NASA Technical Memorandum 86892

The Determination of the Direction of the Optic Axis of Uniaxial Crystalline Materials

(NASA-TM-86892) THE DETERMINATION OF THE DIRECTION OF THE OPTIC AXIS OF UNIAXIAL CRYSTALLINE MATERIALS (NASA) 21 p HC A02/MF A01 CSCL 20F
N86-22915
Unclas
G3/35 05888

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April 1986

NASA

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E-2364

SUMMARY

The birefringence of crystalline substances in general, and of sapphire in particular, is described. A test is described whose purpose is to determine the direction of the optic axis of a cylindrically machined single crystal of sapphire. This test was performed on the NASA Lewis sapphire cylinder and it was found that the optic axis made an angle of 18° with the axis of symmetry of the cylinder.

INTRODUCTION

Amorphous materials such as plexiglass or fused silica and crystals with a simple cubic unit cell such as sodium chloride are optically isotropic. For light incident upon such materials from any direction, the interaction between the incident light and the material is described by a single scalar dielectric constant K and a single index of refraction. For all other types of crystalline materials, incident light from different directions with respect to the unit cell of the crystal interacts with the material in optically unequivalent ways. As a result, this interaction is described by a diagonal three-by-three dielectric constant matrix which is referenced with respect to a special set of axes on the crystal's unit cell. The nonzero elements of this material are unequal. Such nonisotropic materials are called birefrigent or doubly refracting.

For uniaxial crystals (i.e., trigonal, tetragonal, and hexagonal unit cells) of which sapphire is an example, two of the nonzero elements are equal and are denoted by K_o. The third is denoted by K_e. K_o is called the ordinary dielectric constant and K_e is called the extraordinary dielectric constant. Uniaxial crystals possess a single optic axis and all directions in the plane perpendicular to the optic axis are optically equivalent. The direction of this single optic axis with respect to the crystal's geometry will be denoted by $\hat{N}$. For the remainder of this report, any symbol with a carat over it denotes a unit vector.

For biaxial crystals (i.e., triclinic, monoclinic, and orthorhombic unit cells), all three diagonal elements of the dielectric constant matrix are unequal. Such materials possess two different optic axes and will not be considered further in this communication.

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For uniaxial crystals, the ordinary and extraordinary indices of refraction, n_o and n_e respectively, are the square root of K_o and K_e. For any given material, these indices are weakly wavelength dependent, and for sapphire with incident light of $\lambda = 0.546$ μm, it is found (ref. 1) that $n_o = 1.771$ and $n_e = 1.763$. Sapphire is said to be a material of low birefringence since the difference between n_o and n_e (0.008 in this case) is much smaller than n_o or n_e itself.

The standard method for determining the direction of the optic axis of a crystal is to cleave or cut a sample whose thickness is less than 1 mm and examine it between the crossed polaroids of a polarizing microscope (refs. 2 to 4). However, in a number of situations, it is inconvenient or impossible to remove a thin section from the crystal. In these cases, the optical axis direction must be determined by using the entire crystal which is perhaps many inches in diameter. The procedure described in this report represents an adaptation of the polarizing microscope procedure to large samples. Such is the case if one is to perform laser doppler velocimetry (LDV) measurements inside a firing engine cylinder. Traditional materials such as plexiglass or fused silica cannot withstand the large pressures produced during combustion. Thus, fabricating optical access windows from high strength materials such as sapphire overcomes these mechanical problems. But at the same time, the birefringency of sapphire produces new problems in making reliable optical measurements. In order to overcome these optical problems, the direction of the optic axis of the access window must be determined without damaging it or without removing any material from it (ref. 5).

LIST OF SYMBOLS

A	fraction of incident light propagating in ordinary mode
a	crystal thickness
B	fraction of incident light propagating in extraordinary mode
c	speed of light
$\vec{D}$	electric displacement
$\vec{E}$	electric field intensity
f	distance between ordinary and extraordinary rays
$\vec{H}$	magnetic field intensity
I	light intensity
K	dielectric constant
$\vec{k}$	wave vector
$\hat{N}$	optic axis

n	index of refraction
$\hat{n}$	outward normal to crystal surface
Q	$1 - (n_e/n_o)^2$
$\hat{S}$	Poynting vector
$\hat{u}_x, \hat{u}_y, \hat{u}_z$	direction vectors, cartesian
v_p	phase velocity
x, y, z	Cartesian axes
α	polarization angle
γ_i	angle of incidence
γ	angle of refraction
Γ	extraordinary angle of refraction using Snell's law
ϵ_{vac}	permittivity of free space
r, θ, ϕ	spherical coordinates
Θ	angle between $\hat{N}$ and the ray which makes the angle Γ with the surface normal
λ	wavelength
μ	magnetic permeability
ξ	angle between $\hat{S}_e$ and $\hat{k}_i$
Φ	phase difference
ψ	angle between $\hat{k}_e$ and $\hat{N}$

