ANALYSIS OF THE TRANSIENT

BEHAVIOR OF RUBBING COMPONENTS

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ABSTRACT

Finite element equations are developed for studying deformations and temperatures resulting from frictional heating in sliding system. The formulation is done for linear steady state motion in two dimensions. The equations include the effect of the velocity on the moving components. This gives spurious oscillations in their solutions by Galerkin finite element methods. A method called "streamline upwind scheme" is used to try to deal with this deficiency. The finite element program is then used to investigate the friction of heating in gas path seal.

LIST OF SYMBOLS

C_p	Specific heat at constant pressure
E	Young's modulus
F	Body force
H	Hilbert spaces
k	Thermal conductivity
$\hat{k}$	Artificial conductivity
N	Shape function
q	Heat flux
T	Absolute temperature
U	Displacement
V	Velocity
W	Weighting function
α	Thermal expansion coefficient
λ	Lame's constant
μ	Shear modulus
ρ	Density
ξ,η	Natural coordinates
ν	Poisson's ratio
ω	Angular velocity
$( )_{,i}$	$= \frac{\partial( )}{\partial x_i}$

Chapter 1

INTRODUCTION

The first law of thermodynamics expresses the energy balance during mechanical and thermal process. In the analysis of rubbing problem, the loss of mechanical energy (frictional energy) is transformed in its largest percentage to thermal energy. During high speed sliding, contact patches are formed. An analytical treatment of stresses (or displacement) and temperature distribution near the contact patches is necessary. A transient finite element heat conduction analysis has shown (ref. 14) that within a very short time after establishment of the contact zone the temperature distribution approached a steady state relative to a stationary observer. The length of time required to reach this quasi-steady state is so short that it may be concluded that within the contact patches a steady state temperature distribution occurs. Therefore it is not necessary to do a transient temperature analysis. A finite element formulation will be done for linear steady state. The formulation will be given in a form that could be expanded to inelastic, non-linear problems.

The advantage of the Finite Element Method is that it is possible to model finite geometry of complex shapes or different material properties. Both temperature and stress analysis could be done by similar modeling. Indeed, a thermomechanical analysis could

be carried out using one element grid and two linked finite element programs. The major difficulty in applying a Finite Element Method is that the convection operators are nonsymmetric. For instance the Galerkin Finite Element Method is successful when applied to linear symmetric operators, but these methods usually give spurious oscillations in their solutions when applied to convection dominated problems. A "streamline upwind scheme" (ref. 10) is used to deal with this problem by adding an artificial conductivity in a manner which stabilize the solution without destroying the physics of the problem.

In this work, a review of literature about the principal subjects is given, followed by a formulation of the weak form for the heat transfer equation and the thermoelasticity equation. Then a finite element formulation is developped for both thermal and thermoelastic equation for a two dimension solid. The resulting finite element program, which gives the displacement and the temperature distribution, is first compared to an analytical solution such as a semi-infinite plane under a heat flux (ref. 1). The program is then used to a problem of rubbing contact at high velocities in a gas path seal.

Chapter 2

LITERATURE REVIEW

The heat transfer theory started with Fourier's law of heat conduction:

$$q = -k\ A \frac{dT}{dx}$$

When the body is moving with a given velocity, a convection term is added to the equation (ref. 13 & 17). The use of Galerkin Finite Element Method to solve the heat problems for a moving body give rise to spurious oscillations. These oscillations can be removed in this case by severe mesh refinement which undermines the practical utility of the methods (ref. 8, 9 and 10).

New schemes were developped trying to deal with this deficiency. The first scheme appeared by Roache in ref.6 as a classical upwind difference scheme. It has been noted that the Galerkin Finite Element Method produces central difference type approximations to the advection (conduction) term. In finite difference theory, the adverse behavior of central differences in these circumstances has long been noted. But this method was considered as inaccurate.

Heinrich proposed a new scheme (ref. 8). The Finite Element Method is applied using weighted residual formulation with bilinear quadrilateral element shape functions, and non-symmetric weighting functions which are different from the shape functions, and depend on parameters which allow the amount of

"upwinding" to be controlled. An increase in accuracy could be obtained by varying these parameters from element to element. This method is now known as Petro-Galerkin method. But the two dimensionnal quadrilaterals proposed by ref. 8 distord the diffusion (conduction) term when upwinding is applied. It seems very difficult to find an upwinding function that does not disturb the diffusion (conduction) operator, yet upwinds the advection (convection) term. Hansen and Von Flotow (ref. 16) noted that it might be better to apply upwind weighting to the advection (conduction) term only and central to the remainder of the equation.

Another simpler technique proposed by Hushes (ref. 9) called quadrature upwinding was based on moving the integration points in the Galerkin Finite Element Method. But he came later with Brook (ref. 10) to propose a new multi-dimensionnal upwind scheme. The method was applied successfully to one dimension, and then generalized for two dimensions. This method, called "streamline upwind scheme", is applied to the advection-diffusion equation, and then to Navier-Stokes equations.

The thermoelasticity equation is derived from the principles of thermodynamics. A simple formulation of the equation is presented in ref. 2 & 3 as:

$$\mu U_{i,jj} + (\lambda+\mu)\, U_{j,ji} - (3\lambda+2\mu)\,\alpha T_{,i} + F_i = 0$$

Together with the heat transfer equation, the thermoelasticity equation (two equations in dimension two) leads to the determination of the displacement and temperature fields in a thermomechanical

problem such as the problem of rubbing contact at high sliding velocities.

The thermal analysis of bodies in sliding contact has attracted the interest of many investigators because of its importance in many situations in which friction occurs: bearing, seals, brakes, clutches,... Various methods have been proposed, but none has proven universally acceptable. Many surface temperature analyses have been based on heat source methods (ref. 1), in which the solution for temperature distribution due to a point source on a surface is used to develop the solution for a distributed heat flux within a contact patch on the surface of an infinite half space. The difficulties involved with application of heat source techniques to bodies of finite dimensions led to the development of integral transform technique presented by Ling (ref. 4). Although this method have been successfully applied to a number of problems with different geometries, its limitations to simple shapes and its mathematical sophistication have kept this from being widely used by engineers.

Kennedy tried to solve the problem of rubbing contact at high sliding velocities by using the Finite Element Method. He applied the problem to two examples: aircraft disk brakes and gas path seals in turbine engine (ref. 11). The first examples was presented before in the study of transient temperature in disk brakes in ref. 5, considered as one of the first documention that use Finite Element Method in such problems. He retreated the second example experimentally and analytically in ref. 14 & 15. In ref. 11,

Kennedy used one element grid and two linked finite element programs to make a thermomechanical analysis of the contact.

Chapter 3

FORMULATION OF THE PARTIAL DIFFERENTIAL EQUATION

3-1. Heat transfer:

Let k_{ij} , the thermal conductivity, be constant, let ρ, the density, and C_p, the specific heat be constant. Let q be the heat flux. In a steady state, the heat transfer partial differential equation for a moving body with a constant velocity V_i is:

$$k_{ij}\ T_{,ij} - \rho\ C_p\ V_i\ T_{,i} + q = 0$$

The first term represents the conduction heat transfer. The second term represents the convection heat transfer. The problem defined in the equation above could be given after applying Galerkin's method, as:

Find $T \in H^2$ for all $W \in H^0$ scuh that:

$$\int_\Omega [\ W\ k_{ij}\ T_{,ij} - W\ \rho\ C_p\ V_i\ T_{,i} + W\ q\ ]\ d\Omega = 0$$

Where T is assumed to satisfy the essential boundary conditions. An integration by parts allows us to rewrite the problem as:

Find $T \in H^1$ for all $W \in H^1$ such that:

$$\int_\Omega [\ W_{,j}\ k_{ij}\ T_{,i} + W \rho\ C_p\ V_i\ T_{,i}\ ]\ d\Omega = \int_\Omega W\ q\ d\Omega$$

Here homogenous natural boundary conditions have been assumed in those sections of the boundary where T is not specified.

3-2. Thermoelasticity:

The constants λ and μ are respectively Lame's constant and shear modulus. They are related to Young's modulus E and Poisson's ratio ν by:

$$\lambda = \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \quad ; \quad \mu = \frac{E}{2(1 + \nu)}$$

The Navier's equation, with temperature changes, in terms of displacement U is given by:

$$\mu U_{i,jj} + (\lambda + \mu) U_{j,ji} - (3\lambda + 2\mu) \alpha T_{,i} + F_i = 0$$

Where α is the coeficient of the thermal expansion and F_i is the body force.

Applying the Galerkin method to the equation above within an element results in the following formulation:

Find $U \in H^2$, $T \in H^1$ for all $W \in H^0$ such that:

$$\int_\Omega [\, W \mu U_{i,jj} + W (\lambda + \mu) U_{j,ji} - W (3\lambda + 2\mu) \alpha T_{,i} + W F_i \,] \, d\Omega = 0$$

After an integration by parts, we write the formulation as:

Find $U \in H^1$, $T \in H^1$ for all $W \in H^1$ such that:

$$\int_\Omega [\, W_{,j} \mu U_{i,j} + W_{,i} (\lambda + \mu) U_{j,j} + W (3\lambda + 2\mu) \alpha T,i \,] \, d\Omega = \int_\Omega W F_i \, d\Omega$$

This equation, called the weak form, could be obtained if the law of conservation of energy is used.

Chapter 4

FINITE ELEMENT FORMULATION

4-1. Heat transfer:

4-1-1. Streamline upwind scheme:

The heat transfer equation has a convection operator which has been shown to carry spurious oscillations in finite element solution (ref. 10). The conventional approach to mitigate these oscillations is to introduce an artificial diffusion (conduction) term in the heat transfer equation. This method is called "streamline upwind scheme".

The weak form becomes:

$$\int_{\Omega} [\, W_{,i} \, (k_{ij} + \hat{k}_{ij}) \, T_{,j} + W \rho \, C_p \, V_i \, T_{,i} \,] \, d\Omega = \int_{\Omega} W \, q \, d\Omega$$

If the artificial conductivity $\hat{k}_{ij}$ is correctly chosen, no oscillation will occur in the Galerkin Finite Element formulation. In ref. 10, the following technique is presented:

Assume $\hat{k}_{ij} = \hat{k} \, \hat{u}_i \, \hat{u}_j$

where $\hat{u}_i = \dfrac{u_i}{||\underset{\sim}{u}||}$ with $||\underset{\sim}{u}||^2 = u_i \, u_i$ and $u_i = \rho \, C_p \, V_i$

$\hat{k}$ is a scalar artificial conductivity.

Assume that the coordinates are chosen such that locally x_1-direction is aligned with streamlines and the x_2-direction is perpendicular. Then the artificial conductivity matrix in this

coordinate system is:

$$\underset{\sim}{\hat{k}} = \hat{k} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

In case of bilinear quadrilateral, in two dimension, $\hat{k}$ is chosen as:

$$\hat{k} = \tfrac{1}{2} \left(\hat{\xi}\, u_\xi\, h_\xi + \hat{\eta}\, u_\eta\, h_\eta \right)$$

$\hat{\xi}$ and $\hat{\eta}$ define the location of the quadratic point and are given by:

$$\begin{cases} \hat{\xi} = \coth \alpha_\xi - \dfrac{1}{\alpha_\xi} \\ \hat{\eta} = \coth \alpha_\eta - \dfrac{1}{\alpha_\eta} \end{cases}$$

where $\begin{cases} \alpha_\xi = u_\xi\, h_\xi \,/\, (2k) \\ \alpha_\eta = u_\eta\, h_\eta \,/\, (2k) \end{cases}$

h_ξ and h_η are the element length (Fig. 1).

