FINAL REPORT:

EOS RADIOMETER CONCEPTS
FOR SOIL MOISTURE REMOTE SENSING

By Jim Carr, ORI, Inc.

10 February 1986

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ORI, Inc.
8201 Corporate Drive
Suite 350
Landover, Maryland 20785

A Final Report
Under Contract
NAS5-28648

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By Jim Carr, ORI, Inc.

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MEMORANDUM

26 February 1986

N86-23995

TO: Distribution

FROM: J. Carr, ORI, Inc. J. Carr

RE: Erratum for Final Report Under Contract NAS5-28648

Please replace page 151 with the enclosed. We erred in stating that the radiometric threshold defining "useful field-of-view" is 0.1 K. It is, instead, 1.0 K.

Distribution:

Dr. A Chang, Code 624

Dr. D. LeVine, Code 675 (2)

Ms. P. Huey, Code 286

Graphic Arts Branch, Code 253

Patent Counsel, Code 204

I. OVERVIEW

This report is basically a compendium of memos submitted to Dr. A. Chang and Dr. D. LeVine of NASA/GSFC during the course of this contract. Also included are copies of the vugraphs presented by J. Carr of ORI to the NASA Earth Observing System/High resolution Multifrequency Microwave Radiometer (EOS/HMMR) panel. This presentation covers our preliminary work with aperture synthesis concepts for EOS, and appeared in our preliminary report. New to the final report is an exposition of the effects of nonvanishing bandwidths on image reconstruction in apperture synthesis systems. It is found that nonvanishing bandwidths introduce errors in off-axis pixels when naive Fourier processing is used. The net effect is for bandwidth to limit sensor field-of-view. Although known to radio astronomers, this effect has to our knowledge never previously been quantified. To quantify this effect we wrote a computer program (a copy of which is delivered with this report) which is documented in this report. Example runs are included which illustrate the resultant radiometric errors and effective fields-of-view for a plausible simple sensor.

II. REAL APERTURE SYSTEMS

The study began by considering real aperture systems for EOS. These concepts grew out of the considerations for a Shuttle-based instrument examined under a previous contract with ORI.

It was decided that the instrument concepts should address the measurement requirements of:

1) 10 km spatial resolution
2) 3 day temporal resolution
3) 1 K radiometric accuracy.

The requirements were derived from the "EOS Science and Mission Requirements Working Group Report" (NASA-TM-86129) and the published experience of other instruments (e.g., Skylab S-194).

The first memo attached considers an electronically scanned array with a single beam. It was found that the constraints on integration time by the need to form an image did not permit all of the measurement requirements to be simultaneously satisfied. A factor of two improvement in the sensitivity is required to meet .5 K radiometric sensitivity. The remainder of the 1 K radiometric accuracy is budgeted for systematic errors. Several ways to enhance performance were then studied. These are:

1) Increase the reception bandwidth beyond the 27 MHz reserved for passive use only
2) Build a total power radiometer

3) Build a reference averaging radiometer
4) Build a multibeam (wiskbroom) radiometer.

The second attached memo examines the feasibility of bandwidth extension. It is rejected due to the large number and high powers of interferers in neighboring bands. The third memo deals with total power radiometry. It appears that total power radiometry may be a feasible option, but that temperature and power supply stability are crucial. On the performance spectrum between total power and Dicke radiometers is the reference averaging radiometer. This is described in the fourth memo and does appear attractive.

Finally, a wiskbroom system with four beams will certainly provide the necessary sensitivity; however, this introduces antenna design complications. Four beams are necessary since the number of beams increases the integration time linearly and the sensitivity is inversely proportional to the square-root of the integration time; hence the factor of four is required to make-up the factor of two shortfall in sensitivity.

1/9/85
rev. 4/8/85

Baseline Passive Microwave Radiometer for EOS

J. Carr

Guidance from EOS Documents and HMMR pannel

- 5 - 10% accuracy
- 1 - 10 Km spatial resolution
- 1 - 3 day temporal resolution
- 600 - 1000 Km circular 2 pm sun-synchronous orbit
- 20 m apertures considered

Conclusions from Previous Study

- Frequency : 1.4 GHz (21 cm)
- Bandwidth : 27 MHz
- Polarization : H (H & V become desirable when scanning past 20°)
- Accuracy > 1K with .5 K sensitivity
- $\eta_M > 90\%$ (main beam efficiency)
- $\eta_L > 50\%$ (radiation efficiency, implies 3 dB antenna loss)
- Radiometric Accuracy and Sensitivity are consistant with desired soil moisture measurement needs
- Beam width – antenna size relationship for L×L antenna with $\eta_M = 90\%$

$$\beta \doteq (1.6°)\left(\frac{10m}{L}\right)$$

Baseline Real Aperture System Design

- Choose maximum 3 day repeat period to minimize swath requirements

- Choose least altitude exceeding 600 Km giving circular, sun-synchronous, 3 day repeating orbit (see figures). That is —

  $h = 675$ Km
  $i = 98°$
  2 pm ascending node

- Selected orbit has 44 tracks hence the minimum swath required for global coverage is

$$W = \frac{2\pi r_e}{44} = 911 \text{ Km} \qquad r_e = 6378 \text{ Km}$$

- Maximum incidence angle is

$$\theta = \tan^{-1} \frac{W/2}{h} = 34°$$

therefore dual polarization desirable

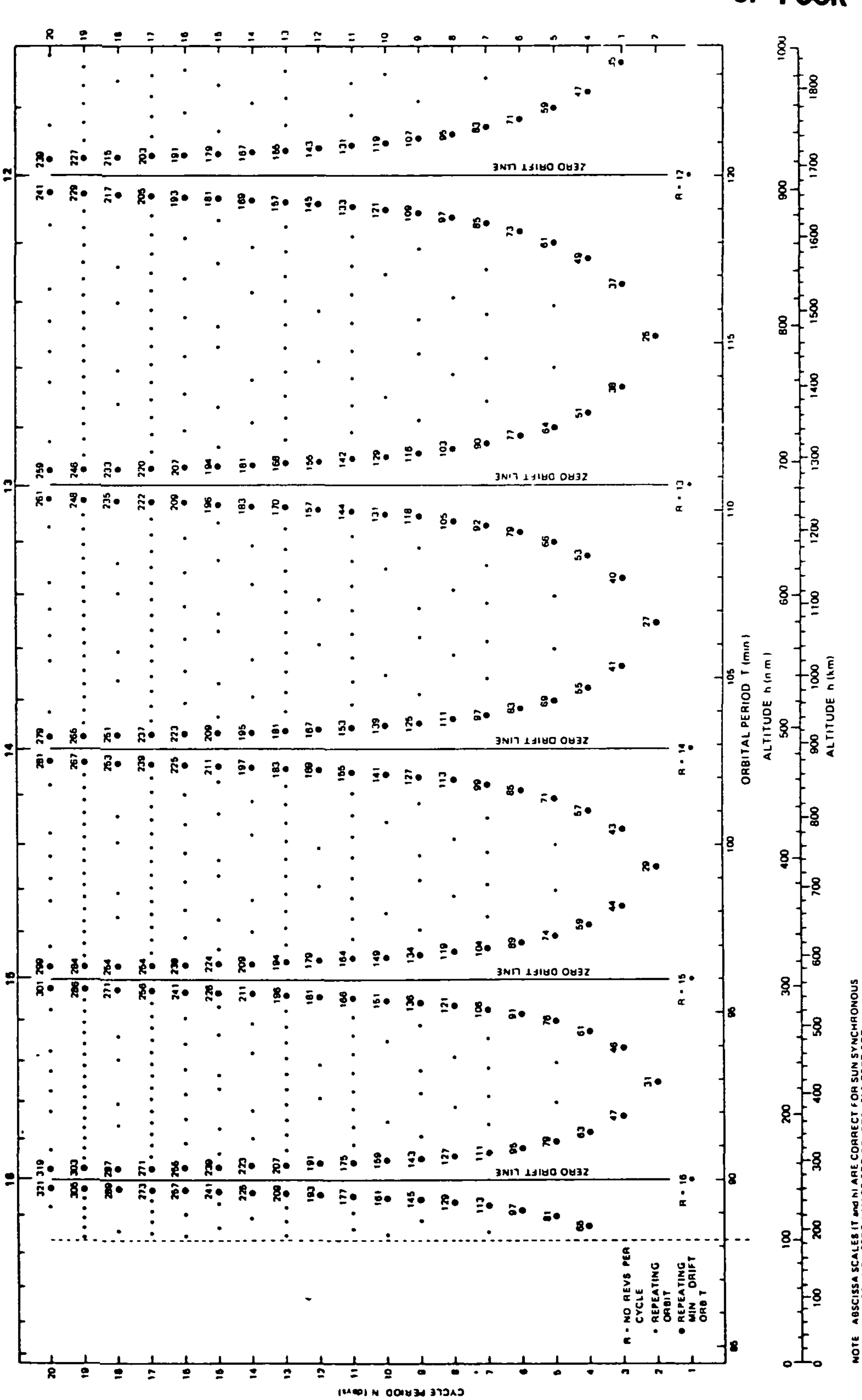

Fig 16-4 Array of periodic coverage patterns

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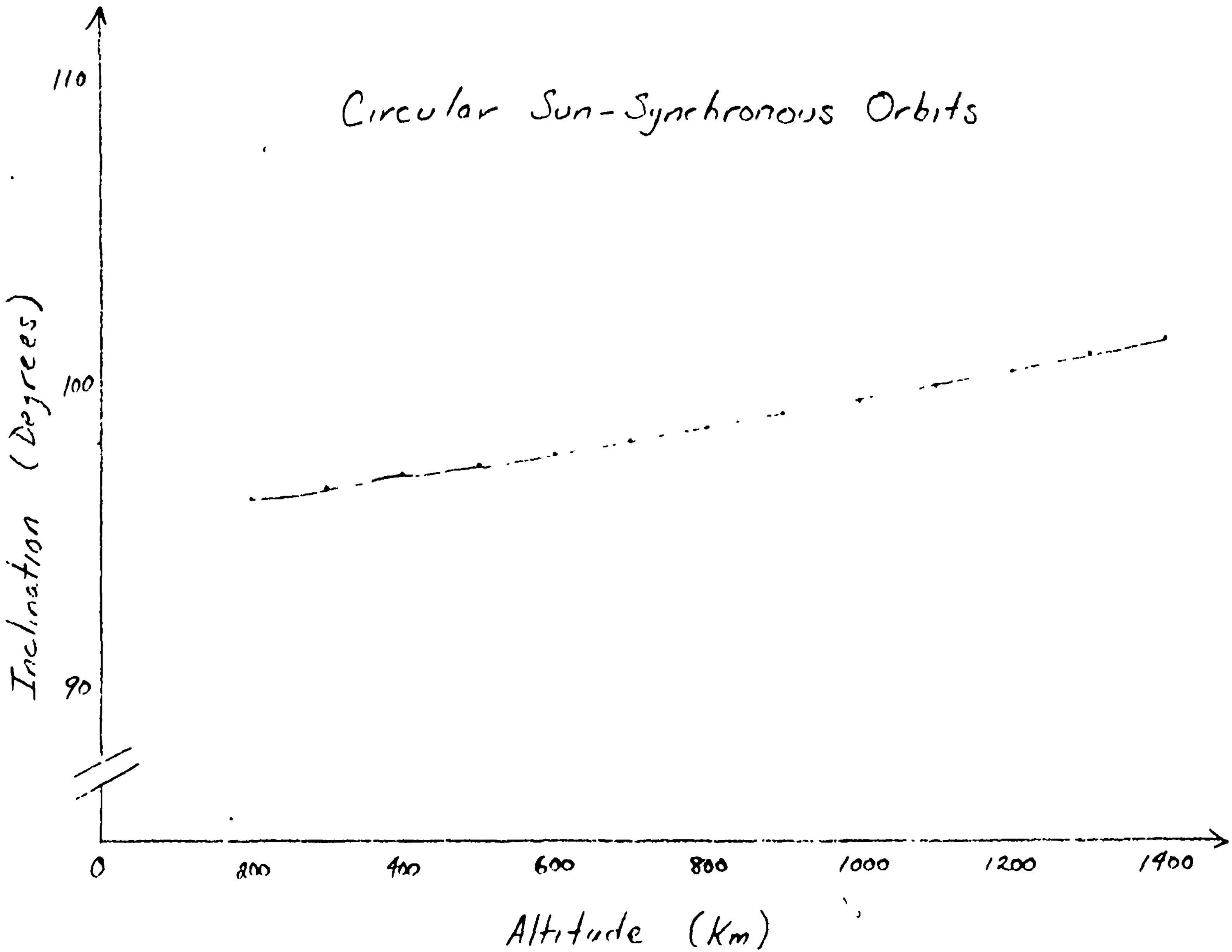

Circular Sun-Synchronous Orbits
Inclination (Degrees)
110
100
90
0
200
400
600
800
1000
1200
1400
Altitude (Km)

- Footprint $\Delta x = 10$ Km implies

$$\beta = \frac{\Delta x}{h} = 1.5 \times 10^{-2} \text{ rad} = .85^\circ$$

which implies an antenna size

$$L = (10m)\left(\frac{1.6^\circ}{\beta}\right) = 19m$$

- Radiometer uncertainty equation:

$$(\Delta T)(\Delta x)\sqrt{B} = 2(T_B + T_N)\sqrt{W u}$$

$$u = \sqrt{\frac{GM}{r_e + h}} \qquad GM = 3.98591 \times 10^5 \text{ Km}^3/s^2$$

$$= 7.5 \text{ Km/s}$$

$T_B \cong 300K$ $\Delta T = .5K$ $\Delta x = 10$ Km $B = 27$ MHz
$W = 911$ Km, solve for T_N:

$$T_N = \frac{(\Delta T)(\Delta x)\sqrt{B}}{2\sqrt{W u}} - T_B$$

$$= -140 \text{ K}$$

Therefore, we must back off on either ΔT or Δx

- Using uncooled parametric amps or GaAs FET amps, $T_N = 75$ K. Solving for $(\Delta T)(\Delta x)$:

$$\frac{(T_B + T_N)(2\sqrt{W_u})}{\sqrt{B}} = 12 \text{ Km-K} = (\Delta T)(\Delta x)$$

$\Delta x = 10$ Km implies backing off ΔT to 1.2 K with comensurate reduction in soil moisture accuracy

$\Delta T = .5$ K implies backing off Δx to 24 Km with the benefit of reduced antenna size

Options

- Frequency : 1.4 GHz
  $B = 27$ MHz
  Polarization : H min.
  $\eta_M = 90\%$
  $\eta_L > 50\%$

  $T_N = 75$ K
  $h = 675$ Km
  $i = 98°$
  2 pm ascending node
  $W = 911$ Km

- Options :

  $\Delta x = 10$ Km, $\Delta T = 1.2$ K } implies $L \cong 20$ m

  $\Delta x \cong 25$ Km, $\Delta T = .5$ K } implies $L \cong 8$ m

  $\Delta x = 20$ Km, $\Delta T = .6$ K } implies $L \cong 10$ m

- Other considerations

  6 day orbit with 911 Km swath so that sites are covered every three day but with different incidence angles

  Lower altitudes

January 25, 1985

TO: File

FROM: J. Carr and B. Candey

RE: Bandwidth Extension

Analysis to establish a baseline real aperture L-band sensor for EOS has found that integration time constraints do not permit both the desired spatial resolution and the desired radiometric resolution. It is also anticipated that this integration time constraint will place more severe restrictions upon synthetic aperture systems. One possible solution is to increase the bandwidth from the nominal 27 MHz (1400-1427 MHz). Initially this seemed feasable since the 1370-1400 MHz band is designated for passive use on a secondary basis. However, as the following analysis will indicate bandwidth extension is infeasable due to the presence of numerous high powered transmitters.

The frequency bands neighboring 1400 MHz have the following allocations[1]:

1) The 1300-1350 MHz band is allocated to radionavigation worldwide except for fixed and mobile services in Indonesia and radiolocation service in Ireland and the United Kingdom. Radionavigation use is limited to ground-based radars and associated airborne transponders. The U.S. government allocations also allow radiolocation but limited to the military services.

[1] Manual of Regulations and Procedures for Federal Radio Frequency Management, NTIA, May 1984. p. 4-53ff.

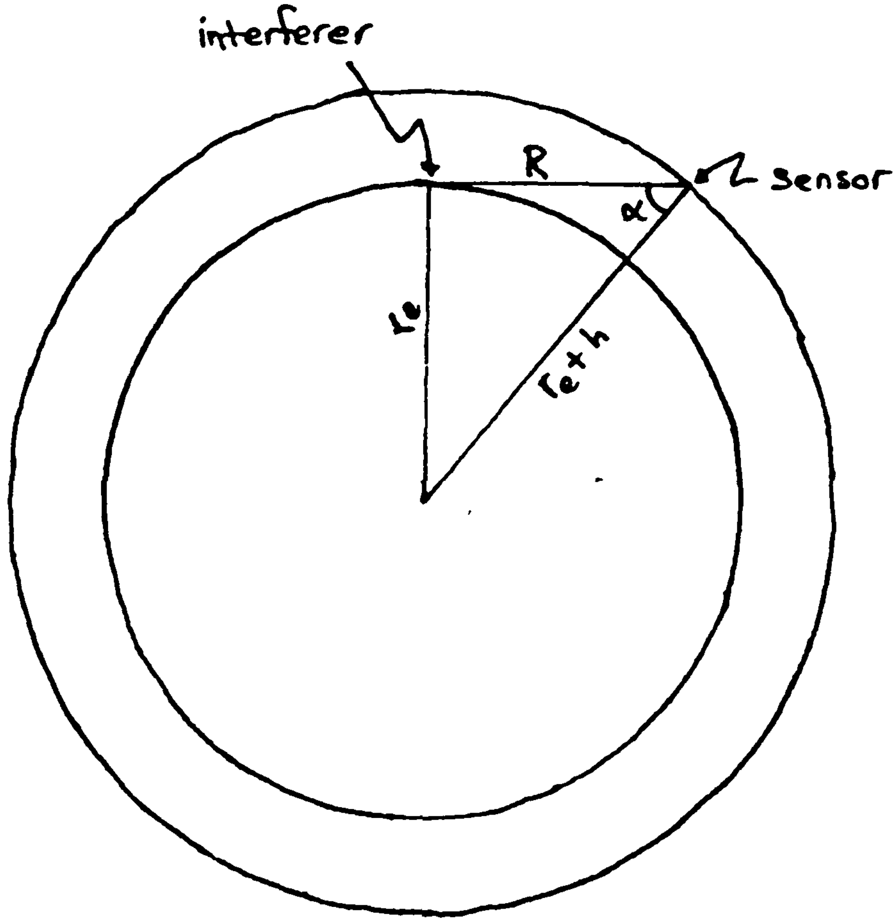
2) The 1350-1400 MHz band is allocated to fixed, mobile, and radiolocation services in Region 1 (North and South America) and radiolocation in Regions 2 and 3 (Europe, Africa and Asia) with some existing radionavigation installations in the Eastern Bloc, U.S., and Canada. The 1370-1400 MHz band is also allocated to passive space and earth research and observations on a secondary basis. In addition, the frequency 1381 MHz is allocated in the US to Fixed and Mobile Satellite Services (Space-to-Earth) for the relay of nuclear burst data. In the U.S., over 100 high powered radars operate in the 1370-1400 MHz band and international listings also indicate extensive use.

3) The 1400-1427 MHz band is reserved solely for passive radio astronomy, space research, and earth exploration-satellite observations. Some second and third harmonics and various intermodulation products of television transmitters and extraband radiation from radars may fall within the 1400-1427 MHz band.

4) The 1427-1525 MHz bands are allocated to fixed, mobile, and space operation and are used heavily.

Ideally one wants a large bandwidth for enhanced sensitivity; however, artificial sources are many orders of magnitude brighter than the thermal emmissions of the earth and must be excluded from the instrument bandpass. We determine the interference threshold as follows. The required receiver sensitivity is $\Delta T = .5$ K which corresponds to a power level of $k(\Delta T)B$ with $k = 1.38 \times 10^{-23}$ J/K and B the bandwidth. The maximum sensor-interferer range will occur just when the sensor rises above the interferer's horizon (see Figure 1). Coincidently this is where radiolocation services (e.g., FAA radars) would have their maximum gain directions and so this is the anticipated worst case geometry dispite the large range. At a nominal altitude of h = 700 Km, the sensor - interferer range is R = 3069 Km with the interferer at a 64° angle from the sensor's nadir. We assume a nominal 10m x 10m sensor aperture which scans $\pm 33^{\circ}$ to cover a 900 Km swath. This places the interferer 31° off the maximum gain direction at the extreme point of the scan. A uniformly weighted aperture would have a power pattern which falls as

$$\left(\frac{\pi L}{\lambda} \sin \phi\right)^{-2}$$



$r_e = 6378$ Km
$h = 700$ Km

$$R = \sqrt{(r_e+h)^2 - r_e^2}$$
$$= \sqrt{2r_eh + h^2}$$
$$= 3069 \text{ Km}$$

$$\alpha = \sin^{-1} \frac{r_e}{r_e+h}$$
$$= 64°$$

Scan angle with 900 Km swath

$$= \tan^{-1} \frac{900 \text{ Km}/2}{h}$$
$$= 33°$$

Figure 1. Sensor - Interferer Geometry

where L is the aperture dimension (10m), λ is the wavelength (21 cm), and ϕ is the angle off the maximum gain direction. For $\phi = 31^o$ this puts the peak sidelobe level at G = -38 dB. However, it is better to taper the aperature weighting in order to trade beam width for lower sidelobes (increased main beam efficiency). Sidelobe levels at $\phi = 31^o$ for a 90% main beam efficient antenna would be more like G = -50 dB (see Figure 2). The power into the sensor's receiver from the interferer is then

$$P = \frac{(EIRP)}{4\pi R^2} L^2 G$$

where EIRP is the interferer's Equivalent Isotropic Radiated Power. In which case, the maximum tolerable EIRP to not exceed a power threshold P is given by

$$(EIRP) = \frac{4\pi R^2 P}{L^2 G}$$

We will conservatively set $P = k(\Delta T)B$ which is the receiver sensitivity. For B = 27 MHz, P = -157 dBW and for B = 100 MHz, P = -152 dBW. Using R = 3069 Km, L = 10 m, G = -50 dB gives EIRP thresholds of + 14 dBW and +19 dBW for the 27 MHz and 100 MHz bandwidths respectively.

Typical EIRPs for fixed stations operating near 1.4 GHz are 37 dBW to 55 dBW [2] which would definitely represent an interference threat. Additional sidelobe rejection of -36 dB would be required to place the 55 dBW transmitters at the received power threshold -152 dBW, and another -10 dB should be added to place the interferer below the recieved power threshold with adequate margin. Then a sidelobe level of -96 dB is required at 31^o off the maximum gain direction. This would be extremely difficult to achieve.

[2]Recommendations and Reports of the CCIR, 1982: XVth Plenary Assembly in Geneva, Volume II: Space Research and Radioastronomy, ITU, p. 386.

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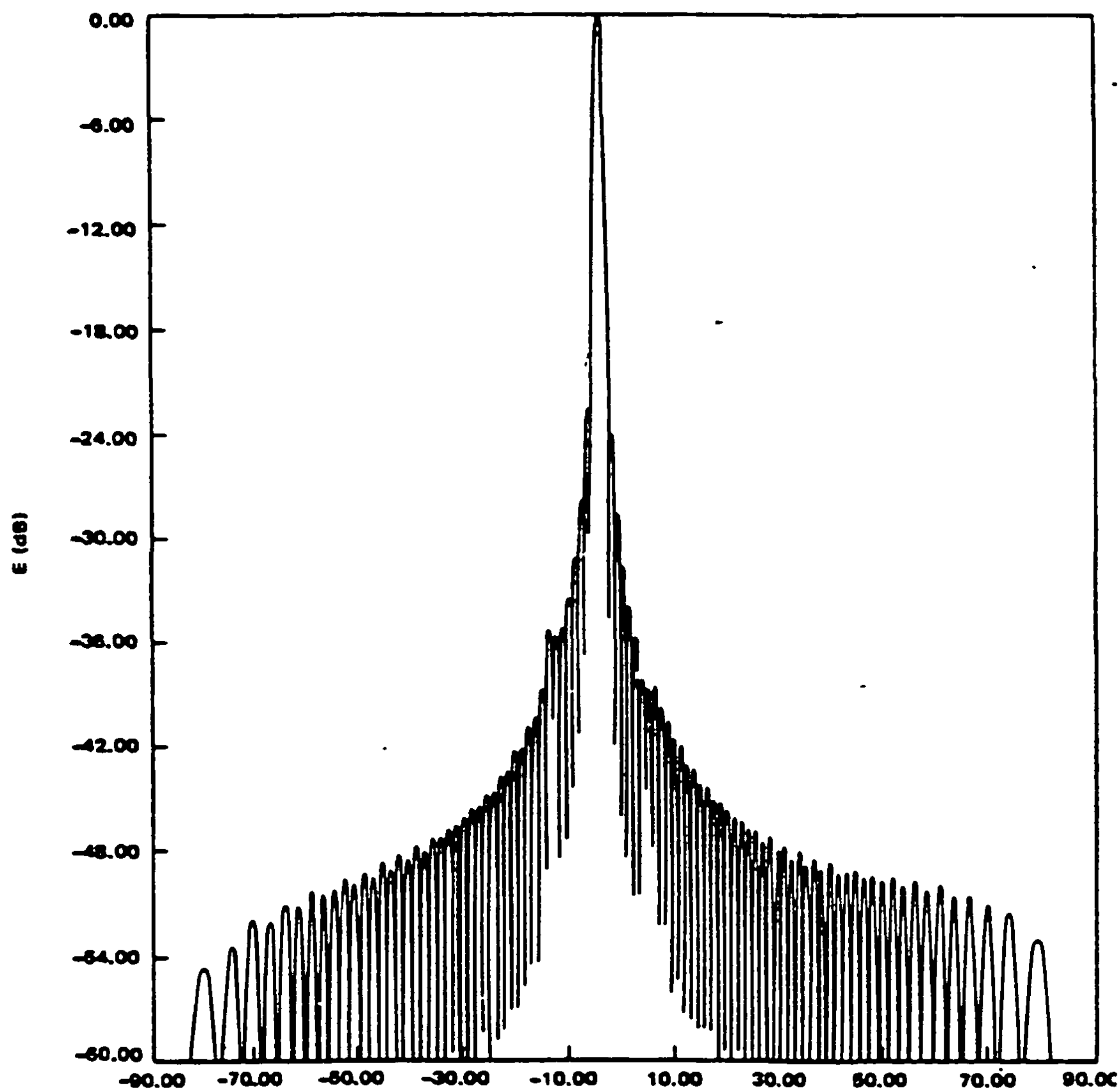


c. 30 DEGREE SCAN

Figure 2. Power Pattern for a 90% Efficient Antenna.

(Source: "L-band Radiometer Antenna Study", Hughes Aircraft under contract NAS5-23418, May 1977)

Using a received power threshold of -165 dBW (compare with -157 dBW) corresponding to an EIRP threshold of 12 dBW (compare with 19 dBW), the International Radio Consultative Committee (CCIR) has performed an interference analysis for a 100 MHz bandwidth sensor[3]. The results are shown in Figure 3 where total data loss occurs in the shaded regions. This analysis includes only the fixed services which populate the 1350-1400 MHz band, so the actual interference environment is in fact worse.

Transmitters in the 1250-1400 MHz band include FAA aeronautical radionavigation radars, DoD long range surveillance and search radars, DoD communication radios, the GPS downlink for relay of nuclear burst data, the TLPS Army transponder and classified military transmitters. A common FAA radar, the ARSR-1, operates at frequencies of 1280-1350 MHz, has an antenna gain of 34.2 dBi and 4 MW peak power over 2 microsecond pulses. DoD communication radios, such as the AN/GRC-50 and AN/GRC-103, broadcast in the 1350-1849.5 MHz band with antenna gain of up to 20 dBi and peak power of 8-30 W. The TLPS transponder uses the 1350-1400 MHz band with a peak power of 120 W.[4]

In light of the above interference analyses, it appears that bandwidth extension beyond the reserved 1400-1427 MHz band is infeasable. This conclusion is corrobated by the opinion of David P. Struba, NASA Headquarters Frequency Management Program Manager, that the secondary allocation for passive use of the 1370-1400 MHz band is almost meaningless.[5] In addition there is also concern that extraband radiation and TV transmitter harmonics could pollute the 1400-1427 MHz band. Although there is no mention of interference problems in the literature reporting the 1973 Skylab L-band radiometer results, this issue should be studied further. If it is determined that such emissions might pose an interference threat then a Shuttle survey with a low cost low resolution antenna might be considered to characterize the L-band environment.

[3]Ibid. pp. 379 - 399.

[4]Andrew Farrar, Spectrum Resource Assessment in the 1215-1400 MHz Band, NTIA Report 81-83, Sept. 1981, pp. 36-8, 50.

[5]Private communications between David P. Struba and Bob Candey.

