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Estimating the R-Curve from Residual Strength Data

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ESTIMATING THE R-CURVE FROM RESIDUAL STRENGTH DATA

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SUMMARY

A method is presented for estimating the crack-extension resistance curve (R-curve) from residual-strength (maximum load against original crack length) data for precracked fracture specimens. The method allows additional information to be inferred from simple test results, and that information can be used to estimate the failure loads of more complicated structures of the same material and thickness.

The paper first reviews the fundamentals of the R-curve concept. Then the analytical basis for the estimation method is presented. The estimation method has been verified in two ways. Data from the literature (involving several materials and different types of specimens) were used to show that the estimated R-curve is in good agreement with the measured R-curve. A recent predictive blind round-robin program offered a more crucial test. When the actual failure loads were disclosed, the predictions were found to be in good agreement.

INTRODUCTION

The crack-extension resistance curve, or R-curve, is one of the most powerful concepts available to the fracture analyst. In this paper, a useful extension to that concept will be described. The R-curve can be used to predict failure loads for any initial crack size in any specimen or structural configuration (of the same material and thickness and in the same environment) for which a stress intensity analysis is available. Now it is possible to reverse the process. That is, if the residual strength curve (fracture stress against original crack size) is known, the R-curve can be determined.

First the fundamentals of the R-curve concept will be reviewed. Although it has been presented earlier, the derivation of the R-curve estimation method will also be reviewed. Then three applications of the estimation method will be described. The method was used to show that several semiempirical fracture analyses are each equivalent to a particular R-curve formulation. The R-curve was estimated to a useful degree from published residual strength data for a variety of specimen configurations. Finally, the estimation method was tested in an ASTM predictive blind round-robin program.

R-CURVE CONCEPT

Four papers are significant in the history of the R-curve concept. Irwin (ref. 1) first introduced the concept in 1954 and corrected it in 1959 (ref. 2). In 1961, Krafft (ref. 3) extended the concept by postulating that (a) for a given material and thickness, crack propagation resistance depends only on crack extension, and (b) an effective crack extension can be inferred from compliance measurements. He stated that "...the information necessary to predict fracture instability over a wide range of dimensions is not a single G_c value but the shape of the entire resistance curve." In 1968, Clausing (ref. 4) showed that "...the complete G_R curve can be determined in one stable specimen by measuring load and crack length as the crack propagates."

The R-curve concept is illustrated schematically in figure 1(a) for an infinite body containing a crack whose original length is $2a_0$. The strain energy release rate, G_a, is given by

$$G_a = \sigma^2 \pi \frac{a}{E'}$$

where σ is the applied stress and E' is the effective modulus. G_a represents the driving force (per unit thickness) tending to cause crack propagation. The material's resistance to crack propagation, G_R, is a function of crack extension Δ. As stress normal to the crack is applied and increased to 90 percent of the critical stress in figure 1(a), the crack must extend only a small distance to develop a large resistance. At this point the crack-extension resistance equals the driving force and the crack is stable. As the stress is increased, progressively larger amounts of crack extension are required to resist the crack driving force. Finally at the critical stress σ_c the driving-force curve and the R-curve are tangent. Beyond the point of tangency the driving force increases faster with crack length than does the material's resistance. This instability condition represents the failure of the body. The point of tangency defines the fracture toughness G_c and the critical crack length $2a_c$. Since the driving force curve for an infinite body is a straight line, both the fracture toughness and the amount of crack extension at instability increase with increasing original crack length. If the R-curve exhibits a plateau, G_c and Δ_c may asymptotically approach limit values.

In simple finite bodies and test specimens the presence of stress-free boundaries results in an additional increase in the crack driving force as the crack extends toward a boundary. Thus the slope of the driving-force curve increases continuously with increasing crack length. The instability condition for a typical finite-width specimen is shown in figure 1(b). As the initial crack length is increased from zero, both G_c and Δ_c increase at first, but reach maximum values that depend on the specimen width and on the forms of both the driving force curve and the R-curve. As the initial crack length is increased still further, both G_c and Δ_c begin to decrease.

Instability predictions can be done graphically or analytically. Graphical methods will not be treated here, but the analytical method is outlined in the following section.

