Two-Fluid Models of Turbulence

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1. **INTRODUCTION**

1. Some defects of turbulence models

It has become customary among engineers and others concerned with practical flow and heat-transfer predictions to use “turbulence models”, ie sets of equations which purport, when coupled with the equations of mean motion, to describe such statistically significant properties as the “effective viscosity”, “length scale” and “energy”. Most frequently used are those models employing two partial differential equations, the dependent variables of which are, for example, the turbulence energy, k, and the dissipation rate, ε.

Although valuable service has been performed by such models, their predictive ability is far from meeting all demands. A brief list of their shortcomings is:

(a) The “intermittency” of turbulent flow, evident to all who look perceptively at, for example, automobile-exhaust plumes, is left out of account.

(b) Attempts to extend the models to chemically-reacting flows have had little success, because chemical reactions take place in regions of steep gradients which find no expression in the equations employed.

(c) Empirical and non-general corrections have had to be made to the “constants” of the equations in order to bring their predictions into line with experimental evidence regarding the influences of mean-streamline curvature and of interactions between gravitational forces and temperature gradients.

(d) The models are totally unable to explain the “unmixing” phenomenon which steepens gradients of average fluid properties rather than diminishing them.

In the view of the present writer, the above defects have a common origin, namely neglect of the “spottiness” of real turbulent flows. The question therefore to be discussed is: How can the major effects of the “spottiness” be described by a mathematical apparatus that is still not too elaborate for practical use?

1.2 **A proposed shift of emphasis**

(a) **The origins**

Modern ideas about turbulence have been greatly influenced by the writings of Osborne Reynolds (eg Ref 1) and Ludwig Prandtl (eg Ref 2), whose observations had led them to conceive of turbulence as involving a semi-random exchange of matter, similar to that envisaged in the kinetic theory of gases, but on a larger scale. The following quotations illustrate this.
Reynolds:
"The heat carried off...is proportional to the rate at which particles or molecules pass backwards and forwards from the surface". (Present author's italics.)

Prandl:
"It is now necessary to make a usable hypothesis for the mixing velocity, w. The transverse momentum associated with this velocity must be constantly destroyed by braking, and constantly re-established". (Present author's italics)

It is a consequence of this pre-occupation with mixing that turbulent fluxes of a scalar quantity are almost invariably expressed, by analogy with Fick's law of diffusion and Fourier's law of heat conduction, in terms of the gradient of the corresponding intensive property. Thus, for heat transfer, the law:

\[ \langle \dot{q} \rangle_d = -\lambda_{eff} \cdot \text{grad} \ T \]  \hspace{1cm} (1.2-1)

is employed, along with the supposition that \( \lambda_{eff} \) is a greater-than-zero measure of the local turbulent motion. Here \( \langle \dot{q} \rangle_d \) is the heat-flux vector, \( \lambda_{eff} \) is the effective conductivity, and \( T \) is the time-mean temperature.

So embedded is this notion in current thinking that, when experimental evidence becomes incontrovertible that the so-defined \( \lambda_{eff} \) is negative, the phrase "contra-gradient diffusion" (Ref 3) is applied to the phenomenon, which is regarded as anomalous.

(b) The splitting phenomenon and its implications

In the view of the present writer, normal, i.e. "co-gradient" diffusion is only one of the two main ways in which turbulent fluxes are caused: the second, although it could be called "non-gradient diffusion", is better described as "splitting". This is the phenomenon according to which intermingled fragments of fluid differing in, say, density, move relative to one another as a consequence of a body-force field.

The splitting process is most easily seen when surface tension keeps the two fluids apart, as when steam bubbles rise from the heated bottom of a kettle. This is not an exchange process: for the bubbles rise continuously; yet it is undoubtedly associated with a flux of energy.
The mathematical expression for the heat flux associated with sifting is conveniently written as:

\[ \langle q \rangle_S = \text{constant} \langle a \rangle \ h' \]  \hspace{1cm} (1.2-2)

which signifies that the flux \( \langle q \rangle_S \) depends upon the local fluctuations of enthalpy, \( h' \), and on the acceleration vector \( \langle a \rangle \). Similarly, if \( w \)-direction momentum transfer is in question, the contribution of sifting to the shear-stress vector \( \tau_S \) may be expressed as:

\[ \langle \tau \rangle_S = \text{constant} \langle a \rangle \ w' \]  \hspace{1cm} (1.2-3)

a term which may or may not have the same sign as the more conventionally defined viscous contribution:

\[ \langle \tau \rangle_d = \mu_{\text{eff}} \ \text{grad} \ w \]  \hspace{1cm} (1.2-4)

Here \( w \) and \( w' \) represent the time-average and fluctuating components of longitudinal velocity.