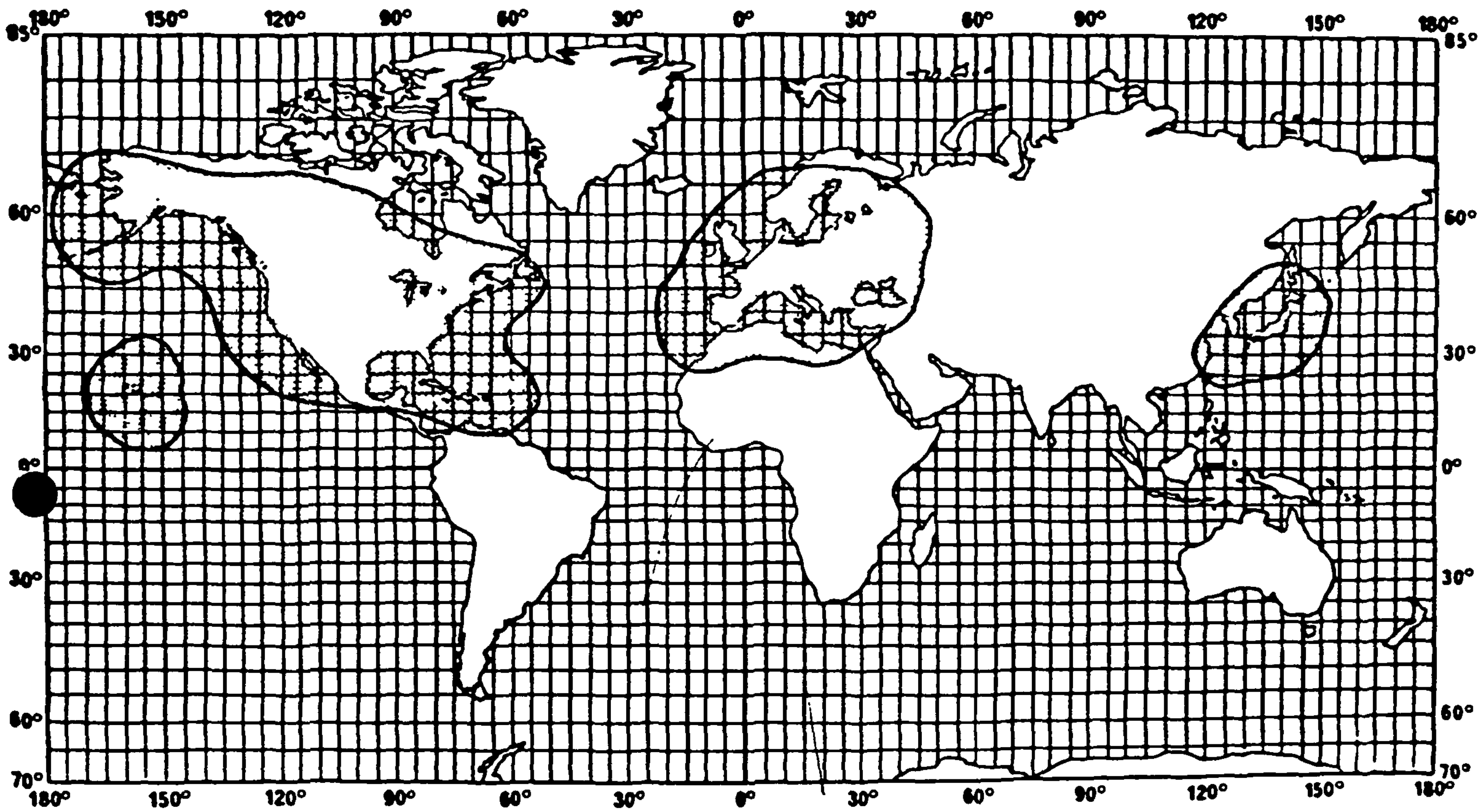


Figure 3. Regions of Total Data Loss
(Source Ibid., p. 387)

25 February 198?

TO: J. Carr

FROM: R. E. Prince RSP

SUBJ: *L - Band Amplifier Drifts with Respect to Total Power Radiometers*

I have made a number of phone calls and managed to talk to about half of the people I attempted to contact.

Two attempted contacts that never returned their calls were:

Chase Hearne, a specialist in GaAs amplifiers.
Hans Blume, a specialist in uncooled parameter amplifiers.

They are both NASA/Langley (LaRC) people whose names were given me by their boss at LaRC, Richard Harrington (804) 865-3631. Harrington said they would call, but . . . So be it. I mention this only because these two were supposedly hot numbers (technically speaking). I got Harrington's name from John Shien at GSFC.

A man named Joe Rogers (GSFC, 344-8816) has tested a number of GaAs L - band amps. He particularly likes units manufactured by California Amplifier and by Berkshire Technologies, Inc. (also in California). Rogers indicates that another GSFC person, David Buhl (344-8810) has a noise figure and gain test setup for these devices based on an HP 8970 instrument. Whether he can measure the small variations we are looking for is another question. Buhl also suggests that we talk to Robert Jones, RF Technology Branch in GSFC, Bldg 19. Apparently building 19 has a test setup based on an AIL instruments.

Although nobody I talked to had test or analytical data for small scale, short term GaAs L - band amplifier gain stability with temperature, all my correspondents agreed that primarily temperature (and only slightly supply voltage) controls amplifier gain. All agreed that with sufficient temperature stabilization a gain stability of 10^{-3} (-30 db) was quite feasible.

Supply voltage requirements are typically $\pm$ 15 volts @ 50 ma. Regulation should be # 10% or better.

All of the amplifier people I spoke to quoted specifications for an L - band unit similar to the following:

frequency range: 1.30 to 1.60 GHz
noise figure: .7 to 1.0 db at room temperature
$\Delta NF/\Delta T$ approx. .05 db/10°c (down to a noise figure of approx. .6 at -70°c)
gain: 35 to 40 db (power) at room temperature
$\Delta G/\Delta T$: -.02 db/°c to -.04 db/°c (gain increases with dropping temperature)

This $\Delta G/\Delta T$ number corresponds to approximately .1° C temperature stability to meet a -30 db $\Delta G/G$ (or .01° C stability for a -40 db $\Delta G/G$). (See attached calculation).

I have enclosed some test data I received from California Amplifier, Inc.

I talked to Jim Bremer for a short while on the telephone concerning his reference - averaging radiometer design. He indicates that in approximately 1977, Hughes Aircarft did a computer simulation and then built a labatory breadboard of the reference - averaging technique. Jim said that the Hughes work verified the Bremer calculations but that simultaneous advances in amplifier stability mode a total-power system feasible. The Hughes work was at millimeter wave lengths.

Jim brought up something else to be remembered about total power systems: a D.C. - Restore capability probably will be necessary. D.C. - Restore is essentially is a calibration - it returns the analog output to the A/D converter to a known D.C. level for a given input every now and then. Typically, it is done once per scan line (every few seconds) or whenever practical. If the amplifier is stable enough that it's baseline plus dynamic range never clamps or extends beyond the A/D converter useful range of inputs, the D.C. - Restore function as a specific analog circuit could be omitted. This function could then be subsumed under the calibration function and accomplished digitally at the output of the A/D converter (or still as an analog function, of course).

People I have attempted to contact include:

Person and Location	Successful	Unsuccessful
R. Harrington, LaRC (804) 865-3631	x	
Chase Hearne, LaRC		x
Hans Blume, LaRC		x
Jim Shieu, GSFC	x	
Roger Ratliff, GSFC	x	
Joe Rogers, GSFC	x	
David Buhl, GSFC	x	
Trontech Corp., Neptune, N.J. (201) 229-4348		x
Miteq, Hauppage, N.Y. (Jared Siddiqui) (516) 543-8873	x	
Applied Microwave Corp., Lawrence, Kansas (Dr. David Brunfield, formerly U.KS Remote Sensing Center) (913) 749-3511	x	
California Amplifier, Newbury Park, Cal. (John Ramsey) (805) 499-8535	x	
Berkshire Technologies, Inc., Oakland, Cal. (415) 655-1986	x	

Although I did not call them, several people suggested Amplica, (805) 498-9671. They are a division of COMSAT.

If the gain variation with temperature, expressed in dB is:

$$\Delta g \ \ ^{db}/_{C^o}$$

the gain change ratio, g_r, is:

$$g_r = \text{antilog}\left(\frac{\Delta g}{10}\right)$$

$$g_r = \frac{g_{new}}{g_{old}}$$

$$\frac{\Delta G}{G} = \frac{g_{new} - g_{old}}{g_{old}}$$

$$\frac{\Delta G}{G} = g_r - 1$$ This is ratio form

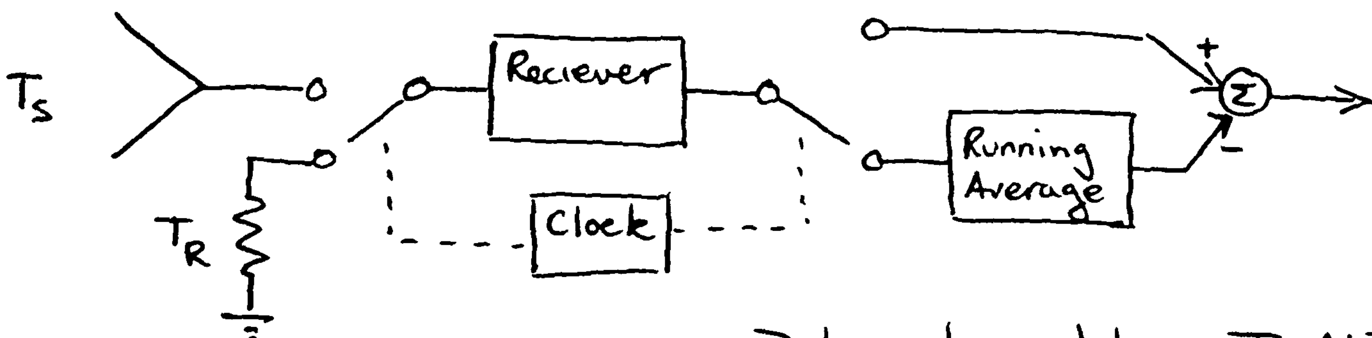
in dB form:

$$\left(\frac{\Delta G}{G}\right)_{db} = 10 \log\left[\left(\text{antilog}\frac{\Delta g}{10}\right) - 1\right]$$

where Δg is the db gain variation with temperature

Bremer (Reference Averaging) Radiometer

Reference : J.C. Bremer, "Improvement of Scanning Radiometer Performance by Digital Reference Averaging", IEEE Trans. Instrumentation and Measurement, IM-28, March 1979.



Balanced condition $T_S \cong T_R$

Radiometer sensitivity

$$\frac{\Delta T}{T} = \frac{M}{\sqrt{B\tau}}$$

M = Radiometer figure of merit
B = Bandwidth
τ = integration time

Without reference averaging

$$M = \sqrt{\tau/\tau_S + \tau/\tau_R}$$

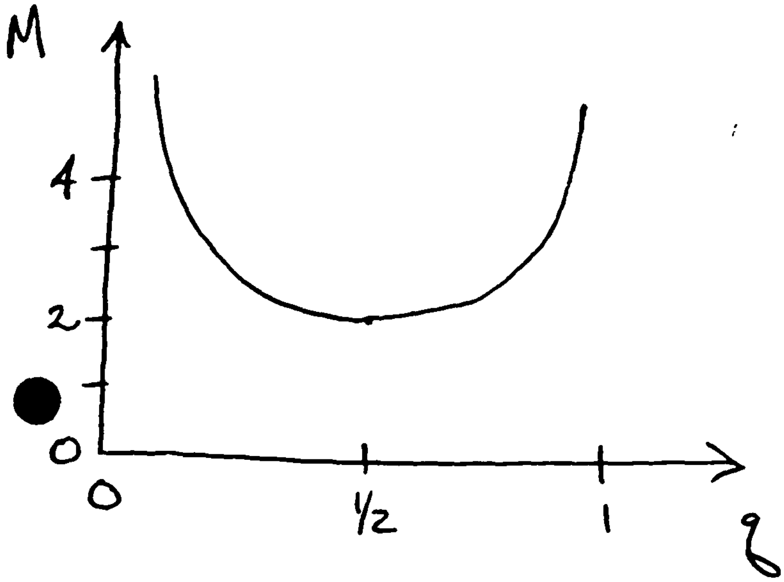
τ_S = scene integration time
τ_R = reference integration time

$$\tau = \tau_S + \tau_R$$

g = duty cycle = 1/2 for Dicke radiometer

$\tau_S = g\tau \qquad \tau_R = (1-g)\tau$

$$M = \sqrt{1/g + 1/(1-g)}$$



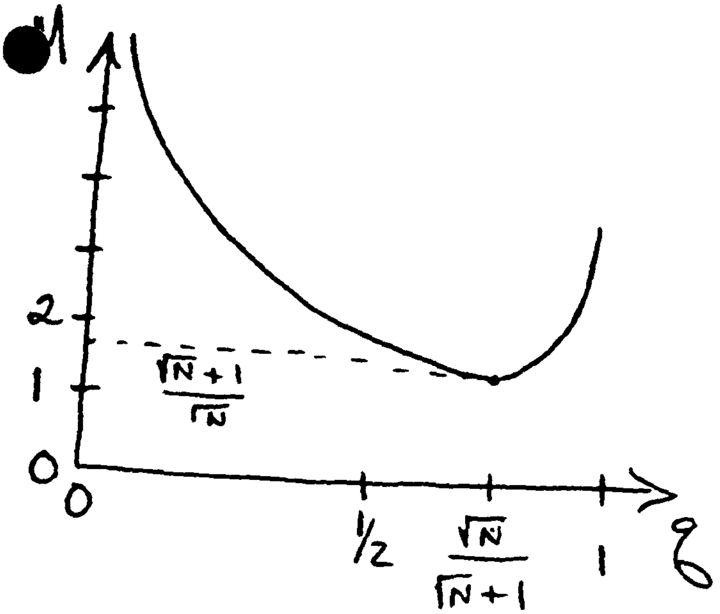
Dicke radiometer (with $g = 1/2$) optimizes sensitivity

With reference averaging

$$M = \sqrt{\tau/\tau_S + \tau/N\tau_R}$$

N = running average length

$$= \sqrt{1/g + 1/N(1-g)}$$



Optimal chopping is asymmetrical

$$q = \frac{\sqrt{N}}{\sqrt{N}+1} \qquad M = \frac{1}{q} = \frac{\sqrt{N}+1}{\sqrt{N}}$$

$$\tfrac{1}{2} \le q \le 1 \qquad 1 \le M \le 2$$

N	q	M
1	.5	2
3	.63	1.58
5	.69	1.45
10	.76	1.32
25	.83	1.20
⋮	⋮	⋮
∞	1	1

N is most easily implemented if $N = 2^{2n}$

III. APERTURE SYNTHESIS SYSTEMS

As an alternative to real aperture systems we have considered synthetic aperture systems. Aperture synthesis can offer high spatial resolution with a sparse antenna array and so it can be economically attractive. Initially the more general case of a synthesis/scanning hybrid was considered (of which a pure synthesis instrument is a degenerate case). Such a hybrid would step-scan its field-of-view across-track. Aperture synthesis provides the imaging within each field-of-view. The first of the following memos describes a one-dimensional scanning/synthesis hybrid. It concludes that such instruments can perform the soil moisture mission. An especially remarkable result is that the necessary radiometric resolution could be achieved despite the diminished aperture area. This is because the collection area loss is offset by both an increased integration time (set by the real field-of-view) and the parallel channel structure of an aperture synthesis instrument.

The second of the following memos details a two-dimensional synthesis/scanning hybrid concept. It concludes that the sensitivity of such an instrument is identical to the one-dimensional case for given aperture area. Therefore, there is no advantage to this more complicated scheme and only one-dimensional concepts were henceforth considered.

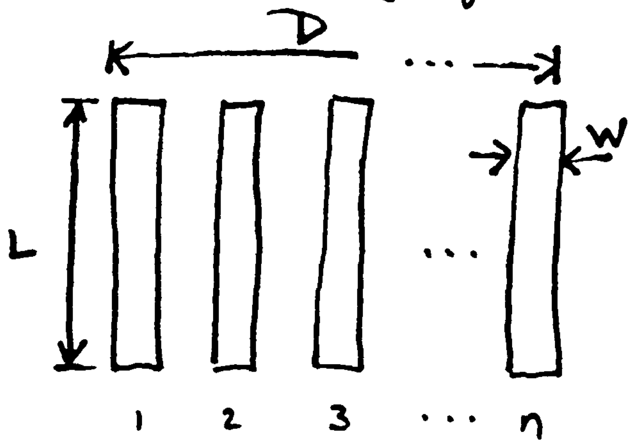
Following this memo is a set of notes detailing the theory of aperture synthesis. It covers the signal processing theory, resolution, grating lobes, and nonlinear effects (fringe washing). The grating lobes and fringe washing effects are particularly important as they restrict the field-of-view.

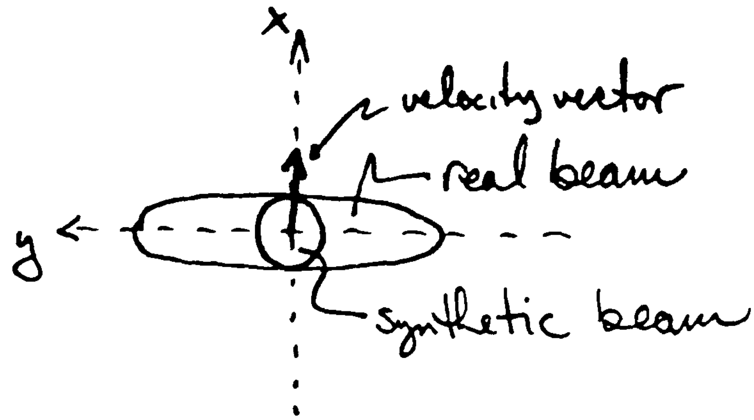
Finally, details of how to construct a pure aperture synthesis instrument were considered. In the last three memos an analog and two digital architectures are presented. All architectures have the same external data rates, whereas the digital architectures (they do correllations digitally as opposed to using analog mixers) can have very significant internal rates.

2/11/85

One-Dimensional Scanning/Aperture Synthesis Hybrid

We have an array of n L×W antennas which partially fill an L×D area as shown





The antennas are not necessarily evenly spaced. We will assume each antenna to have the characteristics of a uniformly illuminated L×W aperature. Each then produces a cigar beam with the widths:

$$\beta_x = (51^\circ)\frac{\lambda}{D} = .9\lambda/L$$

$$\beta_y = (51^\circ)\frac{\lambda}{W} = .9\lambda/W$$

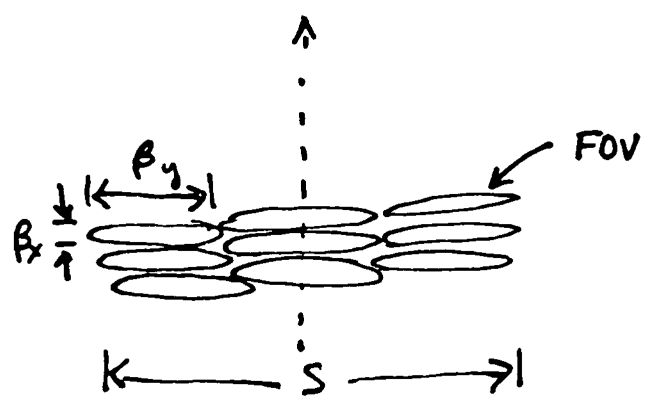
We will also assume that the synthesized beam is characteristic of a uniformly illuminated L×D aperture. The synthesized beam widths are then

$$\beta'_x = .9\lambda/L = \beta_x \qquad \beta'_y = .9\lambda/D$$

The beam width β_y defines the FOV which is step scanned acrosstrack to define a swath of M FOV's. The entire swath has angular extent S, so that

$$S = M\beta_y$$

The dwell time per FOV will be set so that the satellite motion causes the FOV rows to be displaced one along-track synthesized beamwidth β_x' $(=\beta_x)$, as shown.



The time required to advance one synthesized resolution cell along-track is

$$t = \frac{\Delta x'}{u}$$

where u is the subsatellite point velocity and $\Delta x'$ is the synthetic resolution along-track given by

$$\Delta x' \cong h\beta_x' = h\beta_x$$

We apportion t equally amoungst the M FOV's to give the integration time

$$\tau = \frac{t}{M} = \frac{\Delta x'}{Mu} = \frac{h\beta_x}{Mu}$$

the synthetic aperture sensitivity equation is

$$\Delta T' = \frac{T_{sys} A_{syn}}{\sqrt{n(n-1)}\, A_e \sqrt{B\tau}}$$

where $\Delta T'$ is the radiometric resolution, T_{sys} is the system temperature, A_{syn} is the effective synthetic aperature area, A_e is the effective individual aperature area, and B is the bandwidth. We will take

$T_{sys} = 375$ K (300K brightness temp. + 75 K noise temp.)

$B = 27$ MHz (maximum available)

$A_e = LW \qquad A_{syn} = LD$

Substituting for A_e, A_{syn} and τ gives

$$\Delta T' = \frac{T_{sys} D \sqrt{Mu}}{\sqrt{n(n-1)}\, W \sqrt{B\Delta x'}}$$

The ratio D/W is just the beam width ratio β_y / β_y' and using the definition of the angular swath width:

$$\frac{D}{W} = \frac{\beta_y}{\beta_y'} = \frac{S}{M\beta_y'} = \frac{Sh}{M\Delta y'}$$

where $\Delta y' \cong h\beta_y'$ is the synthetic resolution across-track. We then have

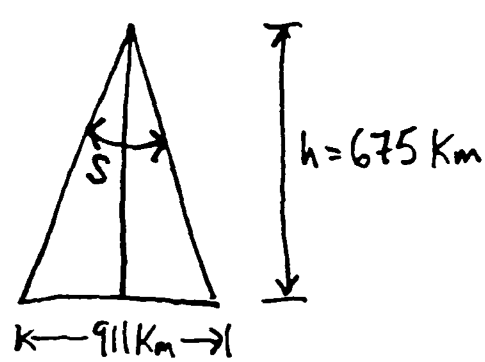
$$\Delta T' = \frac{T_{sys} \sqrt{u}\, S h}{\sqrt{Mn(n-1)}\, \sqrt{B\Delta x'}\, \Delta y'}$$

We will assume a circular orbit with $h = 675$ Km. The subsatellite point then has velocity

$$u = \frac{r_e}{r_e+h}\sqrt{\frac{GM}{r_e+h}} = 6.8 \text{ Km/s}$$

where $r_e = 6378$ Km (Earth radius) and $GM = 3.986 \times 10^5$ Km^3/s^2 (Newton's gravitational constant times the Earth's mass). This orbit was chosen so that global coverage could be provided with 911 Km swaths within 3 days (see Baseline Passive Microwave Radiometer for EOS, 1/9/85). At this altitude the swath angular extent is

$$S = 2\tan^{-1}\frac{911 \text{ Km}/2}{675 \text{Km}} = 68° = 1.19 \text{ radians}$$



The science requirements are for

$$\Delta T' = .5K \quad \Delta x' = \Delta y' = 10\,Km$$

with these requirements all parameters but the integers n and M have been specified. Solving the sensitivity equation for n and M gives

$$Mn(n-1) = \frac{u}{B\Delta x'}\left[\frac{T_{sys}\,S\,h}{\Delta T'\,\Delta y'}\right]^2$$

$$= \frac{(6.8\,Km\,s^{-1})}{(27\times10^6\,s^{-1})(10\,Km)}\left[\frac{(375\,K)(1.19\,rad)(675\,Km)}{(.5\,K)(10\,Km)}\right]^2$$

$$= 91.4$$

Solving for n in terms of M gives

$$n = \frac{1 \pm \sqrt{1 + 365.6/M}}{2}$$

n must be positive so we choose the upper sign. The following table gives n for various values of M. The case $M=1$ corresponds to a pure aperture synthesis instrument.

In addition the antenna width W and fill factor f are entered in the table. The other antenna dimensions L and D are independent of M and n and are set by the resolution requirements

$$L = .9h\lambda/\Delta x' = 12.8m$$

$$D = .9h\lambda/\Delta y' = 12.8m$$

where $\lambda = .21m$ is the wavelength. The antenna width W is set by the FOV size

$$W = .9\lambda M/S = (.16m)M$$

and the fill factor is defined as

$$f = \frac{nLW}{LD} = \frac{nW}{D}$$

We are constrained to have a fill factor less than unity.

M	n	W	f
1	10	.16 m	.13
2	7	.32 m	.18
3	6	.48 m	.23
4	5	.64 m	.27
5	5	.80 m	.30
6	4	.96 m	.33
7	4	1.12 m	.36
8	4	1.28 m	.39
9	4	1.44 m	.42
10	4	1.60 m	.45
15	3	2.40 m	.57
20	3	3.20 m	.67
25	2	4.00 m	.77
30	2	4.80 m	.87
35	2	5.60 m	.96
40	2	6.40 m	1.05
over 40			>1

As indicated in the table, the science requirements can readily be satisfied with many instrument configurations. Depending on economic and engineering considerations, an optimal choice for n and M can be selected. If antenna area is expensive relative to reciever unit costs and scanning complexity then the pure aperture synthesis system ($n = 10$, $M = 1$) would be most attractive. If, on the other hand, the opposite were true or if other constraints come into play (e.g., FOV restrictions due to processing constraints or beam quality constraints) then a hybrid system ($M > 1$) would be most attractive. In either case, it seems that there is no benifit to $M \gtrsim 5$ since increasing M does not effectively reduce n whereas f continues to grow essentially linearly with M.

As the previous analysis indicates, aperture synthesis seems to perform better than the real aperture system previously proposed (see <u>Baseline Passive Microwave Radiometer for EOS</u>, 1/9/85). Initially this seems contrary to our intuition since the amount of energy (and hence the sensitivity) is degraded by a fill factor less than unity. There are, however, several mitigating factors. These are:

1) The superior performance of correlation recievers versus Dicke recievers

2) There are effectively n radiometer working in parallel

3) There is a much longer period available for FOV integration time. Although the per pixel integration time in the same, it is the FOV integration time which appears in the sensitivity equation.

To complete this analysis the following issues should also be examined:

1) Synthetic beam quality

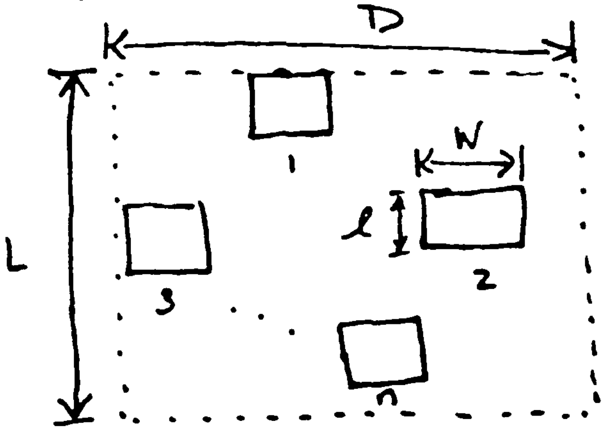
2) Array configurations (minimum redundcy, etc.)

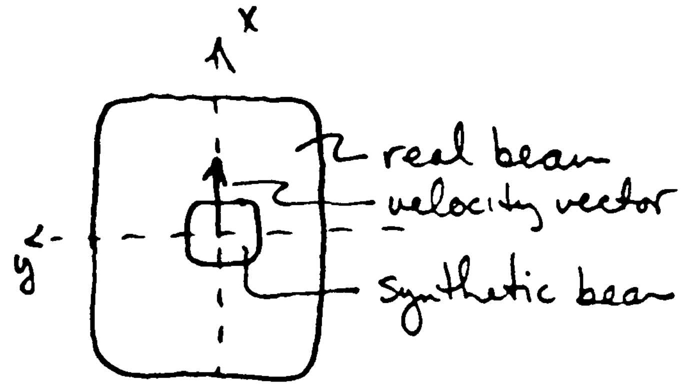
3) Processing requirements

4) FOV limitations imposed by processing requirements

Two-Dimensional Scanning/Aperture Synthesis Hybrid

As a generalization of the one-dimensional scanning/synthesis hybrid we consider an $L \times D$ array thinned in both directions as shown.





The array is composed of n $l \times W$ antennas. For $l = L$ the one-dimensionally thinned array is recovered. Following the previous work (<u>One-dimensional Scanning Synthesis Hybrid</u>, 2/11/85) we set

$$\beta_x = .9\lambda / l \qquad \beta_y = .9\lambda / W$$

$$\beta_x' = .9\lambda / L \qquad \beta_y' = .9\lambda / D$$

We form a swath of angular extent S from M FOV's by step scanning across-track. Then

$$S = M\beta_y$$

For a scan pattern with no over/underlap we require that a scan cycle be completed in the time t it requires for the subsatellite point to advance one along-track synthesized resolution length $\Delta x'$:

$$t = \frac{\Delta x'}{u} \qquad \Delta x' \cong h\beta_x'$$

We apportion t equally amoungst the M FOV's to give the integration time

$$\tau = \frac{t}{M} = \frac{\Delta x'}{Mu}$$

In addition, each synthetic resolution cell will be viewed N times before being scrolled out of view. N is given by the beam width ratio

$$N = \frac{\beta_x}{\beta_x'} = \frac{L}{l}$$

N is the number of synthetic pixels the FOV measures in the along-track direction.

Alternatively, the instrument could step scan across-track in the time required for the satellite to advance one FOV length while tracking the FOV along-track. We will designate this scheme B and the previous scheme A. Scheme B requires beam agility in both dimensions. For scheme B

$$t = \frac{N\Delta x'}{u}$$

$$\tau = \frac{t}{M} = \frac{N\Delta x'}{Mu}$$

and each synthetic resolution cell is viewed only once.

The synthetic aperture sensitivity equation is

$$\Delta T' = \frac{T_{sys} A_{SYN}}{\sqrt{n(n-1)}\, A_e \sqrt{B\tau}}$$

where $T_{sys} = 375\,K$, $A_{SYN} = LD$, $A_e = lW$, and $B = 27\,MHz$. However, with scheme A each synthetic pixel recieves N looks which we average to decrease the radiometric uncertainty by $1/\sqrt{N}$.

We then have

$$\Delta T_A' = \frac{1}{\sqrt{N}} \frac{T_{sys} L D}{\sqrt{n(n-1)}\, \ell W \sqrt{B \tau_A}} \qquad \tau_A = \frac{\Delta x'}{Mu}$$

$$\Delta T_B' = \frac{T_{sys} L D}{\sqrt{n(n-1)}\, \ell W \sqrt{B \tau_B}} \qquad \tau_B = \frac{N \Delta x'}{Mu}$$

However, since $\tau_B = N\tau_A$ we see that $\Delta T_A' = \Delta T_B'$ and so there is no advantage to scheme B. We, henceforth, consider only scheme A. Substituting for τ_A and N:

$$\Delta T' = \frac{T_{sys} D \sqrt{L M u}}{W \sqrt{n(n-1)\, \ell B \Delta x'}}$$

and substitute for one factor of $\sqrt{\frac{D}{W}}$

$$\sqrt{\frac{D}{W}} = \sqrt{\frac{\beta_y}{\beta_y'}} = \sqrt{\frac{S}{M\beta_y'}} \cong \sqrt{\frac{Sh}{M\Delta y'}} \qquad \Delta y' \cong h\beta_y'$$

then

$$\Delta T' = T_{sys} \sqrt{\frac{Sh}{M\Delta y'}} \sqrt{\frac{Mu}{(n-1)B\Delta x'}} \sqrt{\frac{LD}{n\ell W}}$$

The fill factor f is

$$f = \frac{n l W}{L D}$$

hence

$$\Delta T' = T_{sys} \frac{1}{\sqrt{f}} \sqrt{\frac{S h \nu}{(n-1) B \Delta x' \Delta y'}}$$

The parameters $\Delta x'$, $\Delta y'$ depend only on the array exterior dimensions L, D. S depends on M and W but we can vary W by holding S constant and varying M according to:

$$S = M \beta_y = \frac{.9 \lambda M}{W}$$

We then see that for constant fill factor, the sensitivity is independent of the antenna profiles (l to W ratio). There is then no advantage to two-dimensionally thinned arrays (low profile) over one-dimensionally thinned arrays (high profile) in terms of sensitivity.

Indeed the one-dimensionally thinned array requires only one-dimensional Fourier processing and needs the fewests FOV's (M) to cover a given S.