METHOD OF ANALYSIS

Conventional instability calculations require differentiable mathematical expressions for both the crack driving force curve and the R-curve. The former is obtained from elastic fracture mechanics theory, the latter by curve-fitting to experimental data. For instability to occur, the magnitudes and the slopes of both curves must be equal. This requires the simultaneous solution of a pair of equations. As shown in references 5 and 6, following some algebraic manipulation, the instability condition can be written in a particularly useful form as

$$0 = \frac{g(\Delta_c)}{g'(\Delta_c)} - \frac{a_o + \Delta_c}{1 + 2\alpha_c} \tag{1}$$

where
$$\begin{aligned} g(\Delta) &\equiv E'G_R \\ g'(\Delta) &\equiv E'\ dG_R/d\Delta \\ \alpha &\equiv (a/Y)(dY/da) \end{aligned}$$

and Δ is the effective crack extension, a_o is the initial crack length, Y is the stress intensity calibration factor, and the subscript (c) means "evaluated at the instability point." This form is useful for computing since the first term includes only material properties and the second term includes only geometrical parameters. Now if the functions $g(\Delta)$ and $g'(\Delta)$ and the appropriate equation for α are substituted into this equation then, for prescribed values of a_o and W, Δ_c is the least positive root of that equation. This root can be found by any of several numerical methods. Finally, the fracture stress σ_c is calculated from the crack driving force expression. By repeating the calculation for relative crack lengths from near-zero to near-unity, the complete residual strength curve may be developed point-by-point.

Conversely, if an equation for the residual strength curve is available, it is possible to derive an expression for the corresponding R-curve. To do this we must differentiate one of the instability equations with respect to the critical crack extension Δ_c. That operation is described in detail in references 5 and 6, so only the results will be given here. The mathematics are greatly simplified if we reduce the problem to one of a single independent variable. This is done by prescribing the manner in which a_o and W may vary. Three cases will be considered.

Case I: W = Constant

Assume that there is a function f such that we can define

$$\left.\begin{aligned} f(a_o) &\equiv \sigma_c^2 \\ f'(a_o) &\equiv df(a_o)/da_o \end{aligned}\right\} \quad \text{for} \quad W = \text{constant}$$

Following differentiation and algebraic manipulation we have

$$0 = (1 + 2\alpha_c)f(a_o) + (a_o + \Delta_c)f'(a_o) \tag{2a}$$

The use of this equation will be explained shortly.

Case II: a_0/W = Constant

Now assume that there is a function h such that

$$\left.\begin{aligned} h(a_0) &\equiv \sigma_c^2 \\ h'(a_0) &\equiv dh(a_0)/da_0 \end{aligned}\right\} \quad \text{for} \quad a_0/W = \text{constant}$$

and corresponding to equation (2a) we have

$$0 = \left(1 - 2\alpha_c \frac{\Delta_c}{a_0}\right) h(a_0) + (a_0 + \Delta_c)\, h'(a_0) \tag{2b}$$

Case III: a_0 = Constant

This case is of limited usefulness but is included for completeness. Assume that there is a function j such that

$$\left.\begin{aligned} j(W) &\equiv \sigma_c^2 \\ j'(W) &\equiv dj(W)/dW \end{aligned}\right\} \quad \text{for} \quad a_0 = \text{constant}$$

and corresponding to equations (2a) and (2b) we have

$$0 = \frac{-2\alpha_c}{W} + \frac{j'(W)}{j(W)} \tag{2c}$$

The R-curve is developed as follows. First, at a given point on the residual strength curve, the appropriate function (f, h, or j) and its derivative are computed and substituted in the appropriate one of equations (2a, b, and c), which must then be solved for Δ_c. A numerical solution is usually required. Then Δ_c and σ_c^2 are used to calculate G_c. The resulting pair (G_c, Δ_c) define a point on the R-curve. The process is repeated at additional points on the R-curve. If desired, the R-curve can be expressed in terms of $K_R \equiv (E'G_R)^{1/2}$. Also, a suitable function can be fit to describe G_R or K_R as an explicit function of Δ.

APPLICATIONS

Equivalent R-Curves

In reference 5 this method was used to examine the relations between several semiempirical fracture analyses (SEFA) and the R-curve concept. That study explained why a SEFA might yield good results with one set of data and poor results with another. It was shown that for each SEFA there is an equivalent R-curve whose magnitude and shape are determined by the SEFA formulation and its empirical parameters. That R-curve is equivalent in that it predicts exactly the same relation between fracture stress and initial crack size (residual strength) as the SEFA.

This was done by rewriting the SEFA formulation (if necessary), then substituting into equation (2a), which can then be solved to give an explicit formulation for the effective R-curve. The equivalent R-curves were found to differ markedly from one SEFA to another. Figure 2 (from ref. 5) shows (in dimensionless form) equivalent R-curves for four SEFA, (refs. 7 to 10). The Newman equivalent R-curve is asymptotic to unity in figure 5(c). The Bockrath curve is exponential and thus has no asymptote. The shapes of the equivalent R-curves are intrinsic to the specific SEFA formulation.