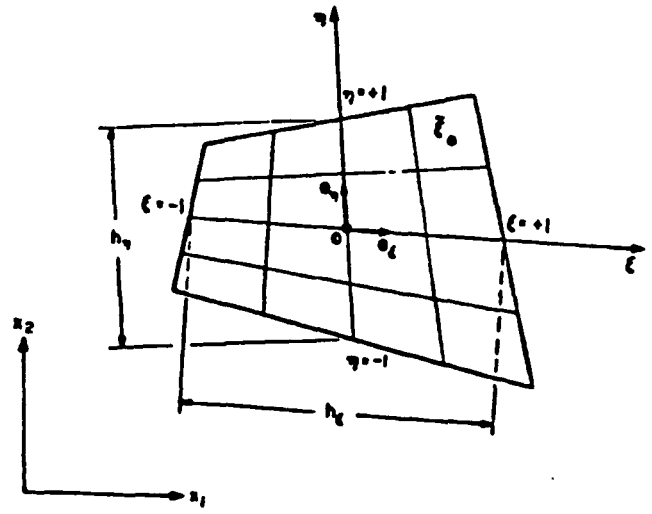

Figure 1. Typical four-node quadrilateral finite element geometry.

u and k are evaluated at the origin of the element in $\xi - \eta$ coordinates. α_i could be expressed by:

$$\alpha_i = \frac{\rho C_p V_i h_i}{2 k} \quad ; \quad i = \xi , \eta$$

4-1-2. Heat transfer finite element formulation:

The weak form is know given by:

$$\int_\Omega [W_{,i} (k_{ij} + \hat{k}_{ij}) T_{,j} + W \rho C_p V_i T_{,i}] d\Omega = \int_\Omega W q \, d\Omega$$

If the body is divided into a number of finite elements, the weighting function W and the temperature distribution T within a discrete element may be approximated by:

$$W = W_I N_I$$

$$T = T_I N_I$$

Where N_I is the shape (interpolation) function of the element. W_I and T_I are constant at the nodes. The majuscule subscripts I indicate the node number.

Substituting these approximations into the weak form we get:

$$\int_\Omega [W_I N_{I,j} (k_{ij} + \hat{k}_{ij}) T_J N_{J,j} + W_I N_I (\rho C_p V_i) T_J N_{J,i}] d\Omega$$

$$= \int_\Omega W_I N_I q \, d\Omega$$

which may be simplified as follows:

$$\{ \int_\Omega [N_{I,i} (k_{ij} + \hat{k}_{ij}) N_{J,j} + N_I (\rho C_p V_i) N_{J,i}] d\Omega \} T_J$$

$$= \int_\Omega N_I q \, d\Omega$$

In the matrix notation, these equations has the following form:

$$[\ K\ ]\ [\ T\ ] = [\ Q\ ]$$

where $[\ K\ ] = \int_{\Omega} [\ N_{I,i}\ (k_{ij} + \hat{k}_{ij})\ N_{J,j} + N_I\ (\rho\, C_p\, V_i)\ N_{J,j}\ ]\ d\Omega$

is the stiffeness matrix.

$$[\ T\ ] = T_J$$

$$[\ Q\ ] = \int_{\Omega} N_I\ q\ d\Omega$$

These equations are the finite element equations which will be formed, assembled, and solved for the temperature distribution through the body. Because of non-symmetry of the second term in the stiffeness matrix [k], its presence require the use of solution routines different from those used in most finite element programs.

To compare the solution obtained with the solution without the use of the upwind scheme, we can just set $\hat{k}_{ij} = 0$. The same finite element equations can also be used for both stationary and moving components of a sliding system by setting V = 0 for elements in the stationary body.

4-2. Thermoelasticity:

The weak form of the thermoelasticity equation was found to be:

$$\int_{\Omega} [\ W_{,j}\ \mu\ U_{i,j} + W_{,i}\ (\lambda + \mu)\ U_{j,j} + W\ (3\lambda + 2\mu)\ \alpha\ T_{,i}\ ]\ d\Omega$$

$$= \int_{\Omega} W F_i \, d\Omega$$

The weighting function W, the displacement distribution U, and the temperature distribution T, within a discrete element may be approximated by:

$$W = W_I N_I$$

$$U = U_I N_I$$

$$T = T_I N_I$$

where the majuscule subscripts indicate the node number and N are the shape function defining the type of element. For some elements, the shape functions are given in the appendix. W_I, U_I and T_I are constants at the nodes.

After substituting these approximations into the weak form, we get:

$$\int_{\Omega} [W_I N_{I,j} \mu U_J N_{J,j} + W_I N_{I,i} (\lambda+\mu) U_J N_{J,j} + W_I N_I (3\lambda+2\mu) \alpha T_J N_{J,i}] \, d\Omega = \int_{\Omega} W_I N_I F_I \, d\Omega$$

which may be simplified as follows:

$$\{ \int_{\Omega} [N_{I,j} \mu N_{J,j} + N_{I,i} (\lambda+\mu) N_{J,j}] \, d\Omega \} U_J + \{ \int_{\Omega} [N_I (3\lambda+2\mu) \alpha N_{J,i}] \, d\Omega \} T_J = \int_{\Omega} N_I F_I \, d\Omega$$

These equations can be written in the matrix notation as:

$$[K_E] [U] + [K_{ET}] [T] = [F]$$

where $[K_E] = \int_\Omega [N_{I,j} \mu N_{J,j} + N_{I,i} (\lambda + \mu) N_{J,j}] d\Omega$ is the elastic stiffeness matrix.

$$[U] = U_J$$

$[K_{ET}] = \int_\Omega [N_I (3\lambda + 2\mu) \alpha N_{J,i}] d\Omega$ is the coupled stiffeness matrix.

$$[T] = T_J$$

$$[F] = \int_\Omega N_I F_I \, d\Omega$$

Based on the equations above and the heat transfer finite element equations, defined in the previous section, a finite element program has been written. This program, which will be used in a thermomechanical analysis, will give the displacement and the temperature distribution through the body. It should be noted that one element grid can be used. With this program simpler problems could be treated. For example:

-Heat transfer without thermal stresses problem (an example of this problem is given later): Set no forces and $\alpha = 0$.

-Elasticity problem without temperature change: Set no flux.

Some basic routines and a typical input data are given in the appendix.

Chapter 5

PROBLEMS AND SOLUTIONS

5-1. Semi-infinite solid under heat friction:

5-1-1. Description of the problem:

The semi-infinite source strip is defined by: $x \in [-b, b]$; $y \in [0, \infty]$; in the plane $z = 0$. The heat is applied at the rate Q per unit time per unit area over the strip. The surrounding media moves across it with velocity V in the direction of the x-axis. This problem was treated by Carslaw and Jaeger (Ref. 1) using the heat source method.

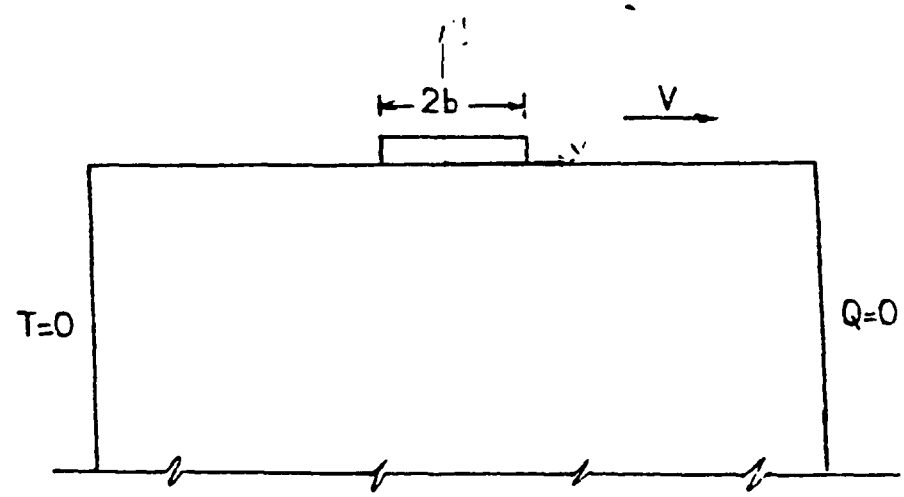

Fig.2-Semi-infinite solid under friction.

5-1-2. Finite element method:

The problem defined above is treated by the finite element program. The elements used in the mesh are quadrilateral (linear or quadratic). The heat flux is applied uniformly distributed on a width of 2b at the nodes. Four meshes were used, which are:

- mesh 1: 200 elements, 231 nodes, four-node quadrilateral element, with upwinding (Fig. 3).
- mesh 2: 200 elements, 231 nodes, four-node quadrilateral element, without upwinding.
- mesh 3: 300 elements, 981 nodes, eight-node quadrilateral element, with upwinding.
- mesh 4: 460 elements, 1493 nodes, eight-node quadrilateral element, with upwinding.

5-1-3. Results:

Carslaw and Jaeger (Ref. 1) gave the temperature as the following integral:

$$T = \frac{2KQ}{\pi kV} \int_{x-B}^{x+B} e^{u} K_0 \left(Z^2 + u^2\right)^{1/2} du$$

where $K_0(x)$ is the modified Bessel function of the second kind of order zero and X, Z, B are dimensionless quantities introduced as:

$$X = \frac{Vx}{2K} \ , \ Z = \frac{Vz}{2K} \ , \ B = \frac{Vb}{2K} \ ; \text{ with } K = \frac{k}{\rho C_p}$$

Some values of the surface temperature of the solid are shown in Fig. 4 for B = 10. The curve represents