J. Giner
2/26/85

Aperture Synthesis Notes

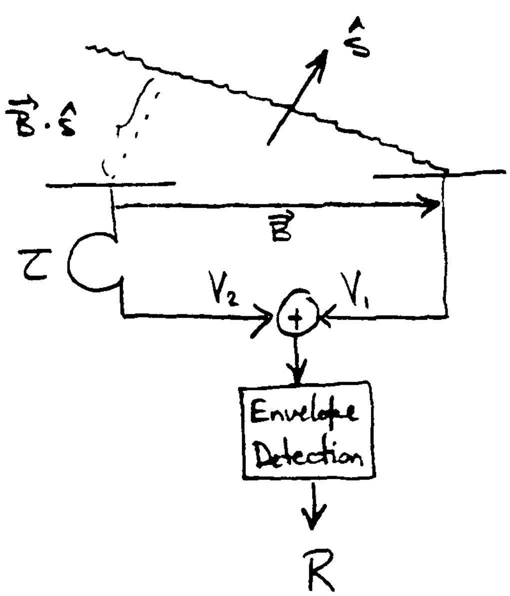
I. Adding Interferometer
   A. Response to Point Source
   B. Response to Extended Source

II. Correlating Interferometer
   A. Response to Point Source
      1. Source at infinite range
      2. Source at finite range
   B. Response to Extended Source
      1. Two point sources
      2. Extended source
      3. In-phase and Quadrature Channels
   C. Response to Polychromatic Source

III. Arrays
   A. Imaging
   B. Restricted Minimum Redundancy
      1. Synthetic beam
      2 Resolution
      3. Grating lobes
      4. FOV

I. Adding Interferometer

A. Response to point source



Signal from a monochromatic point source at infinity

$$V_1(t) = E\, e^{i\omega t}$$
$$V_2(t) = E\, e^{i\omega(t - \frac{\vec{B}\cdot\hat{s}}{c} - \tau)}$$

$$R(t) \sim |V_1(t) + V_2(t)|^2 = |E|^2 \left|1 + e^{-i\omega(\frac{\vec{B}\cdot\hat{s}}{c} + \tau)}\right|^2$$

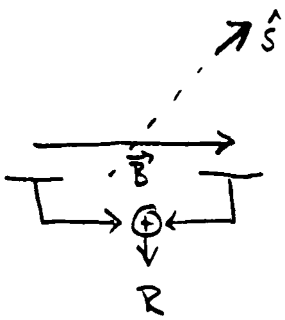
$$= 2|E|^2 \left(1 + \cos \omega\left(\frac{\vec{B}\cdot\hat{s}}{c} + \tau\right)\right)$$

$$\sim I A(\hat{s}) \left(1 + \cos \omega\left(\frac{\vec{B}\cdot\hat{s}}{c} + \tau\right)\right)$$

I = source intensity
A($\hat{s}$) = antenna power pattern

$$= I A(\hat{s}) \left(1 + \cos \frac{2\pi(\vec{B}\cdot\hat{s} + c\tau)}{\lambda}\right)$$

B. Response to Extended Source



1. Two monochromatic point sources at infinity radiating incoherently

$$V_1(t) = E_a e^{i\omega t} + E_b e^{i\omega t}$$
$$V_2(t) = E_a e^{i\omega(t - \frac{\vec{B}\cdot\hat{s}_a}{c} - \tau)} + E_b e^{i\omega(t - \frac{\vec{B}\cdot\hat{s}_b}{c} - \tau)}$$

$$R \sim |V_1 + V_2|^2 = |E_a e^{i\omega t} + E_a e^{i\omega(t - \frac{\vec{B}\cdot\hat{s}_a}{c} - \tau)}|^2$$
$$+ |E_b e^{i\omega t} + E_b e^{i\omega(t - \frac{\vec{B}\cdot\hat{s}_b}{c} - \tau)}|^2$$
$$+ \text{ a \& b cross terms}$$

The cross terms vanish under an ensemble average since they reflect they mutual coherence between a & b. R is then just the incoherent sum of point source responses

2. Extended source

Invoke incoherent summation approximation

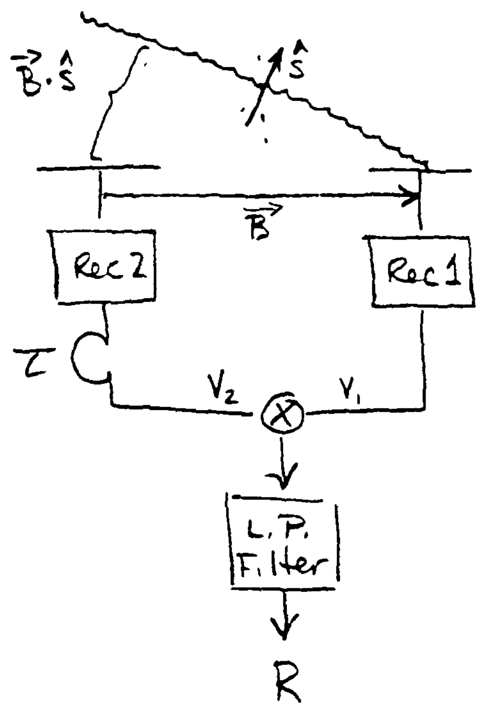
$$R \sim \int I(\hat{s}) A(\hat{s}) \left(1 + \cos \frac{2\pi(\vec{B}\cdot\hat{s} + c\tau)}{\lambda}\right) d\hat{s}$$

$I(\hat{s})d\Omega$ is source intensity within solid angle $d\Omega$

We see that by adjusting τ we can shift the fringes.

II. Correlating Interferometer

A. Response to Point Source



1. Signal from pt source at infinity radiating monochromatically

$$V_1(t) = E\cos\omega t$$

$$V_2(t) = E\cos\omega\left[t - \frac{\vec{B}\cdot\hat{s}}{c} - \tau\right]$$

$$R(t) \sim V_1(t)V_2(t) = E^2\cos\omega\left[\frac{\vec{B}\cdot\hat{s}}{c} + \tau\right] - E^2\sin\omega t\,\sin\omega\left[t - \frac{\vec{B}\cdot\hat{s}}{c} - \tau\right]$$

Rejected by filter

$$\sim E^2\cos\omega\left[\frac{\vec{B}\cdot\hat{s}}{c} + \tau\right]$$

$$\sim \quad I\,A(\hat{s})\cos\frac{2\pi(\vec{B}\cdot\hat{s} + c\tau)}{\lambda}$$

I = intensity of source
A = antenna power pattern

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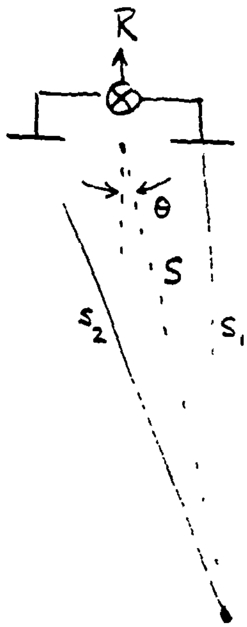
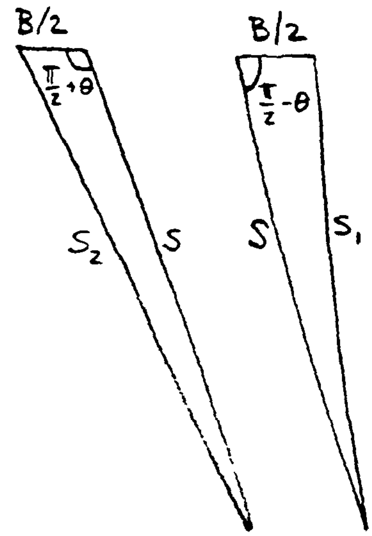
2. Response to a point source at finite range





Law of cosines gives

$$s_1^2 = s^2 + B^2/4 - Bs\cos\left(\tfrac{\pi}{2}-\theta\right) = s^2 + B^2/4 - sB\sin\theta$$

$$s_2^2 = s^2 + B^2/4 - Bs\cos\left(\tfrac{\pi}{2}+\theta\right) = s^2 + B^2/4 + sB\sin\theta$$

$$s_2 - s_1 = \sqrt{s^2 + B^2/4 + sB\sin\theta} - \sqrt{s^2 + B^2/4 - sB\sin\theta}$$

$$= s\left\{\sqrt{1 + B^2/4s^2 + B/s\,\sin\theta} - \sqrt{1 + B^2/4s^2 - B/s\,\sin\theta}\right\}$$

Use the expansion $\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \ldots$

$$s_2 - s_1 = s\left\{1 + \tfrac{1}{2}x_1 - \tfrac{1}{8}x_1^2 + \tfrac{1}{16}x_1^3 + \ldots\right.$$

$$\left. - 1 - \tfrac{1}{2}x_2 + \tfrac{1}{8}x_2^2 - \tfrac{1}{16}x_2^3 + \ldots\right\}$$

$x_1 = B^2/4s^2 + B/s\,\sin\theta$

$x_2 = B^2/4s^2 - B/s\,\sin\theta$

$$x_1 - x_2 = \frac{2B}{s}\sin\theta$$

$$x_1^2 - x_2^2 = (x_1+x_2)(x_1-x_2) = \frac{B^2}{2s^2}\left(\frac{2B}{s}\sin\theta\right) = \frac{B^3}{s^3}\sin\theta$$

$$x_1^3 - x_2^3 = (x_1-x_2)(x_1^2+x_1x_2+x_2^2) = (x_1-x_2)\left((x_1+x_2)^2 - x_1x_2\right)$$

$$= \left(\frac{2B}{s}\sin\theta\right)\left(\frac{B^4}{4s^4} - \left(\frac{B^4}{16s^4} - \frac{B^2}{s^2}\sin^2\theta\right)\right)$$

$$= \left(\frac{2B}{s}\sin\theta\right)\left(\frac{B^2}{s^2}\sin^2\theta + \frac{3B^4}{16s^4}\right)$$

We then have

$$s_2 - s_1 = B\sin\theta - \frac{1}{8}\frac{B^3}{s^2}\sin\theta + \frac{B}{8}\sin\theta\left(\frac{B^2}{s^2}\sin^2\theta + \frac{3B^4}{16s^4}\right) + \cdots$$

$$= B\sin\theta\left\{1 + \frac{1}{8}\frac{B^2}{s^2}(\sin^2\theta - 1) + \frac{3}{128}\frac{B^4}{s^4} + \cdots\right\}$$

$$= B\sin\theta\left\{1 - \frac{1}{8}\frac{B^2}{s^2}\cos^2\theta \qquad + \mathcal{O}(s^{-4})\right\}$$

we drop the $\mathcal{O}(s^{-4})$ term since the next term is of $\mathcal{O}(x^4)$ which is of $\mathcal{O}(s^{-4})$ also.

The phase difference between the two signals is then

$$\phi = \frac{2\pi}{\lambda}(s_2 - s_1) = \qquad \phi_\infty\left\{1 - \frac{1}{8}\frac{B^2}{s^2}\cos^2\theta \qquad + \mathcal{O}(s^{-4})\right\}$$

where $\phi_\infty = \dfrac{2\pi B\sin\theta}{\lambda}$ $\quad \left(\lim_{s\to\infty}\phi = \phi_\infty\right)$

The phase error introduced by the infinite range assumption is to lowest order

$$\frac{\Delta\phi}{\phi_\infty} = \frac{1}{8}\frac{B^2}{s^2}\cos^2\theta \qquad \le \frac{1}{8}\frac{B^2}{s^2}$$

$|\phi_\infty| = \left|\frac{2\pi B \sin\theta}{\lambda}\right| \le \frac{2\pi B}{\lambda}$ so we have a bound on the phase error

$$|\Delta\phi| \le \frac{1}{8}\frac{B^2}{s^2}\frac{2\pi B}{\lambda} = \frac{\pi}{4}\frac{B^3}{s^2\lambda}$$

For earth remote sensing we take

$B = 20\,m \qquad \lambda = .21\,m \qquad s = 500\,Km = 5\times10^5\,m$

then $|\Delta\phi| \le 1\times10^{-7}\,rad = 7\times10^{-6}$ degrees.
This should be compared with the differential phase change corresponding to a resolution cell $d\theta$

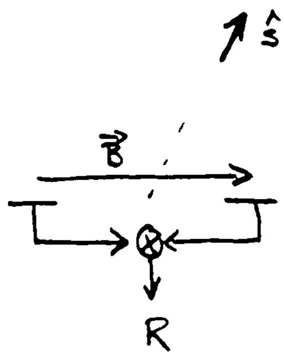
$$d\phi = \frac{2\pi B\cos\theta}{\lambda}\,d\theta$$

For an intended resolution of 10 Km at 500 Km slant range $d\theta = 10\,Km/500\,Km = .02$ radians. Then

$$d\phi = (12 \text{ radians})\cos\theta$$

which suggests that the infinite range approximation is benign in this application.

B. Response to Extended Source



Extended source radiating monochromatically and incoherently at infinity

1. Consider two pt. sources a & b

$$V_1(t) = E_a e^{i\omega t} + E_b e^{i\omega t}$$

$$V_2(t) = E_a e^{i\omega(t - \frac{\vec{B}\cdot\hat{s}_a}{c} - \tau)} + E_b e^{i\omega(t - \frac{\vec{B}\cdot\hat{s}_b}{c} - \tau)}$$

$$R \sim \mathrm{Re}V_1 \, \mathrm{Re}V_2 = \mathrm{Re}(E_a e^{i\omega t})\,\mathrm{Re}(E_a e^{i\omega(t - \frac{\vec{B}\cdot\hat{s}_a}{c} - \tau)}) + \mathrm{Re}(E_b e^{i\omega t})\,\mathrm{Re}(E_b e^{i\omega(t - \frac{\vec{B}\cdot\hat{s}_b}{c} - \tau)})$$

+ a and b cross terms.

Under an ensemble average the cross terms reflect the mutual coherence between a & b. They, therefore, are assumed to vanish and R is just the incoherent sum of pt. source responses

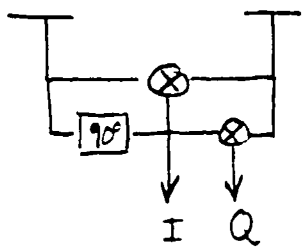
2. For an extended source.

Invoke incoherent summation assumption

$$R \sim \int I(\hat{s})\, A(\hat{s}) \cos \frac{2\pi(\vec{B}\cdot\hat{s} + c\tau)}{\lambda} \, d\Omega$$

$I(\hat{s})\,d\Omega$ is source intensity within solid angle $d\Omega$ about $\hat{s}$.

3. In-phase and Quadrature Channels



90° shift accomplished with delay of $\tau = \lambda/4c = 1/4\nu$

$$R_I \sim \int I(\hat{s})\,A(\hat{s})\cos\frac{2\pi(\vec{B}\cdot\hat{s}+c\tau)}{\lambda}\,d\Omega$$

$$R_Q \sim \int I(\hat{s})\,A(\hat{s})\sin\frac{2\pi(\vec{B}\cdot\hat{s}+c\tau)}{\lambda}\,d\Omega$$

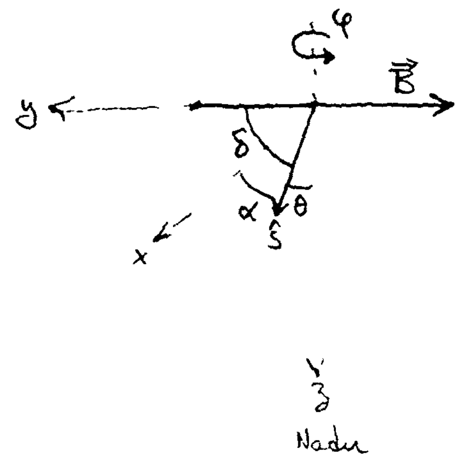
Define the complex response $R \equiv R_I + iR_Q$

$$R \sim \int I(\hat{s})\,A(\hat{s})\,e^{\frac{2\pi i(\vec{B}\cdot\hat{s}+c\tau)}{\lambda}}\,d\Omega$$

Delay τ only changes phase of R and so no beam steering is possible. We set $\tau = 0$

$$R \sim \int I(\hat{s})\,A(\hat{s})\,e^{2\pi i\vec{B}\cdot\hat{s}/\lambda}\,d\Omega$$

Define the coordinate system



$$\frac{1}{\lambda}\vec{B} = u\hat{x} + v\hat{y} \qquad \hat{s} = \cos\alpha\,\hat{x} + \cos\delta\,\hat{y} + \cos\theta\,\hat{z}$$

$$= \cos\alpha\,\hat{x} + \cos\delta\,\hat{y} \pm \sqrt{1-\cos^2\alpha-\cos^2\delta}\;\hat{z}$$

$$= \sin\theta\cos\varphi\,\hat{x} + \sin\theta\sin\varphi\,\hat{y} + \cos\theta\,\hat{z}$$

$$\frac{1}{\lambda}\vec{B}\cdot\hat{s} = u\cos\alpha + v\cos\delta \qquad \cos\alpha = \sin\theta\cos\varphi \qquad \cos\delta = \sin\theta\sin\varphi$$

$$|\cos\theta| = \sqrt{1-\cos^2\alpha-\cos^2\delta}$$

$$\frac{\partial\cos\alpha}{\partial\theta} = \cos\theta\cos\varphi \qquad \frac{\partial\cos\alpha}{\partial\varphi} = -\sin\theta\sin\varphi$$

$$\frac{\partial\cos\delta}{\partial\theta} = \cos\theta\sin\varphi \qquad \frac{\partial\cos\delta}{\partial\varphi} = \sin\theta\cos\varphi$$

$$\Rightarrow \left|\frac{\partial(\cos\alpha,\cos\delta)}{\partial(\theta,\varphi)}\right| = |\sin\theta\cos\theta\cos^2\varphi + \sin\theta\cos\theta\sin^2\varphi|$$

$$= |\sin\theta\cos\theta|$$

$$\sin\theta\, d(\cos\alpha)\, d(\cos\delta) = \sin\theta\left|\frac{\partial(\cos\alpha,\cos\delta)}{\partial(\theta,\varphi)}\right| d\theta\, d\varphi = \left|\frac{\partial(\cos\alpha,\cos\delta)}{\partial(\theta,\varphi)}\right| d\Omega$$

$$\Rightarrow d\Omega = \frac{\sin\theta}{|\sin\theta\cos\theta|}\, d(\cos\alpha)\, d(\cos\delta)$$

$$= \frac{1}{\sqrt{1-\cos^2\alpha-\cos^2\delta}}\, d(\cos\alpha)\, d(\cos\delta)$$

In general the coordinates α and δ do not uniquely identify $\hat{s}$ - rather α and δ select two $\hat{s}$ vectors, one in the upper and one in the lower hemisphere. However, we will assume that the entire contribution to R is from the lower hemisphere and so

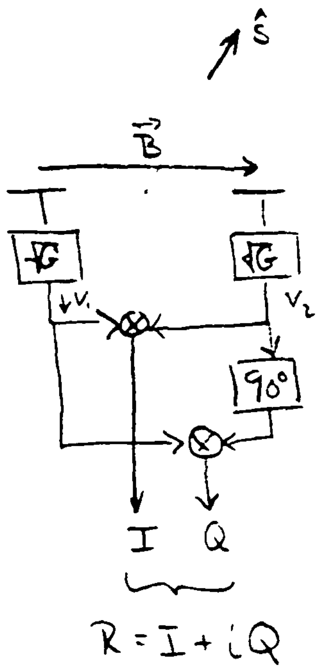
$$R \sim \iint_{x^2+y^2<1} \frac{I(x,y)\,A(x,y)}{\sqrt{1-x^2-y^2}}\, e^{2\pi i(xu+yv)}\, dx\,dy$$

where we have defined x and y to be the direction cosines $x = \cos\alpha$, $y = \cos\delta$. In addition we will define $A(x,y)$ such that it vanishes outside the physical region $x^2+y^2<1$. Hence the integration is extended over the entire xy-plane and we find $R(u,v)$ and $I(x,y)A(x,y)/\sqrt{1-x^2-y^2}$ to be Fourier transform pairs. Absorbing the constant of proportionality:

$$R(u,v) = \iint \frac{I(x,y)A(x,y)}{\sqrt{1-x^2-y^2}}\, e^{2\pi i(xu+yv)}\, dx\,dy$$

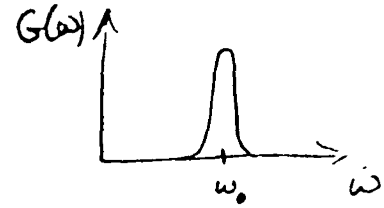
$$\frac{I(x,y)A(x,y)}{\sqrt{1-x^2-y^2}} = \iint R(u,v)\, e^{-2\pi i(xu+yv)}\, du\,dv$$

Response to Polychromatic Extended Source



1. General Case

$\sqrt{G(\omega)}$ is reciever transfer function



Consider a set of mutually incoherent point sources at infinity, each radiating monochromatically

$$E_n e^{i\omega_n t} \qquad \hat{s}_n$$

$$V_1 = \sum E_n e^{i\omega_n t} \qquad V_2 = \sum E_n e^{i\omega_n (t - \frac{\vec{B}\cdot\hat{s}_n}{c})}$$

$$I \sim Re(V_1)\, Re(V_2) = \sum Re(E_n e^{i\omega_n t})\, Re(E_n e^{i\omega_n (t - \vec{B}\cdot\hat{s}_n/c)})$$

+ cross terms
(vanish by mutual incoherence)

$$\xrightarrow[\text{continuous limit}]{} \iiint G(\omega)\, I(\hat{s},\omega)\, A(\hat{s},\omega) \cos(\omega \vec{B}\cdot\hat{s})\, d\omega\, d\Omega$$

and similarly for Q Hence

$$R \sim \iiint G(\omega)\, I(\hat{s},\omega)\, A(\hat{s},\omega)\, e^{i\omega\vec{B}\cdot\hat{s}}\, d\omega\, d\Omega$$

2 Narrow Passband

Assume that the passband is narrow enough so that the below approximation holds.

$$R \simeq \iint I(\hat{s},\omega_0)\, A(\hat{s},\omega_0) \int G(\omega)\, e^{i\omega \vec{B}\cdot\hat{s}}\, d\omega\, d\Omega$$

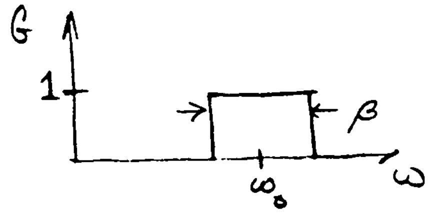
$$= \iint I(\hat{s},\omega_0)\, A(\hat{s},\omega_0)\, e^{i\omega_0 \vec{B}\cdot\hat{s}} \underbrace{\int G(\omega+\omega_0)\, e^{i\omega B\cdot\hat{s}}\, d\omega}_{=W(\hat{s})}\, d\Omega$$

$$= \iint I(\hat{s},\omega_0)\, A(\hat{s},\omega_0)\, W(\hat{s})\, e^{i\omega_0 \vec{B}\cdot\hat{s}}\, d\Omega$$

$$= \iint I(x,y,\omega_0)\, A(x,y,\omega_0)\, W(x,y)\, \frac{1}{\sqrt{1-x^2-y^2}}\, e^{2\pi i(ux+vy)}\, dx\, dy$$

$W(\hat{s})$ is the "fringe washing" function.

For a rectangular passband



$$W(x,y) = \int_{-\beta/2}^{+\beta/2} e^{i\omega\vec{B}\cdot\hat{s}}\, d\omega = \frac{1}{i\vec{B}\cdot\hat{s}}\, e^{i\omega\vec{B}\cdot\hat{s}} \Big|_{-\beta/2}^{+\beta/2}$$

$$= \frac{2c}{\vec{B}\cdot\hat{s}} \sin \frac{\beta}{2}\vec{B}\cdot\hat{s}/c = \frac{2\omega_0}{2\pi(ux+vy)} \sin 2\pi(ux+vy)\frac{\beta}{2\omega_0}.$$

F. Arrays

A Imaging

Each pair of antennas samples $R(u,v)$ at a particular point in the uv-plane. For n antennas there are $m \le n(n-1)/2$ unique baselines (u_i, v_i), $i = 1, \ldots, m$. In addition $W(x,y) I(x,y) A(x,y)$ must be real so $R(-u,-v) = R^*(u,v)$ and we see that we are measuring $R(u,v)$ at (u_i, v_i) and $(-u_i, -v_i)$, $i = 1, \ldots, m$. We take as an estimate of $I(x,y)$:

$$W(x,y) \frac{\hat{I}(x,y) A(x,y)}{\sqrt{1-x^2-y^2}} = \iint S(u,v) R(u,v) e^{-2\pi i(xu-yv)} du\, dv$$

where $S(u,v) = \sum_{i=1}^{m} \{ \delta(u-u_i)\delta(v-v_i) + \delta(u+u_i)\delta(v+v_i) \}$ is the sampling function. Explicitly

$$W(x,y) \frac{\hat{I}(x,y) A(x,y)}{\sqrt{1-x^2-y^2}} = \sum_{\substack{(u_i, v_i) \\ (-u_i, -v_i)}} R(u,v) e^{2\pi i(xu+yv)}$$

Alternatively the convolution theorem gives

$$W(x,y) \frac{\hat{I}(x,y) A(x,y)}{\sqrt{1-x^2-y^2}} = (s * r)(x,y)$$

where

$$A(x,y) = \iint S(u,v)\, e^{-2\pi i(xu+yv)}\, du\, dv$$

$$= \sum_{i=1}^{m} \left\{ e^{-2\pi i(xu_i+yv_i)} + e^{+2\pi i(xu_i+yv_i)} \right\}$$

$$r(x,y) = \iint R(u,v)\, e^{-2\pi i(xu+yv)}\, du\, dv$$

$$= \frac{I(x,y)\, A(x,y)}{\sqrt{1-x^2-y^2}} W(x,y)$$

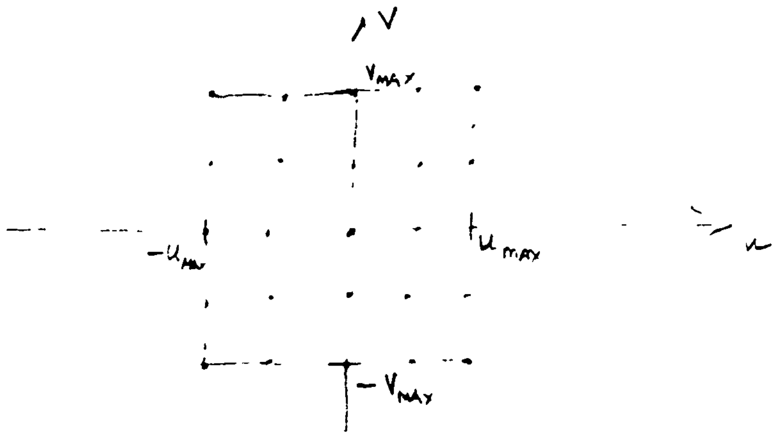
Then

$$\frac{\hat{I}(x,y)\, A(x,y)\, W(x,y)}{\sqrt{1-x^2-y^2}} = \iint A(x-x', y-y')\, \frac{I(x',y')\, A(x',y')\, W(x',y')}{\sqrt{1-x'^2-y'^2}}\, dx'\, dy'$$

We now see that $A(x,y)$ may be thought of as the synthetic beam which we convolve with the scene weighted by $W(x,y) A(x,y) / \sqrt{1-x^2-y^2}$ to get the image weighted by the same function.

