as a limiting factor in transferring the coefficients to adjacent watersheds in which the physical processes are similar.

SECTION V

CONCLUSIONS

Existing snowmelt runoff models, such as the frequently used regression models, are used to make single-valued estimates of runoff for some duration; for example, regression models have provided accurate estimates of snowmelt runoff volumes for periods of 30 days or more. While the single-value estimate can be used in decisionmaking, it does not provide a measure of the risk, i.e., the accuracy of the estimate. Inaccuracies are introduced by the structure of the model, the empiricism of the coefficients, the inaccuracy of the measured input data, and the physical processes that are ignored in the formulation of the model. The inaccuracy of the empirical coefficients are of special concern because models are often calibrated on one watershed and transferred to another watershed to make forecasts. Inaccuracy of the coefficients will result in errors in forecasts. Therefore, the study was conducted to show how such sources of inaccuracy can be evaluated. The emphasis was placed on the methodology rather than analysis of data since data for any one watershed are limited.

The methodology for assessing the accuracy of forecasts consisted of a probabilistic modeling analysis and a sensitivity/error analysis. The snowmelt runoff model (Martinec, et al., 1983) was used because it has a physically based structure, the input data requirements are more diverse, the coefficients have a physical basis, and the model has been applied successfully for forecasts for durations ranging from one day to three months or more. While the coefficients of the snowmelt runoff model are physically based, they must be considered as random variables since they cannot be determined exactly.

Sensitivity equations were derived for each of the four coefficients: the runoff coefficient (C), the degree-day factor (a), the temperature adjustment (ΔT), and the recession coefficient (k). The sensitivity equations are given by Eqs. III-16 to III-19. The sensitivity equations can be used to provide a measure of the relative importance of the coefficients or as part of an error analysis. The results of a simple error analysis showed that the approximate standard error of the coefficients produced the largest error in the recession coefficient, with the errors for the runoff coefficient and the degree-day factor to be much less significant. The error in the runoff due to the error in the value of ΔT was very insignificant. Of course, these results are for one set of data. The effect of error in the coefficients is dependent on the watershed, the duration of the forecasts, the levels of the input variables (i.e., P, T, and S), and the estimated values of the coefficients. However, the methodology presented here can be used to perform a sensitivity analysis for any case.

The sensitivity equations show that the coefficients are interdependent and that error in one coefficient will affect the sensitivity of another coefficient. This suggests that a probabilistic analysis should be made in addition to the sensitivity analysis. A probabilistic analysis was conducted for the Dischma watershed with the data for 1974. The results indicated that errors in the coefficients produced a distribution of errors in the forecasts that was approximately normally distributed; however, there was a slight positive bias in the mean. The results suggest that the input coefficients are reasonably accurate and that the error in the coefficients should not limit the transferability of the model to nearby watersheds.

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