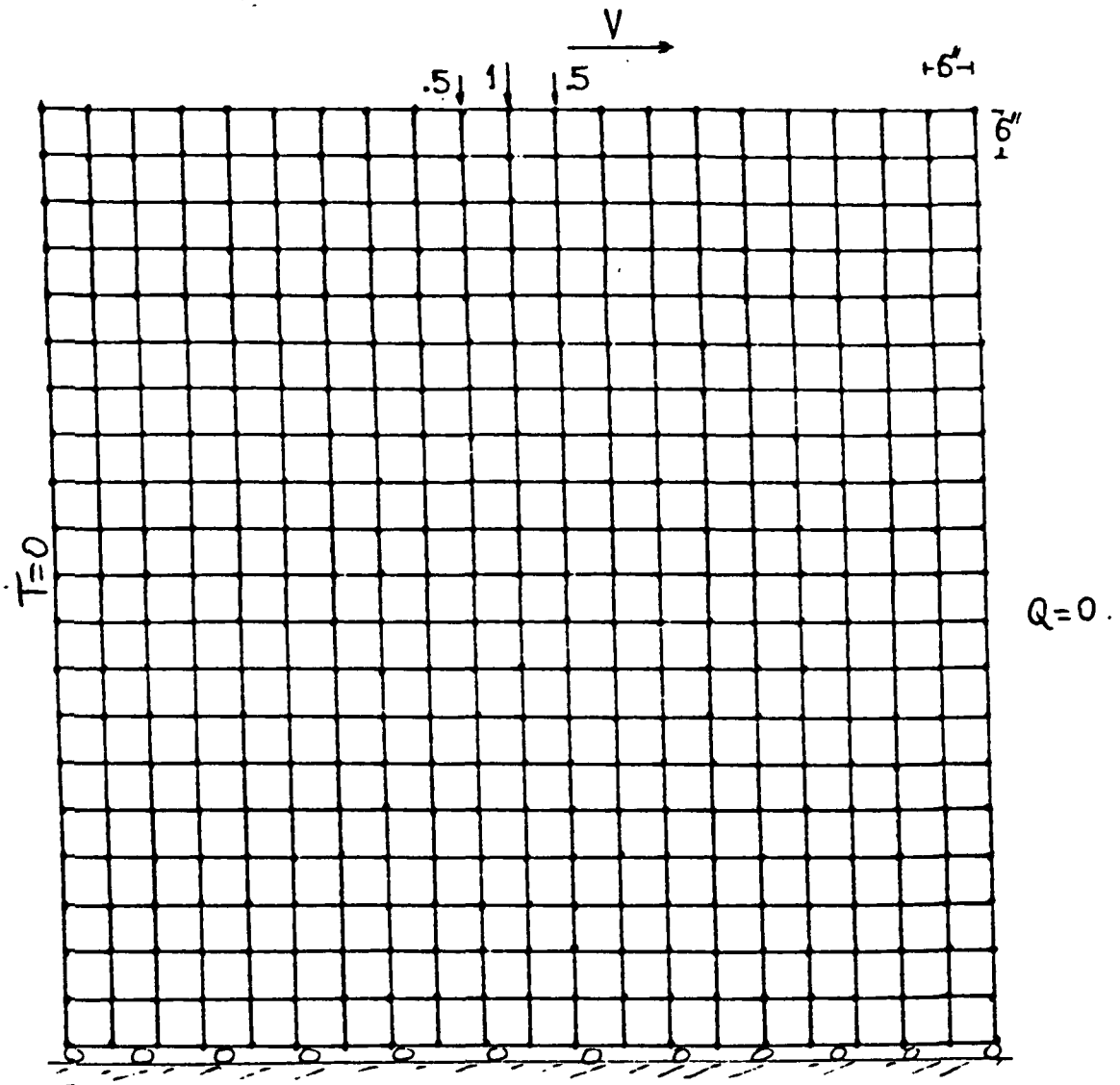

Fig.3-Linear quadrilateral mesh for a "semi-infinite solid under heat friction".

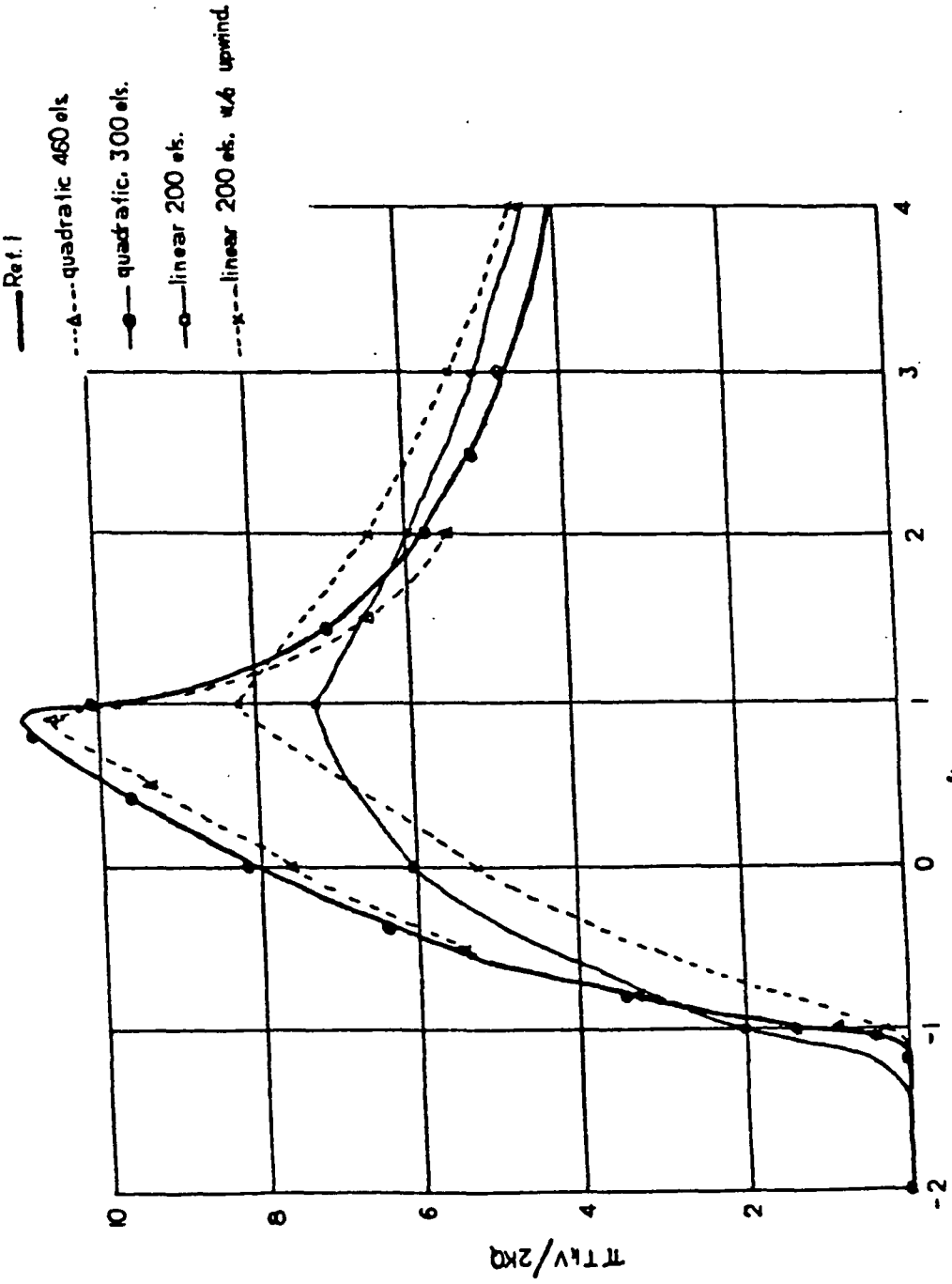

Fig.4-Surface temperature of a semi-infinite solid caused by friction at a contact of width 2b over which it slides with velocity V. (β=10).

(π TkV/2KQ) vs. (x/b) given by Ref. 1. The results by finite element method, for each mesh, is plotted on the same representation.

5-1-4. Discussion:

When the quadratic elements (8 nodes) are used the curve (π TkV/2KQ) vs. (x/b) approaches the one given by the heat source method (Ref. 1). When a smaller number of linear elements (4 nodes) is used the curve is less accurate but has the same pattern. The oscillations in the solution are minimized when the upwinding is used. But it is still necessary to make a mesh refinement in order to get accurate results.

5-2. Gas path seals:

5-2-1. Description of the problem:

The gas path seals are used in the turbine engines of modern aircraft to prevent the axial flow of working fluid (air) around rotating engine components. Reduction of the clearances between rotating and stationary component of these seals can decrease the consumption of the specific fuel and increase the effeciency of the engine. However, such reduction in clearance may result in occasional rubbing between the rotating and the stationary seal components as engine deflection occurs. These rubs, which occurs at very high sliding speeds, can cause high surface temperatures, excessive wear of the seal components, and possible damage

to the engine. The development of gas path seal designs have been retarted by an incomplete understanding of the temperatures, stresses, and deformations which occur during high speed seal rubs (Ref. 11).

An attempt to get the temperature and deformation in this sliding system is done using the finite element analysis. A model has been developed to simulate the rubbing contact which occurs in gas path seals between the rotating knife edge and a stationary seal segment. The outer gas path seal, assumed to have a circular section with 5 layers (Fig. 5 & 6), is rotating at 20,000 r.p.m.. The friction force is taken as 100 lb.. The material properties are given in Table 1. On the external surface the temperature is taken to be 70°F and the displacement is zero. On the internal surface the flux, considered concentrated at the lower point is taken to be:

$$q = N \mu V = 100 \times (0.3) \times (20{,}000 \times 2\pi \times 4.85)$$

$$= 1.8284 \times 10^{7} \text{ BTU/min in}^2$$

where μ is the friction coefficient.

5-2-2. Finite element method:

The thermomechanical program developed before is used to the rubbing contract problem. A finite element mesh based on quadratic quadrilateral (8 nodes) elements, shown in Fig. 7, is used in the development of the model.

5-2-3. Results:

3-1. Temperature:

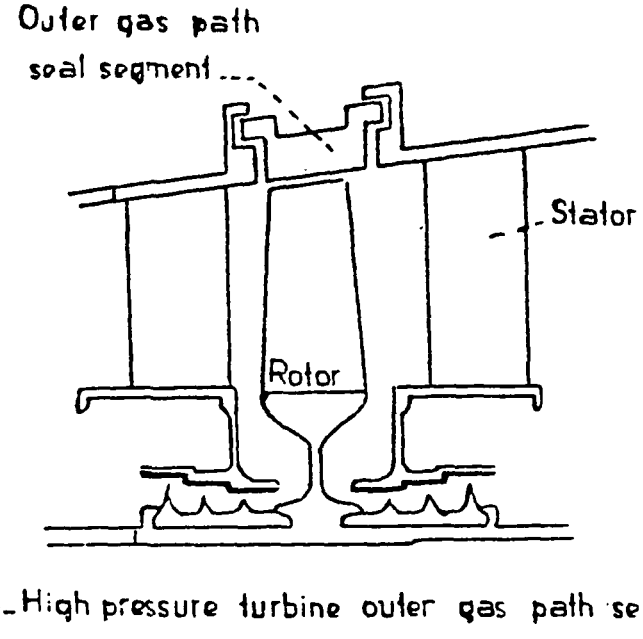

Fig.5_High pressure turbine outer gas path seal.

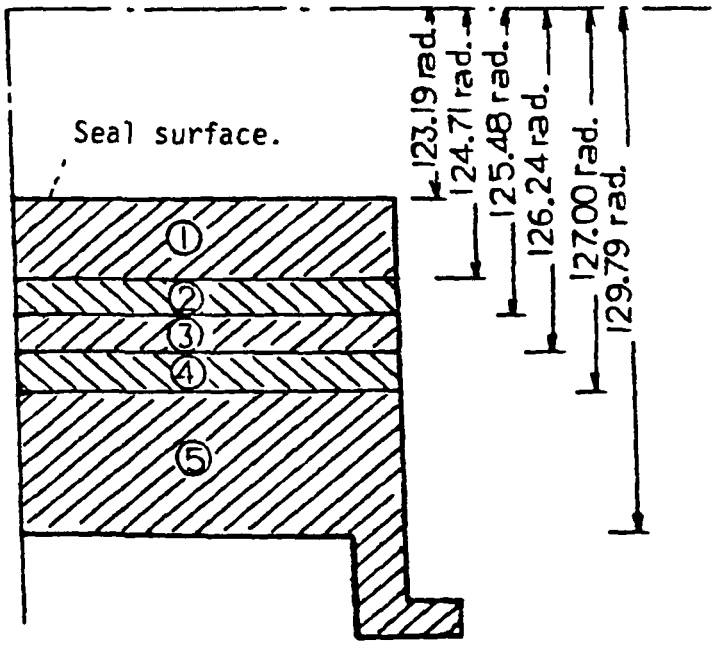

Fig.6-Cross section of outer gas path seal (units mm).