(c) **Circumstances in which "turbulent sifting" is significant**

(i) **Bubble rise in a saucepan**

In the phenomenon just mentioned, it is important to emphasise, the vertical-direction temperature gradient is very small; for both steam and water are at the saturation temperature corresponding to the local pressure, which varies negligibly over the small height change. Nor can enthalpy or concentration gradients account for the bubble motion: for the bubbles are moving towards the region of highest steam concentration, namely the top part of the saucepan, not away from it. The fluid enthalpy there is of course higher than at the bottom.

(ii) **Rise of smoke in a room**

If a fire starts on the floor in the corner of a room, flames and smoke rise in a turbulent manner. often forming a plume of smoke within which the naked eye can distinguish fragments of smoky gas separated by less opaque bodies of clear air; yet the layer of
smoke which flows along the ceiling is usually separated from the air below it by a sharp interface: the lighter smoke fragments have floated upwards leaving the denser air behind.

(iii) Boundary layers on curved surfaces

When a turbulent boundary layer encounters and flows over a convex surface, the pressure gradient along the normal to the surface is positive. As a consequence, fragments of fluid which are flowing at a greater-than-average velocity are flung outward, while slower fragments are forced towards the wall. There is therefore, from this cause, a momentum transfer outwards towards the region of higher velocity.

The momentum flow is reversed when the curved surface is concave: for the higher-velocity fragments move towards the wall. The momentum flux associated with settling has, in this case, the same sign as that associated with turbulent mixing.

(iv) Flow in a cyclone

Similar settling flows occur in cyclone separators. That particles of denser material move preferentially outwards is of course obvious: it is the raison d'être of the apparatus. Less well-recognised is the fact that fluid particles having larger circumferential velocities will migrate to the larger radii, while the slower-moving particles move towards the cyclone axis.

This steady transfer of kinetic energy to the outside is presumably the reason for the observation that, in the Fane-Hilsch tube (which is a kind of cyclone), the larger-diameter cylindrical wall becomes much hotter than the smaller-diameter one.

(v) Turbulent combustion of pre-mixed gases within a duct

When a pre-mixed combustible gas burns within a duct, the density reduction occasioned by the exothermic reaction occasions a reduction in pressure with increase in distance along the duct. This pressure gradient causes the lighter burned gas to accelerate more than the heavier unburned gas, the former is therefore "sifted" towards the duct outlet, where however the average temperature is the highest. This is one of the best-documented examples of "counter-gradient diffusion"; and measurements have clearly revealed the fact that the hot-gas fragments have higher velocities than the cold-gas ones (Refs 3, 4, 5).
(vi) Sand storms

A circumstance in which diffusion and sifting act in opposite directions, and maintain an approximate balance, arises when a strong wind blows over a sandy desert. Then turbulent diffusion of the Reynolds-Prandtl kind carries sand particles from high-concentration regions close to the ground to lower-concentration regions higher up, while sifting effects a flux in the downward direction.

It is worth remarking that, in such a case, the scale of the fragments of upward-moving and downward-moving material is much larger than that of the sand particles: the sand-laden air acts like a single-phase fluid of non-uniform density.

(d) A simple formulation of the sifting process

(i) The sifting flux

Let a turbulent fluid be taken as consisting of fragments and surrounding volumes of two kinds, distinguished by their having different densities, \( \rho_1 \) and \( \rho_2 \), and different values of some scalar quantity \( \phi \).

What will now be proved is that there is a flux of entity \( \phi \) in the direction of the local body-force vector, which is proportional to the difference \( \phi_2 - \phi_1 \), and not to the gradient of either quantity: the relative velocity \( U \), i.e. \( u_2 - u_1 \) also enters. Here \( u \) stands for the absolute velocity.