It was concluded that if, for a given set of data, a SEFA correlates residual strength closely, its equivalent R-curve will closely approximate the effective R-curve of the material. It was further concluded that, of the five SEFA examined, Newman's (ref. 9) appears to be the most generally useful. In Ref. 2, Newman's SEFA was used to generate equivalent R-curves, which were then used to predict the load at 5 percent secant offset for center-crack (fig. 3(a)) and compact (fig. 3(b)) specimens (figures from ref. 5, data from the literature) using only the initial crack length and maximum load data.

Estimation of R-Curve from Residual Strength Data

In reference 6 this method was extended to allow the estimation of the R-curve from residual strength data, without recourse to any SEFA. Here the functions f, h, and j in equation (2) are obtained by numerical approximation and differentiation of residual strength data. This procedure is not always straightforward, but some comments and hints are given in reference 6.

Since residual strength tests are almost always done on center-crack specimens, the examples in reference 6 are limited to that geometry. Figures 4(a) and (b) show R-curves estimated from residual strength data for specimens of 2219-T87 aluminum and AM355 steel respectively (figures from ref. 6, data from the literature). In figure 4(a), essentially the same R-curve is obtained from specimens of two different widths. In figure 4(b), essentially the same R-curve is obtained from a series of specimens having constant crack length as from a series with constant ratio of crack length to specimen width.

In addition to the limitations inherent in numerical analysis, this method has the same limitation as a standard R-curve test run under load control. That is, for a given specimen the R-curve can only be measured (or estimated) to the point at which the crack becomes unstable. Tests run under displacement control allow a much larger portion of the R-curve to be measured.

ASTM Predictive Analytical Round-Robin Program

A recent predictive blind round-robin program (ref. 11) offered a more crucial test. Complete test data (including measured R-curves) for compact tension specimens of three materials were supplied to all participants. However, the R-curves were estimated using only the failure loads and original crack lengths. Failure loads were then predicted for additional compact specimens with different dimensions, for center-crack specimens, and for special three-hole specimens which simulate the behavior of a cracked stiffened panel. Details of the author's participation are given in Appendix K of reference 11. When the actual failure loads were disclosed, the predictions were found to be in good agreement.

Figures 5(a) to (c) show the measured R-curves supplied to the participants and the R-curves estimated using reference 6 for the three test materials, 7075-T651 and 2024-T351 aluminum alloys and AISI 304 stainless steel, respectively. The bold curves were fitted to the data by the round-robin organizers. The fine solid curves represent functions fitted to points on the estimated R-curve, the fine dashed curves represent extrapolation of those functions beyond the point of instability. Within the range of applicability, the estimated R-curves for the aluminum alloys are in good agreement with the measured R-curves. For the very ductile stainless steel, the comparison is not as good. The curves have the same general magnitude but completely different shapes. As discussed in the body of reference 11, the 304 specimens exhibited very large deformations along the crack line during fracture. Clearly the limits of linear elastic fracture mechanics were severely violated, and it is surprising that any R-curve method worked at all.

Because the purpose of my participation was to test the applicability of the estimation method, the limits of the method were strictly adhered to. That is, no attempt was made to predict failure loads by other methods such as a limit load analysis, even when it was obvious that limit load would be the failure load. For cases where predictions were made, these predictions were quite good. As shown in table XVI of reference 11 (as Participant number 11), the standard error for all predictions made was less than 0.1.

SUMMARY

This method of R-curve estimation appears to be quite useful. It allowed an explanation and interpretation of several semiempirical fracture analyses based on the more modern R-curve concept. But more important is the ability to estimate the R-curve from residual strength data. This means that additional information can now be inferred from the simple residual strength tests which were run many years ago. Such data may be found in the literature or may exist in company files. Now the R-curve can be estimated from that data and used to predict the failure loads of more complicated structures (such as reinforced panels or cracked holes) of the same material and thickness.

