DAA/ LANGLEY

(NASA-CR-176476) TRANSFORMATION OF TWO AND THREE-DIMENSIONAL REGIONS BY ELLIPTIC SYSTEMS Semiannual Status Report, 15 May - 14 Nov. 1985 (Mississippi State Univ.) 16 p HC A02/MF A01 CSCL 12A G3/64

N86-19063

Unclas 05311

Semi-Annual Status Report

for the period

May 15, 1985 through November 14, 1985

Submitted Under

NASA Grant NSG 1577

Entitled

Transformation of Two and Three-Dimensional

Regions by Elliptic Systems

by

C. Wayne Mastin and Joe F. Thompson

Mississippi State University

Department of Aerospace Engineering

Mississippi State, MS 39762

January 14, 1986

TRANSFORMATION OF TWO AND THREE-DIMENSIONAL REGIONS BY ELLIPTIC SYSTEMS

This report consists of two manuscripts describing both work which has been completed during this reporting period and work which will be continued into the next year. The paper on parameterization in grid generation is an extended version of the abstract included in our last report. That paper has recently been accepted for publication in Computer-Aided Design. The other paper contains our latest results on systems of overlapping grids. The study of implicit methods is essentially complete, although we may attempt the solution of two-dimensional problems before the results are submitted for publication. The one-dimensional solution of Burgers' equation, which is included, does demonstrate the successful implementation of the algorithms developed in the paper. The major effort during this contract period has been on the derivation of conservative interface conditions. A general formulation of procedures which can be used with any conservative difference scheme has been included. The computation of solutions of a model hyperbolic equation has not been completed and therefore is not included in the paper.

Preliminary results have also been obtained on interface conditions for three-dimensional grids. The extension of the procedures to three-dimensions is presently limited by the lack of a practical and efficient method for computing the integrals that appear in the expressions for the interpolation coefficients. Work is continuing on the solution of this problem.

It is evident that most of the work during this contract period has been in the development of concepts and the derivation of formulas. We are presently beginning to test some of these ideas. The results of those tests will be the subject of our next report.

to appear in <u>Computer-Aided Design</u>

PARAMETERIZATION IN GRID GENERATION*

C. Wayne Mastin
Department of Mathematics and Statistics
Mississippi State University
Mississippi State, MS 39762

Abstract

A natural grid is defined on any parameterized curve or surface by selecting an equi-spaced set of parameter values. Redistributing the grid points can be accomplished by defining a new parameterization. Reparameterization techniques are introduced and applied in the construction of computational grids.

*Research supported by NASA Langley Research Center under Grant No. NSG 1577

I. INTRODUCTION

The mathematical description of physical objects is required in the solution of many problems in numerical simulation and modeling. For problems in computational fluid dynamics or related fields where one must compute the numerical solution of partial differential equations, it is necessary to construct a computational grid on , within, or about various objects [1]. The numerical solution may be very sensitive to the location of the grid points. It is therefore important to be able to specify the distribution of points on a surface or along a curve. It should be noted that the selection of points on which to compute a numerical simulation differs from the geometry definition procedures in computer-aided design. Whereas in the latter case decisions on point locations are based primarily on obtaining an accurate and aesthetically pleasing description of the physical object, the distribution of grid points for calculating the solution of partial differential equations must be chosen so as to include consideration of truncation error, stability, and the resolution of the solution in high gradient regions such as boundary layers and shocks.

The principle of reparameterizing a curve to yield a desired distribution of grid points is certainly not new. Many ad hoc techniques have been devised for solving specific gridding problems. However, in this report, the aim is to cast the problem in a general and a more formal setting. Exactly how the points are to be distributed must ultimately rest with the solution of the physical problem under consideration. The distribution may be based on some intrinsic property of the curve such as arc length or curvature. In more complex cases, such as in the construction of adaptive grids for solving transient problems, the grid distribution may depend on the current values of the numerical solution.