Table 1

Material properties

Layer	1	2	3	4	5
Int. radius (in.)	4.85	4.91	4.94	4.97	5.00
Ext. radius (in.)	4.91	4.94	4.97	5.00	5.11
Material	100 YSZ	85% YSZ 15% CoCr Al Y	70% YSZ 30% CoCr Al Y	40% YSZ 60% CoCr Al Y	MAR-M-50g
Young's modulus $(lb/in^2)x10^6$	2.00	2.00	8.00	17.75	15.60
Poisson's ratio	0.25	0.26	0.27	0.28	0.30
Coef. of expansion $(in/in^oF)x10^{-6}$	4.83	7.70	8.38	9.52	12.20
Thermal cond. $(BTU/min\ in^oF)x10^{-4}$	8.11	16.30	20.95	25.52	422.98
Density (lb/in^3)	0.155	0.180	0.205	0.254	0.320
Specific heat $(BTU/lb^oF)x10^{-2}$	0.161	0.161	0.161	0.158	0.155

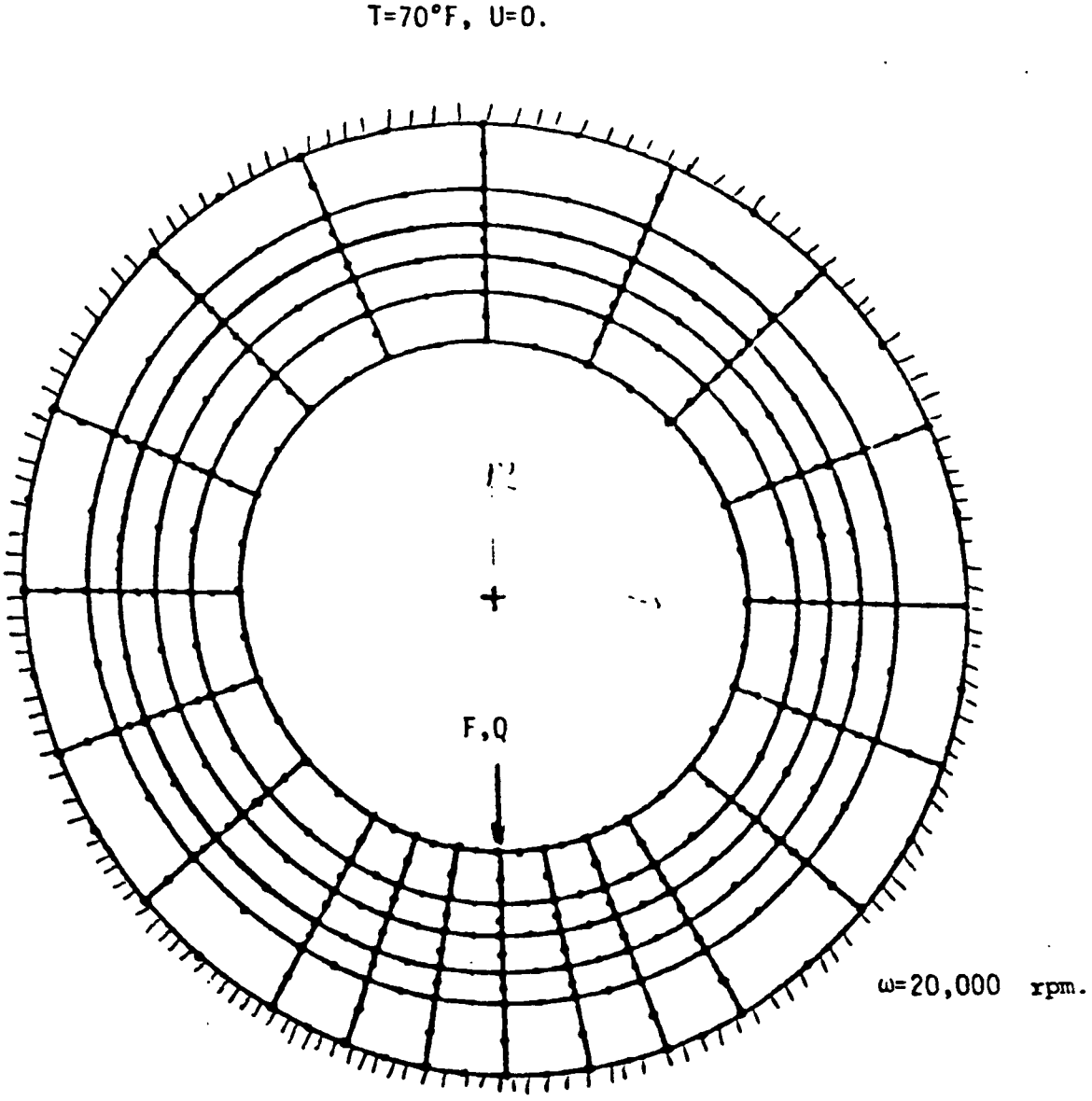

Fig.7- Finite element mesh for gas path seal friction problem.

The finite element results give the temperature at any node. A plot of the temperature on the internal surface vs. the angle near the contact is given in Fig. 8. A peak of temperature occurs at the location of the friction contact.

3-2. Deformation:

The finite element results give the displacement in x-direction and in y-direction of any node. The curve of the deformation in x-direction vs. the angle for the interior surface is given in Fig. 9. The displacement is positive for the nodes at the right of the friction contact and negative for the nodes at its left. The curve of the deformation in y-direction vs. the angle is given in Fig. 10. This curve presents a peak at the friction contact.

5-2-4. Discussion:

At the location of the friction contact, the temperature and the deformation are increased rapidly. The temperature might reach the melting point of the material. The temperature peak was predicted by Marsher (Ref. 12) and confirmed by Kennedy (Ref. 11). The latter conformed also that the maximum amount of deformation occurs on the contact surface, with magnitudes decreasing very rapidly in a direction normal to the surface.

The oscillations that appear clearly in the x-direction displacement (midside node) are due to the presence of the oscillations in the temperature distribution. When the

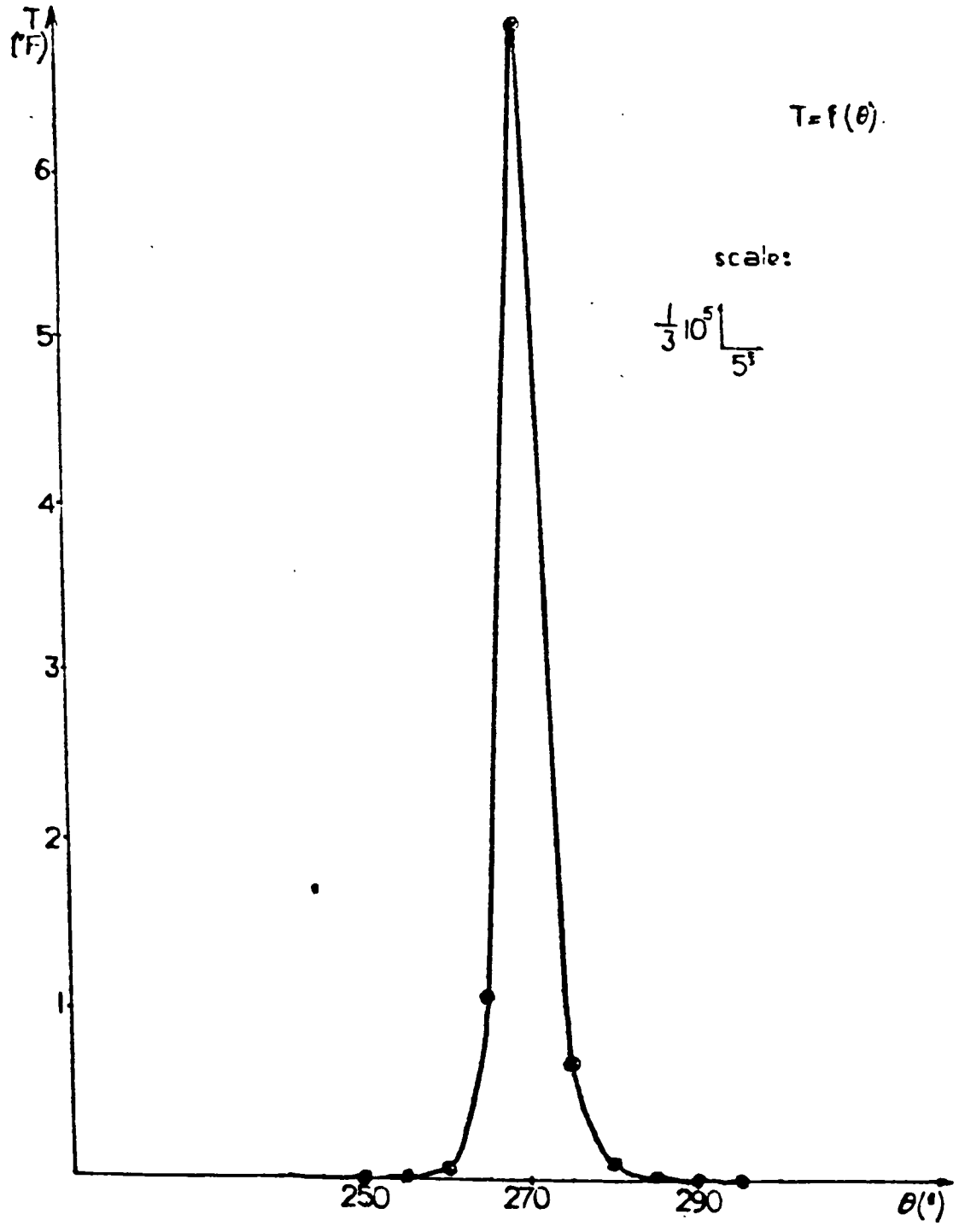

Fig.8-Internal temperature distribution under friction near contact.