The net flux of \( \phi \), viz. \( F_\phi \), can be written as:

\[
F_\phi = r_1 \rho_1 u_1 \phi_1 + r_2 \rho_2 u_2 \phi_2
\]

\[\text{.(1.2-5)}\]

This can be re-written in terms of the relative velocity and of the average velocity, density and \( \phi \), as follows:

\[
F_\phi = \bar{\rho} \bar{u} \phi + (r_1 \rho_1 r_2 \rho_2 / \bar{\rho}) \bar{U} (\phi_2 - \phi_1)
\]

\[\text{.(1.2-6)}\]

wherein use has been made of the definitions:

\[
\bar{\rho} = r_1 \rho_1 + r_2 \rho_2
\]

\[\text{.(1.2-7)}\]
\[ \bar{\rho} \mathbf{u} = r_1 \mathbf{u}_1 + r_2 \mathbf{u}_2 \]
\[ \bar{\rho} \mathbf{w} = r_1 \mathbf{w}_1 + r_2 \mathbf{w}_2 \]
\[ 1 = r_1 + r_2 \]

Equation (1.2-6) is easy to interpret: the first term on the right-hand side represents the transfer of \( \phi \) by the average mass motion, and the second term represents the settling contribution.

Further examination of that term shows that it is linearly dependent upon \( U \) and \( (\phi_1 - \phi_2) \), and also upon the product \( r_1 r_2 \). The latter, by reason of equation (1.2-10), falls to zero when either component is absent, as is readily understood.

(ii) The relative velocity, \( U \)

As will be explained in the next section, the magnitude of \( U \) can be computed by solving momentum equations for \( u_1 \) and \( u_2 \) separately, and then subtracting. However, an approximate formula can be obtained by neglecting all the terms in the momentum equations except the pressure gradient, the body-force acceleration \( a \), and the interfluid friction, as follows. The momentum equations reduce to:

\[ r_1 \left( \frac{d\rho}{dx} - \rho_1 a \right) - C r_1 r_2 \bar{\rho} \frac{IU\mathbf{U}}{I} = 0 \]

and

\[ r_2 \left( \frac{d\rho}{dx} - \rho_2 a \right) + C r_1 r_2 \bar{\rho} \frac{IU\mathbf{U}}{I} = 0 \]

wherein \( C \) represents an inter-fluid friction factor, and \( \bar{I} \) is a measure of the fragment size.

Addition of the two equations and reference to equation (1.2-7) lead to:

\[ \frac{d\rho}{dx} - \bar{\rho} a = 0 \]
whereupon substitution in either equation leads to:

\[
\frac{\eta \mu_j - C^{-1} (p_2 - p_1)}{\eta} \frac{1}{a} \quad (1.2-14)
\]

This equation is also easy to understand: larger fragments move faster, because the body force operates on the volume whereas the friction operates on the surface area; and the velocity difference increases with the density difference.

The "constant" \( C \) can be expected to increase when the fragment size is small, in proportion to the reciprocal of the Reynolds number of the relative motion: but the assumption of constancy is probably correct for high-Reynolds-number flows.

(iii) When \( \rho \) is linear in \( \phi \)

Let \( S_\phi \) stand for the sliding flux of entity \( \phi \), so that:

\[
S_\phi = (\mu \eta \rho \rho_0) \frac{1}{\mu} (\phi_2 - \phi_1) \quad (1.2-15)
\]

and \( \mu, \rho \) be linear in \( \phi \), so that:

\[
(p_2 - p_1) - \rho (\phi_2 - \phi_1) \quad (1.2-16)
\]

Combination of equations \((1.2-15), (1.2-16)\) and \((1.2-14)\) now yields:

\[
S_\phi = r_1 r_2 \frac{\eta \rho}{\rho} \left[ \frac{\eta \phi_3}{C} \right]^{1/2} \left[ \phi_2 - \phi_1 \right]^{3/2} \quad (1.2-17)
\]

Since it is also possible to relate \( \phi_2 - \phi_1 \) to \( \phi' \), the root-mean square value of the fluctuations of \( \phi \) by:

\[
\phi' = (r_1 r_2)^{1/2} |\phi_1 - \phi_2| \quad (1.2-18)
\]
the expression for the silting flux in terms of $\phi'$ is:

$$S_t = (r_1 r_2)^{1/4} \rho_1 \rho_2 \left( \frac{\beta_m}{\gamma} \right)^{1/2} (\phi')^{3/2}$$

(12.19)

This is a term which, in the present author's view, should often appear in the differential equation for $\phi$, when this is employed in a turbulence model.