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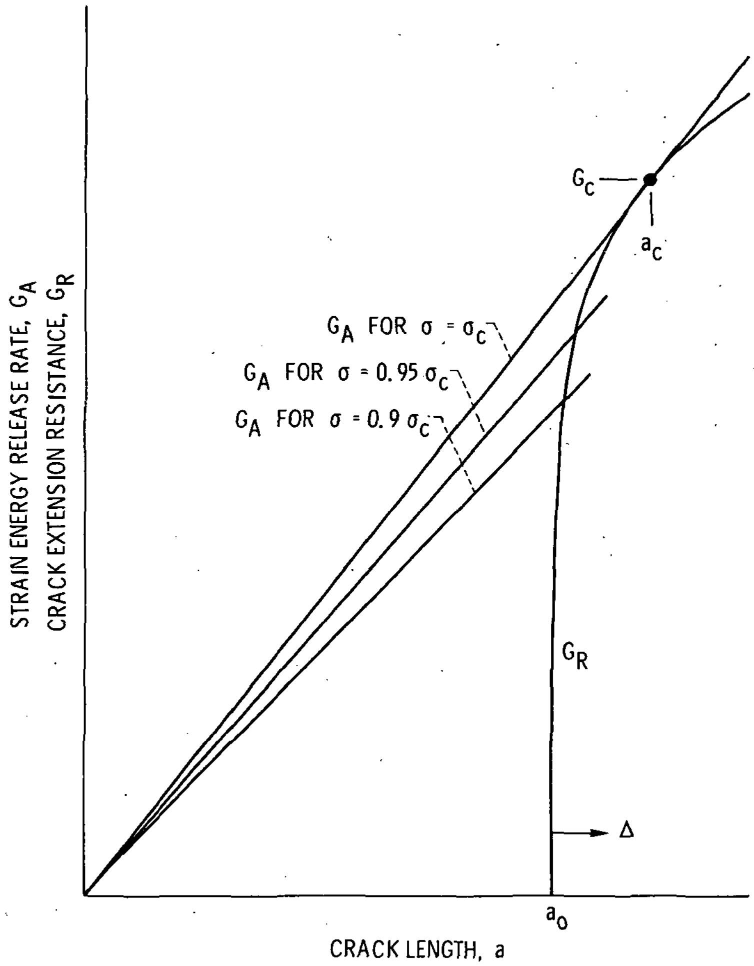
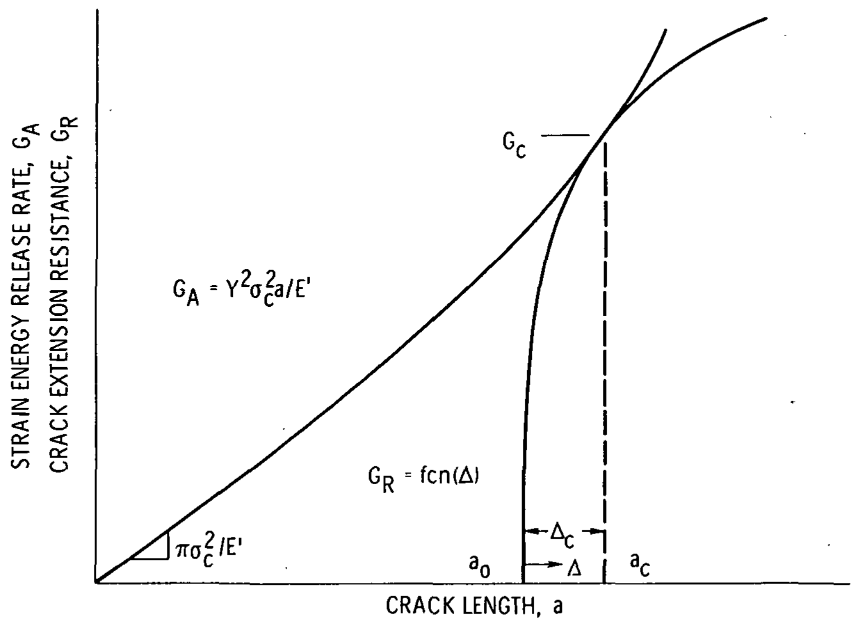


(a) Infinite body.

Figure 1. - Schematic representation of R-curve instability concept for infinite and finite bodies.



(b) Finite body.

Figure 1. - Concluded.