Superscript:

$\hat{}$	unit vector

Subscripts:

o	ordinary ray
e	extraordinary ray

The Geometry of the Ordinary and Extraordinary Rays

Light waves are coupled electric, $\vec{E}$, and magnetic, $\vec{H}$, fields whose surfaces of constant phase are perpendicular to the wavevector $\vec{k}$ and whose direction of energy propagation, the Poynting vector, is given by

$$\hat{S} = \vec{E} \times \vec{H} / |\vec{E} \times \vec{H}| \tag{1.1}$$

The restrictions on the possible modes of propagation of light within a material due to Maxwell's equations alone leave a substantial amount of freedom as to the details of the propagation. All that Maxwell's equations require is that within a material, the wavevector, the electric field, and the electric displacement vector $\vec{D}$ lie in a plane and that $\vec{E}$ and $\vec{D}$ are related by (ref. 6)

$$\vec{D} = \frac{n^2}{\mu} (\vec{E} - \hat{k} (\hat{k} \cdot \vec{E})) \tag{1.2}$$

where μ is the magnetic permeability of the material and n is its index of refraction. The exact details of the propagation of light within a material are provided by the constraints imposed by the constitutive relation between $\vec{E}$ and $\vec{D}$,

$$\vec{D} = [K] \epsilon_{vac} \vec{E} \tag{1.3}$$

For amorphous materials, the combination of equations (1.2 and 1.3) yield only *a single solution, light waves for which* $\vec{E}$ *is transverse to* $\vec{k}$ *and for* which the direction of the wavevector and the direction of energy flow are identical. The phase velocity of such light waves is

$$v_p = \frac{c}{n} \tag{1.4}$$

and is independent of the direction of propagation within the material.

The combination of equations (1.2) and (1.3) for uniaxial crystals whose dielectric constant matrix is

$$[K] = \begin{bmatrix} K_o & 0 & 0 \\ 0 & K_o & 0 \\ 0 & 0 & K_e \end{bmatrix} \tag{1.5}$$

allows two modes of propagation for the light within the crystal, the ordinary and extraordinary modes denoted by the subscripts o and e. The ordinary mode is simple. It is identical to the propagation of light within an amorphous material. The directions of $\hat{k}_o$ and $\hat{S}_o$ are identical, $\vec{E}$ is transverse to this direction, and the phase velocity of the light is

$$v_p = \frac{c}{n_o} \tag{1.6}$$

independent of the direction of propagation within the material.

For the extraordinary mode of propagation, the details of the propagation vary as a function of the angle ψ between the wavevector $\hat{k}_e$ and the optic axis $\hat{N}$. Specifically, the electric field vector is not transverse,

$$\hat{E} \cdot \hat{k}_e = - Q \cos \psi \sin \psi \tag{1.7}$$

where

$$Q = 1 - \frac{n_e^2}{n_o^2} \tag{1.8}$$

Further, the phase velocity of the light is direction dependent (ref. 7).

$$v_p = c \left(\frac{\sin^2 \psi}{n_e^2} + \frac{\cos^2 \psi}{n_o^2} \right)^{1/2} \tag{1.9}$$

A consequence of equation (1.7) is that the direction of energy propagation $\hat{S}_e$ is not identical to the direction of the wavevector $\hat{k}_e$. This presents a difficulty in that although the mathematical formulation of the behavior of the extraordinary mode is most simply written in terms of $\hat{k}_e$, all experimental measurements of energy flow measure $\hat{S}_e$.