Of course, it can be used only when estimates are available of the $r_1 r_2$ product, of the fragment size $d$, and of the fluctuation intensity $\phi'$; but this situation is not rare

(iv) The silting flux of momentum, in flows with curved stream lines

When a fluid having velocity $w$ follows a path having a radius of curvature $R$, it experiences an acceleration $w^2/R$ normal to its trajectory.

If the above equations are re-worked for the situation in which the density is uniform but the two sets of fluid fragments have differing velocities, $w_1$ and $w_2$, there results:

$$S_w = r_1 r_2 \rho (w_2 - w_1)$$

(12.20)

$$U \mid U \mid = C^{-1} (w_2 - w_1) \frac{\ddot{w}}{R}$$

(12.21)

and so:

$$S_w = r_1 r_2 \rho \left( \frac{\ddot{w}}{C \gamma} \right)^{1/2} (w_2 - w_1)^{3/2}$$

(12.22)

or, in terms of $w'$:

$$S_w = (r_1 r_2)^{1/4} \rho \left( \frac{\ddot{w}}{C \gamma} \right)^{1/2} (w')^{3/2}$$

(12.23)

Here $S_w$ represents, of course, a "silting shear stress"; and it is
Interesting to observe that its magnitude depends upon \( w' \) while its direction is always away from the centre of curvature, regardless of the direction of the gradient of \( w \).

Once again, use of the equation requires \( r_{12}, \ell \) and \( w' \) to be estimated, but this is often possible.

2. **The mathematical theory of the flow of two interpenetrating continua**

2.1 **Background**

During recent years, it has been necessary to predict the motion of steam and water in the vessels and pipes of pressurised-water systems experiencing hypothetical loss-of-coolant accidents. This necessity has caused attention to be concentrated upon how the motions of two "interpenetrating continua" can be represented numerically and computed efficiently; and the problem of how to do so has been solved.

The literature of the subject is now too voluminous for summary; but the pioneering paper of Harlow and Amsden deserves special mention (Ref 6); and the present author has also made contributions (eg Ref 7).

As a consequence of these developments, it is possible to compute two-phase flow phenomena as readily as single-phase ones; and indeed the ability to do so is now embodied in a generally-available computer program (Ref 8,9), which has been used for all the numerical work alluded to below.

This has been fortunate: for it has permitted the development of the new two-fluid turbulence model to proceed without awaiting the construction of a new computational tool.

In the following section, a brief introduction to the theory will be provided.

2.2 **Mathematical formulation of the two-fluid model**

(a) **Definitions**

It is supposed that two fluids, 1 and 2, share occupancy of space. The proportions of time during which each can be expected to occupy a particular location are the volume fractions, \( r_1 \) and \( r_2 \), which obey the relation:

\[
\frac{r_1 + r_2}{r_1 + r_2} = 1 \quad \text{(2.2-1)}
\]
How the two fluids are distinguished from one another is, however, at any location and time, all fragments belonging to fluid 1 are supposed to have the same values of temperature, mass fraction of component chemical species, velocity components, and fragment size. This assumption of fluid homogeneity is, of course, an idealisation, at variance with reality; but it may well be closer to the truth than do the more conventional single fluid models of turbulent flows.

All the above fluid properties will be represented by the symbol $\phi_l$, where $l$ takes the value 1 or 2 according to the fluid.

It will be supposed that both fluids have the same pressure at each point, not because the equation system is significantly simplified thereby, but because no mechanism entailing a difference of pressure has been hypothesised so far.

(b) Differential equations

Mass balances. There are two mass conservation laws to express. They can both be written in the form:

$$
\frac{\partial}{\partial t} \left( \rho_l \mathbf{r}_l \right) + \text{div} \left( \rho_l \mathbf{r}_l \langle \mathbf{v}_l \rangle \right) = \mathbf{m}_l \tag{2.2.2}
$$

where $t$ stands for time,
$\rho_l$ stands for the density of fluid $l$,
$\langle \mathbf{v}_l \rangle$ stands for its velocity vector, and
$\mathbf{m}_l$ stands for the rate of transfer of mass into fluid $l$ from the other fluid, with which it is intermingled.