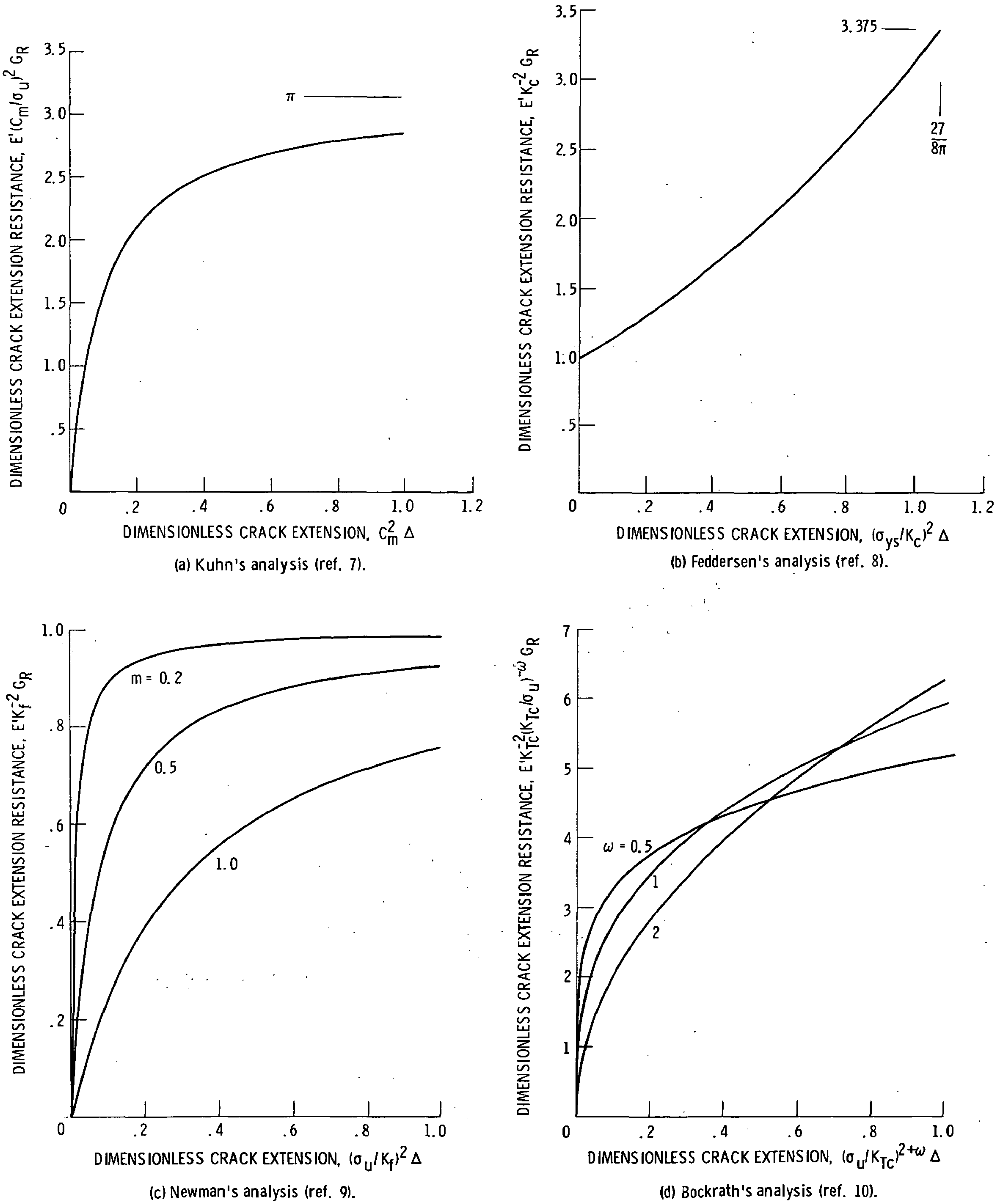
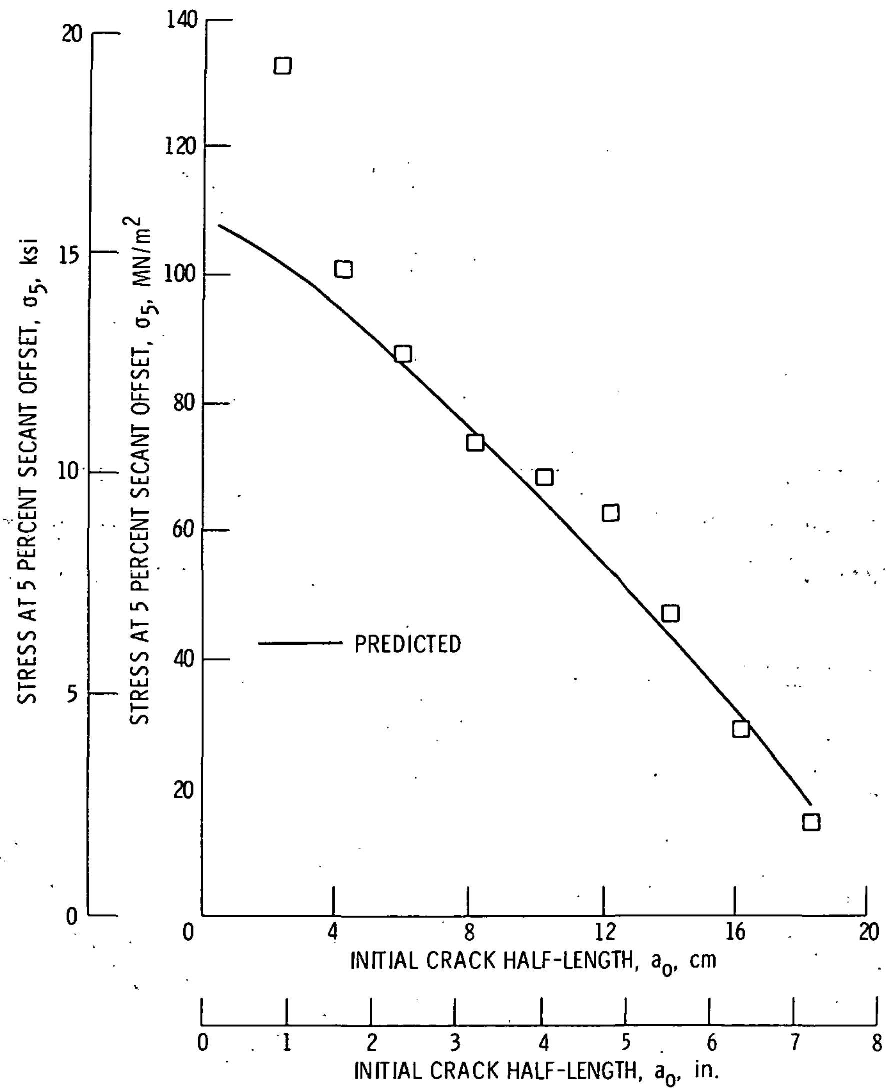