II. ALGORITHM DEVELOPMENT

Suppose a curve is given parametrically by the equation

$$r = r(\eta)\ ,\ 0 \le \eta \le 1\ ,$$

where $r = (x, y, z)$. The objective is to select a set of parameter values so that the corresponding points on the curve are properly distributed. The desired set of values for μ will be defined by introducing a reparameterization of the curve.

$$r = r(\eta\ (t)),\ 0 \le t \le 1.$$

Now for each value of t, the arc length derivative, d(t), will be defined by

$$d(t) = [r_t \cdot r_t]^{\frac{1}{2}}$$

Thus d(t) measures the spacing on the curve relative to the spacing on the parameter axis. Given an equi-spaced set of values for the parameter t, it would be possible to generate any spacing of the points r on the curve provided the proper function d(t) could be found. The function d(t) cannot be completely arbitrary since it must satisfy

$$\int_0^1 d(t)\,dt = L \tag{1}$$

where L is the length of the curve. Further information about the function d can be obtained by noting that r is a composite function of t and therefore

$$[d(t)]^2 = r_t \cdot r_t = (r_\eta \cdot r_\eta)\eta_t^2.$$

Now if the distribution of grid points is defined by specifying the function d, then the function η is the solution of the initial value problem

$$\begin{aligned} &\eta(0) = 0 \\ &\eta_t = F(t, \eta) \\ &F(t, \eta) = \frac{d(t)}{(r_\eta \cdot r_\eta)^{\frac{1}{2}}}. \end{aligned} \tag{2}$$

This problem can be solved accurately and efficiently by various numerical algorithms. A necessary criteria for conditional stability of most popular algorithms is

$$F_\eta \leq 0.$$

In the case of equation (2) with the given value of F, this implies that the initial parameterization of the curve should be chosen so that $r_\eta \cdot r_\eta$ is a nondecreasing function of η. Geometrically, it could be said that stability is improved when the grid spacing resulting from a uniform distribution of η values does not decrease as η increases from 0 to 1.

There are two sources of error in the parameterization algorithm which may lead to a numerical solution of (2) which does not satisfy the condition $\eta(1) = 1$. Since it would usually be desirable to have both end points of the curve as grid points, the numerical solution should be computed up to a value of $t = t_1$ with $\eta(t_1) = 1$. Now whenever t varies over a uniformly spaced set of values between 0 and t_1, the values of η range between 0 and 1. This change in length of the parametric interval will alter the grid spacing, but the ratio of the grid spacing at any two points will be the same. In fact, if only the relative spacings at points along the curve are to be prescribed, no normalization of d(t) is necessary. Therefore, if equal spacing of grid points along the curve is desired, any constant value of d(t) would suffice. In all of the following examples, the initial value problem was solved using a fourth-order Runge-Kutta scheme with variable step length.

III. NUMERICAL EXAMPLES

The parameterization algorithm has been applied to the solution of problems with applications in grid generation. The first example deals with the problem of distributing grid points for the solution of one-dimensional problems with high gradient regions. The graph of a typical solution may be represented by the curve defined by

$$y = \tanh(5x), \; -2 \leq x \leq 2.$$

The distribution of grid points along the x-axis should concentrate points in regions of large truncation error. For a solution represented by the above equation, this is precisely in the regions of high curvature. A natural parameterization of this curve is derived by setting $x = 4\eta - 2$. Three different distributions of points along the curve are illustrated in Figure 1. Figure 1 (a) is the point distribution resulting from equal spacing of the parameter η. Figure 1(b) has the uniform spacing of points along the curve which results when a constant function d is used in (2). In Figure 1(c) the function d was chosen to be a function of curvature so that points are concentrated where the curvature is large. If a partial differential equation with this type of solution is to be solved numerically, the x coordinates of the points from 1 (c) would give a grid which would minimize the smearing or oscillation in the numerical solution.