B. Restricted Minimum Redundancy Arrays

Restricted minimum redundancy arrays uniformly sample the Fourier plane with a minimum number of antennas



$u_{max} = L/\lambda \qquad v_{max} = D/\lambda$ L×D are array exterior dimensions as defined by antenna center-to-center distances

Sampling is at $(u,v) = (m_x u_0, m_y v_0)$ for

$$-M_x \le m_x \le M_x \qquad -M_y \le m_y \le M_y$$

We include (0,0) by taking the autocorrellation of one antenna output (or averaging many)

$$M_x u_0 = u_{max} = L/\lambda$$
$$M_y v_0 = v_{max} = D/\lambda$$

1. Synthetic beam

The unweighted sampling function is

$$S(u,v) = \sum_{(u_i,v_i)} \delta(u-u_i)\delta(v-v_i)$$

where the summation is over all (u_i, v_i) in the sampling domain. This gives a synthetic beam

$$s(x,y) = \sum_{m_x=-M_x}^{+M_x} \sum_{m_y=-M_y}^{+M_y} e^{-2\pi i(m_x x u_0 + m_y y v_0)}$$

$$= \sum_{m_x=-M_x}^{+M_x} \sum_{m_y=-M_y}^{+M_y} \alpha^{m_x} \beta^{m_y} \qquad \alpha = e^{-2\pi i x u_0}, \quad \beta = e^{-2\pi i y v_0}$$

$$= \left(\sum_{m_x=-M_x}^{M_x} \alpha^{m_x}\right)\left(\sum_{m_y=-M_y}^{+M_y} \beta^{m_y}\right)$$

$$= \alpha^{-M_x}\beta^{-M_y}\left(\sum_{m_x=0}^{2M_x} \alpha^{m_x}\right)\left(\sum_{m_y=0}^{2M_y} \beta^{m_y}\right)$$

Use $\sum_{n=0}^{N} z^n = (1-z^{N+1})/(1-z)$

$$s(x,y) = \alpha^{-M_x}\beta^{-M_y} \frac{1-\alpha^{2M_x+1}}{1-\alpha} \frac{1-\beta^{2M_y+1}}{1-\beta}$$

$$= \frac{\alpha^{-M_x} - \alpha^{M_x+1}}{1-\alpha} \frac{\beta^{-M_y} - \beta^{M_y+1}}{1-\beta}$$

$$\mathcal{S}(x,y) = \frac{\alpha^{-(M_x+1/2)} - \alpha^{+(M_x+1/2)}}{\alpha^{-1/2} - \alpha^{1/2}} \; \frac{\beta^{-(M_y+1/2)} - \beta^{+(M_y+1/2)}}{\beta^{-1/2} - \beta^{1/2}}$$

$$= \frac{e^{2\pi i (M_x+1/2) x u_0} - e^{-2\pi i (M_x+1/2) x u_0}}{e^{\pi i x u_0} - e^{-\pi i x u_0}} \cdot \;\ldots\ldots$$

$$= \frac{\sin 2\pi (M_x+1/2) x u_0}{\sin \pi x u_0} \cdot \frac{\sin 2\pi (M_y+1/2) y v_0}{\sin \pi y v_0}$$

2. Resolution

The peak value of $\mathcal{S}(x,y)$ occurs at $x=y=0$

$$\mathcal{S}(0,0) = (2M_x+1)(2M_y+1)$$

The synthetic beam width will be defined to be the x and y widths at half max. We seek $x_{1/2}, y_{1/2}$ such that

$$\mathcal{S}(x_{1/2},0) = \tfrac{1}{2}\,\mathcal{S}(0,0)$$

$$\mathcal{S}(y_{1/2},0) = \tfrac{1}{2}\,\mathcal{S}(0,0)$$

$$A(x_{1/2},0) = \frac{\sin 2\pi (M_x+1/2) x_{1/2} u_0}{\sin \pi x_{1/2} u_0} \cdot (2M_y+1)$$

$$= \frac{1}{2}(2M_x+1)(2M_y+1)$$

Hence

$$\frac{\sin 2\pi (M_x+1/2) x_{1/2} u_0}{\sin \pi x_{1/2} u_0} = M_x + 1/2$$

We expand the sine through third order.

$$2\pi (M_x+1/2) x_{1/2} u_0 - \frac{1}{6}\left[2\pi (M_x+1/2) x_{1/2} u_0\right]^3 + \cdots$$

$$= \left[M_x + 1/2\right]\left\{\pi x_{1/2} u_0 - \frac{1}{6}\left[\pi x_{1/2} u_0\right]^3 + \cdots\right\}$$

Then

$$\pi x_{1/2} u_0 - \frac{1}{6}\left[M_x+1/2\right]^2\left[2\pi x_{1/2} u_0\right]^3 + \frac{1}{6}\left[\pi x_{1/2} u_0\right]^3 \cong 0$$

Or

$$\left\{\frac{4}{3}\left[M_x+1/2\right]^2 - \frac{1}{6}\right\}(\pi x_{1/2} u_0)^2 \cong 1$$

$$\cong \frac{4}{3} M_x^2 \pi^2 u_0^2 x_{1/2}^2$$

We find

$$x_{1/2} \cong \pm \frac{\sqrt{3}}{2} (\pi M_x u_0)^{-1}$$

or the full width

$$\Delta x \cong \sqrt{3} (\pi M_x u_0)^{-1} = \frac{\sqrt{3}}{\pi} \frac{\lambda}{L} = 1.1 \frac{\lambda}{L}$$

Similarly

$$\Delta y \cong 1.1 \frac{\lambda}{D}$$

3. Grating lobes

whenever $x u_0$ is an integer the factor

$$\left| \frac{\sin \pi (2M_x + 1) x u_0}{\sin \pi x u_0} \right|$$

is maximized and similarly for $y v_0$. Grating lobes then occur at $x = \text{integer}/u_0$ and $y = \text{integer}/v_0$. As long as $u_0 < 1$ and $v_0 < 1$ grating lobes occur in the unphysical $x > 1$, $y > 1$ region. Since $u_0 = L/\lambda M_x$ and $v_0 = D/\lambda M_y$ the conditions for no grating lobes are

$$D < \lambda M_y$$

$$L < \lambda M_x$$

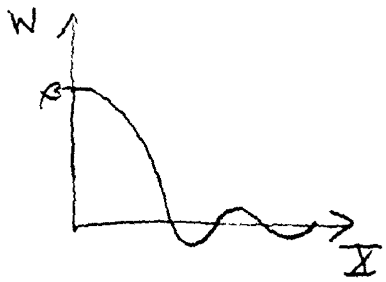
4. FOV

a) bandwidth limited

For a rectangular passband

$$W(x,y) = \frac{2\omega_0}{2\pi(ux+vy)} \sin 2\pi(ux+vy)\frac{\rho}{2\omega_0}$$

$$= \frac{1}{\bar{X}} \sin \rho \bar{X} \qquad \bar{X} = \frac{2\pi(ux+vy)}{2\omega_0}$$



$$\frac{\rho}{2} = \frac{1}{\bar{X}_{1/2}} \sin \rho \bar{X}_{1/2}$$

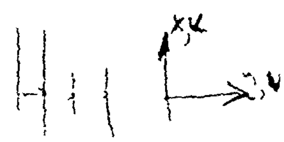
$$= \frac{1}{\bar{X}_{1/2}} \left\{ \rho \bar{X}_{1/2} - \frac{1}{6}\rho^3 \bar{X}_{1/2}^3 + \cdots \right.$$

$$\frac{1}{2}(\rho \bar{X}_{1/2}) \cong \rho \bar{X}_{1/2} - 1/6 (\rho X_{1/2})^3$$

$$\frac{1}{6}(\rho \bar{X}_{1/2})^3 - \frac{1}{2}\rho \bar{X}_{1/2} \cong 0$$

$$(\rho \bar{X}_{1/2})^2 \cong 3$$

$$\bar{X}_{1/2} \cong \pm \frac{\sqrt{3}}{\rho}$$



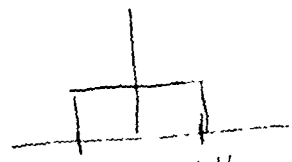
x=0 cut (along-scan)

$$\frac{\pi v}{\omega_0} \sin \theta_{1/2} = \frac{\sqrt{3}}{\rho} \qquad \frac{\sqrt{3}}{2\pi B} \frac{2\pi f_0}{\pi b/\lambda} = \frac{\sqrt{3}}{\pi} \frac{\lambda f_0}{bB} = \frac{\sqrt{3}}{\pi} \frac{c}{bB} = \sin \theta_{1/2}$$

$$= .55 \frac{c}{bB}$$

b) Uniform aperture (1-d)

$$E(\sin\phi) = \int_{-W/2\lambda}^{+W/2\lambda} e^{i2\pi x \sin\phi}\, dx = \frac{1}{i2\pi\sin\phi} e^{i2\pi x \sin\phi}\Big|_{-W/2\lambda}^{+W/2\lambda}$$



$$= \frac{1}{\pi\sin\phi} \sin\left(\frac{\pi W}{\lambda}\sin\phi\right)$$

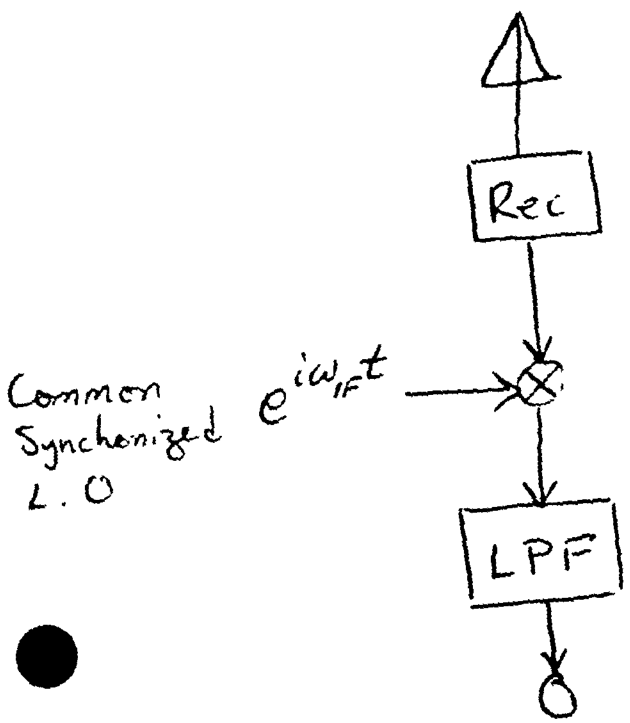
Power pattern (normalized)

$$P = \left(\frac{\lambda}{\pi W \sin\phi}\right)^2 \sin^2\left(\frac{\pi W}{\lambda}\sin\phi\right)$$

$\sin\phi = \frac{.45\lambda}{W}$ is h.p. point

JALOG PROCESSING

There are n antenna/receiver units



$$\longleftarrow \int_0^\infty A(\omega) e^{i\omega t}\, d\omega$$

$$\longleftarrow G\int_{\omega_0-\pi B}^{\omega_0+\pi B} A(\omega) e^{i\omega t}\, d\omega$$

$$\longleftarrow \frac{1}{2} G\int_{\omega_0-\pi B}^{\omega_0+\pi B} A(\omega) \left\{ e^{i(\omega-\omega_{IF})t} + e^{i(\omega+\omega_{IF})t} \right\} d\omega$$

$$\longleftarrow \frac{1}{2} G\int_{\omega_0-\pi B}^{\omega_0+\pi B} A(\omega) e^{i(\omega-\omega_{IF})t}\, d\omega$$

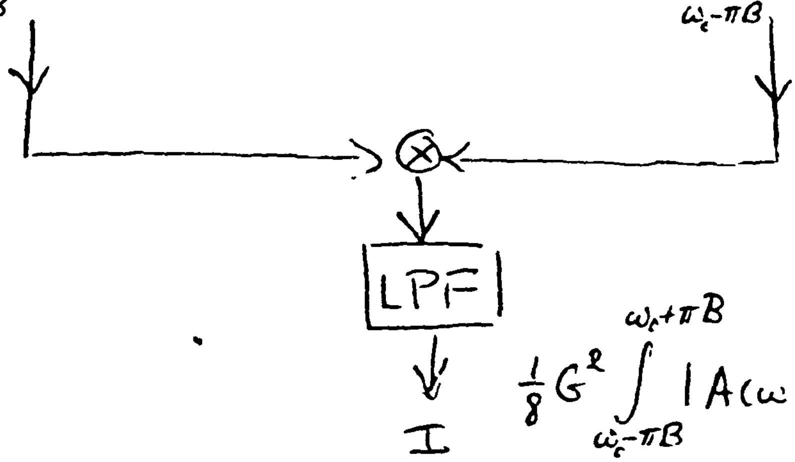
$\omega_0 = 2\pi f_0$ $\quad$ $f_0 = 1413.5$ MHz $\quad$ $B = 27$ MHz

G, ω_{IF} are TBD

● There are N in-phase correlators and N quadrature correlators. Each unit attaches to a pair of antenna/receiver units. Ideally the input signals are identicle except that one has a factor $e^{-i\omega\tau}$ associated with its spectrum where τ is the signal delay at the antenna.

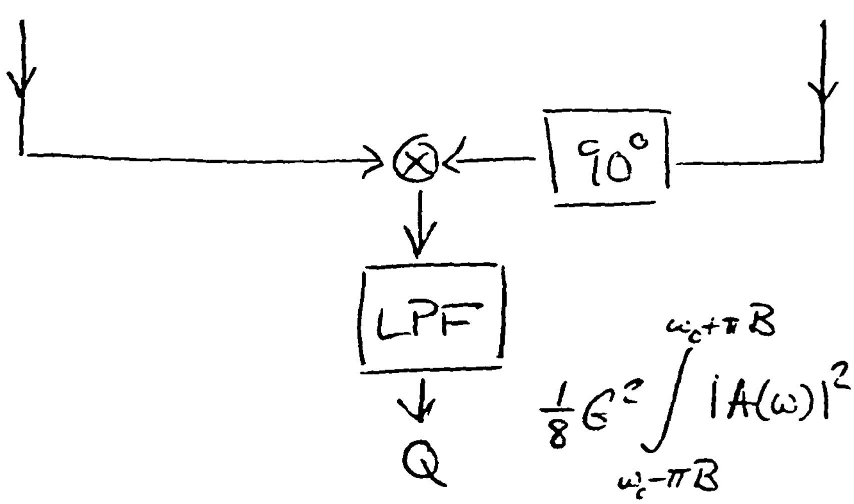
In-phase

$$\frac{1}{2}G\int_{\omega_c-\pi B}^{\omega_c+\pi B} A(\omega)e^{-i\omega\tau}e^{i(\omega-\omega_{IF})t} \qquad \frac{1}{2}G\int_{\omega_c-\pi B}^{\omega_c+\pi B} A(\omega)e^{i(\omega-\omega_{IF})t}$$



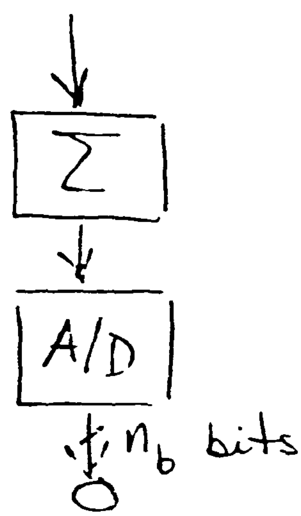
$$\frac{1}{8}G^2\int_{\omega_c-\pi B}^{\omega_c+\pi B} |A(\omega)|^2 e^{i\omega\tau}\, d\omega$$

Quadrature



$$\frac{1}{8}G^2\int_{\omega_c-\pi B}^{\omega_c+\pi B} |A(\omega)|^2 e^{i(\omega\tau-\pi/2)}\, d\omega$$

The output of each I & Q - correlator will be averaged (integrate and dump) over a period $\tau = 1.5$ sec, and then digitized. There will be 2N such units



The antennas will occupy a 20m x 20m area with the receivers colocated with their respective antennas. The correlators may or may not be centrally located.

Issues :

1) How does the signal from the antenna/receiver units get transported to the correlators to minimize losses and phase distortion?

2) What will be the noise temperatures associated with the receivers and mixers?

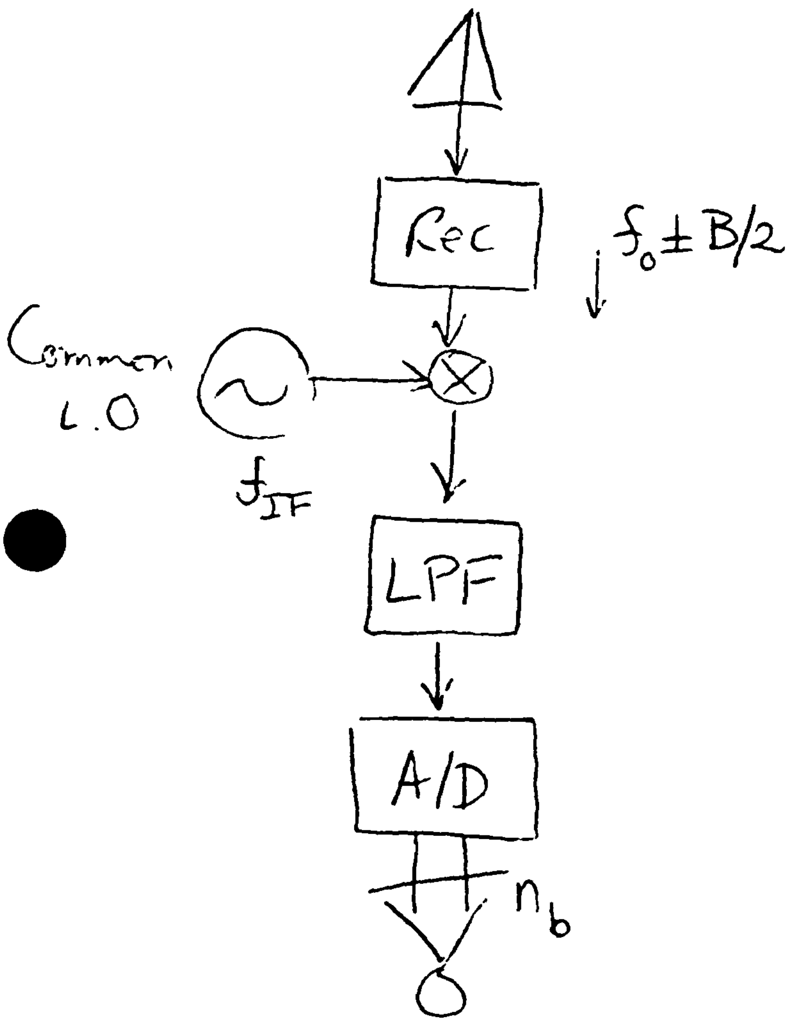
3) What are the characteristics of real phase shifters, mixers, LPF's, etc? (e.g., are there phase shifters that will uniformly shift 90° over the band $\omega - \omega_{IF} \pm \pi B$?)

4) How much power will this system require and how much heat will be generated? Must any of the devices be kept at controlled temperatures?

5) What will be the total output serial data rate?

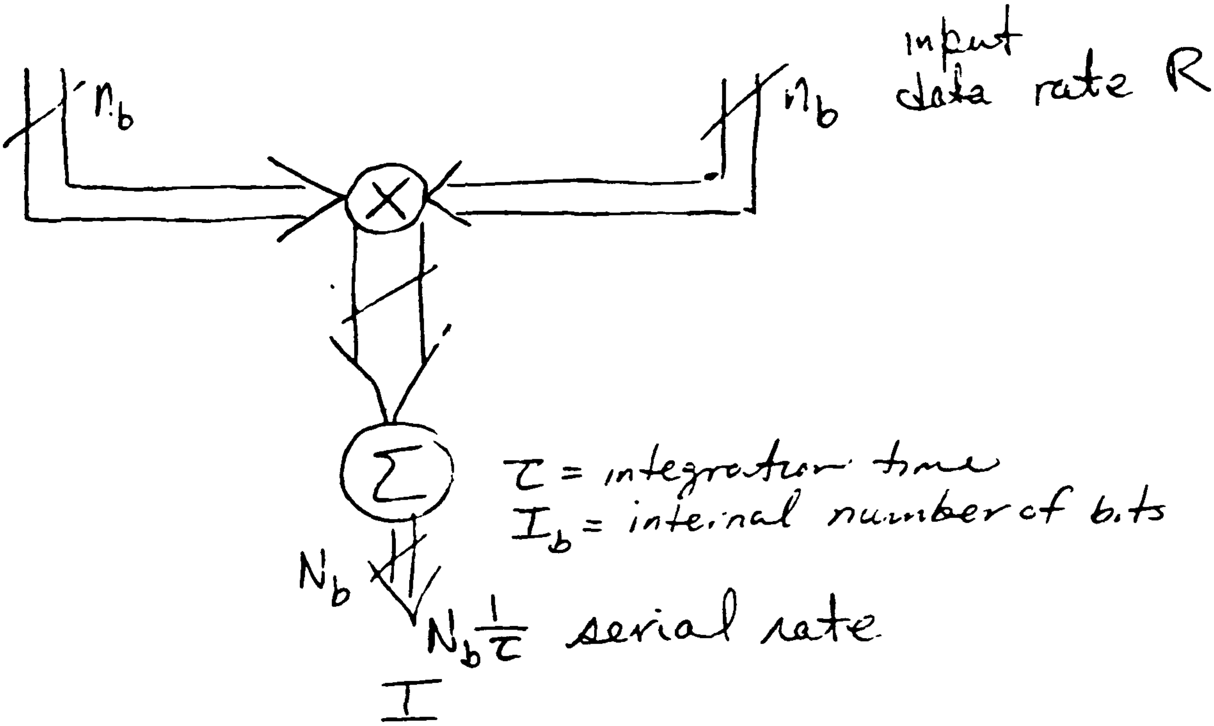
DIGITAL PROCESSING - Architecture I

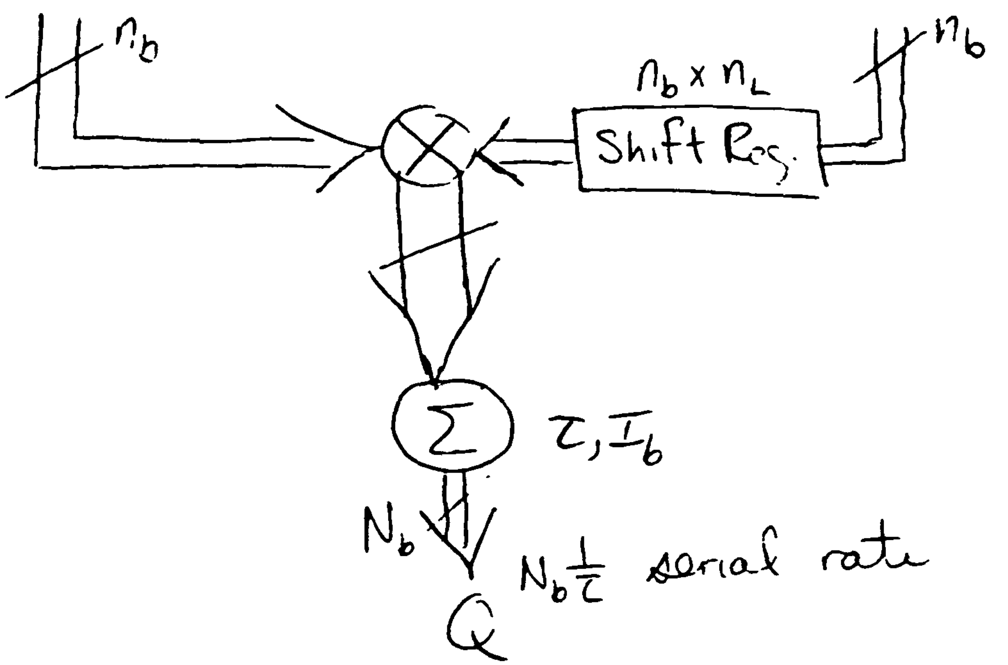
n Antenna/Receiver units



n_b bits at rate R

Inphase and N Quadrature Correlators





A/D output range is $\pm 2^{n_b - 1}$

Correlator output range is $\pm 2^{2n_b - 2}$

Necessary summer range is $\pm 2^{2n_b - 2} R\tau$ which requires $2n_b - 1 + \log_2 R\tau$ (rounded up) internal bits I_b

Signal into shift register represents a $f_0 - f_{IF} \pm B/2$ signal sampled at a rate R. To shift center frequency $90°$ in phase requires a delay of $\tau_S = (4m+1)/(4(f_0 - f_{IF}))$ where m is an integer. The phase shift off-center is

$$\phi = 2\pi f \tau_S = \frac{\pi}{2}(4m+1)\frac{f}{f_0 - f_{IF}}$$

At the high and low ends of the bandpass

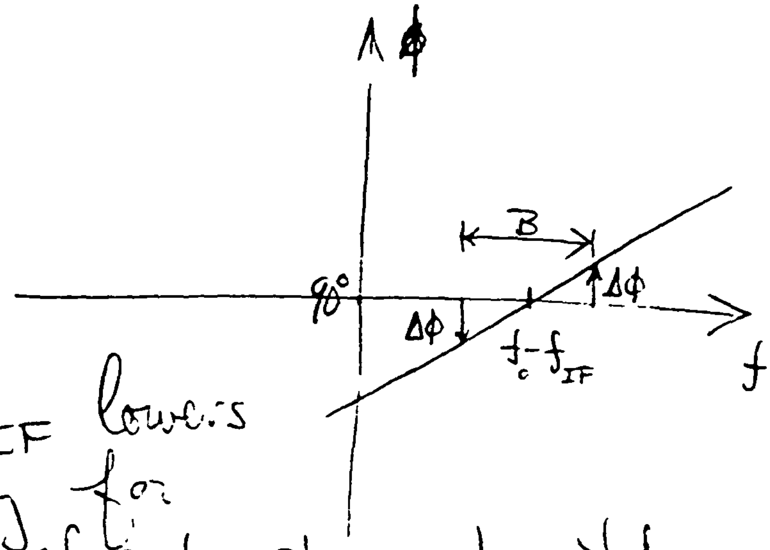
$$\phi = \frac{\pi}{2}(4m+1)\frac{f_0 - f_{IF} \pm B/2}{f_0 - f_{IF}}$$

So that the error is

$$\Delta\phi = \frac{\pi}{2}(4m+1)\frac{B/2}{f_0 - f_{IF}}$$

- Clearly we should have $m=0$ then

$$\Delta\phi = \frac{\pi}{4}\frac{B}{f_0 - f_{IF}}$$



Although a high f_{IF} lowers the Nyquist frequency for signal sampling, it clearly degrades the phase shift performance.

- To achieve a 90° phase shift requires that we sample at least at $R = 4(f_0 - f_{IF})$ then the length of the shift register is $n_L = 1$. In general $R = 4n_L(f_0 - f_{IF})$. $4(f_0 - f_{IF})$ is approximately $2 \times$ Nyquist so we will select $n_L = 1$ to keep rates low.

The output word of the I & Q channels should have l.s.b. which corresponds to $\frac{1}{2}$ the minimum detectable signal. The output of a correlating receiver has sensitivity

- $$\Delta T = \frac{T_{sys}}{\sqrt{2B\tau}} \qquad T_{sys} = T_A + T_N$$

The minimum detectable signal then ~~occurs~~ occurs when $T_A = 0$ (cold calibration source):

$$\Delta T_{min} = \frac{T_N}{\sqrt{2B\tau}}$$

The maximum output (signal level $\pm 2^{N_b - 1}$) corresponds to $T_{A\,max} + T_N$ so we need

$$2^{N_b - 1} = \frac{T_{A\,MAX} + T_N}{\frac{1}{2}\Delta T_{min}} = \frac{T_{A\,MAX} + T_N}{T_N}\sqrt{2B\tau}$$

So

$$N_b = \log_2 \left\{ \frac{T_{A\,MAX} + T_N}{T_N}\sqrt{2B\tau} \right\}$$

(round up)

Example configurations

Restrict the phase error to 5°

Then

$$\Delta\phi = 5^\circ = (45^\circ)\frac{B}{f_0 - f_{IF}}$$

$B = 27\ \text{MHz}$
$f_0 = 1413.5\ \text{MHz}$
$\tau = 1.5\ \text{s}$

$$\Rightarrow f_{IF} = 1170.5\ \text{MHz}$$

$$\Rightarrow f_0 - f_{IF} = 243\ \text{MHz}$$

Set $R = 2\times$ Nyquist $= 4(f_0 - f_{IF}) = 972\ \text{MHz}$

Summer internal bits $I_b = 2n_b - 1 + \underset{\uparrow \log_2 R\tau}{31} = 2n_b + 30$

External bits, assume $T_{A_{max}} = 300K$, $T_N = 300K$

$$N_b = \log_2 2\sqrt{2B\tau} = 15$$

Total serial rate for $N = 37$ units

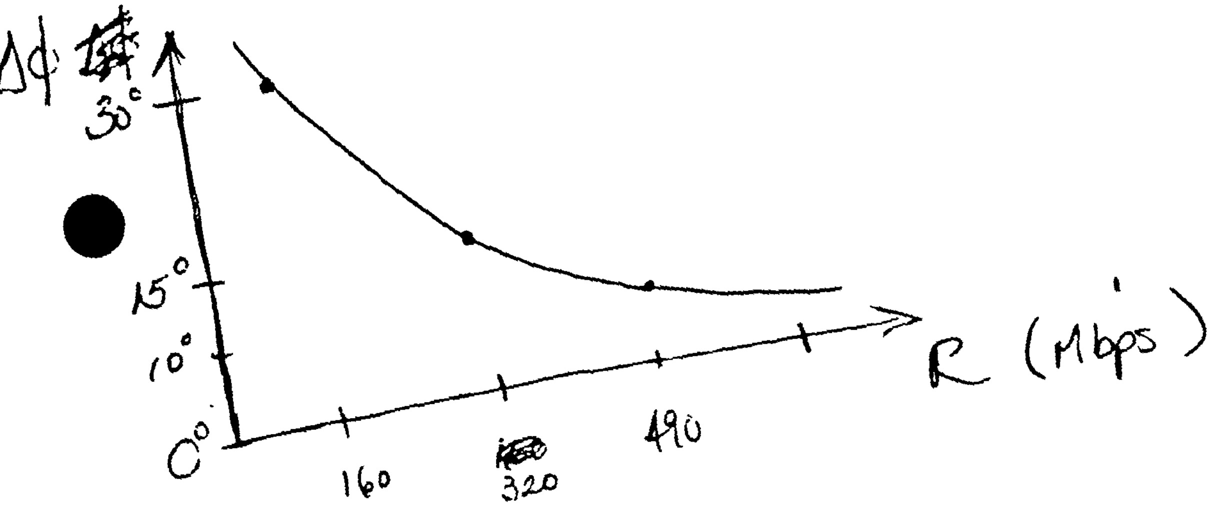
$$2 \times 37 \times 15 \frac{1}{1.5\ s} = 740\ \text{bps}$$

e see that the internal data rate R
driven by the phase error limit

$$4(f_0 - f_{IF}) = R$$

$$\Delta\phi = (45^\circ)\frac{B}{f_0 - f_{IF}}$$

$$= (180^\circ)\frac{B}{R}$$



Issues:

1) Can these data rates be reasonably accomplished aboard a spacecraft?

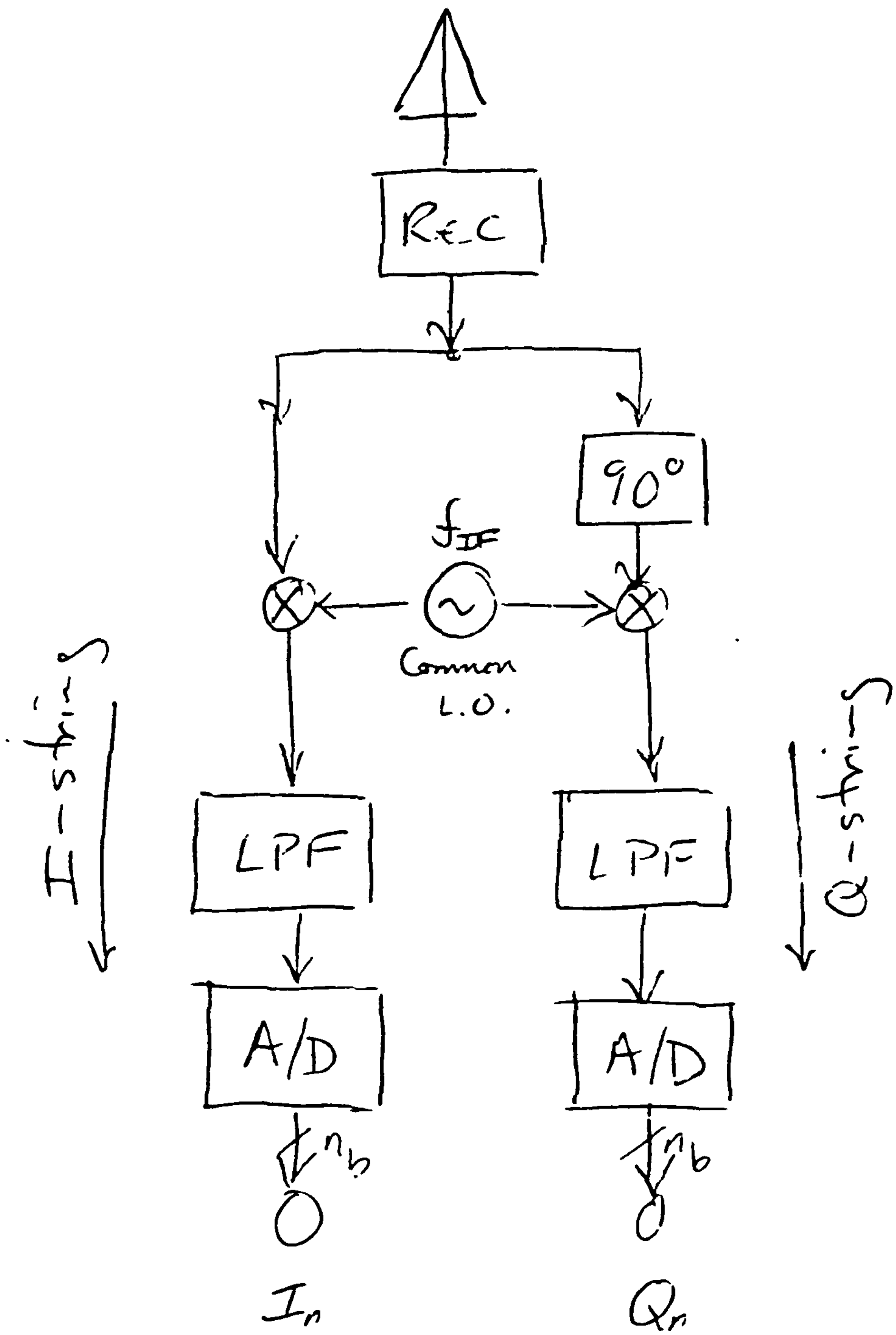
2) Are there A/D, shift registers, multipliers, etc. that will function at these rates?