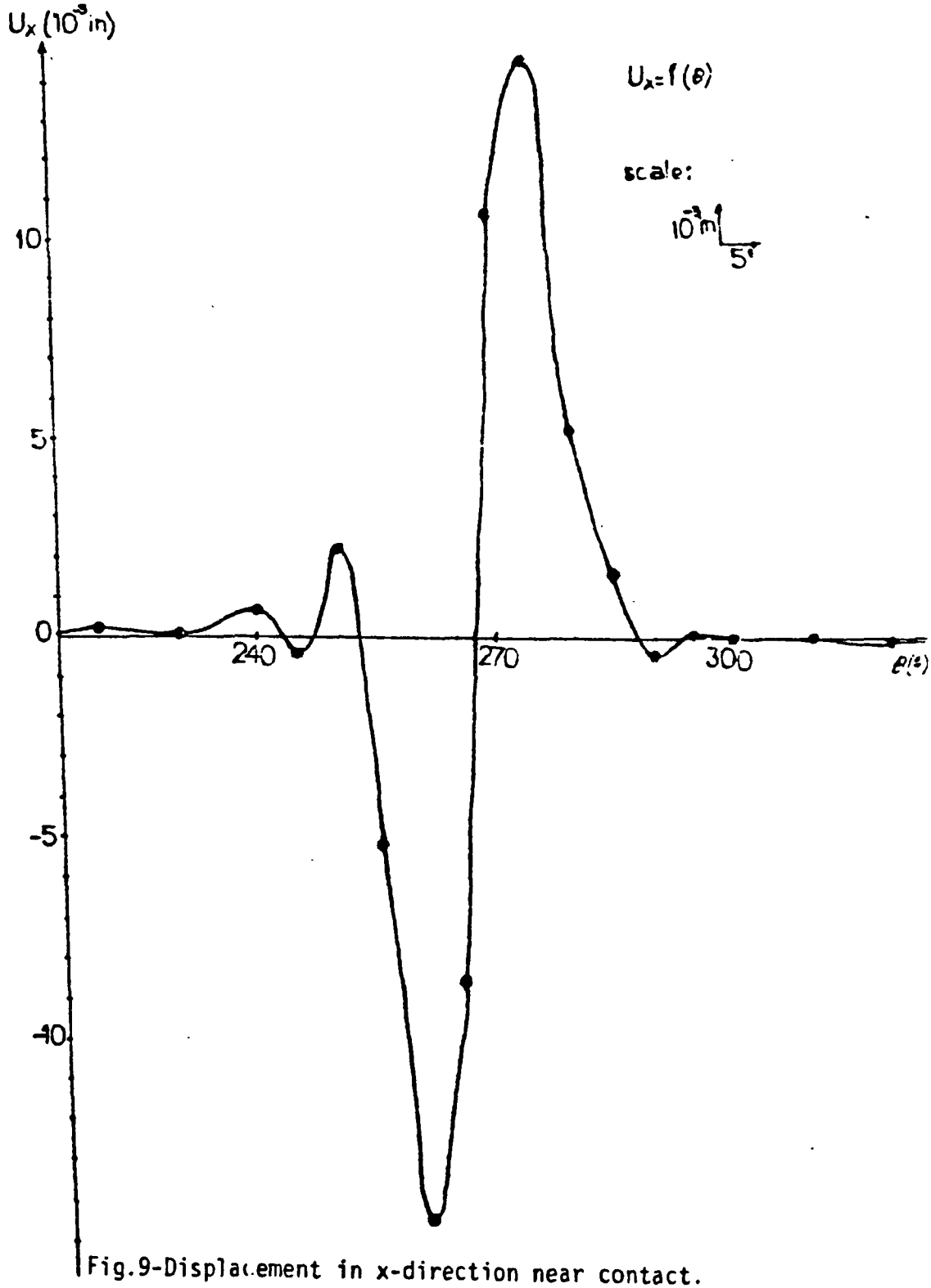

Fig.9-Displacement in x-direction near contact.

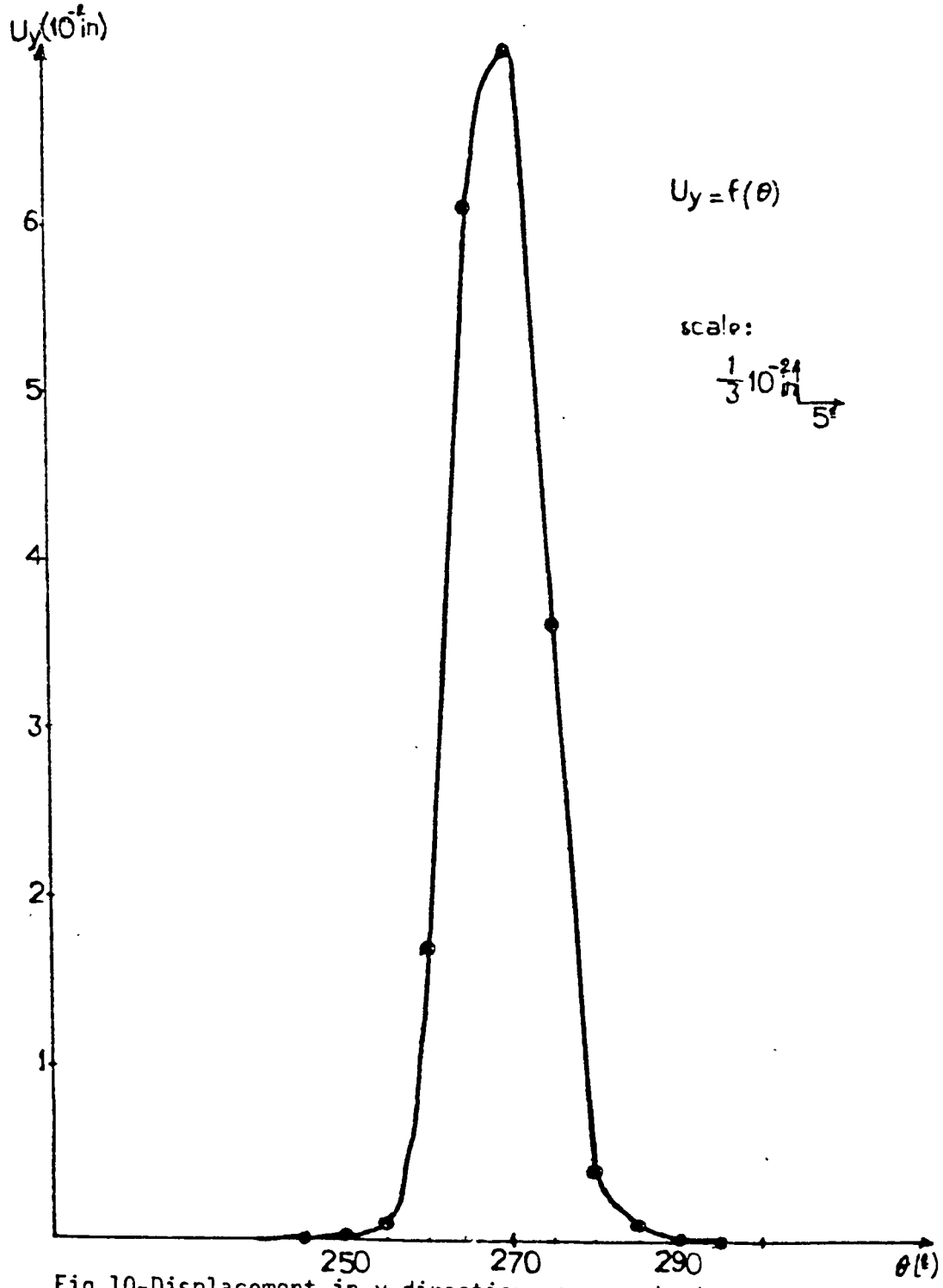

Fig.10-Displacement in y-direction near contact.

upwinding term is increased, the magnitude of temperature and displacement decrease but the oscillations don't vanish.

Chapter 6

CONCLUSIONS

A finite element program has been developed which can efficiently and accurately predict temperature and deformation distribution in a thermomechanical problem such as sliding systems. The method includes velocity effects with the use of the streamline upwind scheme which eliminates spurious oscillations and is therefore very useful in cases involving high speed sliding. The comparison with some results that used heat source method shows the accuracy of the method. A special application to gas path seal problem shows the existence of a peak in the temperature and deformation near the contact, indicating a possibility of melting and excessive wear which might provoke an early thermomechanical failure. Further studies could investigate the importance of thermal conductivity of the moving and stationary component on decreasing surface temperature and deformation.

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Appendix I

SHAPE FUNCTIONS N_I

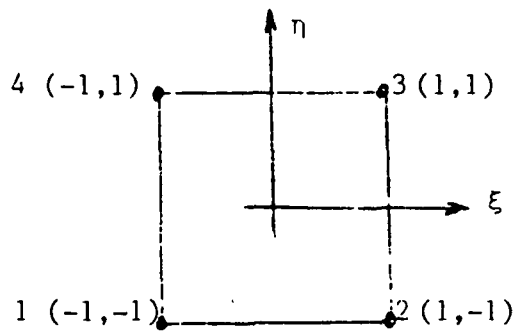
1-Linear quadrilateral element:

$$N_1 = \tfrac{1}{4}(1-\xi)(1-\eta)$$

$$N_2 = \tfrac{1}{4}(1+\xi)(1-\eta)$$

$$N_3 = \tfrac{1}{4}(1+\xi)(1+\eta)$$

$$N_4 = \tfrac{1}{4}(1-\xi)(1+\eta)$$



2-Quadratic quadrilateral element:

$$N_1 = -\tfrac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta)$$

$$N_2 = -\tfrac{1}{4}(1+\xi)(1-\eta)(1-\xi+\eta)$$

$$N_3 = -\tfrac{1}{4}(1+\xi)(1+\eta)(1-\xi-\eta)$$

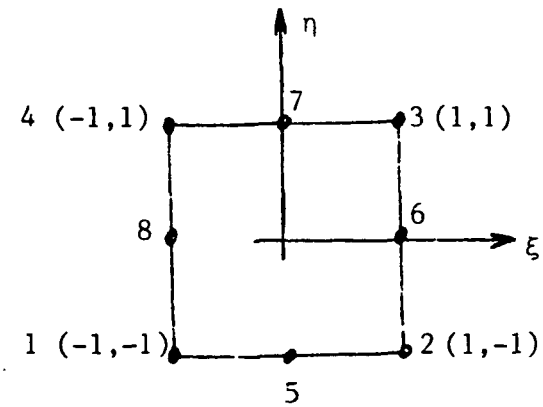
$$N_4 = -\tfrac{1}{4}(1-\xi)(1+\eta)(1+\xi-\eta)$$