In equation (2.2.2), instantaneous values of $\rho$, $\mathbf{r}$, $\langle \mathbf{v} \rangle$ and $\mathbf{m}$ are in question. Because it is impractical, in numerical calculation, to compute the most rapid fluctuations of these quantities, time-average quantities are preferable, in which case the quantity operated upon by div must be augmented by $D_l$, an effective diffusion flux. The equation then becomes:

$$
\frac{\partial}{\partial t} \left( \rho_l \mathbf{r}_l \right) + \text{div} \left( \rho_l \mathbf{r}_l \mathbf{v}_l + D_l \right) = \mathbf{m}_l \tag{2.2.3}
$$

from which, however, overbars signifying time averaging have been
illed so as to avoid needless clutter.

The first term represents the difference of outflow from inflow along the line dimension; div represents that difference along the ace dimension; and $m_1$ can be regarded as the negative of the resistance between outflow and inflow along an "interfluid reaction" if all these fluxes are denoted by $G_1$. And div is defined as a generalisation of div which makes reference to all air dimensions. Equation (2.2-3) can be finally expressed as:

$$\text{DIV} (G_1) = 0 \quad \text{.(2.2-4)}$$

Changes of other quantities. The above nomenclature facilitates a compact writing of the differential equation governing the distribution of the fluid property $\phi$. The equation becomes:

$$\text{DIV} (\phi G_1) = \phi \quad \text{.(2.2-5)}$$

Herein $\phi$ is the volumetric source of the entity $\phi$ in phase 1, and $\phi_1$ is the value of $\phi$ associated with the flux which it multiplies.

Of especial importance are the momentum equations, in which $\phi_1$ stands for a velocity component. For these equations, the volumetric-source quantity $\phi_1$ consists primarily of pressure-gradient, body-force, interfluid-friction and wall-friction terms. Thus, for the velocity $u_1$, the equation becomes:

$$\text{DIV} (u_1 G_1) = - \frac{\partial \rho_1}{\partial x} + g \rho_1 \rho_1 + \frac{f_{ij} (u_j - u_i) - f_{lw}}{u_1} \quad \text{.(2.2-6)}$$

Here: $x$ is the distance coordinate aligned with $u_1$.
$p$ is the pressure.
$g_b$ is the body-force component in the $x$-direction.
$f_{ij}$ is the volumetric inter-fluid-friction coefficient.
$u_1$ is the local velocity of the other component. and
$f_{lw}$ is the volumetric coefficient of friction between fluid 1 and a nearby wall.

"Pressure-gradient" and other kinds of diffusion. Because of the association of the fluid density $\rho_1$ with each of the $G_1$ on the left-hand side of equation (2.2-6), and of its non-association with
the pressure-gradient term on the right hand side, the effect of
the pressure gradient on the velocity components of the two fluids
is inversely proportional to their densities. Pressure gradients
cause the two fluids therefore to have different velocities, so that
they move relatively to one another.

Relativley motion between intermingled fluids can also result from
turbulent diffusion, i.e. from the random movements of fluid
fragments, the net magnitudes of which are usually proportional to
the gradient of concentration (i.e. volume fraction). To distinguish
the two phenomena, that caused by the pressure gradient will be
here called "sliding", as explained above.

(c) Auxiliary relations

Interfluid friction. In order that the equilbrium system should be
"closed", i.e. rendered complete enough for solution, mathematical
expressions must be devised for such inter-fluid transport quantities
as $m_j$ and $l_j$. Since the present writer has discussed these
questions elsewhere (Refs 10, 11, 12), it suffices here to indicate
what formulae have been used in the computations which will be
presented below.

The formula which appears in equation (2.2.6), is:

$$f_{ij} = c_i \beta^{-1} \rho \nu_i - \nu_j \mu_{ij}$$  \hspace{1cm} (2.2.7)

In this formula, $c_i$ is a dimensionless constant. $\rho$ is the density
of the lighter of the two fluids; $\beta^{-1}$ is the amount of
fluid-fragment interface area per unit volume of space; and
$\nu_i - \nu_j$ is the local time average relative speed of the two
fluids.

The area/volume quantity, $\beta^{-1}$, is taken as being proportional to
the volume-fraction product $\nu_1 \nu_2$, so that it vanishes when either
fluid disappears.

In more advanced work, a differential equation governing $\beta^{-1}$ is
formulated and solved. In the present work however $\beta^{-1}$ has
been taken as proportional to the reciprocal of the lateral
dimension of the flow.