3) What is the necessary number of bits n_b to meet the instrument specs?

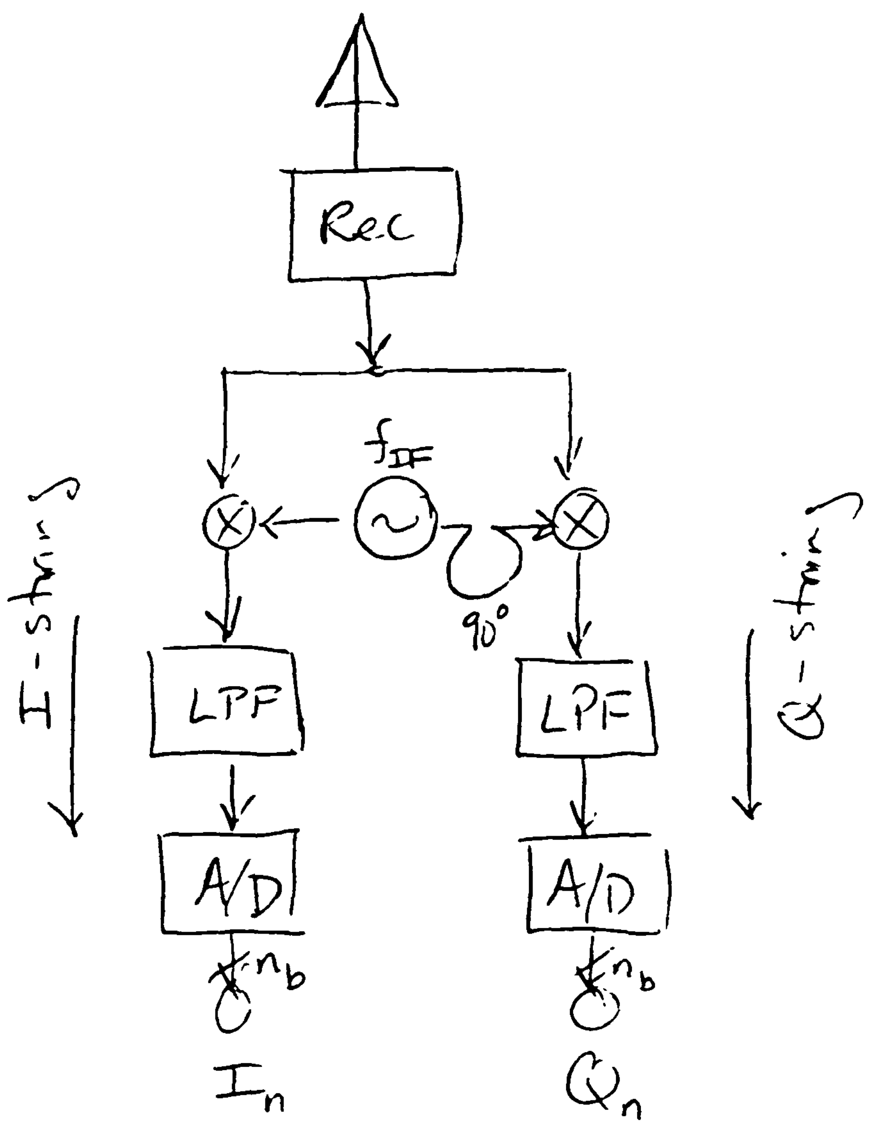
4) What will be the power and heat rejection needs?

DIGITAL PROCESSING — ARCHITECTURE II

n Antenna/Receiver units



Alternatively



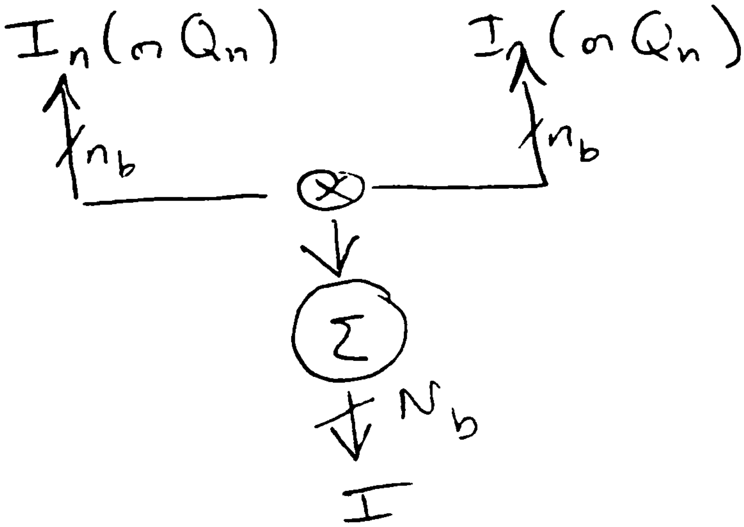
This is probably preferable for two reasons:

1) Phase shift can be done with delay lines without substantial introduction of errors due to nonvanishing signal bandwidth

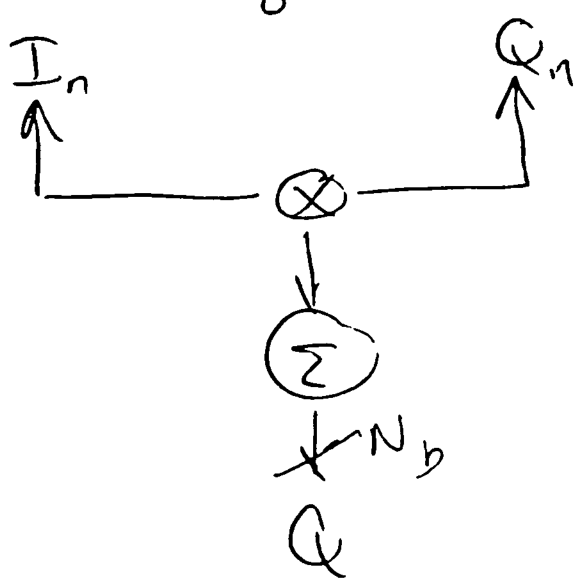
2) Impedance of two strings should be more similar because phase shifter is out of Q-string hence power splitting is less of a problem.

2N Correlation units

N configured as In-phase Correlators



N configured as Quadrature correlators



Here, since mixing is done after or coincident with phase shifting, we may take $f_{IF} = f_0 - B/2$ so that the spectrum of the signal into the A/D is from 0 to B. The Nyquist frequency is then 2B. Set the sampling rate $R = 2B = 54$ MHz.
Assume $\tau = 1.5$ s.

The summer internal # of bits is then

$$I_b = 2n_b - 1 + \log_2 R\tau$$

$$= 2n_b + 26$$

The external bit rate and precision (N_b) is not architecture dependent

$N_b = 15$ External serial rate = 740

(see Architecture I)

• This architecture has the obvious advantage of lower data rates but at the price of greater Antenna/Rec. unit complexity

Issues:

1) What is the required n_b to meet instrument requirements?

2) What will be the power and heat rejection needs?

IV. EOS/HMMR PANEL PRESENTATION

What follows are the vugraphs from our presentation to the EOS/HMMR panel. This presentation summarizes some of our results and was intended to generate some confidence that aperture synthesis could be used to perform the soil moisture mission. We took as science requirements:

1) 10 km spatial resolution
2) 3 day temporal resolution
3) 1 K radiometric resolution.

We took the 3 day temporal resolution requirement to mean that both dayside and nightside overpasses count as revisits. At a presumed 700 Km sun-synchronous circular orbit, a 450 km swath implies three day coverage (at least in a statistical sense, i.e., mean revisit time). Since swath width is the most difficult parameter to achieve due to grating lobes and fringe washing, and since we wanted to present a "minimal" instrument, the swath width goal was degraded to 400 km. This implies 3 day (statistical) coverage in the temperate latitudes above 30^0 and 4 day (statistical) coverage at the equator. Increasing the swath width to 450 km (or even 900 km which gives precisely periodic 3 day coverage rather than statistical 3 day coverage) is possible at the cost of more antennas.

The system presented consists of 11 13.2 m x .34 m antennas occupying a 13.2 m x 16.2 m array. The array fill-factor is 23 percent. It provides 10 km resolution both along and across-track with a .6 K radiometric sensitivity. With this one-dimensional concept, aperture synthesis provides the across-track resolution; whereas, the width along-track of the antennas' fields-of-view provides the along-track resolution.

A digital system architecture was presented. This architecture joins 11 antenna/receiver units, arranged to form a minimum redundancy linear array, with about 40 in-phase and 40 quadrature digital correlators. Each pair of antennas forming a unique baseline are joined with both in-phase and quadrature correlators. In addition, several autocorrelation channels would be required to measure the zero-baseline or DC component of the scene spatial frequency distribution. Each channel need not physically correspond to a unique piece of hardware, as different baselines could be time-phase multiplexed on a common correlator. The instrument internal data rate would be fairly high at 20 Mbps which is determined by the Nyquist sampling rate of the signals being digitized by the antenna/receiver units. The external data rates would, however, be quite modest at about 1 Kbps which is determined by the dynamic range and noise levels in the measurements. Fourier processing to construct an image could be easily done on the ground in real-time.

APERTURE SYNTHESIS

FOR SOIL MOISTURE REMOTE SENSING

A Presentation to the EOS/HMMR Panel

25 March 1985

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OUTLINE

I. Soil Moisture Science Requirements

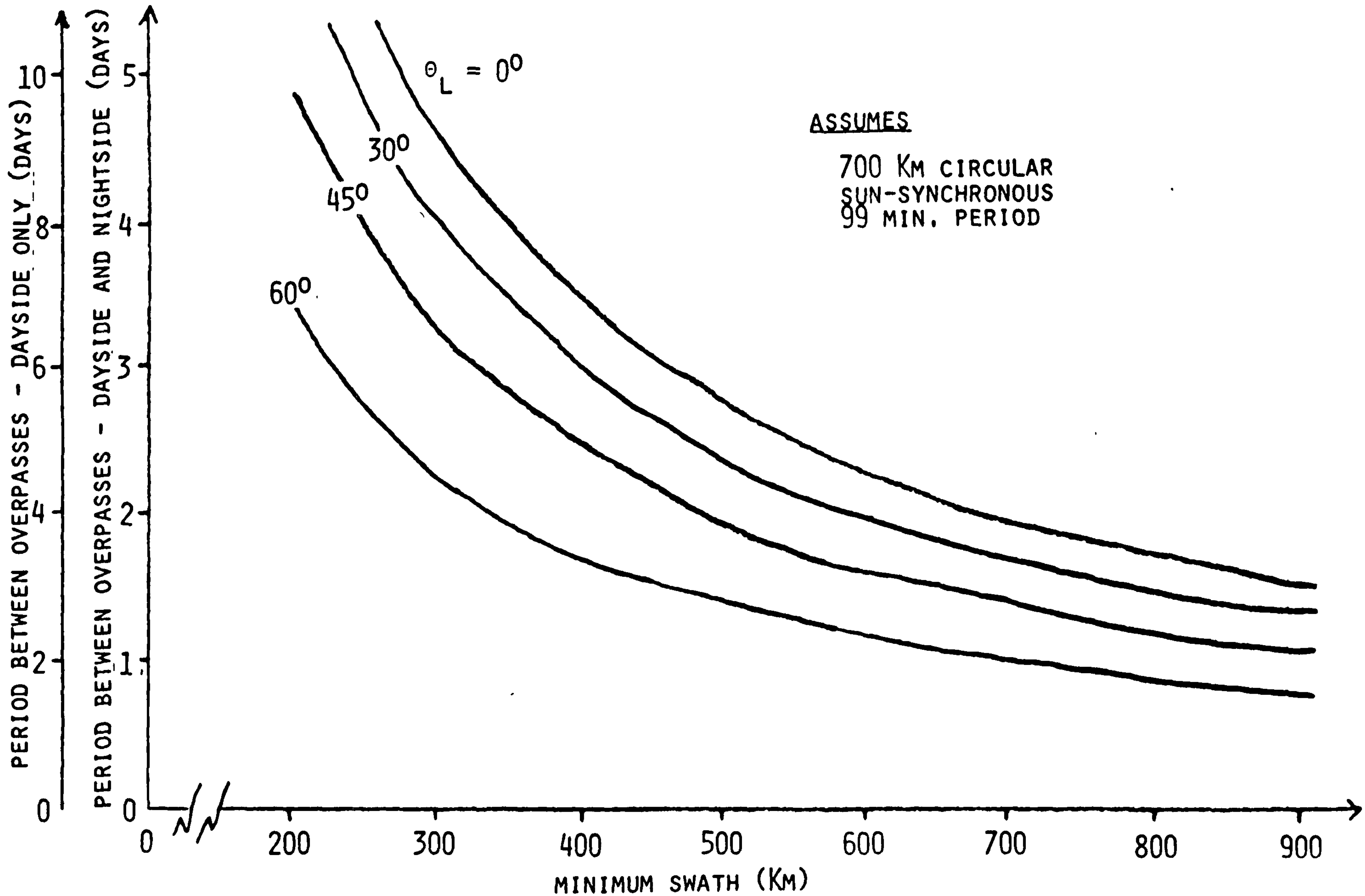
II. Candidate L-Band Aperture Synthesis Instrument

III. Research Areas and Other Applications

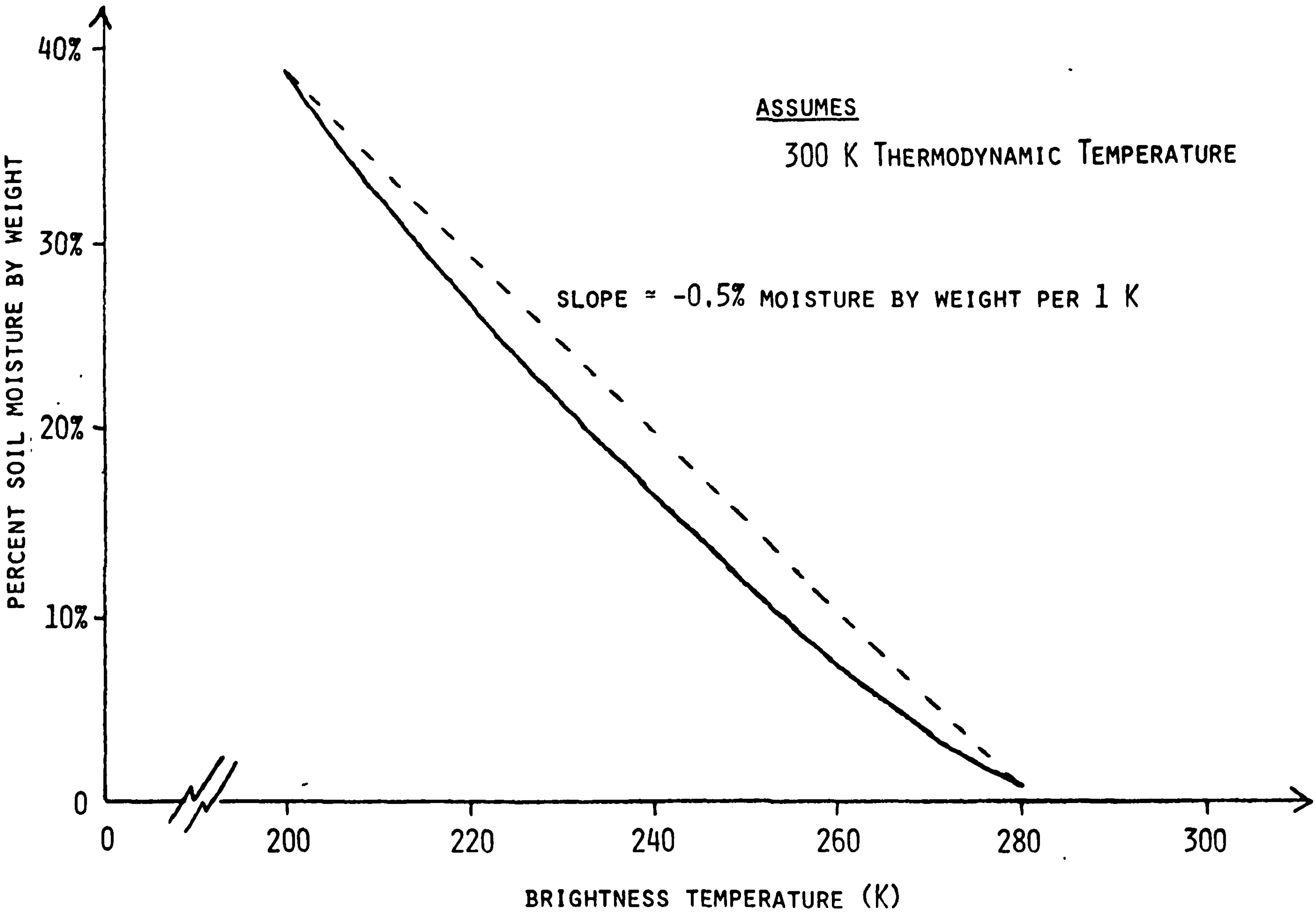
SOIL MOISTURE SCIENCE REQUIREMENTS

- Spatial Resolution: 10 Km
- Temporal Resolution: 3 days
- Radiometric Resolution: 1 K

TEMPORAL RESOLUTION AND SWATH WIDTH



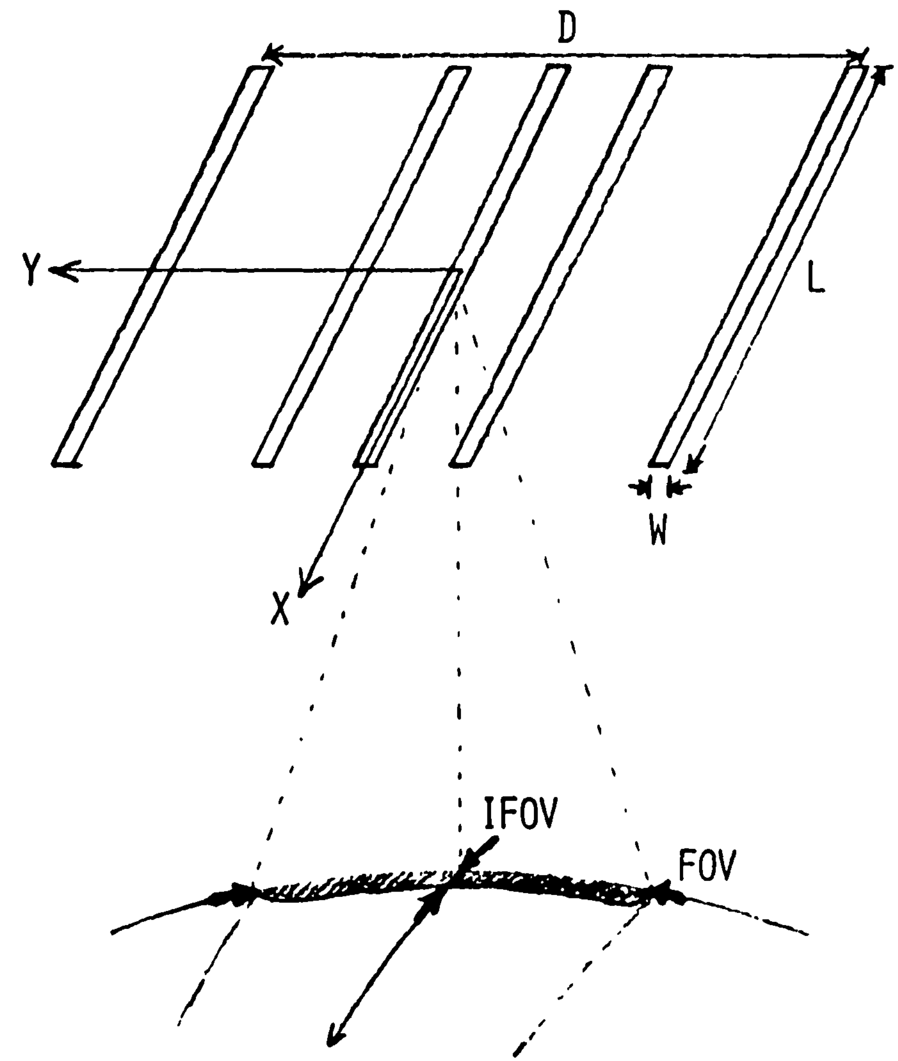
SOIL MOISTURE-BRIGHTNESS TEMPERATURE RELATIONSHIP
(As Reported for Skylab S-194)



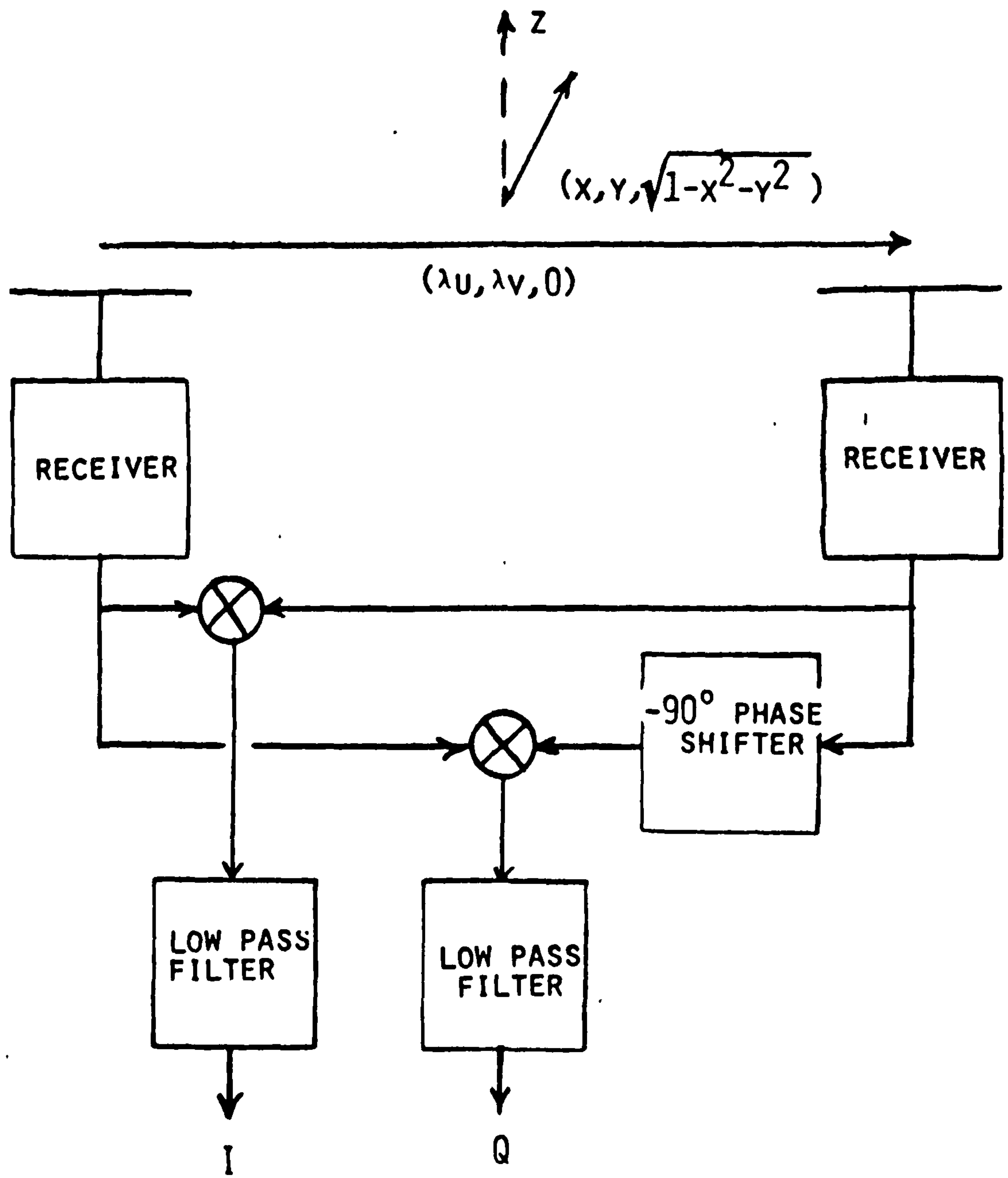
CANDIDATE APERATURE SYNTHESIS INSTRUMENT

- Orbit: h=700 Km Circular, Sun-synchronous
- Spatial Resolution: 10 Km
- Swath Width: 400 Km ($\pm 16^{o}$)
    - 4 Day Period Between Overpasses - Dayside and Nightside
    - 3 Day Period Above 30^{o} Latitude - Dayside and Nightside
- Radiometric Sensitivity: .5 K
  (Overall Accuracy of 1 K)
- Center Frequency: 1413.5 MHz (λ=21 cm)

INSTRUMENT CONCEPT



CORRELATION INTERFEROMETER



$$R = I + iQ$$

APERTURE SYNTHESIS

- Each Baseline in Array Measures a Fourier Component

$$R(u,v) = \iint \frac{I(x,y)P(x,y)}{\sqrt{1-x^2-y^2}} \, \mathrm{EXP}\{\, 2\pi i(ux + vy) \,\} \, dx\,dy$$

where

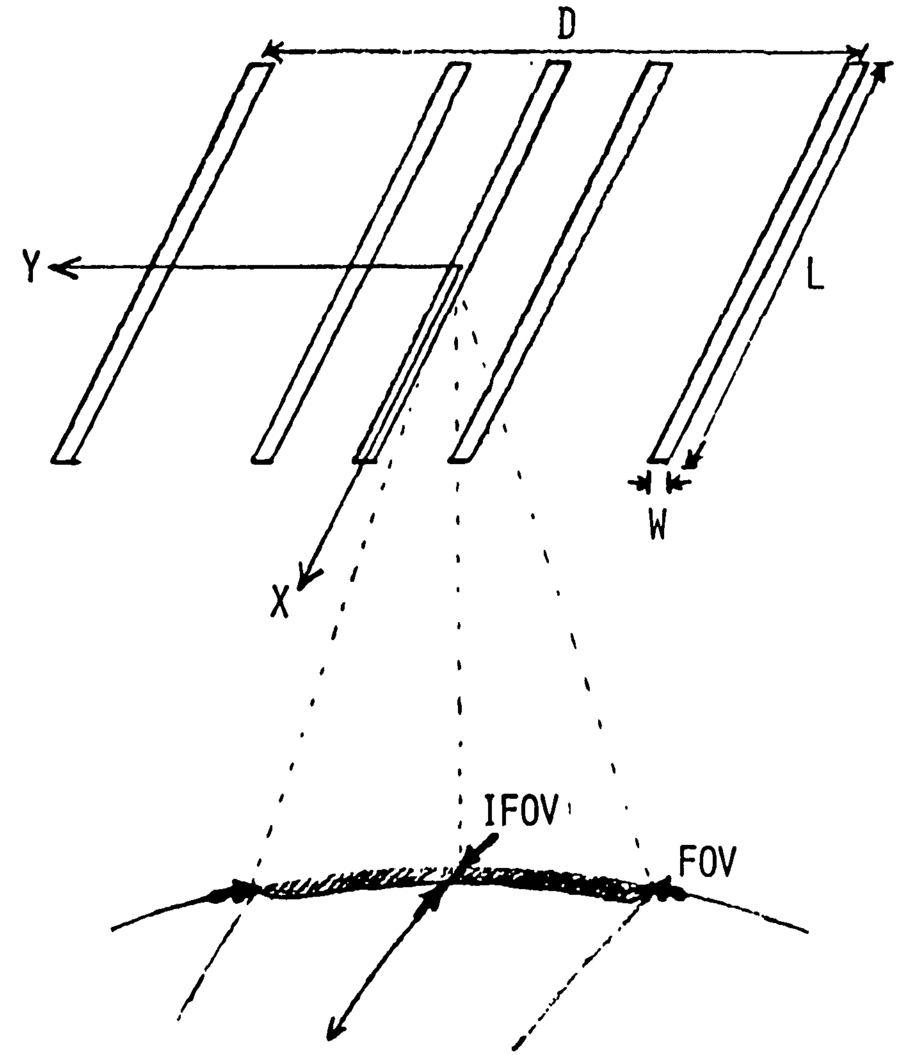
I(x,y) is Scene Intensity Distribution
P(x,y) is Individual Antenna Power Pattern