$$N_5 = \tfrac{1}{2}(1-\xi^2)(1-\eta)$$

$$N_6 = \tfrac{1}{2}(1+\xi)(1-\eta^2)$$

$$N_7 = \tfrac{1}{2}(1-\xi^2)(1+\eta)$$

$$N_8 = \tfrac{1}{2}(1-\eta^2)(1-\xi)$$



Appendix II

STIFFENESS MATRIX ROUTINE

```
      SUBROUTINE CSTIF(X,Y,IEX,NGX,MTYPE,SK)
      IMPLICIT REAL*8 (A-H,O-Z)
C*****************************************************************
C**** S U B R O U T I N E  C S T I F                          ****
C****                                                        ****
C**** THIS SUBROUTINE CALCULATES THE VALUE OF THE STIFFNESS MATRIX  ****
C**** FOR ANY ORDER TWO DIMENSIONAL ISOPARAMETRIC ELEMENT    ****
C****                                                        ****
C**** NAME      TYPE          USAGE                          ****
C****                                                        ****
C**** DXDE      REAL*8        P (X)/E (ETA)                  ****
C**** DXDP      REAL*8        P (X)/P (XI)                   ****
C**** DYDE      REAL*8        P (Y)/E (ETA)                  ****
C**** DYDP      REAL*8        P (Y)/P (XI)                   ****
C**** F         REAL*8        NODAL BODY FORCES              ****
C**** FAC       REAL*8        AXISYMETRIC INTEGRATION FACTOR ****
C**** I1        INTEGER       DO LOOP INDEX                  ****
C**** I2        INTEGER       DO LOOP INDEX                  ****
C**** I3        INTEGER       DO LOOP INDEX                  ****
C**** IE        INTEGER       CURRENT ELEMENT TYPE           ****
C**** IEX       INTEGER       REQUESTED ELEMENT TYPE         ****
C**** JACOB     REAL*8        DETERMINATE OF JACOBIAN MATRIX ****
C**** MAXD      INTEGER       MAXIMUM INCORE SPACE           ****
C**** NGX       INTEGER       REQUESTED NUMBER OF INTEGRATION POINTS ****
C**** MTYPE     INTEGER       CURRENT MATERIAL TYPE          ****
C**** NTYPE     INTEGER       REQUESTED MATERIAL TYPE        ****
C**** S         REAL*8        RESULTING S MATRIX             ****
C****                                                        ****
C**** SUBROUTINES CALLED:                                    ****
C****               CMAT      RETURNS THE C MATRIX           ****
C****               CSHPI     INITIALIZE THE SHAPE FUNCTIONS ****
C****               SMPRD     MATRIX MULTIPLICATION          ****
C****               STPRD     MATRIX TRANSPOSE MULTIPLICATION ****
C****                                                        ****
C*****************************************************************
      REAL*8 JACOB,JMIN,JMAX,L
      DIMENSION ST(6,24),DFEE(3,24),RU(6,24),TE(6,24)
      DIMENSION X(8),Y(8),SK(576),SK11(576),F(6,24),SK12(576),F(6,24)
     1,SK13(576)
      COMMON /FEP3/ NTYPE,N(6,6),RLEV(6,6),AL
      COMMON /FEP5/ RJMIN,RJMAX,EJMIN,EJMAX
      COMMON /SHAPE/ IE,NG,FLE(6,9),DSDF(6,9),DSDE(6,9),WAIT(9)
      COMMON /TITLE/ TITLE(20),MAXD,LPIG,NLLE,NNJE
      DATA NDOFEE /3/
      DATA JE/0/
      JMIN=9.99D+20
      JMAX=-JMIN
C
C**** SET UP ELEMENT INFORMATION ****
C
      ND=NDOFEE*IEX
      IF(IE .NE. IEX .OR. NG .NE. NGX) CALL CSHPI(IEX,NGX)
      ND2=ND*ND
      DO 1 I1=1,ND2
    1 SK(I1)=0.0
C
C**** ZERO OUT BODY FORCE VECTOR ****
C
      DO 22 I1=1,IE
      DO 22 I2=1,NDOFEE
   22 F(I2,I1)=0.0
      IF(MTYPE .NE. MTYPE) CALL CMAT(MTYPE)
      DO 10 I1=1,NG
      R=0.0
      DXDP=0.0
      DXDE=0.0
      DYDP=0.0
      DYDE=0.0
      DO 5 I2=1,IE
      R=R+X(I2)*FLE(I2,I1)
      DXDP=X(I2)*DSDF(I2,I1)+DXDP
      DXDE=X(I2)*DSDE(I2,I1)+DXDE
      DYDP=Y(I2)*DSDF(I2,I1)+DYDP
    5 DYDE=Y(I2)*DSDE(I2,I1)+DYDE
      DO 6 I2=1,IE
      DFEE(1,I2)=DSDF(I2,I1)*DYDE-DSDE(I2,I1)*DYDP
    6 DFEE(2,I2)=DXDP*DSDE(I2,I1)-DXDE*DSDF(I2,I1)
      JACOB=DXDP*DYDE-DXDE*DYDP
```

```
0078          IF (JACOB .LT. JMIN) JMIN=JACOB
0079          IF (JACOB .GT. JMAX) JMAX=JACOB
00E0          IF(JACOB .LE. 0.0) STOP 4
00E1      C
00E2      C***** CALCULATE THE 'K' MATRIX *****
00E3      C
0084          WW=WAIT(II)/JACOB
0085          I3=1
0086          DO 7 I2=1,IE
0087          BT(1,I3)=0.0
0088          BT(2,I3)=0.0
0089          BT(3,I3)=0.0
0090          BT(4,I3)=0.0
0091          BT(5,I3)=0.0
0092          BT(6,I3)=0.0
0093          B(1,I3)=DFEE(1,I2)
0096          B(2,I3)=0.0
0095          B(3,I3)=0.0
0096          B(4,I3)=DFEE(2,I2)
0097          B(5,I3)=0.0
0098          B(6,I3)=0.0
0099          F(1,I3)=0.0
0100          F(2,I3)=0.0
0101          F(3,I3)=0.0
0102          F(4,I3)=0.0
0103          F(5,I3)=0.0
0104          F(6,I3)=0.0
0105          I3=I3+1
0106          BT(1,I3)=0.0
0107          BT(2,I3)=0.0
0108          BT(3,I3)=0.0
0109          BT(4,I3)=0.0
0110          BT(5,I3)=0.0
0111          BT(6,I3)=0.0
0112          B(1,I3)=0.0
0113          B(2,I3)=DFEE(2,I2)
0114          B(3,I3)=0.0
0115          B(4,I3)=DFEE(1,I2)
0116          B(5,I3)=0.0
0117          B(6,I3)=0.0
0118          F(1,I3)=0.0
0119          F(2,I3)=0.0
0120          F(3,I3)=0.0
0121          F(4,I3)=0.0
0122          F(5,I3)=0.0
0123          F(6,I3)=0.0
0124          I3=I3+1
0125          BT(1,I3)=FEE(I2,I1)*AL
0126          BT(2,I3)=FEE(I2,I1)*AL
0127          BT(3,I3)=FEE(I2,I1)*AL
0128          BT(4,I3)=0.0
0129          BT(5,I3)=0.0
0130          BT(6,I3)=0.0
0131          B(1,I3)=0.0
0132          B(2,I3)=0.0
0133          B(3,I3)=0.0
0134          B(4,I3)=0.0
0135          B(5,I3)=DFEE(1,I2)
0136          B(6,I3)=DFEE(2,I2)
0137          F(1,I3)=0.0
0138          F(2,I3)=0.0
0139          F(3,I3)=0.0
0140          F(4,I3)=0.0
0141          F(5,I3)=FEE(I2,I1)
0142          F(6,I3)=FEE(I2,I1)
0143          I3=I3+1
0144        7 CONTINUE
0145      C***** SET UP THE THERMAL STRAIN (COUPLED TERM) ****
0146          CALL GTFRD(B,BT,SKT3,6,ND,ND)
0147      C***** SET UP THE CONDUCTION-STATIC ELASTICITY TERM, WITH UPWINDING ****
0148          CALL GMPRD(K,B,BT,6,6,ND)
0149          CALL GTFRD(B,BT,SKT1,6,ND,ND)
0150      C***** SET UP THE CONVECTION TERM , WITH THE THERMAL STRAIN ****
0151          CALL GMPRD(RCPV,B,RV,6,6,ND)
0152          CALL GTFRD(F,RV,SKT2,6,ND,ND)
0153          FAC=1.0
0154          WW=WW*FAC
0155          DO 8 I2=1,ND2
0156        8 SR(I2)=SR(I2)+SKT1(I2)*WW+(SKT2(I2)+SKT3(I2))*WAIT(II)
0157       10 CONTINUE
0158          IF (JE.NE.0) RETURN
0159          JE=1
0160          CONTINUE
0161          RETURN
0162          END
```

```
      SUBROUTINE CMAT(MTYPE)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 K,NU,KHAT,KEI
      COMMON /MAT/ EY(6),PR(6),ALPHA(6),XK(6),YK(6),RO(6),CP(6),
     1XV(6),YV(6),GM(6)
      COMMON /REF3/ NTYPE,K(6,6),RCPV(6,6),AL
      DIMENSION A(6,6),B(6,6),UPW(6,6),B1(6,6)
      DO 1 J=1,6
      DO 1 I=1,6
      K(I,J)=0.0
      UPW(I,J)=0.0
      RCPV(I,J)=0.0
      A(I,J)=0.0
    1 B(I,J)=0.0
      NTYPE=MTYPE
      H=GM(NTYPE)
      NU=(XV(NTYPE)**2+YV(NTYPE)**2)**.5
      IF (H) 15,25,15
   25 KHAT=0.0
      GO TO 16
   15 IF (NU.EQ.0.0) GO TO 20
      IF (XV(NTYPE)) 30,30,50
   30 KSI=0.0
      GO TO 5
   50 ALKSI=(H*RO(NTYPE)*CP(NTYPE)*XV(NTYPE))/(2.*XK(NTYPE))
      KSI=(1./TANH(ALKSI))-(1./ALKSI)
    5 IF (YV(NTYPE)) 60,40,60
   40 ETA=0.0
      GO TO 6
   60 ALETA=(H*RO(NTYPE)*CP(NTYPE)*YV(NTYPE))/(2.*YK(NTYPE))
      ETA=(1./TANH(ALETA))-(1./ALETA)
    6 KHAT=(H/2.)*((KSI*KO(NTYPE)*CP(NTYPE)*YV(NTYPE))+(ETA*RO(NTYPE)*
     $CP(NTYPE)*YV(NTYPE)))
   16 WRITE(6,100) KHAT
  100 FORMAT(4X,'KHAT=',2X,G10.5)
      A(5,5)=KHAT
      B(5,5)=XV(NTYPE)/NU
      B(5,6)=YV(NTYPE)/NU
      B(6,5)=-YV(NTYPE)/NU
      B(6,6)=XV(NTYPE)/NU
      CALL GMPRD(A,B,B1,6,6,6)
      CALL GTPRD(K,B1,UPW,6,6,6)
   20 CONTINUE
C
C***** PLANE STRAIN *****
C
      X=EY(NTYPE)/(1.+PR(NTYPE))/(1.-2.*PR(NTYPE))
      X1=X*(1.-PR(NTYPE))
      X2=X*PR(NTYPE)
      AL=-0.5*EY(NTYPE)*ALPHA(NTYPE)/(1.-2.*PR(NTYPE))
      WRITE(6,200) AL
  200 FORMAT(4X,'AL=',2X,G10.5)
      K(1,1)=X1
      K(2,2)=X1
      K(2,1)=X2
      K(1,2)=X2
      K(4,4)=X*(1.-2.*PR(NTYPE))*.5
      K(5,5)=XK(NTYPE)+UPW(5,5)
      K(5,6)=UPW(5,6)
      K(6,5)=UPW(6,5)
      K(6,6)=YK(NTYPE)+UPW(6,6)
      RCPV(5,5)=RO(NTYPE)*CP(NTYPE)*XV(NTYPE)
      RCPV(6,6)=RO(NTYPE)*CP(NTYPE)*YV(NTYPE)
      RETURN
      END
```

Appendix III

INPUT DATA FORMAT

Card type	Columns	Format	Description
1	1-80	20A4	Title to be printed at beginning of output.
2	1-5	I5	Number of elements in this problem.
	5-10	I5	Number of nodes in this problem.
3	1-10	G10.4	Young's modulus for material type 1.
	11-20	G10.4	Yount's modulus for material type 2.
	.	.	. . .
	.	.	. . .
	.	.	. . .
	51-60	G10.4	Young's modulus for material type 6.
4	1-10	G10.4	Poisson's ratio for material type 1.
	11-20	G10.4	Poisson's ratio for material type 2.
	.	.	. . .
	.	.	. . .
	.	.	. . .
	51-60	G10.4	Poisson's ratio for material type 6.
5	1-10	G10.4	Expansion coefficient for material type 1.
	11-20	G10.4	Expansion coefficient for material type 2.
	.	.	. . .
	.	.	. . .
	.	.	. . .
	51-60	G10.4	Expansion coefficient for material type 6.
6	1-10	G10.4	Conductivity in x-direction for material type 1.
	11-20	G10.4	Conductivity in x-direction for material type 2.
	.	.	. . .
	.	.	. . .
	.	.	. . .
	51-60	G10.4	Conductivity in x-direction for

			material type 6.
7	1-10	G10.4	Conductivity in y-direction for material type 1.
	11-20	G10.4	Conductivity in y-direction for material type 2.
	.	.	. . .
	.	.	. . .
	.	.	. . .
	51-60	G10.4	Conductivity in y-direction for material type 6.
8	1-10	G10.4	Density for material type 1.
	11-20	G10.4	Density for material type 2.
	.	.	. . .
	.	.	. . .
	.	.	. . .
	51-60	G10.4	Density for material type 2.
9	1-10	G10.4	Specific heat for material type 1.
	11-20	G10.4	Specific heat for material type 2.
	.	.	. . .
	.	.	. . .
	.	.	. . .
	51-60	G10.4	Specific heat for material type 6.
10	1-10	G10.4	Velocity in x-direction for material type 1.
	11-20	G10.4	Velocity in x-direction for material type 2.
	.	.	. . .
	.	.	. . .
	.	.	. . .
	51-60	G10.4	Velocity in x-direction for material type 6.
11	1-10	G10.4	Velocity in y-direction for material type 1.
	11-21	G10.4	Velocity in y-direction for material type 2.
	.	.	. . .
	.	.	. . .
	.	.	. . .
	51-60	G10.4	Velocity in y-direction for material type 6.
12	1-10	G10.4	Element height for material type 1.
	11-21	G10.4	Element height for material type 2.
	.	.	. . .
	.	.	. . .