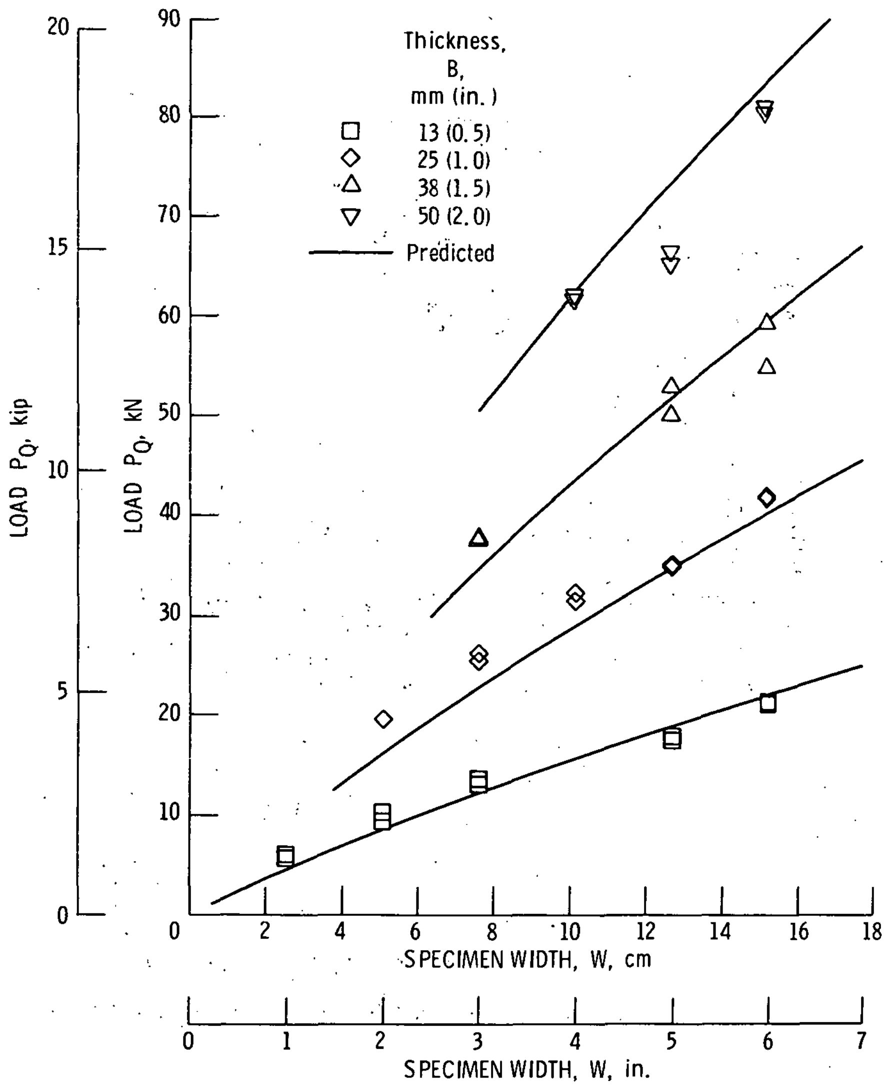