The remaining examples will demonstrate the utility of the parameterization algorithm in the construction of two-dimensional grids. Three-dimensional grids have also been constructed, but the relative spacing of grid points is difficult to depict using two-dimensional plots.

For the next example a function d of the form

$$d(t) = a e^{t} + b e^{-t} \tag{3}$$

was selected. The constants a and b may be chosen so that the normalization (1) holds and also so that the grid spacing is specified at one end of the curve. This form was used to construct the grid between the ellipse and circle in Figure 2. The grid points along the grid lines connecting the ellipse and circle were distributed so that a small fixed grid spacing occurred at the ellipse. The same parameterization was used on all circumferential grid lines. The distribution of that parameter was selected to concentrate grid lines where the curvature of the ellipse is greatest. There is nothing unique about the function d(t) in (3). Many other two parameter families of distribution functions could be used. One should check to make sure that d(t) is positive or else the function η (t) would not be one-to-one.

In the construction of two-dimensional grids, it would be desirable to control the distribution of grid points along both sets of grid lines. This is a more difficult problem since a redistribution of the grid points in one direction changes the distribution in the other direction. Some success in solving this problem has been achieved by alternately reparameterizing in each direction. Suppose, for example, that the plane region is defined in terms of parameters ξ and η. First, an η = constant curve is derived from a specified distribution function. Next the region is reparameterized by connecting corresponding points on the η = constant curves to form a new family of ξ = constant curves. Now a new set of ξ parameter values along these curves is defined by the second distribution function. This procedure is then repeated in the other direction. A new family at η = constant curves is defined and a set of ξ values is defined by the first distribution function. This alternating direction procedure is continued until there is no noticeable change in the location of the grid points. A grid constructed using this technique, with constant values for the distribution functions, is plotted in Figure 3. In this case each family of new grid lines was defined to be piecewise linear. In other cases where the distribution functions depended on curvature, a higher order interpolation polynomial would be necessary. As a further example a grid was constructed using a constant distribution function in one direction and a function of the form (3) in the other direction. Figure 4 clearly demonstrates the concentration of the grid points at a boundary contour in one direction and the uniform distribution in the other direction.

IV. CONCLUSIONS AND RECOMMENDATIONS

Reparameterizing a curve can be an effective technique for obtaining a desired distribution of grid points in one, two and three-dimensional regions. Only a few applications of reparameterization in grid generation have been given. Perhaps the most attractive application is the generation of adaptive grids by selecting d(t) to be a solution dependent function. With this or any grid redistribution scheme, the skewness of the grid lines and the overall smoothness of the grid should be examined as they have an effect on the accuracy of the numerical solution of partial differential equations [1].

By casting the parameterization problem as an initial value problem, the solution procedure is similar to the methods which construct orthogonal grids by orthogonal trajectories. No doubt only a glimpse of the potential of grid generation techniques based on the solution of ordinary differential equations has been seen. These methods might be classed as the method of lines algorithms for grid generation. They are intended to complement the currently popular algebraic and elliptic techniques [1].