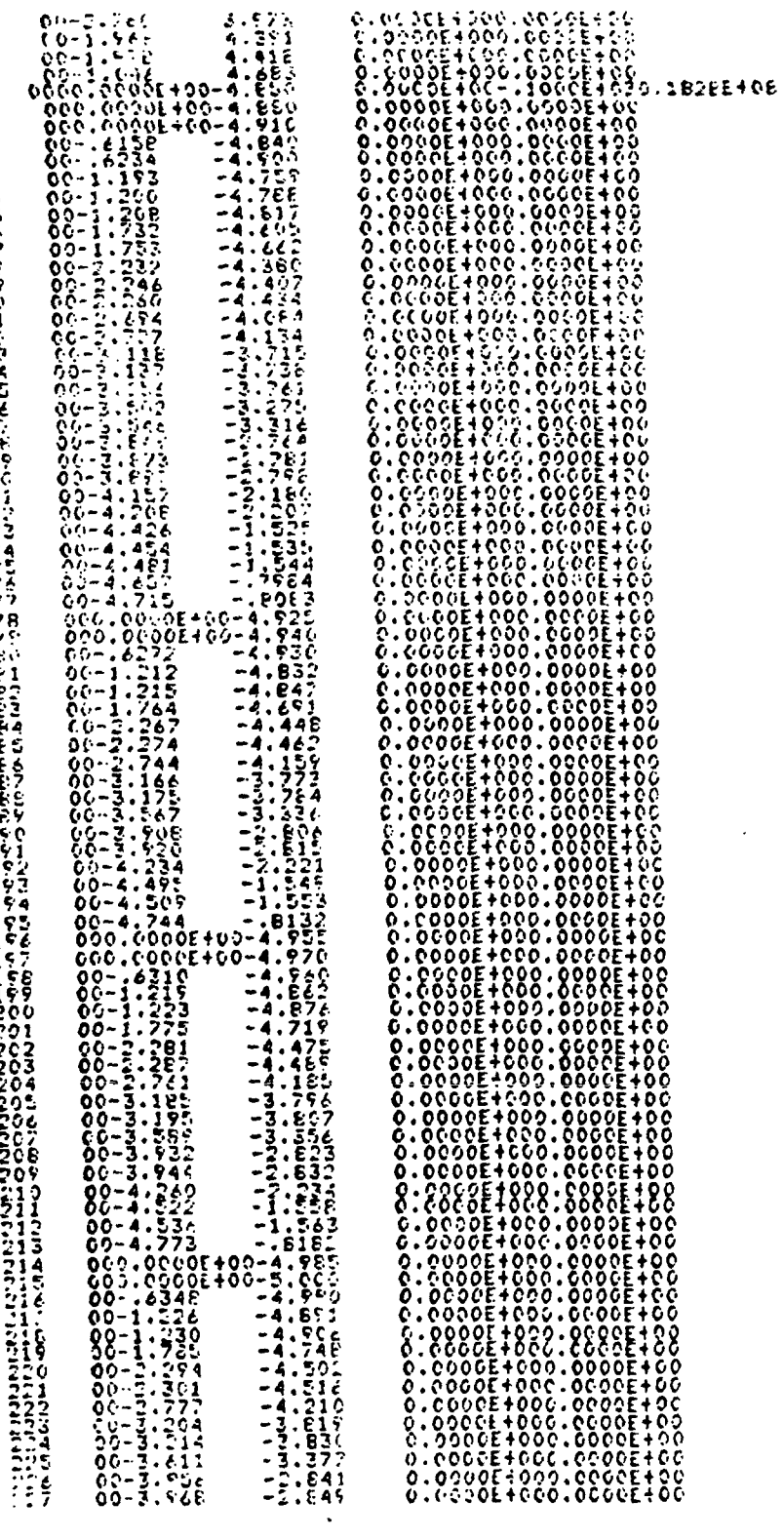
	51-60	G10.4	Element heigh for material type 6.
13	1-5	I5	Node number
	9-9	I1	Displacement fixity in x-direction at this node = 0 applied traction = 1 applied displacement
	10-10	I1	Displacement fixity in y-direction at this node.
	11-11	I1	Temperature fixity at this node = 0 given flux = 1 no temperature
	12-21	G10.4	x coordinate of node.
	22-31	G10.4	y coordinate of node
	32-41	G10.4	Force at node in x-direction
	42-51	G10.4	Force at node in y-direction
	52-61	G10.4	Flux or temperature at node
14	1-5	I5	Element number
	6-10	I5	Node number for first node on element
	11-15	I5	Node number for second node on element
	41-50	I5	Node number for eigth node on element
	57-57	I1	Material type for element
	64-64	I1	Element type

Data cards can be omitted for nodes equispaced between node N_i and N_j as long as the data for nodes N_i and N_j are included.

Example of input data

```
GAS PATH SEAL
 100  340    1
.2000E+070.2000E+070.8000E+070.1775E+080.1560E+080.0000E+00
.2500     0.2600     0.2700     0.2800     0.3000     0.0000E+00
.4830E-050.7700E-050.8360E-050.9520E-050.1220E-040.0000E+00
.E110E-030.1630E-020.2095E-020.2552E-020.4230E-010.0000E+00
.P110E-070.1630E-020.2095E-020.2552E-020.4230E-010.0000E+00
.150      0.18       0.200      0.254      0.32
.1410E-020.1630E-020.1410E-020.1580E-020.1550E-02
 6.0        6.0        6.0        6.0        6.0
   1    000.0000E+00 4.850      0.0000E+000.0000E+00
   2    000.0000E+00 4.880      0.0000E+000.0000E+00
   3    000.0000E+00 4.910      0.0000E+000.0000E+00
   4    00  1.046    4.683      0.0000E+000.0000E+00
   5    00  1.059    4.241      0.0000E+000.0000E+00
   6    00  1.966    4.391      0.0000E+000.0000E+00
   7    00  1.976    4.418      0.0000E+000.0000E+00
   8    00  1.990    4.445      0.0000E+000.0000E+00
   9    00  2.002    4.472      0.0000E+000.0000E+00
  10    00  2.812    4.047      0.0000E+000.0000E+00
  11    00  3.483    3.483      0.0000E+000.0000E+00
  12    00  3.493    3.493      0.0000E+000.0000E+00
  13    00  4.047    2.812      0.0000E+000.0000E+00
  14    00  4.459    1.966      0.0000E+000.0000E+00
  15    00  4.472    2.002      0.0000E+000.0000E+00
  16    00  4.770    1.065      0.0000E+000.0000E+00
  17    00  4.925    0.0000E+000.0000E+000.0000E+00
  18    00  4.940    0.0000E+000.0000E+000.0000E+00
  19    000.0000E+00 4.955      0.0000E+000.0000E+00
  20    000.0000E+00 4.970      0.0000E+000.0000E+00
  21    00  1.065    4.799      0.0000E+000.0000E+00
  22    00  2.008    4.486      0.0000E+000.0000E+00
  23    00  2.014    4.499      0.0000E+000.0000E+00
  24    00  2.825    4.071      0.0000E+000.0000E+00
  25    00  3.504    3.504      0.0000E+000.0000E+00
  26    00  3.514    3.514      0.0000E+000.0000E+00
  27    00  4.071    2.825      0.0000E+000.0000E+00
  28    00  4.486    2.008      0.0000E+000.0000E+00
  29    00  4.499    2.014      0.0000E+000.0000E+00
  30    00  4.799    1.071      0.0000E+000.0000E+00
  31    00  4.955    0.0000E+000.0000E+000.0000E+00
  32    00  4.970    0.0000E+000.0000E+000.0000E+00
  33    000.0000E+00 4.985      0.0000E+000.0000E+00
  34    000.0000E+00 5.000      0.0000E+000.0000E+00
  35    00  1.078    4.828      0.0000E+000.0000E+00
  36    00  2.021    4.513      0.0000E+000.0000E+00
  37    00  2.027    4.527      0.0000E+000.0000E+00
  38    00  2.836    4.096      0.0000E+000.0000E+00
  39    00  3.525    3.525      0.0000E+000.0000E+00
  40    00  3.536    3.536      0.0000E+000.0000E+00
```