Figure 2. - Dimensionless R-curves equivalent to various semiempirical fracture analyses for the case of a crack in an infinite plate.



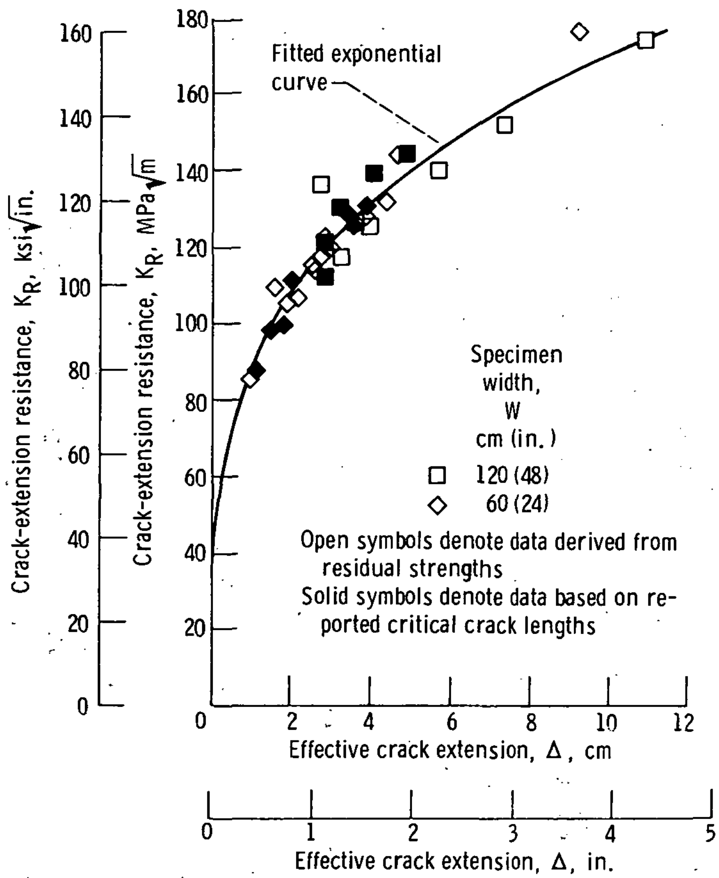
(a) Stress at 5 percent secant offset for 7075-T7351 aluminum alloy center-crack specimens.

Figure 3. - 5% secant-offset loads for center-crack and compact specimens.



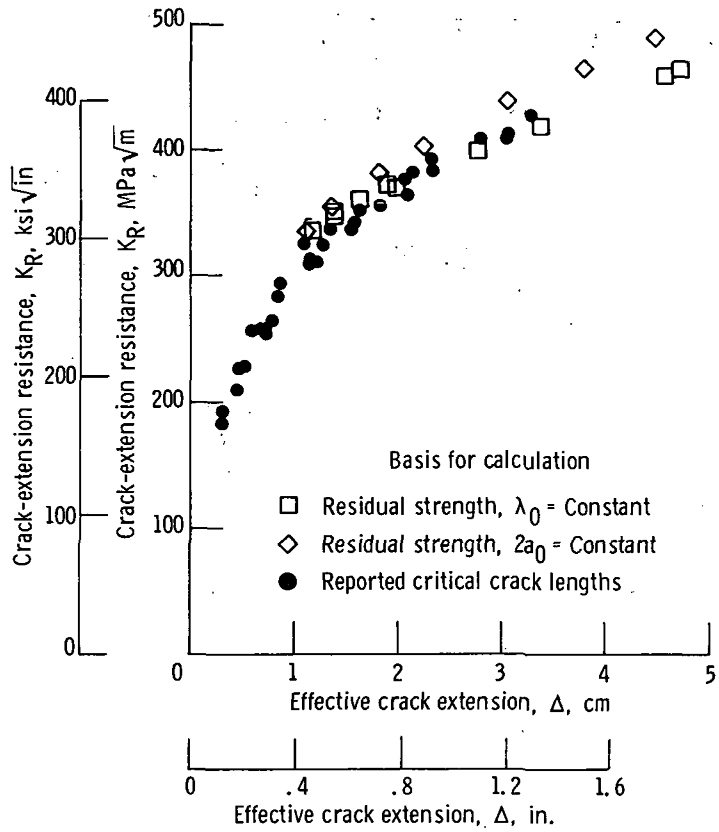
(b) Load at 5 percent secant offset for 2219-T851 aluminum alloy compact specimen.