Both the ordinary and extraordinary light waves traveling within a uniaxial crystal have definite states of polarization. By solving equations (1.2) and (1.3) simultaneously for the components of $\hat{D}$, we obtain the polarization directions of the ordinary and extraordinary rays,

$$\hat{D}_o = \hat{k}_o \times \hat{N}/|\hat{k}_o \times \hat{N}| \tag{1.10}$$

and

$$\hat{D}_e = \hat{k}_e \times (\hat{k}_e \times \hat{N})/|\hat{k}_e \times (\hat{k}_e \times \hat{N})| \tag{1.11}$$

Solving for the components of the electric field for the extraordinary ray and computing $\hat{S}_e$ via equation (1.1) we obtain a relationship between $\hat{S}_e$ and $\hat{k}_e$

$$\hat{S}_e = \hat{k}_e - Q(\hat{N} \cdot \hat{k}_e)\hat{N}/|\hat{k}_e - Q(\hat{N} \cdot \hat{k}_e)\hat{N}| \tag{1.12}$$

This result is significant since it allows the determination of the experimentally measured propagation direction using the mathematically convenient wavevector $\hat{k}_e$.

The actual problem at hand is the following. Consider a uniaxial crystal with a flat surface, the outward normal to the surface being $\hat{n}$ and the direction of the optic axis being $\hat{N}$. Consider light of an arbitrary polarization incident upon the surface with the wavevector $\vec{k}_i$ at an angle of incidence γ_i as in figure 1. At the surface, the light wave breaks into two components, the ordinary one with a certain polarization direction, and the extraordinary one with another polarization. The problem of interest is to determine in which directions the light waves travel within the crystal and what is the percentage of initial light energy in each polarization mode.

The calculation which determines this information proceeds as follows. The continuity of the fields at the crystal surface requires that (ref. 8)

$$\frac{\hat{k}_{crystal}}{v_p} - \frac{\hat{k}_i}{c} \quad \alpha \; \hat{n} \tag{1.13}$$

For the ordinary ray, equation (1.13) determines that the ordinary wavevector $\hat{k}_o$ has the angle of refraction γ_o as in figure 1, which obeys the Snell's law relation

$$\sin \gamma_i = n_o \sin \gamma_o \tag{1.14}$$

and that $\hat{k}_i$, $\hat{n}$, and $\hat{k}_o = \hat{S}_o$ all lie in the same plane. For the extraordinary ray, equation (1.13) becomes

$$\sin \gamma_i = \left(\frac{\sin^2 \psi}{n_e^2} + \frac{\cos^2 \psi}{n_o^2}\right)^{-1/2} \sin \gamma_e \tag{1.15}$$

where γ_e is the angle of refraction of the wavevector $\hat{k}_e$ in figure 2 and where $\hat{k}_i$, $\hat{n}$, and $\hat{k}_e$ all lie in the same plane. However, since Ψ is the angle between $\hat{k}_e$ and the optic axis $\hat{N}$, it is a function of γ_e, and as a result, equation (1.15) represents a fourth degree polynomial in $\sin \gamma_e$. The refraction of the extraordinary ray does not obey Snell's law.

For materials of large birefringence, there is no choice but to solve this fourth order equation numerically. But for low birefringence materials such as sapphire where

$$Q = 0.0090 \tag{1.16}$$

an accurate approximation to the solution of the equation may be obtained as follows. Let the angle Γ in the plane of $\hat{k}_i$, $\hat{n}$, and $\hat{k}_e$ be defined as in figure 2 by the Snell's law relation

$$\sin \gamma_i = n_e \sin \Gamma \tag{1.17}$$

Then the refraction angle γ_e may be written in terms of a power series in Q,

$$\gamma_e = \Gamma - \frac{Q \sin \gamma_i \cos^2 \theta}{2 n_e \cos \Gamma} + \mathcal{O}(Q^2) \tag{1.18}$$

where θ is the angle between the ray which makes the angle Γ with the surface normal and $\hat{N}$. Then once $\hat{k}_e$ is determined from equation (1.18), the propagation direction for the extraordinary ray is given by equation (1.12).

For normally incident light, equations (1.14), (1.17), and (1.18) give $\hat{k}_i = \hat{k}_o = \hat{k}_e$ and the polarization directions of the ordinary and extraordinary rays of equations (1.10) and (1.11) reduce to

$$\hat{D}_o = \hat{k}_i \times \hat{N}/|\hat{k}_i \times \hat{N}| \tag{1.19}$$

and

$$\hat{D}_e = \hat{k}_i \times \hat{D}_o/|\hat{k}_i \times \hat{D}_o| \tag{1.20}$$

This geometry is indicated in figure 3. The geometry of this figure is especially appropriate to the problem of the cylindrical sapphire crystal.