- Inverting R(u,v) Measurements Using Discrete Fourier Transform Produces Image

- Array Exterior Dimensions Determine Synthesized Resolution

- Individual Antenna Patterns Determine Field-of-View

INSTRUMENT CONCEPT



C-2

ARRAY EXTERIOR DIMENSIONS

- Along-Track Resolution Determined By Length L

$$\Delta X = (.9)\lambda H/L$$

ΔX = 10 Km Implies L = 13.2 m

- Across-Track Resolution Determined By D

$$\Delta Y = (1.1)\lambda H/D$$

ΔY = 10 Km Implies D = 16.2 m

ANTENNA WIDTHS

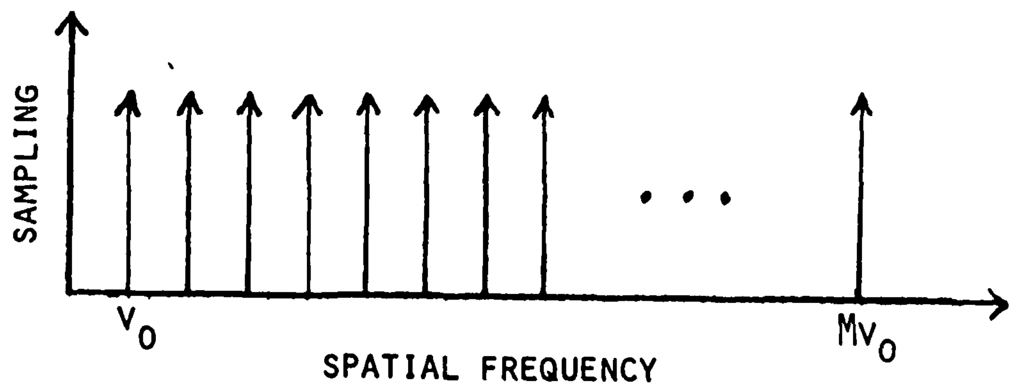
- Define Field-of-View By Half-Power Points of Individual Antenna Patterns

$$\mathrm{SIN}\ \Theta_{HP} = (.45)\lambda/W$$

400 Km Swath Implies $\Theta_{HP} = 16^{o}$, Implying $W = .34$ m

GRATING LOBES

- Minimum Redundancy Linear Array Uniformly Sampling In Spatial Frequency Domain



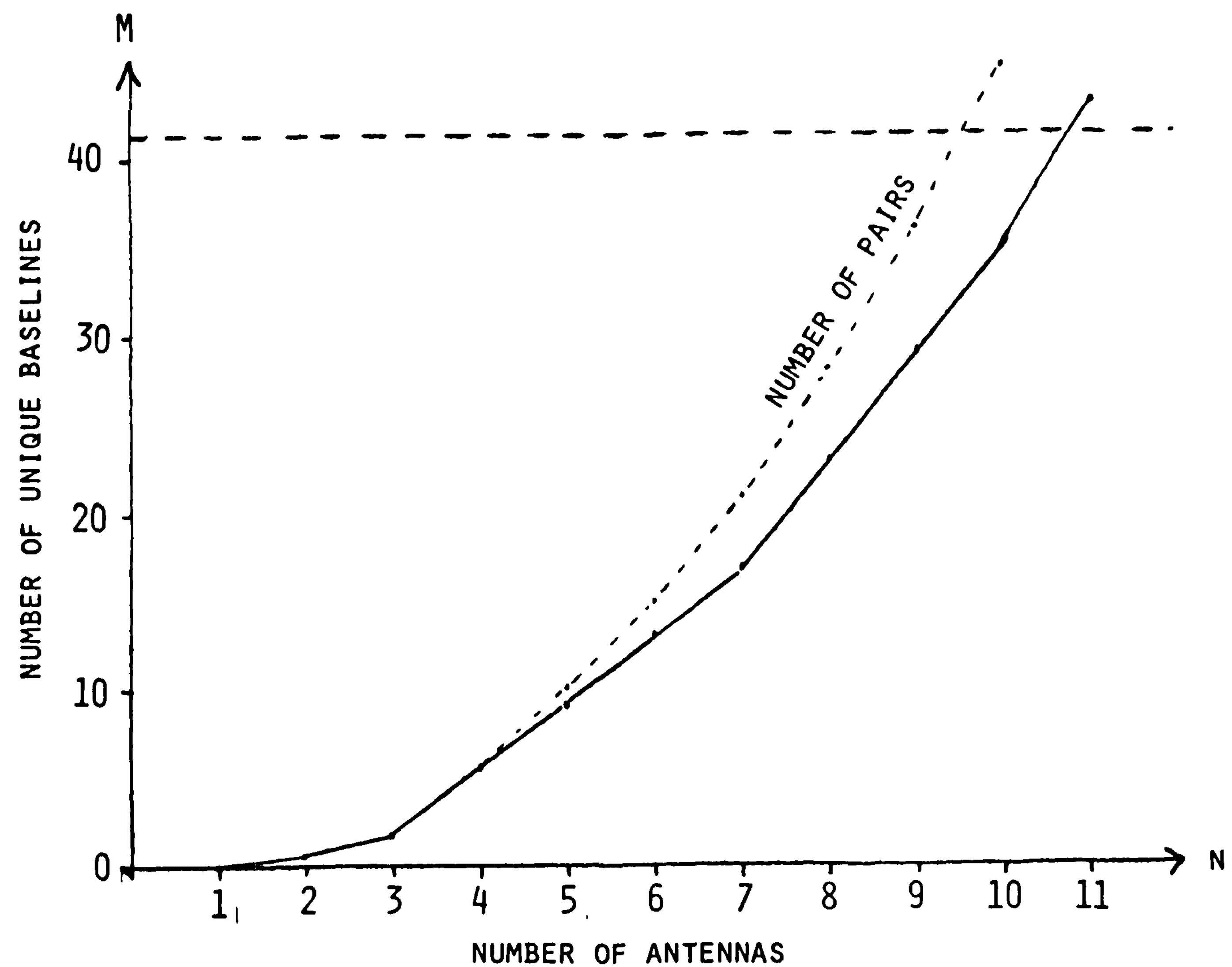
- First Grating Lobe Occurs When

$$\sin \Theta_{GL} = \lambda M/D$$

- To Avoid Folding Require $\Theta_{GL} > \Theta_{FOV}$ (=32°), Implying

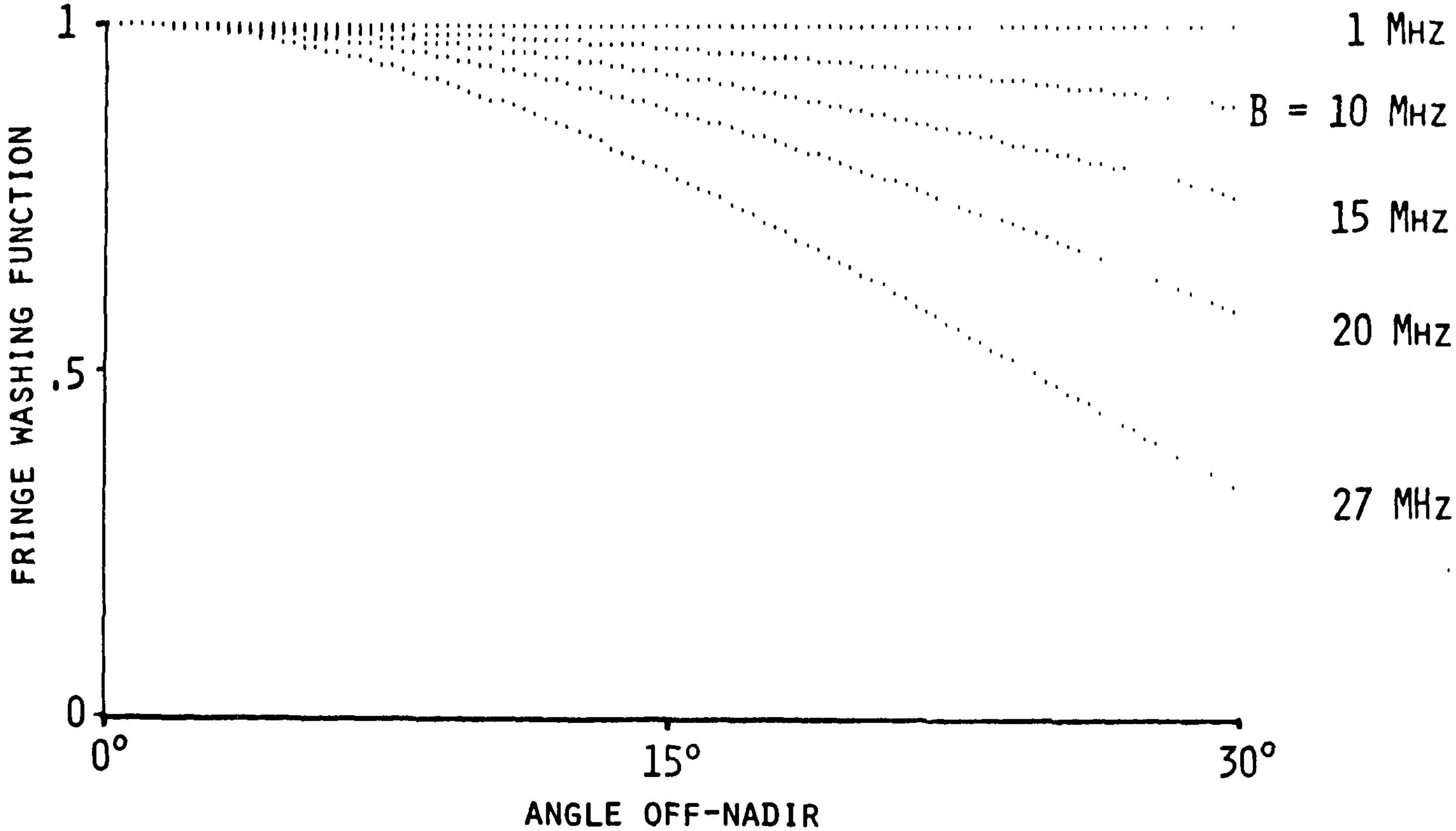
$$M > 41$$

MINIMUM REDUNDANCY LINEAR ARRAYS



o N = 11 Puts First Grating Lobe at $\Theta_{GL} = 34^\circ$

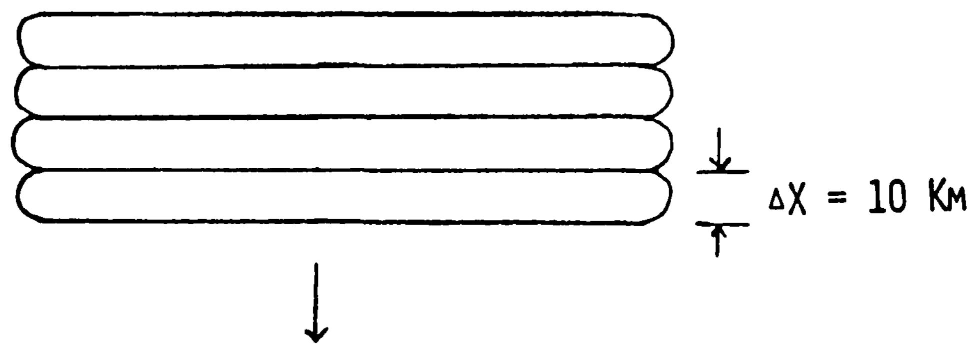
BANDWIDTH AND LOSS OF CORRELATION



- Fix B = 10 MHz
- Bandwidth up to 27 MHz is protected for Radio Astronomy

INTEGRATION TIME

- Integration Time Is Set As Time Required To Advance One Resolution Element Along-Track



$\tau = \Delta X/u = 1.5$ s

SENSITIVITY

- Use Sensitivity Formula

$$\Delta T = \frac{T_{SYS}\, A_{SYNTHETIC}}{\sqrt{2M}\, A_{REAL} \sqrt{B\tau}}$$

$A_{synthetic} = LD = (13.2\ m)\ (16.2\ m) = 214\ m^2$

$A_{real} = LW = (13.2\ m)\ (.34\ m) = 4.5\ m^2$

$M = 43 \qquad B = 10\ MHz \qquad \tau = 1.5\ s$

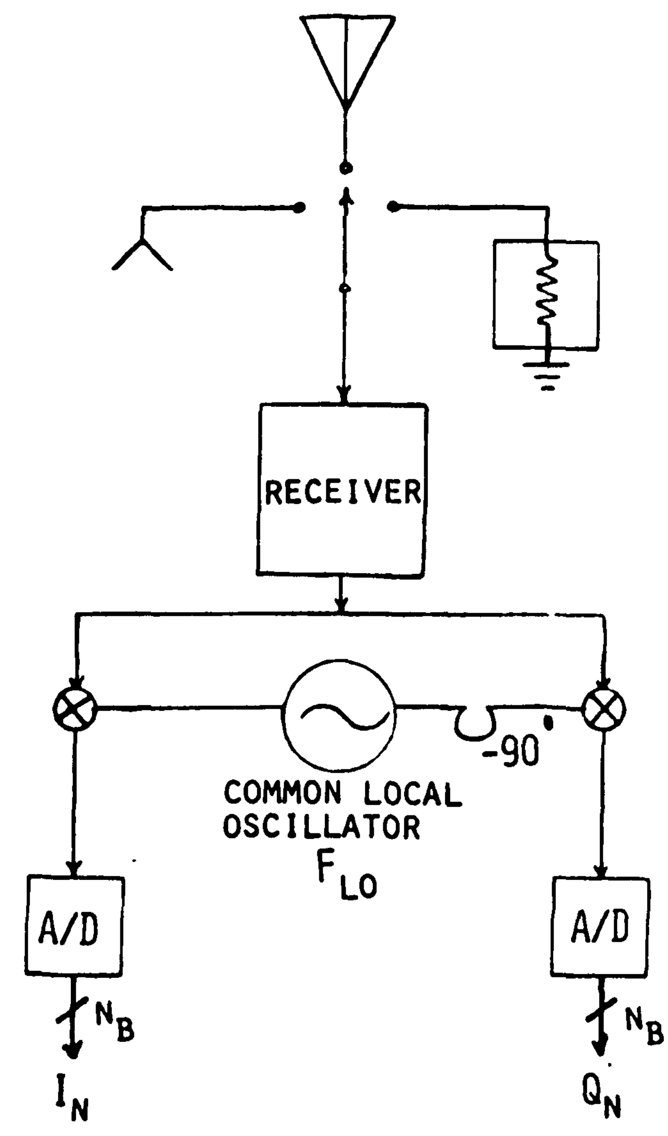
- For $T_{sys} = 450$ K (300 K Antenna Temperature, 150 K Noise Temperature), $\Delta T = .6$ K
- Necessary Sensitivity Achieved With Fill-Factor

$$F = \frac{NLW}{LD} = .23$$

SYSTEM ARCHITECTURE:

DIGITAL IMPLEMENTATION

o 11 Antenna/Receiver Units



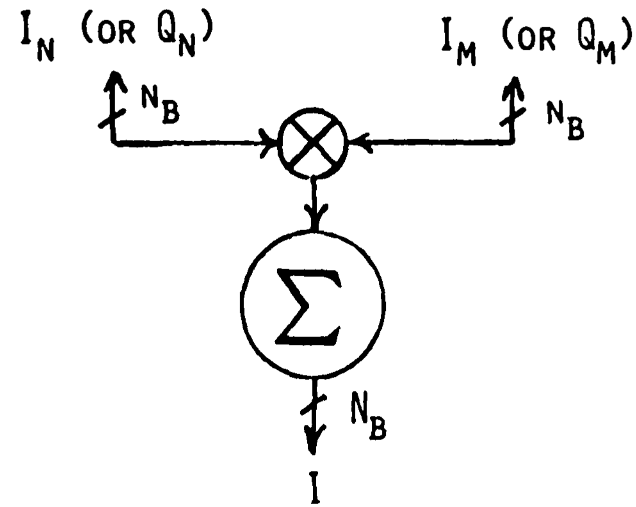
SAMPLE AT $R = 2(F_{IF} + B/2)$

$F_{IF} = F - F$

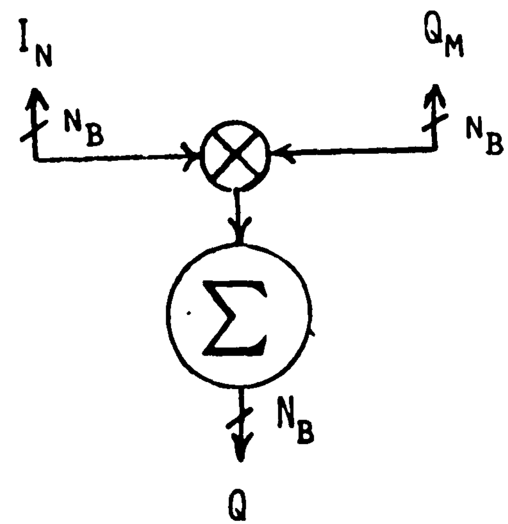
SYSTEM ARCHITECTURE:

DIGITAL IMPLEMENTATION

o > 43 In-phase Correlation Units



o > 43 Quadrature Correlation Units



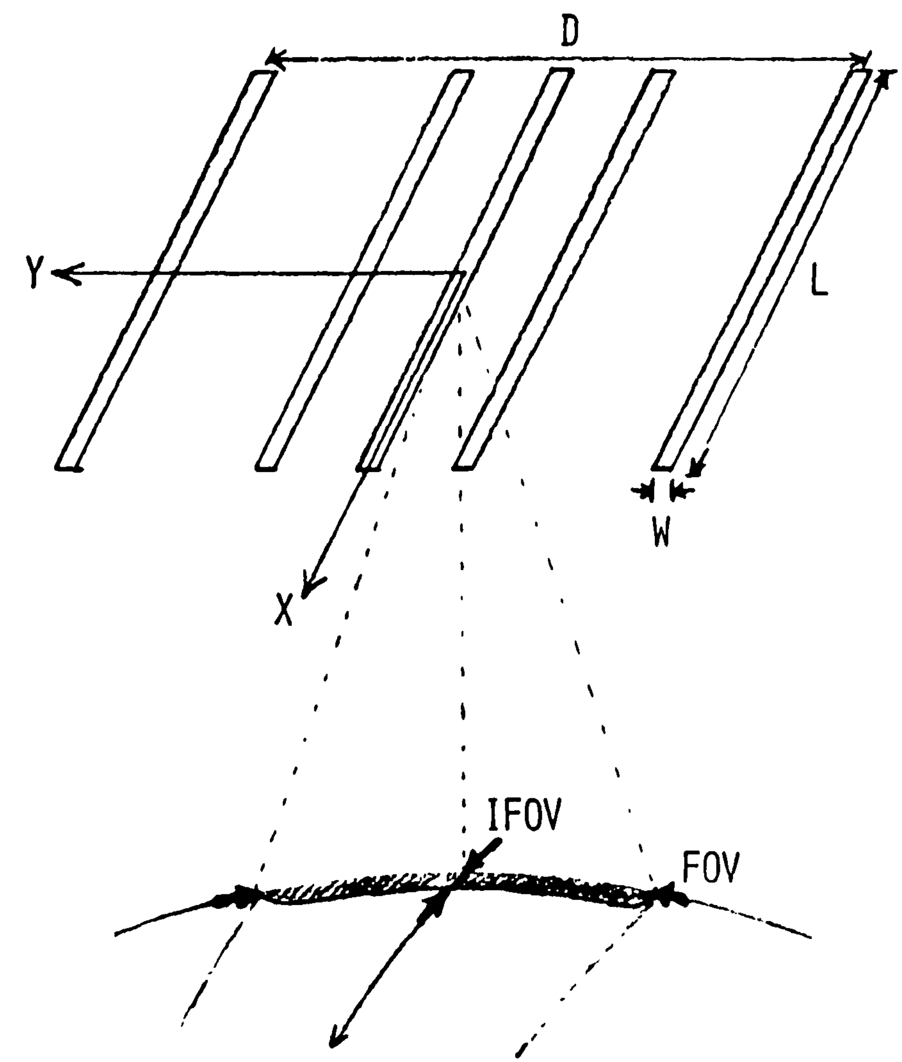
DATA RATES

- INTERNAL
  - QUANTIZATION: TBD
    (VLA USES 3 BITS)
  - RATES SET BY A/D SAMPLING RATE WHICH CAN BE AS LOW AS R = 2B = 20 MHZ

- EXTERNAL
  - QUANTIZATION: 10 BITS - 11 BITS
    (SET BY DYNAMIC RANGE AND SENSITIVITY)
  - TOTAL SERIAL RATE CAN BE AS LOW AS

    2 X NUMBER OF CORRELATIONS X 10 BITS / 1.5 S
    = 600 BPS

INSTRUMENT CONCEPT



SUMMARY

- o An Aperture Synthesis Instrument With
  - 11 13.2 m x .34 m Antennas
  - Filling 23% of 13.2 m x 16.2 m Array
  - In a 700 Km Circular Sun-Synchronous Orbit
  - With a 600 bps Data Rate

  Can Address Soil Moisture Science Requirements
  - Spatial Resolution: 10 Km
  - Swath: 400 Km (4 day temporal resolution at equator, 3 day temporal resolution at 30^0 latitude; counting both dayside and nightside overpasses)
  - Radiometric Resolution: 1 K

RESEARCH AREAS

1. Can the Architectures be Implemented with Resonable Technologies and if so what are the Power Requirements?

2. How well can these Instruments be Calibrated?

3. What are the Antenna and Array Structural Tolerances?

4. What are Cost/Benefit Tradeoffs Between Real and Synthetic Aperture Systems?

RESEARCH AREAS

5. The Field-of-View Requirement Drives the Number of Antennas. Would a Hybrid Synthesis/Scanner or Synthesis/Pushbroom Instrument then be Attractive?

6. The Greater Integration Time and Parallel Channel Structure of Aperture Synthesis Instruments gives them a Sensitivity as well as a Resolution Advantage. Can this be Exploited in other Applications such as C-Band Oceanographic Remote Sensing?

V. BANDWIDTH AND RECONSTRUCTION

As noted earlier, a nonvanishing system bandwidth results in imaging errors ("fringe washing") off-axis when naive Fourier processing is used for image reconstruction. The net effect is for system bandwidth to limit field-of-view. In this section we exhibit the theory behind this effect. This is exploited in the computer simulation described in section VI.

The first attached memo derives the relationship between system response and scene brightness distribution for a polychromatic extended source. The problem of image reconstruction is cast as inverting an integral equation:

$$R(u) = \int_0^{\infty}\int_{-\infty}^{+\infty} G(\omega)\, K(x)\, e^{2\pi i \frac{\omega}{\omega_0} u x}\, dx\, d\omega$$

where

$R(u)$ is the response at spatial frequency (or baseline) u

$G(\omega)$ is the filter power gain at radian frequency ω

ω_0 is the center or fiducial radian frequency

$K(x)$ is the integral kernal to be estimated at scene coordinate x (sine of angle across track).

ORIGINAL PAGE IS
OF POOR QUALITY

The kernal K(x) is the product of the antenna power pattern, scene brightness distribution and Jacobian factor $(1-x^2)^{-1/2}$. This is almost a Fourier relationship between R(u) and K(x).

To see the connection between bandwidth and field-of-view, we perform a change of variables $\xi = \frac{(\omega-\omega_0)}{\omega_0}$ and integrate over ξ. Then

$$R(u) = \int_{-\infty}^{+\infty} h(u,x)\, K(x)\, e^{2\pi i u x}\, dx$$

where

$$h(u,x) = \int_{-\infty}^{+\infty} g(\xi)\, e^{2\pi i \xi u x}\, d\xi$$

$$g(\xi) = \begin{cases} \omega_0 G((1+\xi)\omega_0) & \xi \geq -1 \\ 0 & \xi < -1 \end{cases}$$

Here h(u,x) multiplies K(x) which contains the antenna power pattern. Then h(u,x) can be thought of as an additional but spatial frequency dependent pattern factor. For a single frequency bandpass h(u,x) in a constant, but as the bandwidth increases, it becomes tapered away from x=0.

The second memo describes how this formalism is translated into the discrete problem of inverting a finite sampling of spatial frequencies to estimate scene brightness. There we have an almost discrete Fourier relationship:

$$R_m = \sum_{n=1}^{N} K_n W_{nm}\, e^{2\pi i (m-1)(n-1)/N}$$

where

R_m is R(u) sampled at $u=(m-1)\Delta u$ and weighted by a phase factor
K_n is K(x) sampled at $x=(n-\frac{N+1}{2})\Delta x$
W_{nm} is a filter design dependent weighting

The samplings Δx, Δu are such that $\Delta x \Delta u = \frac{1}{N}$ where N is the number of baselines.

For a real sensor some baselines may occur with multiplicities other than 1, in which case, we multiply W_{nm} by the multiplicity factor μ_{nm}.

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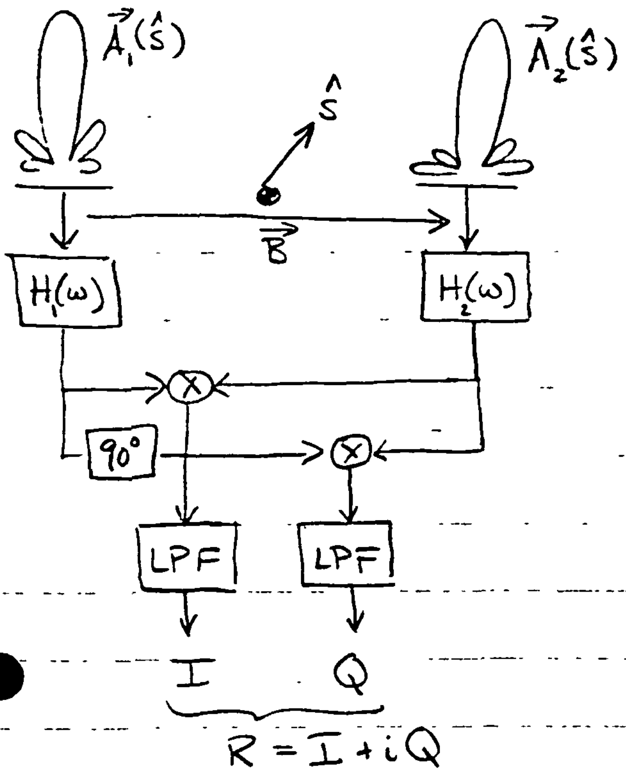
Correlation Interferometer Response To A Polychromatic Extended Source



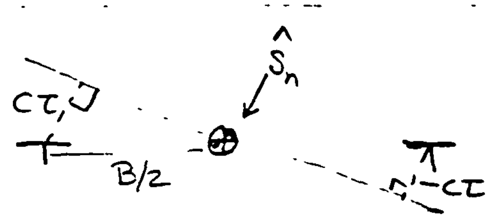
$\vec{A}$ - antenna voltage pattern.
H - receiver transfer function (one-sided)
$\vec{B}$ - baseline
$\hat{s}$ - line-of-sight vector
⊕ - phase center

Model the source as a collection of mutually incoherent point sources radiating monochromatically. At the phase center the radiation from the n^{th} source is

$$\vec{E}_n e^{i\omega_n t}$$

This wave is retarded (advanced) at antenna 1 (2) by the delay

$$\tau = \pm \frac{1}{2} \vec{B} \cdot \hat{s}_n / c$$



The radiation from the n^{th} source at each antenna is then

antenna 1

$$\vec{E}_n e^{i\omega_n(t - \frac{1}{2}\vec{B}\cdot\hat{s}_n/c)}$$

antenna 2

$$\vec{E}_n e^{i\omega_n(t + \frac{1}{2}\vec{B}\cdot\hat{s}_n/c)}$$

The signals into each receiver are then

receiver 1

$$\sum_n \vec{A}_1(\hat{s}_n)\cdot\vec{E}_n e^{i\omega_n(t - \frac{1}{2}\vec{B}\cdot\hat{s}_n/c)}$$

receiver 2

$$\sum_n \vec{A}_2(\hat{s}_n)\cdot\vec{E}_n e^{i\omega_n(t + \frac{1}{2}\vec{B}\cdot\hat{s}_n/c)}$$

The receiver voltage outputs are then

receiver 1 $V_1 = \sum_n H_1(\omega_n)\vec{A}_1(\hat{s}_n)\cdot\vec{E}_n e^{i\omega_n(t - \frac{1}{2}\vec{B}\cdot\hat{s}_n/c)}$

receiver 2 $V_2 = \sum_n H_2(\omega_n)\vec{A}_2(\hat{s}_n)\cdot\vec{E}_n e^{i\omega_n(t + \frac{1}{2}\vec{B}\cdot\hat{s}_n/c)}$

The in-phase channel input to the LPF is

$$Re(V_1)Re(V_2)$$

The quadrature channel input to the LPF is

$$Re(iV_1)Re(V_2) = -Im(V_1)Re(V_2)$$

We regard each component and phase of $\vec{E}_n$ to be an independent draw from some distribution (mutual incoherence) with $\langle\vec{E}_n\rangle = 0$. The process of summation represents a weighted averaging process; so that all cross terms in the above products vanish

$$Re(V_1)Re(V_2) = \sum_n Re\left(H_1\vec{A}_1\cdot\vec{E}_n e^{i\omega_n(t-\frac{1}{2}\vec{B}\cdot\hat{s}_n/c)}\right)Re\left(H_2\vec{A}_2\cdot\vec{E}_n e^{i\omega_n(t+\frac{1}{2}\vec{B}\cdot\hat{s}_n/c)}\right)$$

$$Re(iV_1)Re(V_2) = -\sum_n Im(\quad\cdots\quad)Re(\quad\cdots\quad)$$

Each term in the above has two frequency components with frequencies 0 and $2\omega_n$. Assume that H_1 and H_2 are such that contributions from the $2\omega_n$ frequencies is not extend below the LPF cut off. Thus

$$I = \frac{1}{2}Re\left(\sum_n \left(H_1\vec{A}_1\cdot\vec{E}_n e^{-i\omega_n\frac{1}{2}\vec{B}\cdot\hat{s}_n/c}\right)\left(H_2\vec{A}_2\cdot\vec{E}_n e^{i\omega_n\frac{1}{2}\vec{B}\cdot\hat{s}_n/c}\right)^*\right)$$

$$Q = -\frac{1}{2}Im\left(\sum_n(\quad\cdots\quad)(\quad\cdots\quad)^*\right)$$

Hence

$$R^* = \tfrac{1}{2}\sum_n \left(H_1 \vec{A}_1 \cdot \vec{E}_n e^{-i\omega_n \frac{1}{2}\vec{B}\cdot\hat{s}_n/c}\right)\left(H_2 \vec{A}_2 \cdot \vec{E}_n e^{i\omega_n \frac{1}{2}\vec{B}\cdot\hat{s}_n/c}\right)^*$$

$$= \tfrac{1}{2}\sum_n H_1 H_2^* \vec{A}_2^* \cdot \vec{E}_n^* \vec{E}_n \cdot \vec{A}_1 e^{-i\omega_n \vec{B}\cdot\hat{s}_n/c}$$

We now specialize to the case where

$H_1 = H_2 = H$ and $|H|^2 = G$ = power gain for rec.

$\vec{A}_1 = \vec{A}_2 = \hat{e} A$ and $|A|^2 = P$ = power gain for ant.
$\hat{e}$ = pol. for ant.