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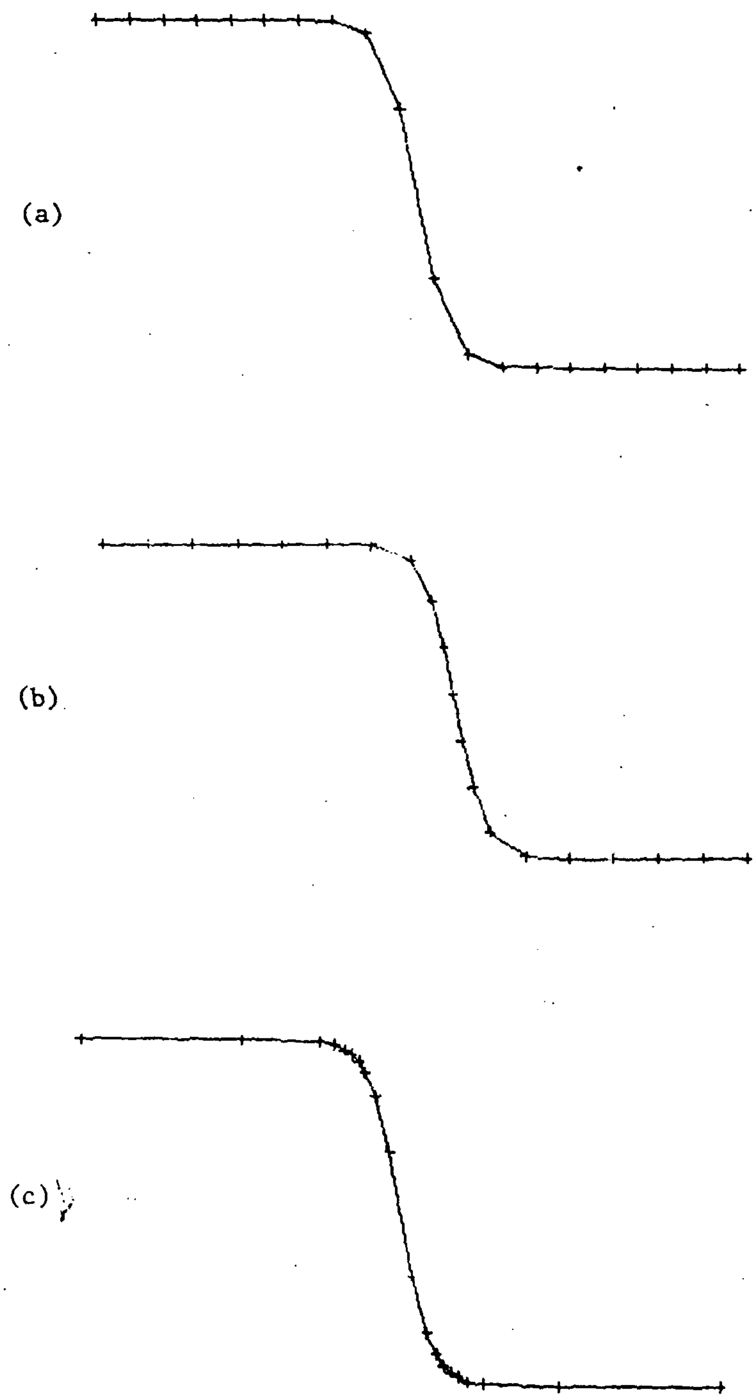

Figure 1. Grid point distributions for (a) uniform η, (b) $d(t) = c$, c = constant, and (c) $d(t) = c/(1 + 5|\kappa|)$, κ = curvature.

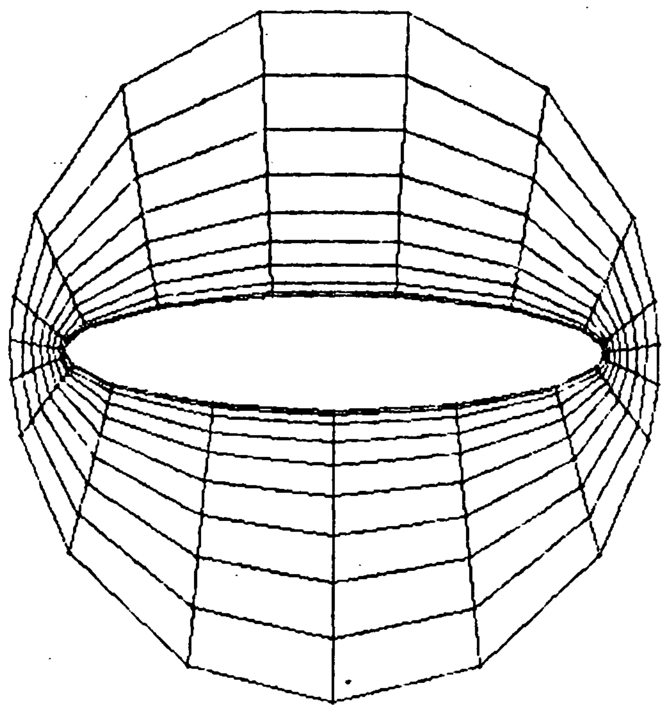

Figure 2. Grid concentration near inner boundary.