Let the axis of symmetry of the cylinder be the Z axis and let $\hat{N}$ be at an angle θ with respect to the Z axis in the XZ plane. Let the laser beam be normally incident upon the cylindrical surface in the XY plane at an angle of ϕ with respect to the X axis. For this case, the unit vector $\hat{D}_o$ is

$$\hat{D}_o = \frac{\sin \phi \cos \theta \hat{u}_x - \cos \phi \cos \theta \hat{u}_y - \sin \phi \sin \theta \hat{u}_z}{(\cos^2 \theta + \sin^2 \theta \sin^2 \phi)^{1/2}} \tag{1.21}$$

and the unit vector $\hat{D}_e$ is

$$\hat{D}_e = \frac{-\sin^2 \phi \sin \theta \hat{u}_x + \sin \phi \cos \phi \sin \theta \hat{u}_y - \cos \theta \hat{u}_z}{(\cos^2 \theta + \sin^2 \theta \sin^2 \phi)^{1/2}} \tag{1.22}$$

The Determination of the Direction of the Optic Axis

Consider light normally incident upon the cylinder which is polarized at an angle α clockwise from the Z axis as is shown in figure 4. Then the direction of the polarization of the initial light is

$$\hat{D}_1 = -\sin \alpha \sin \phi \hat{u}_x + \sin \alpha \cos \phi \hat{u}_y + \cos \alpha \hat{u}_z \tag{2.1}$$

When this light is incident upon a hollow crystalline cylinder, a certain fraction A of the incident light amplitude propagates in the ordinary mode, and a certain fraction B propagates in the extraordinary mode as is shown in figure 5. Continuity of the tangential components of $\vec{E}$ at the interface requires that

$$\hat{D}_1 = A\hat{D}_o + B\hat{D}_e \tag{2.2}$$

Combining this with equations (1.21), (1.22), and (2.1) we obtain

$$A = \frac{\sin \alpha \cos \theta + \cos \alpha \sin \phi \sin \theta}{(\cos^2 \theta + \sin^2 \theta \sin^2 \phi)^{1/2}} \tag{2.3}$$

and

$$B = \frac{\sin \alpha \sin \phi \sin \theta - \cos \alpha \cos \theta}{(\cos^2 \theta + \sin^2 \theta \sin^2 \phi)^{1/2}} \tag{2.4}$$

The light amplitude leaving the front section of the hollow crystalline cylinder has the form

$$\vec{D}_{\text{leaving front section}} = A\hat{D}_o + e^{i\Phi}B\hat{D}_e \tag{2.5}$$

where the $e^{i\Phi}$ term represents the phase delay of the extraordinary ray relative to the ordinary ray. This is due to the fact that within the cylinder, the ordinary and extraordinary rays have different wavelengths and travel different paths. Thus, the two rays oscillate through a different number of cycles and are out of phase when they leave the cylinder, i.e.,

$$\Phi \approx \frac{2\pi a}{\lambda} (n_e - n_o) \sin^2 \Theta \tag{2.6}$$

For the geometry of figure 3, θ reduces to the angle between $\hat{k}_1$ and $\hat{N}$ so that

$$\sin^2 \Theta = 1 - \sin^2 \theta \cos^2 \phi \tag{2.7}$$

When the light that has passed through the front sector of the hollow cylinder traverses the rear sector of the cylinder, that fraction which was in the ordinary mode continues solely in the ordinary mode, and that fraction which was in the extraordinary mode continues solely in the extraordinary mode. The phase difference between the ordinary and extraordinary rays increases from Φ to 2Φ. If this light is then passed through a polaroid crossed with the direction $\hat{D}_1$, the resulting normalized transmitted intensity is

$$I_{final} = [\sin^2 \Phi] \left\{ \frac{\sin 2\alpha (\sin^2 \phi \sin^2 \theta - \cos^2 \theta) - 2\sin \phi \sin \theta \cos \theta \cos 2\alpha}{\sin^2 \phi \sin^2 \theta + \cos^2 \theta} \right\}^2 \tag{2.8}$$

The first term in this intensity expression is commonly called the isochromat pattern and the second term is commonly called the isogyre pattern (ref. 9). If the light passing through a thick section of the crystal is projected onto a screen, the isochromat pattern will appear as a series of from 2 to 20 fringes across the field of view. The isogyre pattern will appear as an overall illumination level or contrast level modulating the isochromat pattern.