60	00	4.094	3.846	0.0000E+000	0.0000E+00
61	00	4.512	5.021	0.0000E+000	0.0000E+00
62	00	4.527	2.027	0.0000E+000	0.0000E+00
63	00	4.828	1.078	0.0000E+000	0.0000E+00
64	00	4.985	0.0000E+000	0.0000E+000	0.0000E+00
65	00	5.000	0.0000E+000	0.0000E+000	0.0000E+00
66	000	0.0000E+00	5.055	0.0000E+000	0.0000E+00
67	1120	0.0000E+00	5.110	0.0000E+000	0.0000E+0070.0
68	112	1.102	4.934	0.0000E+000	0.0000E+0070.0
69	00	2.049	4.576	0.0000E+000	0.0000E+00
70	112	2.071	4.626	0.0000E+000	0.0000E+0070.0
71	112	2.908	4.186	0.0000E+000	0.0000E+0070.0
72	00	3.574	3.574	0.0000E+000	0.0000E+00
73	112	3.613	3.613	0.0000E+000	0.0000E+0070.0
74	112	4.186	2.908	0.0000E+000	0.0000E+0070.0
75	00	4.576	2.049	0.0000E+000	0.0000E+00
76	112	4.626	2.071	0.0000E+000	0.0000E+0070.0
77	112	4.934	1.102	0.0000E+000	0.0000E+0070.0
78	00	5.055	0.0000E+000	0.0000E+000	0.0000E+00
79	112	5.110	0.0000E+000	0.0000E+000	0.0000E+0070.0
80	00	5.000	0.0000E+000	0.0000E+000	0.0000E+00
81	00	5.055	0.0000E+000	0.0000E+000	0.0000E+00
82	112	5.110	0.0000E+000	0.0000E+000	0.0000E+0070.0
83	00	4.828	1.078	0.0000E+000	0.0000E+00
84	112	4.934	1.102	0.0000E+000	0.0000E+0070.0
85	00	4.527	2.027	0.0000E+000	0.0000E+00
86	00	4.576	2.049	0.0000E+000	0.0000E+00
87	112	4.626	2.071	0.0000E+000	0.0000E+0070.0
88	00	4.094	3.846	0.0000E+000	0.0000E+00
89	112	4.186	2.908	0.0000E+000	0.0000E+0070.0
90	00	3.536	3.536	0.0000E+000	0.0000E+00
91	00	3.574	3.574	0.0000E+000	0.0000E+00
92	112	3.613	3.613	0.0000E+000	0.0000E+0070.0
93	00	2.846	4.096	0.0000E+000	0.0000E+00
94	112	2.908	4.186	0.0000E+000	0.0000E+0070.0
95	00	2.027	4.527	0.0000E+000	0.0000E+00
96	00	2.049	4.576	0.0000E+000	0.0000E+00
97	112	2.071	4.626	0.0000E+000	0.0000E+0070.0
98	00	1.078	4.828	0.0000E+000	0.0000E+00
99	112	1.102	4.934	0.0000E+000	0.0000E+0070.0
100	00	4.970	0.0000E+000	0.0000E+000	0.0000E+00
101	00	4.985	0.0000E+000	0.0000E+000	0.0000E+00
102	00	4.799	1.072	0.0000E+000	0.0000E+00
103	00	4.499	2.014	0.0000E+000	0.0000E+00
104	00	4.513	2.021	0.0000E+000	0.0000E+00
105	00	4.071	2.829	0.0000E+000	0.0000E+00
106	00	3.514	3.514	0.0000E+000	0.0000E+00
107	00	3.525	3.525	0.0000E+000	0.0000E+00
108	00	2.829	4.051	0.0000E+000	0.0000E+00
109	00	2.014	4.499	0.0000E+000	0.0000E+00
110	00	2.021	4.513	0.0000E+000	0.0000E+00
111	00	1.072	4.799	0.0000E+000	0.0000E+00
112	00	4.940	0.0000E+000	0.0000E+000	0.0000E+00
113	00	4.955	0.0000E+000	0.0000E+000	0.0000E+00
114	00	4.770	1.065	0.0000E+000	0.0000E+00
115	00	4.472	2.002	0.0000E+000	0.0000E+00
116	00	4.486	2.008	0.0000E+000	0.0000E+00
117	00	4.047	2.812	0.0000E+000	0.0000E+00
118	00	3.493	3.493	0.0000E+000	0.0000E+00
119	00	3.504	3.504	0.0000E+000	0.0000E+00
120	00	2.812	4.047	0.0000E+000	0.0000E+00
121	00	2.002	4.472	0.0000E+000	0.0000E+00
122	00	2.008	4.486	0.0000E+000	0.0000E+00
123	00	1.065	4.770	0.0000E+000	0.0000E+00
124	00	4.910	0.0000E+000	0.0000E+000	0.0000E+00
125	00	4.925	0.0000E+000	0.0000E+000	0.0000E+00
126	00	4.741	1.059	0.0000E+000	0.0000E+00
127	00	4.445	1.990	0.0000E+000	0.0000E+00
128	00	4.459	1.996	0.0000E+000	0.0000E+00
129	00	4.022	2.795	0.0000E+000	0.0000E+00
130	00	3.472	3.472	0.0000E+000	0.0000E+00
131	00	3.483	3.483	0.0000E+000	0.0000E+00
132	00	2.795	4.022	0.0000E+000	0.0000E+00
133	00	1.990	4.445	0.0000E+000	0.0000E+00
134	00	1.996	4.459	0.0000E+000	0.0000E+00
135	00	1.059	4.741	0.0000E+000	0.0000E+00
136	00	4.850	0.0000E+000	0.0000E+000	0.0000E+00
137	00	4.880	0.0000E+000	0.0000E+000	0.0000E+00
138	00	4.683	1.046	0.0000E+000	0.0000E+00
139	00	4.391	1.966	0.0000E+000	0.0000E+00
140	00	4.418	1.978	0.0000E+000	0.0000E+00
141	00	3.973	2.760	0.0000E+000	0.0000E+00
142	00	3.429	3.429	0.0000E+000	0.0000E+00
143	00	3.451	3.451	0.0000E+000	0.0000E+00



[illegible]	00	-4.285	-1.248	0.0000E+000.0000E+00
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230	00	-4.563	-1.572	0.0000E+000.0000E+00
231	00	-4.801	-1.635	0.0000E+000.0000E+00
232	000	.0000E+00	-5.055	0.0000E+000.0000E+00
233	1120	.0000E+00	-5.110	0.0000E+000.0000E+0070.0
234	112	-.6488	-5.100	0.0000E+000.0000E+0070.0
235	00	-1.243	-4.660	0.0000E+000.0000E+00
236	112	-1.257	-5.014	0.0000E+000.0000E+0070.0
237	112	-1.825	-4.852	0.0000E+000.0000E+0070.0
238	00	-2.317	-4.565	0.0000E+000.0000E+00
239	112	-2.352	-4.615	0.0000E+000.0000E+0070.0
240	112	-2.839	-4.203	0.0000E+000.0000E+0070.0
241	00	-3.249	-3.872	0.0000E+000.0000E+00
242	112	-3.285	-3.914	0.0000E+000.0000E+0070.0
243	112	-3.690	-3.451	0.0000E+000.0000E+0070.0
244	00	-4.013	-2.880	0.0000E+000.0000E+00
245	112	-4.055	-2.912	0.0000E+000.0000E+0070.0
246	112	-4.380	-2.297	0.0000E+000.0000E+0070.0
247	00	-4.613	-1.590	0.0000E+000.0000E+00
248	112	-4.664	-1.607	0.0000E+000.0000E+0070.0
249	112	-4.907	-.8412	0.0000E+000.0000E+0070.0
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252	00	4.563	-1.572	0.0000E+000.0000E+00
253	00	4.613	-1.590	0.0000E+000.0000E+00
254	112	4.664	-1.607	0.0000E+000.0000E+0070.0
255	00	4.285	-2.248	0.0000E+000.0000E+00
256	112	4.380	-2.297	0.0000E+000.0000E+0070.0
257	00	3.946	-2.849	0.0000E+000.0000E+00
258	00	4.013	-2.880	0.0000E+000.0000E+00
259	112	4.055	-2.912	0.0000E+000.0000E+0070.0
260	00	3.611	-3.377	0.0000E+000.0000E+00
261	112	3.690	-3.451	0.0000E+000.0000E+0070.0
262	00	3.214	-3.830	0.0000E+000.0000E+00
263	00	3.249	-3.872	0.0000E+000.0000E+00
264	112	3.285	-3.914	0.0000E+000.0000E+0070.0
265	00	2.777	-4.210	0.0000E+000.0000E+00
266	112	2.839	-4.303	0.0000E+000.0000E+0070.0
267	00	2.301	-4.516	0.0000E+000.0000E+00
268	00	2.317	-4.565	0.0000E+000.0000E+00
269	112	2.352	-4.615	0.0000E+000.0000E+0070.0
270	00	1.785	-4.748	0.0000E+000.0000E+00
271	112	1.825	-4.852	0.0000E+000.0000E+0070.0
272	00	1.230	-4.906	0.0000E+000.0000E+00
273	00	1.242	-4.960	0.0000E+000.0000E+00
274	112	1.257	-5.014	0.0000E+000.0000E+0070.0
275	000	.6348	-4.990	0.0000E+000.0000E+00
276	1120	.6488	-5.100	0.0000E+000.0000E+0070.0
277	00	4.773	-.8182	0.0000E+000.0000E+00
278	00	4.536	-1.563	0.0000E+000.0000E+00
279	00	4.550	-1.568	0.0000E+000.0000E+00
280	00	4.260	-2.234	0.0000E+000.0000E+00
281	00	3.944	-2.832	0.0000E+000.0000E+00
282	00	3.956	-2.841	0.0000E+000.0000E+00
283	00	3.585	-3.356	0.0000E+000.0000E+00
284	00	3.195	-3.807	0.0000E+000.0000E+00
285	00	3.204	-3.819	0.0000E+000.0000E+00
286	00	2.761	-4.185	0.0000E+000.0000E+00
287	00	2.287	-4.489	0.0000E+000.0000E+00
288	00	2.294	-4.502	0.0000E+000.0000E+00
289	00	1.775	-4.719	0.0000E+000.0000E+00
290	00	1.223	-4.876	0.0000E+000.0000E+00
291	00	1.226	-4.891	0.0000E+000.0000E+00
292	000	.6310	-4.960	0.0000E+000.0000E+00
293	00	4.744	-.8132	0.0000E+000.0000E+00
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297	00	3.920	-2.815	0.0000E+000.0000E+00
298	00	3.932	-2.823	0.0000E+000.0000E+00
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305	00	1.764	-4.691	0.0000E+000.0000E+00
306	00	1.215	-4.847	0.0000E+000.0000E+00
307	00	1.219	-4.862	0.0000E+000.0000E+00
308	000	.6272	-4.930	0.0000E+000.0000E+00
309	00	4.715	-.8083	0.0000E+000.0000E+00
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311	00	4.495	-1.549	0.0000E+000.0000E+00

312	00 4.208	-2.207	0.0000E+000.0000E+00
313	00 3.897	-2.758	0.0000E+000.0000E+00
314	00 3.908	-2.806	0.0000E+000.0000E+00
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317	00 3.166	-3.773	0.0000E+000.0000E+00
318	00 2.727	-4.134	0.0000E+000.0000E+00
319	00 2.260	-4.434	0.0000E+000.0000E+00
320	00 2.267	-4.448	0.0000E+000.0000E+00
321	00 1.753	-4.662	0.0000E+000.0000E+00
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337	00 1.732	-4.605	0.0000E+000.0000E+00
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7	18	34	31	13	33	32	30	15	2	5
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13	42	56	53	39	55	54	52	40	3	5
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15	48	62	59	45	61	60	58	46	3	5
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67	221	242	239	221	240	240	238	22	4	
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69	227	248	245	227	246	246	244	28	4	
70	80	86	248	230	81	249	247	31	4	
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78	281	283	277	278	282	279	280	55	4	
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81	290	292	287	287	291	291	289	64	4	
82	192	295	272	290	294	294	292	67	4	
83	294	298	51	37	297	50	293	70	4	
84	297	281	278	294	298	297	296	73	4	
85	300	284	281	297	301	300	299	76	4	
86	303	287	284	300	304	303	302	79	4	
87	306	290	287	303	307	306	305	82	4	
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89	310	294	37	23	311	93	309	88	4	
90	313	297	294	310	314	296	311	91	4	
91	316	300	297	313	317	299	314	94	4	
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93	322	306	303	319	323	305	320	100	4	
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95	326	310	23	21	327	309	325	106	4	
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99	338	322	319	335	339	321	336	118	4	
100	148	150	322	338	149	324	339	340	1	

PUBLICATIONS

NASA Grant NAG3-369

R. L. Mullen and R. C. Hendricks, "A 3-Body Approach to Friction Contact Modelling", to be presented 22nd Annual Meeting of the Society of Engineering Sciences, Pennsylvania State University, University Park, October, 1985.

R. C. Hendricks, M. J. Braun, R. L. Wheeler III and R. L. Mullen "Two-Phase Flows With Ambient Pressure above the Thermodynamic Critical Pressure", Bently Roter Dynamics Research Symposium of Instability in Rotating Machinery, Carson City NV, 1985.

R. C. Hendricks, M. J. Braun ,R. L. Mullen, R. E. Bucham and W. A. Diamond, "Analysis of Experimental Shaft Seal Data for High-Performance Turbomachines As for Space Shuttle Main Engines", Proceedings Workshop on Heat and Mass Transfer in Rotating Systems, 1985.

M. J. Braun, R. L. Mullen, Andre Prekwas and R. C. Hendricks, "Finite Difference Solution for a Generalized Reynolds Equation with Homogeneous Two-Phase Flow", Proceedings Workshop on Heat and Mass Transfer in Rotating Systems, ASME,1985.

R. L. Mullen, Andre Prekwas, M. J. Braun and R. C. Hendricks, "Finite Element and Finite Difference Methods for a Reynolds Equation using a Power Law Fluid", Workshop on Heat and Mass Transfer in Rotating Systems, ASME,1985.

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