Hence

$$R = \tfrac{1}{2}\sum_n G(\omega_n) P(\hat{s}_n) |\hat{e}\cdot\vec{E}_n|^2 e^{+i\omega_n \vec{B}\cdot\hat{s}_n/c}$$

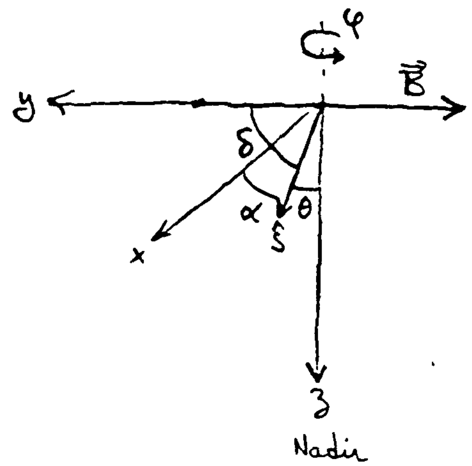
In the limit of a continuous source

$$R \sim \iint G(\omega) P(\hat{s})\, I(\omega,\hat{s})\, e^{+i\omega\vec{B}\cdot\hat{s}/c}\, d\omega\, d\Omega$$

$$= \iint G(\omega) P(\hat{s})\, I(\omega,\hat{s})\, e^{i\frac{\omega}{\omega_0} 2\pi \vec{B}\cdot\hat{s}/\lambda}$$

λ = center wavelength

Define the coordinate system



$$\frac{1}{\lambda}\vec{B} = u\hat{x} + v\hat{y} \qquad \hat{s} = \cos\alpha\,\hat{x} + \cos\delta\,\hat{y} + \cos\theta\,\hat{z}$$
$$= \cos\alpha\,\hat{x} + \cos\delta\,\hat{y} \pm \sqrt{1-\cos^2\alpha-\cos^2\delta}\,\hat{z}$$
$$= \sin\theta\cos\varphi\,\hat{x} + \sin\theta\sin\varphi\,\hat{y} + \cos\theta\,\hat{z}$$

$$\frac{1}{\lambda}\vec{B}\cdot\hat{s} = u\cos\alpha + v\cos\delta \qquad \cos\alpha = \sin\theta\cos\varphi \qquad \cos\delta = \sin\theta\sin\varphi$$
$$|\cos\theta| = \sqrt{1-\cos^2\alpha-\cos^2\delta}$$

$$\frac{\partial\cos\alpha}{\partial\theta} = \cos\theta\cos\varphi \qquad \frac{\partial\cos\alpha}{\partial\varphi} = -\sin\theta\sin\varphi$$

$$\frac{\partial\cos\delta}{\partial\theta} = \cos\theta\sin\varphi \qquad \frac{\partial\cos\delta}{\partial\varphi} = \sin\theta\cos\varphi$$

$$\Rightarrow \left|\frac{\partial(\cos\alpha,\cos\delta)}{\partial(\theta,\varphi)}\right| = |\sin\theta\cos\theta\cos^2\varphi + \sin\theta\cos\theta\sin^2\varphi|$$
$$= |\sin\theta\cos\theta|$$

$$\sin\theta\, d(\cos\alpha)\,d(\cos\delta) = \sin\theta\left|\frac{\partial(\cos\alpha,\cos\delta)}{\partial(\theta,\varphi)}\right| d\theta\, d\varphi = \left|\frac{\partial(\cos\alpha,\cos\delta)}{\partial(\theta,\varphi)}\right| d\Omega$$

$$\Rightarrow d\Omega = \frac{\sin\theta}{|\sin\theta\cos\theta|}\, d(\cos\alpha)\,d(\cos\delta)$$

$$= \frac{1}{\sqrt{1-\cos^2\alpha-\cos^2\delta}}\, d(\cos\alpha)\,d(\cos\delta)$$

Let $x = \cos\alpha$, $y = \cos\varepsilon$. Then

$$R \sim \int_0^{+\infty}\iint_{\sqrt{x^2+y^2}<1} G(\omega) P(x,y) I(\omega,x,y) \frac{1}{\sqrt{1-x^2-y^2}} e^{i\frac{\omega}{\omega_0}2\pi(ux+vy)}\, dx\, dy\, d\omega$$

For the case when $P(x,y)$ is very narrow in the y-direction say $P(x,y) = P(x)\,\delta(y)$

$$R \sim \int_0^{+\infty}\int_{-1}^{+1} G(\omega) P(x) I(\omega,x) \frac{1}{\sqrt{1-x^2}} e^{i\frac{\omega}{\omega_0}2\pi ux}\, dx\, d\omega$$

Extend the domain of x-integration by defining $I(\omega,x) = 0$ for $|x| \geq 1$, and absorb the proportionality constant into G:

$$R(u) = \int_0^{\infty}\int_{-\infty}^{+\infty} G(\omega) P(x) I(\omega,x) \frac{1}{\sqrt{1-x^2}} e^{i\frac{\omega}{\omega_0}2\pi ux}\, dx\, d\omega$$

If $G(\omega)$ is very narrow (ideally $G(\omega) = G\,\delta(\omega-\omega_0)$) there is a Fourier transform relation between R and the integral kernal

$$K(x) = P(x) I(\omega_0, x)/\sqrt{1-x^2}$$

Otherwise this Fourier relation is [illegible].

J. Check
10/22/85

FRINGE WASHING WITH A RECTANGULAR PASSBAND FOR 1-DIMENSIONAL APERTURE SYNTHESIS

Correllation receiver response is

$$R(u) = \int_0^{\infty}\int_{-\infty}^{+\infty} G(\omega) K(x)\, e^{2\pi i \frac{\omega}{\omega_0} u x}\, dx\, d\omega$$

For a rectangular passband

$$G(\omega) = \begin{cases} \frac{1}{2\pi B} & \text{if } |\omega - \omega_0| < \pi B \\ 0 & \text{otherwise} \end{cases}$$

$$\int_0^{\infty} G(\omega)\, e^{2\pi i \frac{\omega}{\omega_0} u x}\, d\omega$$

$$= \int_{\omega_0 - \pi B}^{\omega_0 + \pi B} \frac{1}{2\pi B}\, e^{2\pi i \frac{\omega}{\omega_0} u x}\, d\omega$$

$$= \frac{1}{2\pi B}\, \frac{\omega_0}{2\pi i u x}\, e^{2\pi i u x \frac{\omega}{\omega_0}} \Bigg|_{\omega = \omega_0 - \pi B}^{\omega = \omega_0 + \pi B}$$

$$= \frac{1}{2\pi B}\, \frac{\omega_0}{2\pi i u x}\, e^{2\pi i u x}\, 2i \sin\left(2\pi u x \frac{\pi B}{\omega_0}\right)$$

$$= \left[\frac{\omega_0}{2\pi^2 u x B} \sin\left(\frac{2\pi^2 u x B}{\omega_0}\right)\right] e^{2\pi i u x}$$

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Hence

$$R(u) = \int_{-\infty}^{+\infty} \left[\frac{\omega_o}{2\pi^2 u x B} \sin\left(\frac{2\pi^2 u x B}{\omega_o} \right)\right] K(x)\, e^{2\pi i u x}\, dx$$

We assume that $K(x)$ is slowly varying so that we may take the integral over into a sum

$$R(u) \longrightarrow \sum_{n=1}^{N} K_n \left[\frac{\omega_o}{2\pi^2 u x B} \sin\left(\frac{2\pi^2 u x B}{\omega_o} \right)\right] e^{2\pi i u (n - \frac{1}{2} - \frac{N}{2})\Delta x}$$

where $K_n \cong K((n-\frac{1}{2}-\frac{N}{2})\Delta x)\,\Delta x$. We have sampled K at

$$x = -\frac{(N-1)}{2}\Delta x, \ldots, \frac{(N-1)}{2}\Delta x$$

Recall that $K(x)$ vanishes for $|x| > 1$ so we should choose $\Delta x \geq 2/N$.

$R(u)$ is sampled at the baselines

$$u = 0, \ldots, (N-1)\Delta u$$

We choose $\Delta x = 1/N\Delta u$, so that we should have $\Delta u \leq 1/2$ for adequate sampling. We have

$$R_m = R((m-1)\Delta u) = \sum_{n=1}^{N} K_n W_{nm}\, e^{2\pi i (m-1)(n-\frac{1}{2}-\frac{N}{2})/N}$$

where

$$W_{nm} = \left(\frac{\omega_o N}{2\pi^2(m-1)(n-\frac{1}{2}-\frac{N}{2})B}\right)\sin\left(\frac{2\pi^2(m-1)(n-\frac{1}{2}-\frac{N}{2})B}{\omega_o N}\right)$$

If we define $\tilde{R}_m$

$$\tilde{R}_m = e^{2\pi i(m-1)(N-1)/2N} R_m$$

Then

$$\tilde{R}_m = \sum_{n=1}^{N} K_n W_{nm} e^{2\pi i(m-1)(n-1)/N}$$

If $B=0$, $W_{nm}=1$ and there is an explicit discrete Fourier relationship between $\tilde{R}$ and K

VI. COMPUTER SIMULATION

This section documents our aperture synthesis computer simulation designed to quantify the nonvanishing bandwidth and reconstruction problem. The source listing follows, it is written in Microsoft FORTRAN for the IBM PC. The source and object codes for which are delivered on a floppy disk. Two main programs appear - APSYNSIM and BASELINE. APSYNSIM calls subprograms FOURT, SINC, and PLOT.

APSYNSIM is the main program for the aperture synthesis simulation. It reads the run input data from file MASTER.DAT (listed after source listings). The inputs are as in table VI-1. System parameters specified are the number of baselines, null-to-null field-of-view, center frequency, and bandwidth. The test scene is assumed to be of constant nominal brightness temperature and limited to a specified scene width ($< 180^0$) to avoid the Jacobian factor in K(x) raising a division by zero error. APSYNSIM reads the baseline multiplicities and processing weights from input specified files (also listed). Input data sets can be concatenated for multiple runs.

The first step in the program is the creation of a scene weighted by a $(\text{sinc})^2$ antenna power pattern with the necessary null-to-null FOV, and the Jacobian factor. This is stored in array SCENE corresponding to K_n in section V. The unweighted scene has a constant nominal brightness temperature. Receiver responses are then calculated according to section V and stored in complex array R. The weightings W_{nm} are based on an ideal rectangular bandpass filter. A scene estimate SHAT is generated by Fourier transforming R. FOURT is a Fourier transform subroutine from the ORI program library. The difference between SHAT and SCENE and the fractional difference are computed as measures of estimation error. The fractional difference

TABLE VI - 1

APSYNSIM Input Variables

Name	Meaning	Format
RIDNAM	Run ID name	A20
N	Number of baselines (power of 2)	I3
SWDTH	Scene full width (degrees)	F8.2
FOV	Null-to-null FOV (degrees)	F8.2
FO	Center frequency (GHz)	F8.2
BW	Bandwidth (MHz)	F8.2
TEMP	Constant scene temperature (K)	F8.2
MULTFI	Multiplicity file name	A20
WGTFIL	Weighting file name	A20
OUTFIL	Output file name	A20

(SCENE-SHAT)/SCENE is a measure of the radiometric error induced since SCENE and SHAT both are brightness distributions weighted by the same antenna and Jacobian factors. Since the scene nominal temperature is specified, multiplying the fractional difference by the nominal temperature gives the radiometric error.

Output at the various stages of the calculation are displayed on printer plots generated by PLOT. In addition, an output file is created with the input data as a header and four columns containing the abscissa (scene coordinate x) and three ordinates-the test scene, scene and estimate difference, and radiometric error. The output file is in a format suitable for importation to LOTUS from which graphics can be readily generated. Simulation results are discussed in the next section.

A second main program, named BASELINE, is included here. It reads from a keyboard specified input file and writes to a keyboard specified output file. The input file contains integer antenna locations in a one-dimensional array. The output file lists the multiplicity for each baseline in the array.

MAIN PROGRAM - APSYNSIM


```
D Line# 1      7                                    Microsoft FORTRAN77 V3.30 March 1985
      C   1-D APERATURE SYNTHESIS SIMULATION
            PARAMETER (NMAX = 256)
            DIMENSION SCENE(NMAX), X(NMAX), SHAT(NMAX),  ...T(NMAX),MULT(NMAX)
            COMPLEX R(NMAX), SUM, WOF , WEIGHT(NMAX)
            CHARACTER*20 RIDNAM,MULTFI,WGTFIL,OUTFIL,...,WGTHED
            PI = 3.141592654
            D2R = PI / 180.0
      C     READ THE INPUT PARAMETERS
            OPEN(21,FILE='MASTER.DAT',STATUS='OLD')
         10 READ(21,20,END=240)RIDNAM,N,SWDTH,FOV,FO,BW,TEMP,MULTFI,WGTFIL,
           1OUTFIL
         20 FORMAT(A20/,I3/,5(F8.2/),2(A20/),A20)
            OPEN(22,FILE=MULTFI,STATUS='OLD')
            READ(22,30)MLTHED,K
         30 FORMAT(A10/,I3)
            DO 40 I = 1, K
1              READ(22,'(I5)')MULT(I)
1        40 CONTINUE
            OPEN(23,FILE=WGTFIL,STATUS='OLD')
            READ(23,45)WGTHED,L
         45 FORMAT(A20/,I3)
            DO 50 I = 1, L
1              READ(23,'(2E12.5)')WEIGHT(I)
1        50 CONTINUE
      C     INITIALIZE THE SCENE
            V0 = PI /  SIN(.5 * FOV )
            XMIN = -SIN(.5 * SWDTH * D2R)
            DX = (-2 * XMIN) / (N - 1)
            DO 70 I = 1, N
1                X(I) = (I-1) * DX + XMIN
1                P = SINC(V0 * X(I))
1                SCENE(I) = (P * P) / SQRT(1 - (X(I) * X(I))) * TEMP
1        70 CONTINUE
      C     DISPLAY SCENE
            WRITE(*,'(1H1)')
            CALL PLOT (N,X,SCENE)
            WRITE(*,90)N,FOV,SWDTH
         90 FORMAT(/1X,'TEST SCENE WITH'/3X,'N =',I3/3X,'FOV =',F6.2,1X,
           6'DEGREES'/3X,'SWDTH = ',F6.2,1X,'DEGREES')
      C     COMPUTE RECEIVER RESPONSES
            PHIO = 2 * PI / N
            V0 = (PI * BW) / (FO * 1000.0 * N)
            DO 110 I = 1, N
1                PHI = (I - 1) * PHIO
1                SUM = CMPLX(0,0)
1                   DO 130 J = 1, N
2                      U = (I - 1) * ((J - 0.5) - (0.5 * N)) * V0
2                      W = SINC(U)
2                      SUM = SUM + SCENE(J) * W * EXP(CMPLX(0,(PHI * (J-1))))
2       130         CONTINUE
1                R(I) = SUM / N * MULT(I)
1       110 CONTINUE
      C    WEIGHT RESPONSES CALCULATED
            DO 115 I = 1, N
1              R(I) = R(I) * WEIGHT(I)
1       115 CONTINUE
```

```
     C       COMPUTE THE SCENE ESTIMATE
             CALL FOUFT(R,3,1,-1,1,WORK)
             DO 150 I = 1, N
1               SHAT(I) = REAL(R(I))
1        150 CONTINUE
     C       DISPLAY THE SCENE ESTIMATE
             WRITE(*,'(1H1 ')
             CALL PLOT(N,X,SHAT)
             WRITE(*,170)FO,BW
         170 FORMAT(/1X,'SCENE ESTIMATE WITH'/3X,'FO =',F6.2,1X,'GHz'/3X, BW ='
            7,F8.2,1X,'MHz')
     C       COMPUTE WEIGHTED SCENE ERROR
             DO 200 I = 1, N
1                SHAT(I) = SCENE(I) - SHAT(I)
1        200 CONTINUE
     C       DISPLAY WEIGHTED SCENE ERROR
             WRITE(*,'(1H1)')
             CALL PLOT(N,X,SHAT)
             WRITE(*,210)
         210 FORMAT(/1X,'SCENE ESTIMATE ERROR')
     C       COMPUTE SCENE ERROR
             DO 220 I = 1, N
1                THAT(I) = (SHAT(I) / SCENE(I)) * TEMP
1        220 CONTINUE
     C       DISPLAY SCENE ERROR
             WRITE(*,'(1H1)')
             CALL PLOT(N,X,THAT)
             WRITE(*,230)
         230 FORMAT(/1X,'SCENE ESTIMATE PERCENTAGE ERROR')
             OPEN(24,FILE=OUTFIL,STATUS='NEW')
             WRITE(24,236)RIDNAM,N,SWDTH,FOV,FO,BW,TEMP,MULTFI,WGTFIL,MLTHED,
            2WGTHED
         236 FORMAT(1H",A20,1H",/,I3,5F8.2,/,1H",A20,1H",1H",A20,1H",/,
            31H",A20,1H",1H",A20,1H")
             DO 238 I = 1, N
1               WRITE(24,237)X(I),SCENE(I),SHAT(I),THAT(I)
1        237    FORMAT(4E20.5)
1        238 CONTINUE
             GO TO 10
         240 STOP
             END
```

Name	Type	Offset	P	Class
BW	REAL	9276		
CMPLX				INTRINSIC
D3R	REAL	9236		
DX	REAL	9464		
EXP				INTRINSIC
FO	REAL	9272		
FOV	REAL	9268		
I	INTEGER*4	9410		
J	INTEGER*4	9624		
K	INTEGER*4	9396		
L	INTEGER*4	9438		
MLTHED	CHAR*20	9376		

```
MULT     INTEGER*4          8206
MULTFI   CHAR*20            9284
N        INTEGER*4          9260
NMAX     INTEGER*4                   PARAMETER
OUTFIL   CHAR*20            9324
P        REAL               9472
PHI      REAL               9612
PHIO     REAL               9604
PI       REAL               9232
R        COMPLEX            6160
REAL                                 INTRINSIC
RIDNAM   CHAR*20            9240
SCENE    REAL                 16
SHAT     REAL               2064
SIN                                  INTRINSIC
SINC     REAL                        FUNCTION
SQRT                                 INTRINSIC
SUM      COMPLEX            9616
SWDTH    REAL               9264
TEMP     REAL               9280
THAT     REAL               3088
U        REAL               9632
VO       REAL               9456
W        REAL               9636
WEIGHT   COMPLEX            4112
WGTFIL   CHAR*20            9304
WGTHED   CHAR*20            9418
WORK     COMPLEX            9696
X        REAL               1040
XMIN     REAL               9460


     98
     99
    100

Name     Type               Size    Class

FOURT                               SUBROUTINE
MAIN                                PROGRAM
PLOT                                SUBROUTINE
SINC     REAL                       FUNCTION


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             100 Source Lines

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SINC


```
D Line# 1                                   Microsoft FORTRAN77 V3.20 March 1983
      C       THIS IS A FUNCTION TO COMPUTE SINE(X)/X
              FUNCTION SINC(U)
                IF (U .EQ. 0) GO TO 1
              SINC = SIN(U) / U
              RETURN
            1   SINC = 1
              RETURN
              END

Name      Type          Offset P Class

SIN                                    INTRINSIC
U        REAL                 0 *

Name      Type            Size    Class

SINC     REAL                     FUNCTION


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               8 Source Lines

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PLOT


```
      C     THIS SUBROUTINE PLOTS TO IBM-PC SCREENS
            SUBROUTINE PLOT (N,X,Y)
            PARAMETER (NX = 70,NY = 20)
            DIMENSION X(1),Y(1)
            DIMENSION IXT(5),IYT(5),XLAB(5),YLAB(5)
            CHARACTER*1 SCREEN(NX,NY)
            XLO = X(1)
            YLO = Y(1)
            XHI = X(1)
            YHI = Y(1)
                  DO 30 I = 2, N
   1                 IF (X(I) .LT. XLO) XLO = X(I)
   1                 IF (X(I) .GT. XHI) XHI = X(I)
   1                 IF (Y(I) .LT. YLO) YLO = Y(I)
   1                 IF (Y(I) .GT. YHI) YHI = Y(I)
   1         30   CONTINUE
            XSCALE = (NX - 1) / (XHI - XLO)
            YSCALE = (NY - 1) / (YHI - YLO)
                  DO 35 K = 1, NY
   1                 DO 35 J = 1, NX
   2                    SCREEN(J,K) = ' '
   2         35      CONTINUE
            DO 36 I = 1, NX
   1              SCREEN(I,NY) = '-'
   1         36 CONTINUE
            DO 37 I = 1, NY
   1              SCREEN(1,I) = 'I'
   1         37 CONTINUE
            XTIC = (XHI - XLO) / 4.0
            YTIC = (YHI - YLO) / 4.0
            DO 38 I = 1, 5
   1              XLAB(I) = XTIC * (I - 1) + XLO
   1              YLAB(I) = YTIC * (I - 1) + YLO
   1              K = (XLAB(I) - XLO) * XSCALE + 1
   1              IXT(I) = K
   1              SCREEN(K,NY) = '+'
   1              K = (YHI - YLAB(I)) * YSCALE + 1
   1              IYT(I) = K
   1              SCREEN(1,K) = '+'
   1         38 CONTINUE
            DO 40 I = 1, N
   1              IX = (X(I) - XLO) * XSCALE + 1
   1              IY = (YHI - Y(I)) * YSCALE + 1
   1              SCREEN(IX,IY) = '.'
   1         40 CONTINUE
            DO 50 J = 1, NY
   1           DO 51 K = 1, 5
   2              IF (J .EQ. IYT(K)) GO TO 45
   2         51    CONTINUE
   1        WRITE(*,100)(SCREEN(I,J),I = 1, NX)
   1        100 FORMAT(10X,70A1)
   1        GO TO 50
   1         45 WRITE(*,101) YLAB(K), (SCREEN(I,J),I = 1, NX)
   1        101 FORMAT(1X,G9.2,70A1)
   1         50 CONTINUE
            WRITE(*,102)XLAB
```

```
57    100 FORMAT(6X,G10.2,4G17.2)
58          RETURN
59          END
```

Name	Type	Offset	P	Class
I	INTEGER*4	1516		
IX	INTEGER*4	1552		
IXT	INTEGER*4	1460		
IY	INTEGER*4	1556		
IYT	INTEGER*4	1480		
J	INTEGER*4	1536		
K	INTEGER*4	1532		
N	INTEGER*4	0	*	
NX	INTEGER*4			PARAMETER
NY	INTEGER*4			PARAMETER
SCREEN	CHAR*1	60		
X	REAL	4	*	
XHI	REAL	1508		
XLAB	REAL	40		
XLO	REAL	1500		
XSCALE	REAL	1524		
XTIC	REAL	1540		
Y	REAL	8	*	
YHI	REAL	1512		
YLAB	REAL	20		
YLO	REAL	1504		
YSCALE	REAL	1528		
YTIC	REAL	1544		

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Name	Type	Size	Class
PLOT			SUBROUTINE

Pass One   No Errors Detected
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```
      C SUBROUTINE FOURT FOLLOWS
      C
              SUBROUTINE FOURT(DATA,NN,NDIM,ISIGN,IFORM,WORK)
              DIMENSION DATA(1),NN(1),IFACT(32),WORK(1)
              TWOPI=6.283185307
              IF(NDIM-1)920,1,1
        1     NTOT=2
              DO 2 IDIM=1,NDIM
1             IF(NN(IDIM))920,920,2
1       2     NTOT=NTOT*NN(IDIM)
        C
        C     MAIN LOOP FOR EACH DIMENSION
        C
              NP1=2
              DO 910 IDIM=1,NDIM
1             N=NN(IDIM)
1             NP2=NP1*N
1             IF(N-1)920,900,5
1       C
1       C     FACTOR N
1       C
1       5     M=N
1             NTWO=NP1
1             IF=1
1             IDIV=2
1       10    IQUOT=M/IDIV
1             IREM=M-IDIV*IQUOT
1             IF(IQUOT-IDIV)50,11,11
1       11    IF(IREM)20,12,20
1       12    NTWO=NTWO+NTWO
1             M=IQUOT
1             GO TO 10
1       20    IDIV=3
1       30    IQUOT=M/IDIV
1             IREM=M-IDIV*IQUOT
1             IF(IQUOT-IDIV)60,31,31
1       31    IF(IREM)40,32,40
1       32    IFACT(IF)=IDIV
1             IF=IF+1
1             M=IQUOT
1             GO TO 30
1       40    IDIV=IDIV+2
1             GO TO 30
1       50    IF(IREM)60,51,60
1       51    NTWO=NTWO+NTWO
1             GO TO 70
1       60    IFACT(IF)=M
1       C
1       C     SEPARATE FOUR CASES--
1       C        1. COMPLEX TRANSFORM OR REAL TRANSFORM FOR THE 4TH, 5TH,ETC.
1       C           DIMENSIONS.
1       C        2. REAL TRANSFORM FOR THE 2ND OR 3RD DIMENSION.  METHOD--
1       C           TRANSFORM HALF THE DATA, SUPPLYING THE OTHER HALF BY CON-
1       C           JUGATE SYMMETRY.
1       C        3. REAL TRANSFORM FOR THE 1ST DIMENSION, N ODD.  METHOD---
```

```
D Line# 1      7                         Microsoft FORTRAN77 V3.30 March 1985
1       C          TRANSFORM HALF THE DATA AT EACH STAGE, SUPPLYING THE OTHER
1       C          HALF BY CONJUGATE SYMMETRY.
1       C       4. REAL TRANSFORM FOR THE 1ST DIMENSION. N EVEN.  METHOD--
1       C          TRANSFORM A COMPLEX ARRAY OF LENGTH N/2 WHOSE REAL PARTS
1       C          ARE THE EVEN NUMBERED REAL VALUES AND WHOSE IMAGINARY PARTS
1       C          ARE THE ODD NUMBERED REAL VALUES.  SEPARATE AND SUPPLY
1       C          THE SECOND HALF BY CONJUGATE SYMMETRY.
1       C
1       70      NON2=NP1*(NP2/NTWO)
1               ICASE=1
1               IF(IDIM-4)71,90,90
1       71      IF(IFORM)72,72,90
1       72      ICASE=2
1               IF(IDIM-1)73,73,90
1       73      ICASE=3
1               IF(NTWO-NP1)90,90,74
1       74      ICASE=4
1               NTWO=NTWO/2
1               N=N/2
1               NP2=NP2/2
1               NTOT=NTOT/2
1               I=3
1               DO 80 J=2,NTOT
2               DATA(J)=DATA(I)
2       80      I=I+2
1       90      I1RNG=NP1
1               IF(ICASE-2)100,95,100
1       95      I1RNG=NP0*(1+NPREV/2)
1       C
1       C       SHUFFLE ON THE FACTORS OF TWO IN N.  AS THE SHUFFLING
1       C       CAN BE DONE BY SIMPLE INTERCHANGE, NO WORKING ARRAY IS NEEDED
1       C
1       100     IF(NTWO-NP1)600,600,110
1       110     NP2HF=NP2/2
1               J=1
1               DO 150 I2=1,NP2,NON2
2               IF(J-I2)120,130,130
2       120     I1MAX=I2+NON2-2
2               DO 125 I1=I2,I1MAX,2
3               DO 125 I3=I1,NTOT,NP2
4               J3=J+I3-I2
4               TEMPR=DATA(I3)
4               TEMPI=DATA(I3+1)
4               DATA(I3)=DATA(J3)
4               DATA(I3+1)=DATA(J3+1)
4               DATA(J3)=TEMPR
4       125     DATA(J3+1)=TEMPI
2       130     M=NP2HF
2       140     IF(J-M)150,150,145
2       145     J=J-M
2               M=M/2
2               IF(M-NON2)150,140,140
2       150     J=J+M
1       C
1       C       MAIN LOOP FOR FACTORS OF TWO.  PERFORM FOURIER TRANSFORMS OF
1       C       LENGTH FOUR, WITH ONE OF LENGTH TWO IF NEEDED.  THE TWIDDLE FACTOR
```