With this general geometry, the determination of the optic axis proceeds as follows. First the initial polarization $\hat{D}_1$ is set to be vertical, i.e., $\alpha = 0°$. Then the isogyre contribution to the normalized intensity becomes

$$I_{isogyre} = \left\{ \frac{2 \sin \phi \sin \theta \cos \theta}{\cos^2 \theta + \sin^2 \theta \sin^2 \phi} \right\}^2 \tag{2.9}$$

If the laser source is stationary and the cylinder is rotated, then θ remains constant and ϕ rotates with the cylinder. The total intensity on the viewing screen is zero, i.e., extinction occurs, when $\phi = 0°$ or $\phi = 180°$. This is when the optic axis is in the plane of the laser beam and the symmetry axis of the cylinder as is shown in figure 6. The isogyre intensity contribution reaches its maximum value at $\sin \phi = \pm\cot \theta$, or simply at $\phi = 90°$ and $\phi = 270°$ when $|\theta| < 45°$. Rotation of the cylinder to the maximum isogyre intensity position produces the maximum bright to dark contrast of the isochromat intensity pattern which takes the appearance shown in figure 7.

In this $\phi = 90°$ or $270°$ position, with $|\theta| < 45°$ and variable polarization direction α, the isogyre contribution to the normalized intensity becomes

$$I_{isogyre} = [\sin 2(\alpha + \theta)]^2 \tag{2.10}$$

This has extinctions when $\alpha = -\theta$ or when $\alpha + \theta = 90°$. The first alternative occurs when the initial polarization $\hat{D}_1$ points in the direction of the optic axis $\hat{N}$ and the second alternative occurs when the final polaroid which is crossed with $\hat{D}_1$ points in the direction of the optic axis. To distinguish between these two alternatives one returns to the configuration for the isochromat intensity of figure 7, and rotates the cylinder a few degrees. This will cause the isochromat fringe pattern to migrate as in figure 8. The direction of this migration is the direction of the optic axis.

Experimental Procedure

The following procedure is used to align the optical system in order to prepare for the determination of the optic axis.

1. Set the laser in the single line mode and project it onto the viewing screen. Mark the laser spot on the screen. This spot will serve as a reference location for all further alignments.

2. Set the cylinder on a rotating table in the laser beam. To ensure that the laser beam passes through the center of the cylinder, make sure that the light reflected from the front surface of the cylinder appears on the mouth of the laser. Also make sure that the laser light transmitted through the cylinder is centered on the marked spot on the viewing screen.

3. Set the microscope objective to be used as a beam expander immediately in front of the mouth of the laser. The laser light on the viewing screen will now be very diffuse and the illumination region will have a large diameter. Center it as best as is possible on the marked spot.

4. Set the beam stop with a roughly 1/8 in. diameter hole between the microscope objective and the cylinder. The brightest center portion of the light from the microscope objective should pass through the hole in the beam stop. The resulting light pattern on the viewing screen should be centered on the marked spot. This step in the procedure serves to filter the laser beam.

5. Set the polarization rotator between the microscope objective and the beam stop near the focal point of the objective lens. The light pattern on the viewing screen should be centered on the marked spot. Set the polarization to be vertical. This vertical alignment is important because, as will be seen in the next section, the location of the optic axis is very sensitive to misalignments in the initial polarization. At this point note also that for many polarization rotators every 1° rotation of the polarization rotator dial produced a 2° rotation of the polarization of the laser light. It is also important to note that if a series of mirrors are used to translate the laser beam, the polarization should be aligned after the last mirror. This is because when polarized light diagonally reflects off of a mirror, the direction of the polarization of the light is changed.

6. Set a polaroid between the cylinder and the viewing screen. The polaroid should be crossed with the polarization rotator. This is accomplished by rotating the polaroid until the fringe pattern on the viewing screen has the poorest bright to dark contrast. It is important not to place the polaroid in an unexpanded laser beam. If this is done, some types of polaroids burn out in about 2 sec.

This completes the alignment of the optical system, and the geometry of steps 1 to 6 is shown in figure 9. At this point, the direction of the optic axis is determined in the following way.

7. Observe the interference fringe pattern and its bright to dark contrast on the viewing screen. Rotate the cylinder until extinction occurs. The extinction lasts for a rotation range of about 5°. Choose the center of this range as the $\phi = 0°$ position. The plane of the laser beam and the axis of symmetry of the cylinder now contains the optic axis.