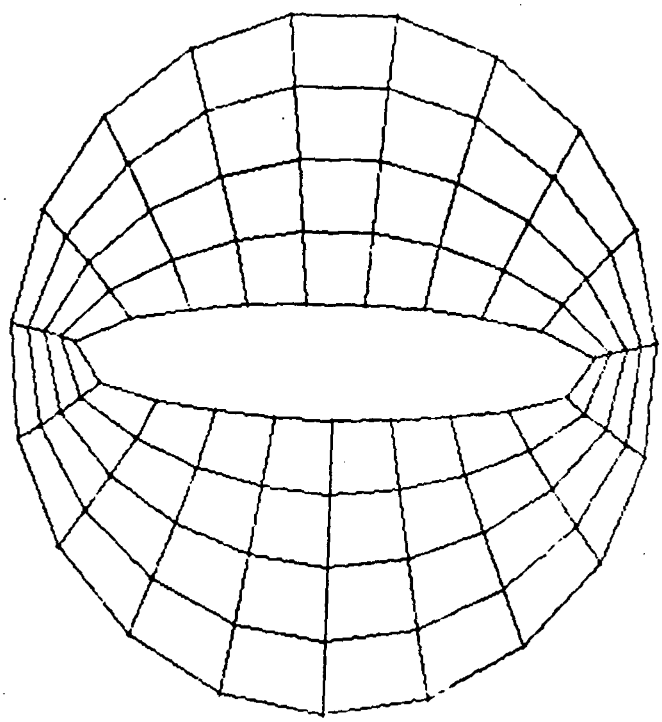

Figure 3. Uniform spacing in both directions.

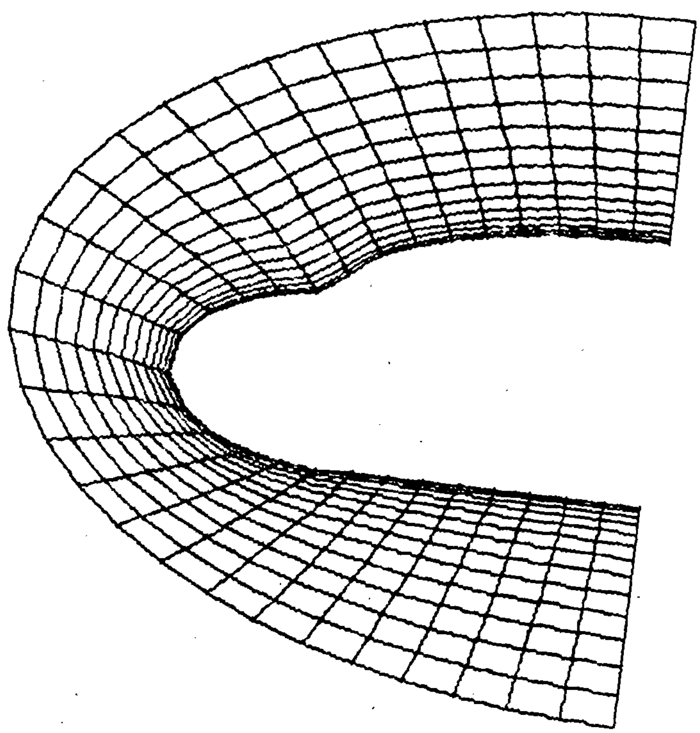

Figure 4. Grid concentration along a boundary contour with uniform spacing on one set of grid lines.