Interfluid mass transfer. The inter-fluid mass transfer process has
been presumed to proceed only in one direction, namely from
teasier (fluid 1) to slower (fluid 2). This is not the only possible
formulation; and more symmetrical ones are being investigated by
the present writer in other studies.

The rate of mass transfer has been taken as directly proportional to the inter fluid friction factor, i.e., the proportionality factor being in excess of unity.

Interfluid, heat transfer and diffusion of chemical species. Conductive heat transfer and diffusive transfer of chemical species between the two fluids have been taken as obeying a law similar to equation (2.7-7).

(d) Solution procedure

The solutions which are to be presented of the above system of equations have been obtained by straightforward application of the PHOENICS computer program (Refs. 8, 9). There is therefore no need to do more here, by way of description of the solution procedure, than to recap the main features of what is embodied in PHOENICS. These are:

- The differential equations are replaced by fully-conservative, fully-implicit finite domain equations, valid for a staggered grid.

- Convection fluxes (i.e., those represented by Gj in equation (2.2-4)), are computed from "upwind" values of rj, qj and Φj.

- An iterative solution procedure is adopted which results in the diminution to below pre-set limits of all the imbalances in all the finite-domain equations.

- A sufficient sample of the computations is repeated on successively finer grids (i.e., with smaller and smaller intervals of space and time) until the physically-significant results are found to be independent of the interval size.

A full account of the computational details will be provided in "PHOENICS Demonstration Reports" from the Imperial College Computational Fluid Dynamics Unit.

2.3 Some typical results from work in progress

(a) The plane wake

M. A. Mallin has been studying, under the author's direction, the application of the two-fluid model of turbulence to various unbounded turbulent shear flows. These are dominated by mixing rather than settling; but it is nevertheless important to establish that the model will fit the available experimental data.

One report of this work (Ref 13) relates to the plane turbulent
wake, for which experimental data are available. Fig 2.3.1 is reproduced from the report, which must be referred to for full details.

Three temperature profiles are represented in the diagram. The higher represents the temperature of the hotter of the two fluids, which is also (although this is not shown) the slower moving one. The lower represents the temperature distribution of the faster-moving fluid which is drawn into the wake from the surroundings, and the curve between them represents the average temperature, weighted by the computed volume fractions.

Comparison is possible between measurements and predictions in respect of the upper two curves; and it is rather satisfactory. The measurements relating to the upper curve have been made, it should be mentioned, by "conditioned sampling".

(b) The axi-symmetrical jet

Fig 2.3-2. from the same report, provides corresponding comparisons for the axi-symmetrical jet. In this case measurements are available for the temperature profiles of the hotter (now faster-moving) fluid, of the cooler (slower) fluid, and of the average.

The agreement between predictions and experiments is rather poorer than before; but it is premature to attempt to ascribe a cause.

(c) A combustion study

The two-fluid model of turbulence is especially well suited to the representation of combustible processes: for it permits explicit allowance to be made for the fact that flames consist of interspersions of hotter and colder fragments of gas and that the chemical reaction rate is especially active at the interfaces between fragments.

Figs 2.3-4, 5, 6, 7, 8 and 9 contain some results of a computation of what may happen when a shock wave, induced by the obstructing of flow in a pipe, travels through an interspersion of hot-gas fragments in a surrounding colder combustible gas. Fig 2.3-3 sketches the apparatus in question.

Although reference must be made to a more complete report (Ref 14) for a full explanation, it can perhaps be perceived, by inspection of the diagrams and their legends, that:

- a pressure wave of growing amplitude passes along the
duct:

- the velocity difference between the hot and cold gas increases, resulting in a "slit" of hot gas into the power-pressure region;
- it is in the region where the velocity difference is greatest that the inter-fluid mass transfer, and the reaction rate, are most vigorous.

(d) Other studies

The author's students and co-workers are pursuing further studies in this area. Attention is being paid to:

- turbulent flows near walls;
- turbulent diffusion flames under free-convection conditions;
- flows in hydrocyclones;
- flows induced by Rayleigh-Taylor instabilities.

Reports on this work are now beginning to appear in the first instance in the Imperial College CFDU Report series.

The work proceeds slowly, because the aim is to develop a single two-fluid model which is capable of fitting a large number of diverse experimental data. However, progress is being made.