8. Rotate the cylinder 90° until the resulting fringe pattern appears as in figure 7. Notice the way the fringes migrate under small rotations of the cylinder as in figure 8. Now in the 90° position, rotate the polarization rotator and the crossed polaroid until extinction occurs. The polarization of the incident laser light is now parallel or perpendicular to the optic axis. The direction of the fringe migration of figure 8 enables one to choose the direction of the optic axis from these two alternatives.

When this procedure was performed on the NASA Lewis sapphire cylinder, it was found that the optic axis made an angle of roughly 17° to 19° to the vertical as is shown in figure 10. This cylinder has an outer radius of 2 3/8 in. and a wall thickness of 5/8 in.

Polarization and Stability Questions

Referring to the geometry of figure 3, if the optic axis were along the symmetry axis of the cylinder, $\hat{D}_e$ would always be in the Z direction and $\hat{D}_o$ would always be in the XY plane normal to the laser beam direction for all angles ϕ. Thus, vertically polarized incident light would propagate solely in the extraordinary mode and horizontally polarized incident light would propagate solely in the ordinary mode. As a result, in the crossed polaroid experiment of Section 4, extinction would occur for all rotations of the cylinder ϕ for vertically or horizontally polarized light. Similarly, if the optic axis of the cylinder were at $\theta = 90°$ along the X axis as in figure 3, $\hat{D}_e$ would always be in the XY plane normal to the laser beam direction and $\hat{D}_o$ would always be along the Z axis for all angles ϕ. Thus, vertically polarized incident light would propagate solely in the ordinary mode and horizontally polarized incident light would propagate solely in the extraordinary mode. Again for the crossed polaroid experiment, extinction would occur for all rotations of the cylinder ϕ for vertically or horizontally polarized light.

For the situation at hand when θ is neither 0° nor 90° the polarization directions are in general not as simple as for the two previous cases. The only case where simplicity arises is when the cylinder is rotated to the $\phi = 0°$ position as in step 7 of the previous section. In this case, the plane of the laser beam and the symmetry axis of the cylinder contains the optic axis as in figure 6. For this geometry, $\hat{D}_e$ is along the Z axis and $\hat{D}_o$ is along the Y axis. Thus vertically polarized incident light propagates solely in the extraordinary mode and horizontally polarized incident light propagates solely in the ordinary mode. For this geometry, although birefringence is possible, with vertically or horizontally polarized light it in fact does not occur. It is expected that this geometry will be of central importance to LDV applications.

In the previous section, instabilities due to initial polarization misalignments were alluded to. These are calculated in the following way. For the angles θ, ϕ, and α in figure 3 being arbitrary, the isogyre intensity is given by equation (2.8). Extinction occurs when that expression is equated to zero, i.e., when

$$\sin \phi = - \frac{\tan \alpha}{\tan \theta} \tag{3.1}$$

As seen in the previous section, for vertical polarization, $\alpha = 0°$ and thus the $\phi = 0°$ plane which contains the optic axis is determined. For small misalignments α and with $\theta = 18°$ for the NASA Lewis cylinder, for every degree that α is misaligned, the value of ϕ where extinction occurs is roughly 3° away from the $\phi = 0°$ position. Thus any misalignment in the initial polarization is magnified by a factor of about three in the determination of the direction of the optic axis.

CONCLUSIONS

For small, thin samples of a birefringent material, the polarizing microscope is a useful tool for determining the direction of the optic axis. For large samples, a variant of the polarizing microscope procedure has been described which allows one to identify the optic axis direction to within ±1°.

APPENDIX

An interesting consequence of equation (1.12) is that although the ordinary ray always propagates in the plane of incidence within the crystal, the extraordinary ray almost never does. This is especially noticeable for the case of normal incidence. For normal incidence, the ordinary ray continues to propagate straight ahead undeflected while the extraordinary ray is deflected at the surface of the crystal towards, or directly away from the direction of the optic axis. As long as the ordinary and extraordinary rays propagate within the crystal, the distance between them increases. However, once they leave the crystal through another flat surface parallel to the entrance surface, they propagate parallel to each other in the direction of $\hat{k}_1$ with the separation f. For normal incidence, the distance f is calculated as follows. Let θ be the angle between $\hat{k}_1$ and $\hat{N}$. Let ξ be the angle between $\hat{S}_e$ and $\hat{k}_1$. Then equation (1.11) gives

$$\cos \xi = \frac{1 - Q \cos^2 \theta}{[1 + (Q^2 - 2Q) \cos^2 \theta]^{1/2}} \tag{A.1}$$