Figure 3. - Concluded.



(a) R-curve for 2219-T87 aluminum alloy.

Figure 4. - R-curves estimated from residual strength data.



(b) R-curve for AM355 steel.

Figure 4. - Concluded.

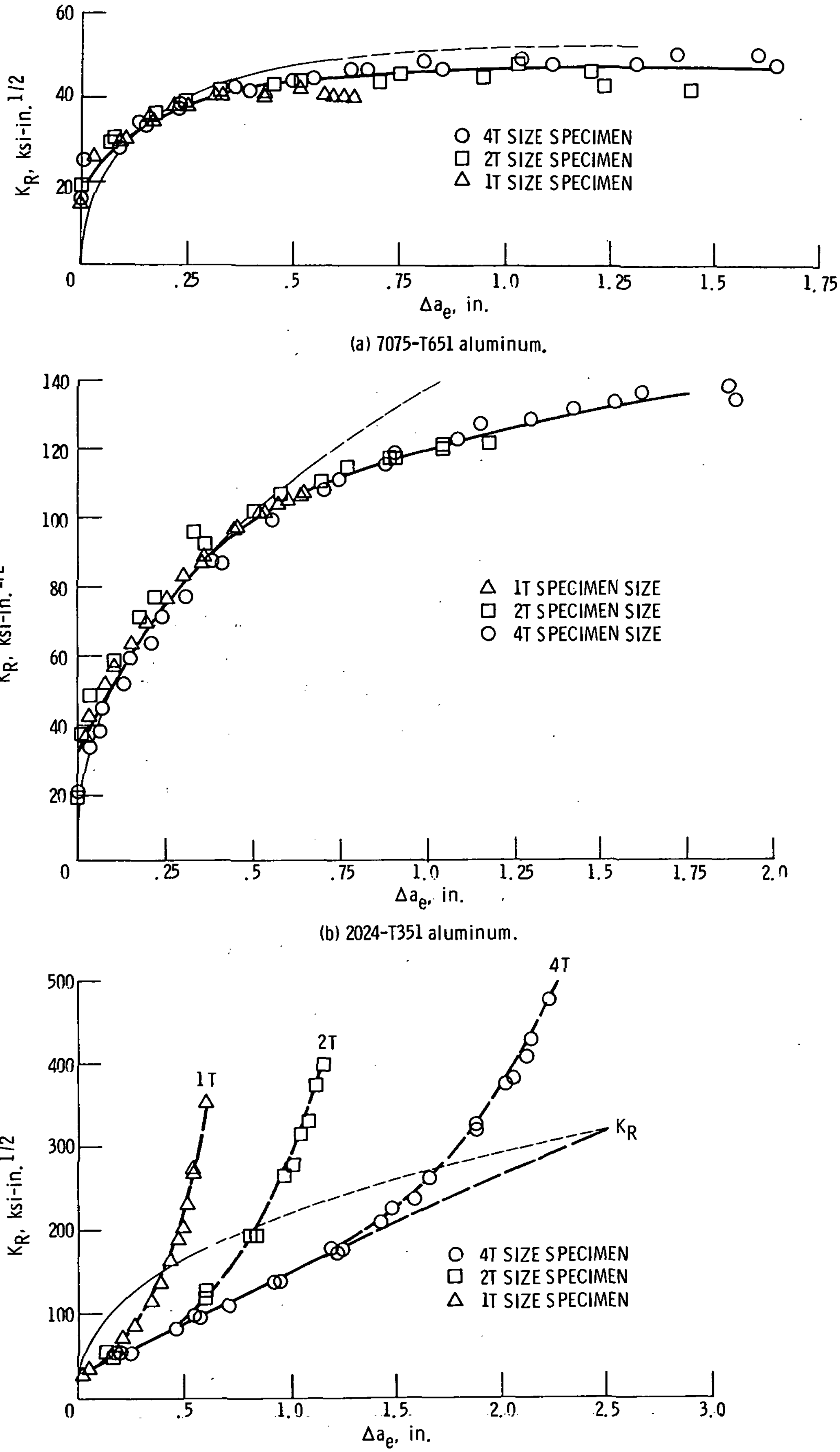


Figure 5. - Effective K_R curves supplied to round-robin participants.

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16. Abstract

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