3. SOME OPEN QUESTIONS

The two-fluid model of turbulent flows is not yet ready for recommendation to practising engineers as a reliable predictive tool: for although plausible-seeming predictions have been made, and qualitative agreement with experiment has been achieved in respect of features which other models cannot predict at all, there are many questions still to be answered. Some of the questions to which the present author attaches importance will here be listed. They are:

(a) What is the most suitable formulation for the inter-fluid mass transfer law? It seems certain that early presumptions of one-way transfer (eg from slower to faster, or from low-vorticity to high-vorticity) cannot be universally satisfactory: it should the mass-transfer rate change sign when the volume fraction of a fluid attains 0.5, or some other number dependent upon density ratio or other dimensionless quantity?

(b) "Will it suffice to employ a single length scale which is characteristic of both fluids? And, if so, how should the length scale be formulated to allow for the experimental observations that large fragments "swallow" small ones when
the settling process is dominant, and also in some shear-flows (e.g. the plane mixing layer). whereas at very high shear rates the average fragment size diminishes.

Although it seems wise to develop the model by use of the full set of two-fluid equations, as described in section 2C. it would be very desirable to provide a model employing only the conventional single-fluid equations with additional terms for the settling flux. If this is to be done, what means should be used for calculating $\varphi$, etc., which appear in the expressions for this flux?

(d) What experimental data already exist, and what new experiments can usefully be devised, which will enable the as-yet-unknown constants and functions of the two-fluid model to be most easily and accurately deduced?

(e) What role can computational fluid dynamics play, when applied to fragment formation, distortion and interaction processes. to elucidate the unknowns in the two-fluid model equations? And how can such a program of study best be formulated — and funded?

(f) Will it indeed be possible to advance to the point of providing engineers with a valuable design tool without increasing the number of fluids considered beyond two? And, if not. what is the smallest number of fluids which will suffice?

4. CONCLUDING REMARKS

It is the present author's opinion that. despite the slowness of the progress and the number of still unanswered questions. the two-fluid idea can form a good basis for further advances.

As to slowness of progress. it must be observed that turbulence-model development at the single-fluid level has stagnated in recent years. An increasing number of experimental studies is showing the inadequacy of the $k-e$ model as formulated in the early 1970s: yet generally valid refinements and improvements have not been forthcoming; and the Reynolds-stress-modelling approach. in the writer's opinion, increases the difficulties of making predictions without significantly improving their quality.

Because ability to perform two-fluid calculations has. until recently. been confined to a few persons and institutions heavily engaged in other work; the man-power devoted to pursuing two-fluid-modelling has remained small. Now however it is worth
mentioning, the free availability of PHOENICS to academic institutions makes it possible for many more individuals and teams to join in the exploration of this new branch of turbulence research, should they wish to do so.

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Figure 2.3-1. Plane wake: temperature similarity profiles.

Figure 2.3-2. Axisymmetric jet: temperature similarity profiles.
Figure 2.3-3.

(a) Sketch of a tube containing an interspersion of hot combustion products in cooler pre-mixed combustible gas. Initially both gases flow at equal velocity from left to right: then a valve at the right-hand end closes, sending a pressure wave along the duct. Combustion is thereby intensified.

(b) Sketch of the pressure and combustion wave trajectories on a distance-time plane.

(c) Sketch of instantaneous profiles of pressure $p$, mass-transfer rate $m$, cold-gas velocity $u_1$, and hot-gas velocity $u_2$, 1 ms after closing of the valve.
Figure 2.3-4. The flame of Fig 2.3-3, 0.2 milliseconds after the closure of the right-hand end.

Figure 2.3-5. The flame of Fig 2.3-3, 0.4 milliseconds after closure of the right-hand end.
Figure 2.3-6. The flame of Fig 2.3-3. 0.6 milliseconds after closure of the right-hand end.

Figure 2.3-7. The flame of Fig 2.3-3. 0.8 milliseconds after closure of the right-hand end.
Figure 2.3-8. Distributions of pressure, velocities, the mass-transfer rate and other quantities, 1 millisecond after right-hand-end closure.

Figure 2.3-9. Variations with time of pressure, reactedness, mass-transfer rate, volume fraction of hotter fluid (V), velocities of hotter (H) and colder (C) fluids, all for a location 3/4 of the distance along the pipe, i.e. 25 cm from the closed end.