For small Q, the denominator may be expanded to give

$$\sin \frac{\xi}{2} \approx \frac{Q}{4} \sin 2\theta \tag{A.2}$$

Finally, the separation f is

$$f = a \tan \xi \approx \frac{Qa}{2} \sin 2\theta \tag{A.3}$$

For normal incidence through a 1-in. thickness of sapphire

$$f \leq (1 \text{ in.})\ (25\,400\ \mu\text{m/in.})\ \frac{0.0090}{2} = 114.2\ \mu\text{m} \tag{A.5}$$

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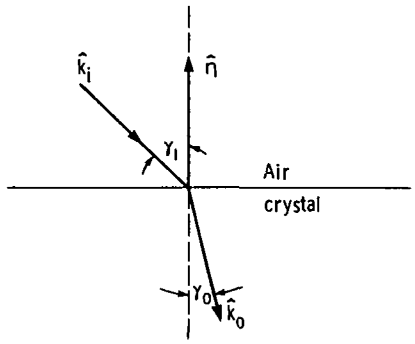

Figure 1. - The geometry for the refracted ordinary light ray produced at the interface of a uniaxial crystal

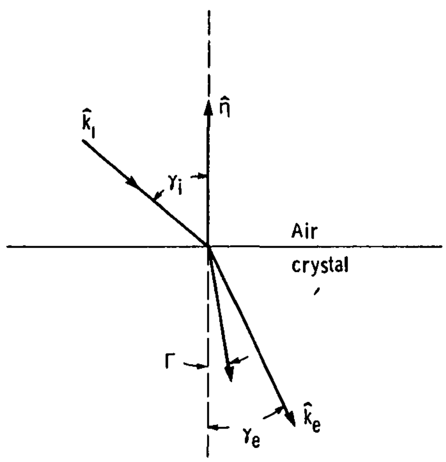

Figure 2. - The geometry for the refracted extraordinary light ray produced at the interface of a uniaxial crystal.

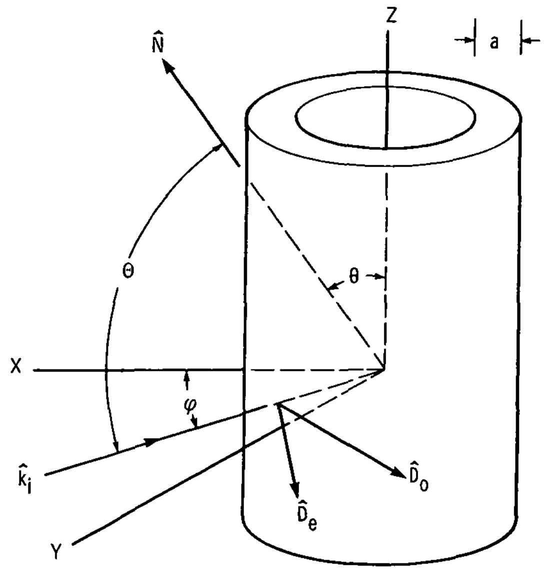

Figure 3. - The geometry for light normally incident upon a hollow cylindrical crystal and the ordinary and extra-ordinary polarization directions within the crystal.

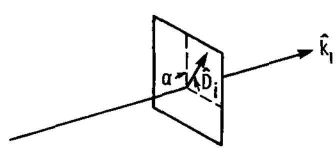

Figure 4. - The initial polarization of the normally incident light ray.

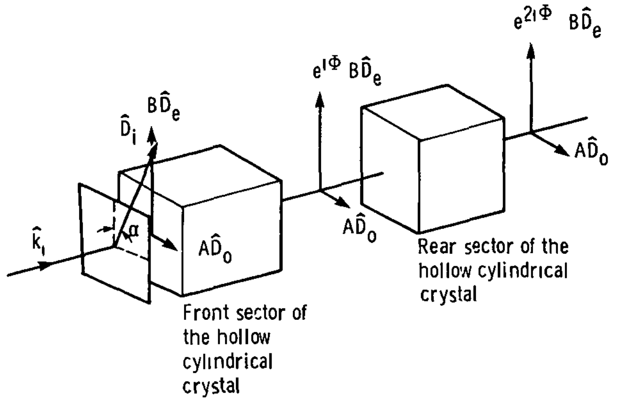

Figure 5. - The accumulation of phase delay of the extraordinary ray relative to the ordinary ray.

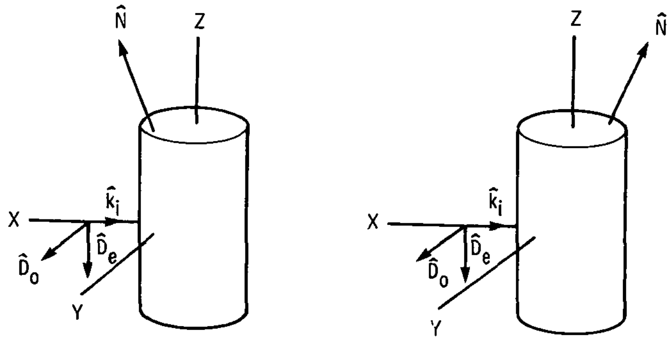

Figure 6. - The directions of the ordinary and extraordinary ray polarizations when the optic axis is in the plane of incidence.

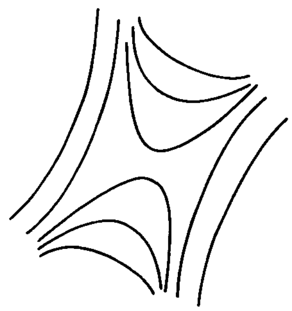

Figure 7. - The isochromat intensity pattern when the optic axis is perpendicular to the plane of incidence.

Figure 8. - The migration of the isochromat intensity pattern.

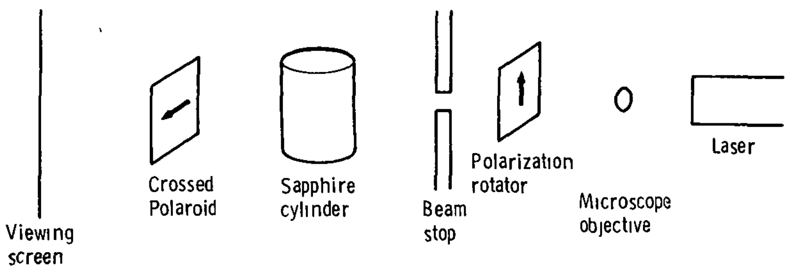

Figure 9. - The optical system schematic.

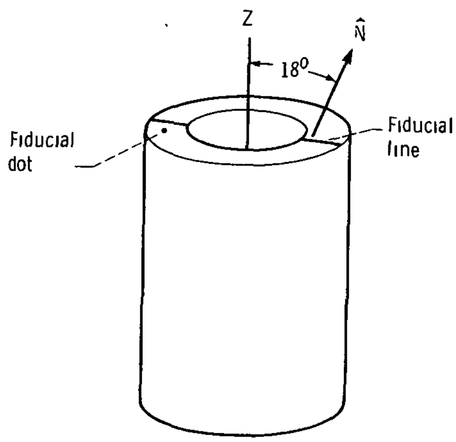

Figure 10. - The direction of the optic axis for the sapphire cylinder used by the NASA Intermittent Combustion Engine Branch.

1. Report No.	2 Government Accession No.	3. Recipient's Catalog No.
NASA TM-86892		

4. Title and Subtitle	5. Report Date
The Determination of the Direction of the Optic Axis of Uniaxial Crystalline Materials	April 1986
	6. Performing Organization Code 505-40-68
7 Author(s) James A. Lock, Harold J. Schock, and Carolyn A. Regan	8. Performing Organization Report No. E-2364
	10 Work Unit No.
9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135	11 Contract or Grant No.
	13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546	14 Sponsoring Agency Code

15. Supplementary Notes

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16. Abstract

The birefringence of crystalline substances in general, and of sapphire in particular, is described. A test is described whose purpose is to determine the direction of the optic axis of a cylindrically machined single crystal of sapphire. This test was performed on the NASA Lewis sapphire cylinder and it was found that the optic axis made an angle of 18° with the axis of symmetry of the cylinder.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Optics; Sapphire; Windows	Unclassified - unlimited STAR Category 35

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price*
Unclassified	Unclassified		

*For sale by the National Technical Information Service, Springfield, Virginia 22161

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