```
D Line# 1      7                                Microsoft FORTRAN77 V3.20 March 1985
          C    W=EXP(ISIGN*2*PI*SQRT(-1)*M/(4*MMAX)).  CHECK FOR W=ISIGN*SQRT(-1.
          C    AND REPEAT FOR W=ISIGN*SQRT(-1)*CONJUGATE(W).
          C
1              NON2T=NON2+NON2
1              IPAR=NTWO/NP1
1         310  IF(IPAR-2)350,330,320
1         320  IPAR=IPAR/4
1              GO TO 310
1         330  DO 340 I1=1,I1RNG,2
2              DO 340 J3=I1,NON2,NP1
3              DO 340 K1=J3,NTOT,NON2T
4              K2=K1+NON2
4              TEMPR=DATA(K2)
4              TEMPI=DATA(K2+1)
4              DATA(K2)=DATA(K1)-TEMPR
4              DATA(K2+1)=DATA(K1+1)-TEMPI
4              DATA(K1)=DATA(K1)+TEMPR
4         340  DATA(K1+1)=DATA(K1+1)+TEMPI
1         350  MMAX=NON2
1         360  IF(MMAX-NP2HF)370,600,600
1         370  LMAX=MAX0(NON2T,MMAX/2)
1              IF(MMAX-NON2)405,405,380
1         380  THETA=-TWOPI*FLOAT(NON2)/FLOAT(4*MMAX)
1              IF(ISIGN)400,390,390
1         390  THETA=-THETA
1         400  WR=COS(THETA)
1              WI=SIN(THETA)
1              WSTPR=-2.*WI*WI
1              WSTPI=2.*WR*WI
1         405  DO 570 L=NON2,LMAX,NON2T
2              M=L
2              IF(MMAX-NON2)420,420,410
2         410  W2R=WR*WR-WI*WI
2              W2I=2.*WR*WI
2              W3R=W2R*WR-W2I*WI
2              W3I=W2R*WI+W2I*WR
2         420  DO 530 I1=1,I1RNG,2
3              DO 530 J3=I1,NON2,NP1
4              KMIN=J3+IPAR*M
4              IF(MMAX-NON2)430,430,440
4         430  KMIN=J3
4         440  KDIF=IPAR*MMAX
4         450  KSTEP=4*KDIF
4              DO 520 K1=KMIN,NTOT,KSTEP
5              K2=K1+KDIF
5              K3=K2+KDIF
5              K4=K3+KDIF
5              IF(MMAX-NON2)460,460,480
5         460  U1R=DATA(K1)+DATA(K2)
5              U1I=DATA(K1+1)+DATA(K2+1)
5              U2R=DATA(K3)+DATA(K4)
5              U2I=DATA(K3+1)+DATA(K4+1)
5              U3R=DATA(K1)-DATA(K2)
5              U3I=DATA(K1+1)-DATA(K2+1)
5              IF(ISIGN)470,475,475
5         470  U4R=DATA(K3+1)-DATA(K4+1)
```

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```
5          U4I=DATA(K4)-DATA(K3)
5          GO TO 510
5    475   U4R=DATA(K4+1)-DATA(K3+1)
5          U4I=DATA(K3)-DATA(K4)
5          GO TO 510
5    480   T2R=W2R*DATA(K2)-W2I*DATA(K2+1)
5          T2I=W2R*DATA(K2+1)+W2I*DATA(K2)
5          T3R=WR*DATA(K3)-WI*DATA(K3+1)
5          T3I=WR*DATA(K3+1)+WI*DATA(K3)
5          T4R=W3R*DATA(K4)-W3I*DATA(K4+1)
5          T4I=W3R*DATA(K4+1)+W3I*DATA(K4)
5          U1R=DATA(K1)+T2R
5          U1I=DATA(K1+1)+T2I
5          U2R=T3R+T4R
5          U2I=T3I+T4I
5          U3R=DATA(K1)-T2R
5          U3I=DATA(K1+1)-T2I
5          IF(ISIGN)490,500,500
5    490   U4R=T3I-T4I
5          U4I=T4R-T3R
5          GO TO 510
5    500   U4R=T4I-T3I
5          U4I=T3R-T4R
5    510   DATA(K1)=U1R+U2R
5          DATA(K1+1)=U1I+U2I
5          DATA(K2)=U3R+U4R
5          DATA(K2+1)=U3I+U4I
5          DATA(K3)=U1R-U2R
5          DATA(K3+1)=U1I-U2I
5          DATA(K4)=U3R-U4R
5    520   DATA(K4+1)=U3I-U4I
4          KMIN=4*(KMIN-J3)+J3
4          KDIF=KSTEP
4          IF(KDIF-NP2)450,530,530
4    530   CONTINUE
2          M=MMAX-M
2          IF(ISIGN)540,550,550
2    540   TEMPR=WR
2          WR=-WI
2          WI=-TEMPR
2          GO TO 560
2    550   TEMPR=WR
2          WR=WI
2          WI=TEMPR
2    560   IF(M-LMAX)565,565,410
2    565   TEMPR=WR
2          WR=WR*WSTPR-WI*WSTPI+WR
2    570   WI=WI*WSTPR+TEMPR*WSTPI+WI
1          IPAR=3-IPAR
1          MMAX=MMAX+MMAX
1          GO TO 360
1     C
1     C    MAIN LOOP FOR FACTORS NOT EQUAL TO TWO.  APPLY THE TWIDDLE FACTOR
1     C    W=EXP(ISIGN*2*PI*SQRT(-1)*(J2-1)*(J1-J2)/(NP2*IFP1)), THEN
1     C    PERFORM A FOURIER TRANSFORM OF LENGTH IFACT(IF), MAKING USE OF
1     C    CONJUGATE SYMMETRIES.
```

```
1     C
1     600   IF(NTWO-NP2)605,700,700
      605   IFP1=NON2
1           IF=1
1           NP1HF=NP1/2
1     610   IFP2=IFP1/IFACT(IF)
1           J1RNG=NP2
1           IF(ICASE-3)612,611,612
1     611   J1RNG=(NP2+IFP1)/2
1           J2STP=NP2/IFACT(IF)
1           J1RG2=(J2STP+IFP2)/2
1     612   J2MIN=1+IFP2
1           IF(IFP1-NP2)615,640,640
1     615   DO 635 J2=J2MIN,IFP1,IFP2
2           THETA=-TWOPI*FLOAT(J2-1)/FLOAT(NP2)
2           IF(ISIGN)625,620,620
2     620   THETA=-THETA
2     625   SINTH=SIN(THETA/2.)
2           WSTPR=-2.*SINTH*SINTH
2           WSTPI=SIN(THETA)
2           WR=WSTPR+1.
2           WI=WSTPI
2           J1MIN=J2+IFP1
2           DO 635 J1=J1MIN,J1RNG,IFP1
3           I1MAX=J1+I1RNG-2
3           DO 630 I1=J1,I1MAX,2
4           DO 630 I3=I1,NTOT,NP2
5           J3MAX=I3+IFP2-NP1
5           DO 630 J3=I3,J3MAX,NP1
6           TEMPR=DATA(J3)
6           DATA(J3)=DATA(J3)*WR-DATA(J3+1)*WI
6     630   DATA(J3+1)=TEMPR*WI+DATA(J3+1)*WR
3           TEMPR=WR
3           WR=WR*WSTPR-WI*WSTPI+WR
3     635   WI=TEMPR*WSTPI+WI*WSTPR+WI
1     640   THETA=-TWOPI/FLOAT(IFACT(IF))
1           IF(ISIGN)650,645,645
1     645   THETA=-THETA
1     650   SINTH=SIN(THETA/2.)
1           WSTPR=-2.*SINTH*SINTH
1           WSTPI=SIN(THETA)
1           KSTEP=2*N/IFACT(IF)
1           KRANG=KSTEP*(IFACT(IF)/2)+1
1           DO 698 I1=1,I1RNG,2
2           DO 698 I3=I1,NTOT,NP2
3           DO 690 KMIN=1,KRANG,KSTEP
4           J1MAX=I3+J1RNG-IFP1
4           DO 680 J1=I3,J1MAX,IFP1
5           J3MAX=J1+IFP2-NP1
5           DO 680 J3=J1,J3MAX,NP1
6           J2MAX=J3+IFP1-IFP2
6           K=KMIN+(J3-J1+(J1-I3)/IFACT(IF))/NP1HF
6           IF(KMIN-1)655,655,665
6     655   SUMR=0.
6           SUMI=0.
6           DO 660 J2=J3,J2MAX,IFP2
```

```
7       SUMR=SUMR+DATA(J2)
7  660  SUMI=SUMI+DATA(J2+1)
6       WORK(K)=SUMR
6       WORK(K+1)=SUMI
6       GO TO 680
6  665  KCONJ=K+2*(N-KMIN+1)
6       J2=J2MAX
6       SUMR=DATA(J2)
6       SUMI=DATA(J2+1)
6       OLDSR=0.
6       OLDSI=0.
6       J2=J2-IFP2
6  670  TEMPR=SUMR
6       TEMPI=SUMI
6       SUMR=TWOWR*SUMR-OLDSR+DATA(J2)
6       SUMI=TWOWR*SUMI-OLDSI+DATA(J2+1)
6       OLDSR=TEMPR
6       OLDSI=TEMPI
6       J2=J2-IFP2
6       IF(J2-J3)675,675,670
6  675  TEMPR=WR*SUMR-OLDSR+DATA(J2)
6       TEMPI=WI*SUMI
6       WORK(K)=TEMPR-TEMPI
6       WORK(KCONJ)=TEMPR+TEMPI
6       TEMPR=WR*SUMI-OLDSI+DATA(J2+1)
6       TEMPI=WI*SUMR
6       WORK(K+1)=TEMPR+TEMPI
6       WORK(KCONJ+1)=TEMPR-TEMPI
6  680  CONTINUE
4       IF(KMIN-1)685,685,686
4  685  WR=WSTPR+1.
4       WI=WSTPI
4       GO TO 690
4  686  TEMPR=WR
4       WR=WR*WSTPR-WI*WSTPI+WR
4       WI=TEMPR*WSTPI+WI*WSTPR+WI
4  690  TWOWR=WR+WR
3       IF(ICASE-3)692,691,692
3  691  IF(IFP1-NP2)695,692,692
3  692  K=1
3       I2MAX=I3+NP2-NP1
3       DO 693 I2=I3,I2MAX,NP1
4       DATA(I2)=WORK(K)
4       DATA(I2+1)=WORK(K+1)
4  693  K=K+2
3       GO TO 698
3  C
3  C    COMPLETE A REAL TRANSFORM IN THE 1ST DIMENSION, N ODD, BY CON-
3  C    JUGATE SYMMETRIES AT EACH STAGE.
3  C
3  695  J3MAX=I3+IFP2-NP1
3       DO 697 J3=I3,J3MAX,NP1
4       J2MAX=J3+NP2-J2STP
4       DO 697 J2=J3,J2MAX,J2STP
5       J1MAX=J2+J1RG2-IFP2
5       J1CNJ=J3+J2MAX+J2STP-J2
```

```
5          DO 697 J1=J2,J1MAX,IFP2
6          K=1+J1-I3
6          DATA(J1)=WORK(K)
6          DATA(J1+1)=WORK(K+1)
6          IF(J1-J2)697,697,696
6     696  DATA(J1CNJ)=WORK(K)
6          DATA(J1CNJ+1)=-WORK(K+1)
6     697  J1CNJ=J1CNJ-IFP2
3     698  CONTINUE
1          IF=IF+1
1          IFP1=IFP2
1          IF(IFP1-NP1)700,700,610
1     C
1     C    COMPLETE A REAL TRANSFORM IN THE 1ST DIMENSION, N EVEN, BY CON-
1     C    JUGATE SYMMETRIES.
1     C
1     700  GO TO (900,800,900,701),ICASE
1     701  NHALF=N
1          N=N+N
1          THETA=-TWOPI/FLOAT(N)
1          IF(ISIGN)703,702,702
1     702  THETA=-THETA
1     703  SINTH=SIN(THETA/2.)
1          WSTPR=-2.*SINTH*SINTH
1          WSTPI=SIN(THETA)
1          WR=WSTPR+1.
1          WI=WSTPI
1          IMIN=3
1          JMIN=2*NHALF-1
1          GO TO 725
1     710  J=JMIN
1          DO 720 I=IMIN,NTOT,NP2
2          SUMR=(DATA(I)+DATA(J))/2.
2          SUMI=(DATA(I+1)+DATA(J+1))/2.
2          DIFR=(DATA(I)-DATA(J))/2.
2          DIFI=(DATA(I+1)-DATA(J+1))/2.
2          TEMPR=WR*SUMI+WI*DIFR
2          TEMPI=WI*SUMI-WR*DIFR
2          DATA(I)=SUMR+TEMPR
2          DATA(I+1)=DIFI+TEMPI
2          DATA(J)=SUMR-TEMPR
2          DATA(J+1)=-DIFI+TEMPI
2     720  J=J+NP2
1          IMIN=IMIN+2
1          JMIN=JMIN-2
1          TEMPR=WR
1          WR=WR*WSTPR-WI*WSTPI+WR
1          WI=TEMPR*WSTPI+WI*WSTPR+WI
1     725  IF(IMIN-JMIN)710,730,740
1     730  IF(ISIGN)731,740,740
1     731  DO 735 I=IMIN,NTOT,NP2
2     735  DATA(I+1)=-DATA(I+1)
1     740  NP2=NP2+NP2
1          NTOT=NTOT+NTOT
1          J=NTOT+1
1          IMAX=NTOT/2+1
```

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```
1     745  IMIN=IMAX-2*NHALF
1          I=IMIN
1          GO TO 755
1     750  DATA(J)=DATA(I)
1          DATA(J+1)=-DATA(I+1)
1     755  I=I+2
1          J=J-2
1          IF(I-IMAX)750,760,760
1     760  DATA(J)=DATA(IMIN)-DATA(IMIN+1)
1          DATA(J+1)=0.
1          IF(I-J)770,780,780
1     765  DATA(J)=DATA(I)
1          DATA(J+1)=DATA(I+1)
1     770  I=I-2
1          J=J-2
1          IF(I-IMIN)775,775,765
1     775  DATA(J)=DATA(IMIN)+DATA(IMIN+1)
1          DATA(J+1)=0.
1          IMAX=IMIN
1          GO TO 745
1     780  DATA(1)=DATA(1)+DATA(2)
1          DATA(2)=0.
1          GO TO 900
1     C
1     C    COMPLETE A REAL TRANSFORM FOR THE 2ND OR 3RD DIMENSION BY
1     C    CONJUGATE SYMMETRIES.
1     C
1     800  IF(I1RNG-NP1)805,900,900
1     805  DO 860 I3=1,NTOT,NP2
2          I2MAX=I3+NP2-NP1
2          DO 860 I2=I3,I2MAX,NP1
3          IMIN=I2+I1RNG
3          IMAX=I2+NP1-2
3          JMAX=2*I3+NP1-IMIN
3          IF(I2-I3)820,820,810
3     810  JMAX=JMAX+NP2
3     820  IF(IDIM-2)850,850,830
3     830  J=JMAX+NP0
3          DO 840 I=IMIN,IMAX,2
4          DATA(I)=DATA(J)
4          DATA(I+1)=-DATA(J+1)
4     840  J=J-2
3     850  J=JMAX
3          DO 860 I=IMIN,IMAX,NP0
4          DATA(1)=DATA(J)
4          DATA(I+1)=-DATA(J+1)
4     860  J=J-NP0
1     C
1     C    END OF LOOP ON EACH DIMENSION
1     C
1     900  NP0=NP1
1          NP1=NP2
1     910  NPREV=N
      920  RETURN
           END
```

Name	Type	Offset	P	Class
CMS				INTRINSIC
DATA	REAL	0	*	
DIFI	REAL	704		
DIFR	REAL	700		
FLOAT				INTRINSIC
I	INTEGER*4	208		
I1	INTEGER*4	252		
I1MAX	INTEGER*4	248		
I1RNG	INTEGER*4	220		
I2	INTEGER*4	236		
I2MAX	INTEGER*4	640		
I3	INTEGER*4	260		
ICASE	INTEGER*4	204		
IDIM	INTEGER*4	152		
IDIV	INTEGER*4	188		
IF	INTEGER*4	184		
IFACT	INTEGER*4	16		
IFORM	INTEGER*4	16	*	
IFP1	INTEGER*4	472		
IFP2	INTEGER*4	480		
IMAX	INTEGER*4	716		
IMIN	INTEGER*4	684		
IPAR	INTEGER*4	288		
IQUOT	INTEGER*4	192		
IREM	INTEGER*4	196		
ISIGN	INTEGER*4	12	*	
J	INTEGER*4	212		
J1	INTEGER*4	520		
J1CNJ	INTEGER*4	668		
J1MAX	INTEGER*4	580		
J1MIN	INTEGER*4	516		
J1RG2	INTEGER*4	492		
J1RNG	INTEGER*4	484		
J2	INTEGER*4	500		
J2MAX	INTEGER*4	600		
J2MIN	INTEGER*4	496		
J2STP	INTEGER*4	488		
J3	INTEGER*4	272		
J3MAX	INTEGER*4	544		
JMAX	INTEGER*4	736		
JMIN	INTEGER*4	688		
K	INTEGER*4	604		
K1	INTEGER*4	304		
K2	INTEGER*4	316		
K3	INTEGER*4	408		
K4	INTEGER*4	412		
KCONJ	INTEGER*4	624		
KDIF	INTEGER*4	392		
KMIN	INTEGER*4	388		
KRANG	INTEGER*4	556		
KSTEP	INTEGER*4	396		
L	INTEGER*4	348		
LMAX	INTEGER*4	324		

/

M	INTEGER*4	176		
MAXO				INTRINSIC
NMAX	INTEGER*4	320		
N	INTEGER*4	168		
NDIM	INTEGER*4	8	*	
NHALF	INTEGER*4	680		
NN	INTEGER*4	4	*	
NON2	INTEGER*4	200		
NON2T	INTEGER*4	284		
NP0	INTEGER*4	224		
NP1	INTEGER*4	160		
NP1HF	INTEGER*4	476		
NP2	INTEGER*4	172		
NP2HF	INTEGER*4	232		
NPREV	INTEGER*4	228		
NTOT	INTEGER*4	148		
NTWO	INTEGER*4	180		
OLDSI	REAL	632		
OLDSR	REAL	628		
SIN				INTRINSIC
SINTH	REAL	512		
SUMI	REAL	612		
SUMR	REAL	608		
T2I	REAL	452		
T2R	REAL	448		
T3I	REAL	460		
T3R	REAL	456		
T4I	REAL	468		
T4R	REAL	464		
TEMPI	REAL	280		
TEMPR	REAL	276		
THETA	REAL	328		
TWOPI	REAL	144		
TWOWR	REAL	636		
U1I	REAL	420		
U1R	REAL	416		
U2I	REAL	428		
U2R	REAL	424		
U3I	REAL	436		
U3R	REAL	432		
U4I	REAL	444		
U4R	REAL	440		
W2I	REAL	364		
W2R	REAL	360		
W3I	REAL	372		
W3R	REAL	368		
WI	REAL	336		
WORK	REAL	20	*	
WR	REAL	332		
WSTPI	REAL	344		
WSTPR	REAL	340		

Name   Type   Size   Class

```
Pass One     No Errors Detected
             447 Source Lines
```

A

BASELINE


```
D Line# 1       7                                    Microsoft FORTRAN77 V3.30 March 1985
     C PROGRAM READS (OPEN) DATA BY KEYBOARD DEFINITION
           DIMENSION IANT(32),MULT(496)
           CHARACTER*64 INFIL,OUTFIL,FNAME
           WRITE(*,'(3X,A\)')'INPUT FILE NAME IANT(N).DAT ?'
           READ(*,'(A)')INFIL
           OPEN(12,FILE=INFIL,STATUS='OLD')
           READ(12,10)FNAME,NANT
        10 FORMAT(A10/,I5)
           DO 15 I = 1, NANT
 1         READ(12,12)IANT(I)
 1      12 FORMAT(I5)
 1      15 CONTINUE
           NBASE = NANT*(NANT - 1) / 2
           DO 20 I = I, NBASE
 1         MULT(1) = 0
 1      20 CONTINUE
           IMAX = NANT - 1
           DO 30 I = 1, IMAX
 1         JMIN = I + 1
 1         DO 25 J = JMIN, NANT
 2         IBASE = ABS(IANT(I) - IANT(J))
 2         MULT(IBASE) = MULT(IBASE) + 1
 2      25 CONTINUE
 1      30 CONTINUE
           IMAX = NBASE
           DO 32 I = IMAX, 1, -1
 1            IF(MULT(I) .GT. 0) THEN
 1             GO TO 33
 1            END IF
 1            IF(MULT(I) .EQ. 0) THEN
 1             NBASE = NBASE - 1
 1            END IF
 1      32 CONTINUE
        33 WRITE(*,'(3X,A\)') OUTPUT FILE NAME MULT(N).DAT ?'
           READ(*,'(A)')OUTFIL
           OPEN(14,FILE=OUTFIL,STATUS='NEW')
           WRITE(14,35)FNAME,NBASE
        35 FORMAT(1X,'DATA TAKEN FROM ',A10,'POSSIBLE BASELINE COMBOS IS 'I5)
           WRITE(14,36)
        36 FORMAT(1X,'BASELINE',5X,'MULTIPLICITY')
           WRITE(14,37)0,NANT
        37 FORMAT(4X,I3,10X,I5)
           DO 40 I = 1, NBASE
 1         WRITE(14,38)I,MULT(I)
 1      38 FORMAT(4X,I3,10X,I5)
 1      40 CONTINUE
           STOP
           END

Name     Type          Offset P Class

ABS                             INTRINSIC
FNAME    CHAR*64          2196
I        INTEGER*4        2274
IANT     INTEGER*4          20
IBASE    INTEGER*4        231[illegible]
```

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```
IMAX    INTEGER*4        2294
INFIL   CHAR*64          2132
        INTEGER*4        2306
JMIN    INTEGER*4        2302
MULT    INTEGER*4         148
NANT    INTEGER*4        2260
NBASE   INTEGER*4        2286
OUTFIL  CHAR*64          2318


     49

Name     Type           Size    Class

MAIN                            PROGRAM


 Pass One     No Errors Detected
              49 Source Lines

A
```

```
TYPE MASTER.DAT
DEMONSTRATION RUN 1
 32
90.
60.
1.4
0.
250.
MULT.DAT
WGT.DAT
OUTPUT1.PRN
DEMONSTRATION RUN 2
 32
90.
60.
1.4
10.
250.
MULT.DAT
WGT.DAT
OUTPUT2.PRN
DEMONSTRATION RUN 3
 32
90.
60.
1.4
20.
250.
MULT.DAT
WGT.DAT
OUTPUT3.PRN
DEMONSTRATION RUN 4
 32
90.
60.
1.4
30.
250.
MULT.DAT
WGT.DAT
OUTPUT4.PRN
DEMONSTRATION RUN 5
 32
90.
60.
1.4
40.
250.
MULT.DAT
WGT.DAT
OUTPUT5.PRN
DEMONSTRATION RUN 6
 32
90.
60.
1.4
50.
250.
MULT.DAT
WGT.DAT
OUTPUT6.PRN

A
```

```
TYPE MULT.DAT
TEST
 32
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1
      1

A>
```

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WGT.DAT

```
TYPE WGT.DAT
TEST
22
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
        1.            0.
$
```

```
[illegible] MULT.DAT TYPE OUTPUT.DAT TYPE OUTPUT.[illegible]
'TEST DATA                    "
 [illegible] 100.00   [illegible].00    1.40   50.[illegible]   300.00
'MULT.DAT                     '"WGT.DAT                  "
'TEST                         ""TEST                     "
```

-.98481E+00	.40[illegible]89E+00	-.84[illegible]59E+00	-.61962E+03
-.92127E+00	.51[illegible]84E+01	-.1[illegible]781E+00	-.73905E+01
-.85774E+00	.12[illegible]08E+02	.7[illegible]703E+00	.18111E+02
-.79420E+00	.18[illegible]6E+02	.[illegible]2507E+01	.20463E+02
-.73066E+00	.20542E+02	.11626E+01	.16978E+02
-.66713E+00	.17248E+02	.54469E+00	.94742E+01
-.60359E+00	.96039E+01	-.26470E+00	-.82685E+01
-.54006E+00	.19198E+01	-.84790E+00	-.13250E+03
-.47652E+00	.82250E+00	-.93209E+00	-.33997E+03
-.41298E+00	.13224E+02	-.52885E+00	-.11998E+02
-.34945E+00	.43689E+02	.84499E-01	.58023E+00
-.28591E+00	.92154E+02	.52903E+00	.17222E+01
-.22238E+00	.15291E+03	.54013E+00	.10597E+01
-.15884E+00	.21546E+03	.11467E+00	.15966E+00
-.95304E-01	.26703E+03	-.49298E+00	-.55385E+00
-.31768E-01	.29619E+03	-.92316E+00	-.93504E+00
.31768E-01	.29619E+03	-.92307E+00	-.93495E+00
.95304E-01	.26703E+03	-.49280E+00	-.55365E+00
.15884E+00	.21546E+03	.11443E+00	.15932E+00
.22238E+00	.15291E+03	.54010E+00	.10596E+01
.28591E+00	.92154E+02	.52922E+00	.17228E+01
.34945E+00	.43689E+02	.84633E-01	.58115E+00
.41298E+00	.13224E+02	-.52876E+00	-.11996E+02
.47652E+00	.82251E+00	-.93185E+00	-.33988E+03
.54006E+00	.19198E+01	-.84776E+00	-.13247E+03
.60359E+00	.96039E+01	-.26428E+00	-.82554E+01
.66713E+00	.17248E+02	.54475E+00	.94751E+01
.73066E+00	.20542E+02	.11626E+01	.16979E+02
.79420E+00	.18336E+02	.12508E+01	.20464E+02
.85774E+00	.12208E+02	.73694E+00	.18109E+02
.92127E+00	.51884E+01	-.12771E+00	-.73845E+01
.98481E+00	.40990E+00	-.84685E+00	-.61980E+03

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VII. SIMULATION RESULTS

In this section we present our simulation results using a simple but realistic sensor as an example. The sensor parameters are listed in table VII-1. A scene width of 90^0 and a nominal temperature of 250K are used.

The sensor resolution is approximately the scene sampling interval ($90^0/32$). At an altitude of 300 km this corresponds to 15 Km resolution with an antenna array dimension given approximately by

$$D = \frac{(.21\,m)(300\,Km)}{15\,Km} = 4.2\,m$$

The value .21 m is the wavelength. There are no grating lobes as the value

$$\sin \Theta_{GL} = \frac{(.21m)N}{D} > 1$$

Figure VII-1 shows the constant test scene weighted by the antenna power and Jacobian factors as a function of scene coordinate (sine of across track angle). Figures VII-2 to 7 show the weighted errors in the scene estimates. By this we mean the error before the antenna and Jacobian factor are removed. Figure 2 is for zero bandwidth, in this case the Fourier processing is perfect and only random round-off error is present. Figures VII - 3 to 7 have bandwidths of 10, 20, 30, 40, and 50 MHz respectively. As the bandwidth increases the error amplitude envelope increases.

TABLE VII - 1

Sensor Parameters

Name	Parameter	Value
N	Number of baselines	32
FOV	Field-of-view	60^o
FO	Center frequency	1.4 GHz
BW	Bandwidth	0, 10, 20, 30, 40, 50 MHz

Uniform multiplicities

Uniform processing weights

The most important information is contained in figures VII - 8 to 12 which show the radiometric (or unweighted) errors. These errors are small but nonzero near nadir (scene coordinate 0). Off-nadir they become large, especially near the pattern nulls. In general, a region around nadir exists where radiometric errors are uniformly tolerable. Defining this region to have radiometric errors less than say 1.0 K specifies the useful field-of-view of this sensor. In figure VII - 13 we have plotted useful field-of-view versus bandwidth by interpolating the scene coordinate for which the radiometric error exceeds 1.0 K. At zero bandwidth the entire 60° field-of-view is usable, but this diminishes as the bandwidth increases. Finally, at 50 MHz there is no usable field-of-view since nadir radiometric errors exceed 1.0 K. The bandwidth of 30 MHz (just greater than that allocated for passive use) has a useful field-of-view of about 30° by this definition. Obviously there is a tradeoff here between radiometric errors induced by this effect and the receiver radiometric sensitivity, since the former increases with bandwidth and the latter decreases.

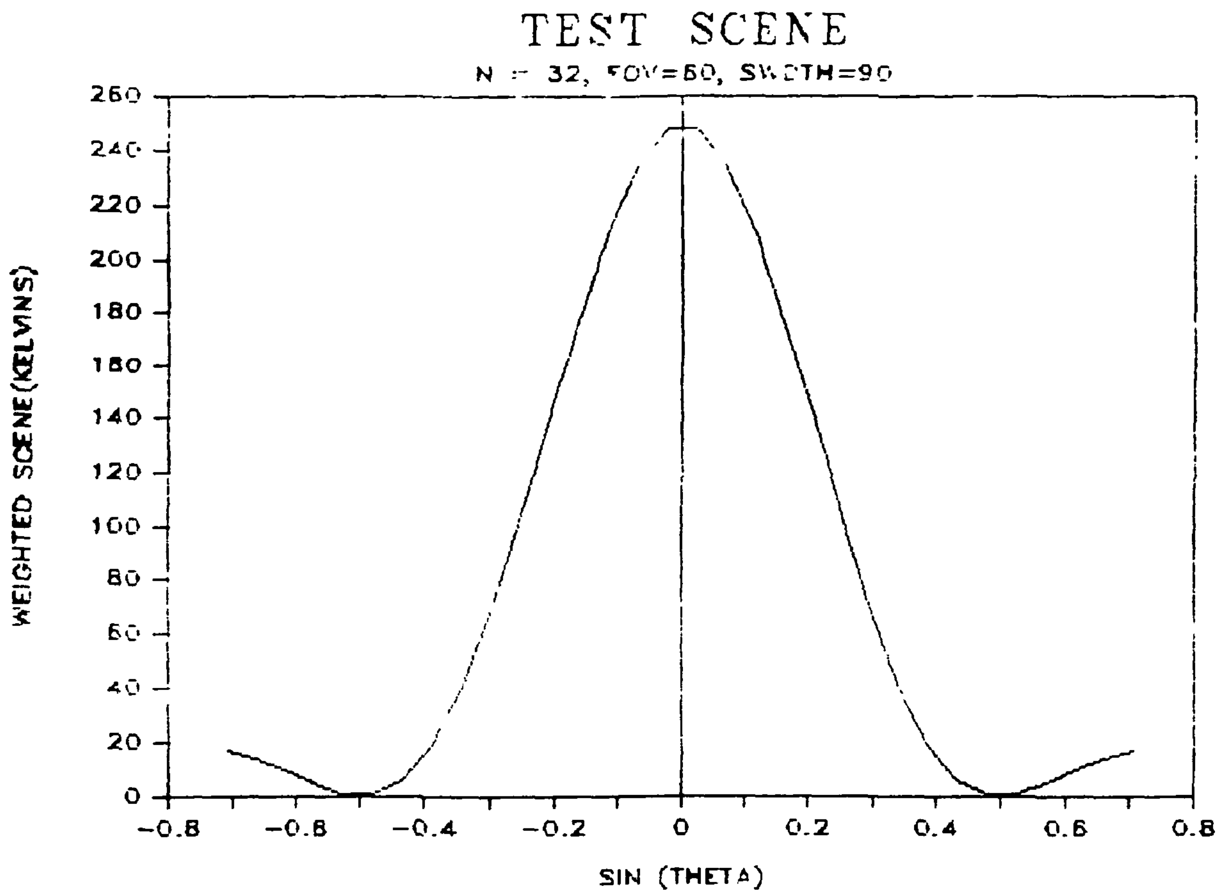

Figure VII-1. Test Scene

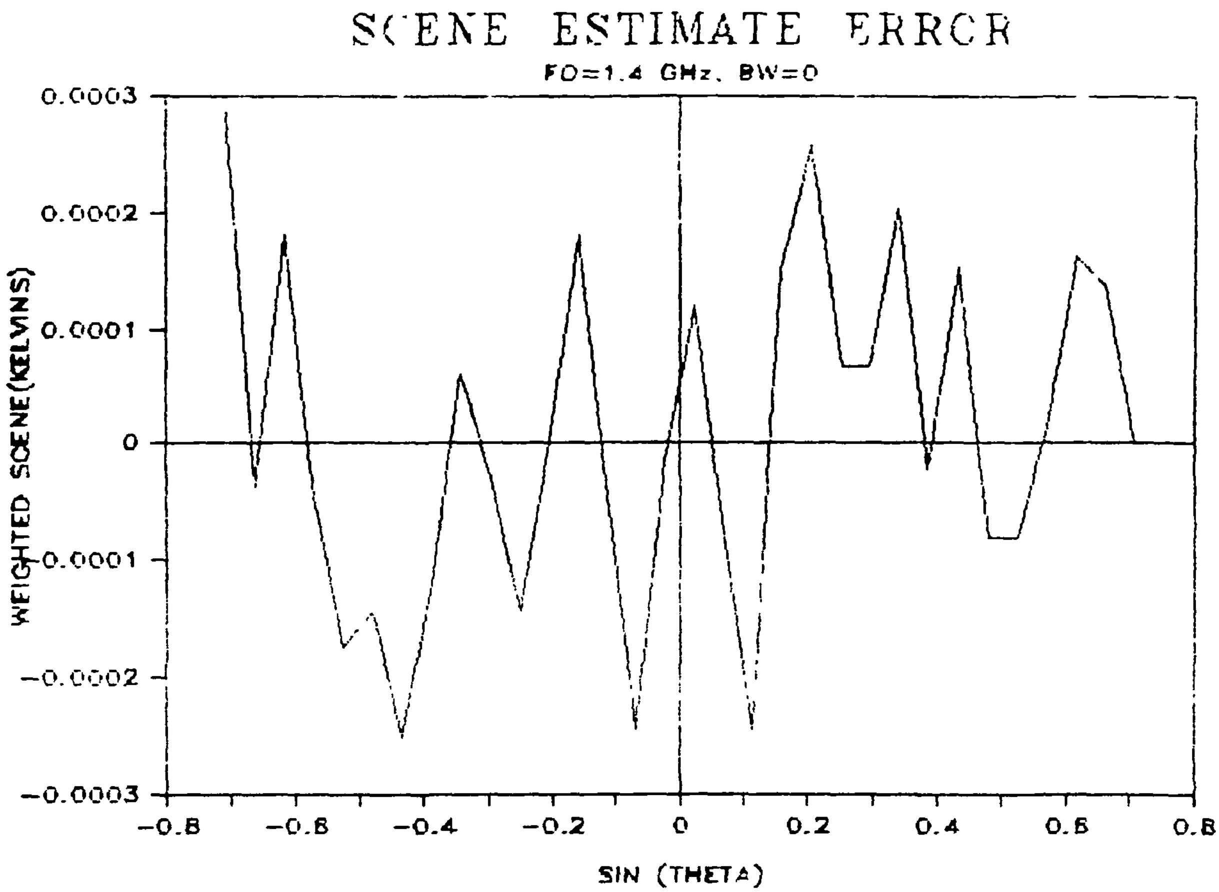

Figure VII-2. Scene Estimate Error, BW = 0.

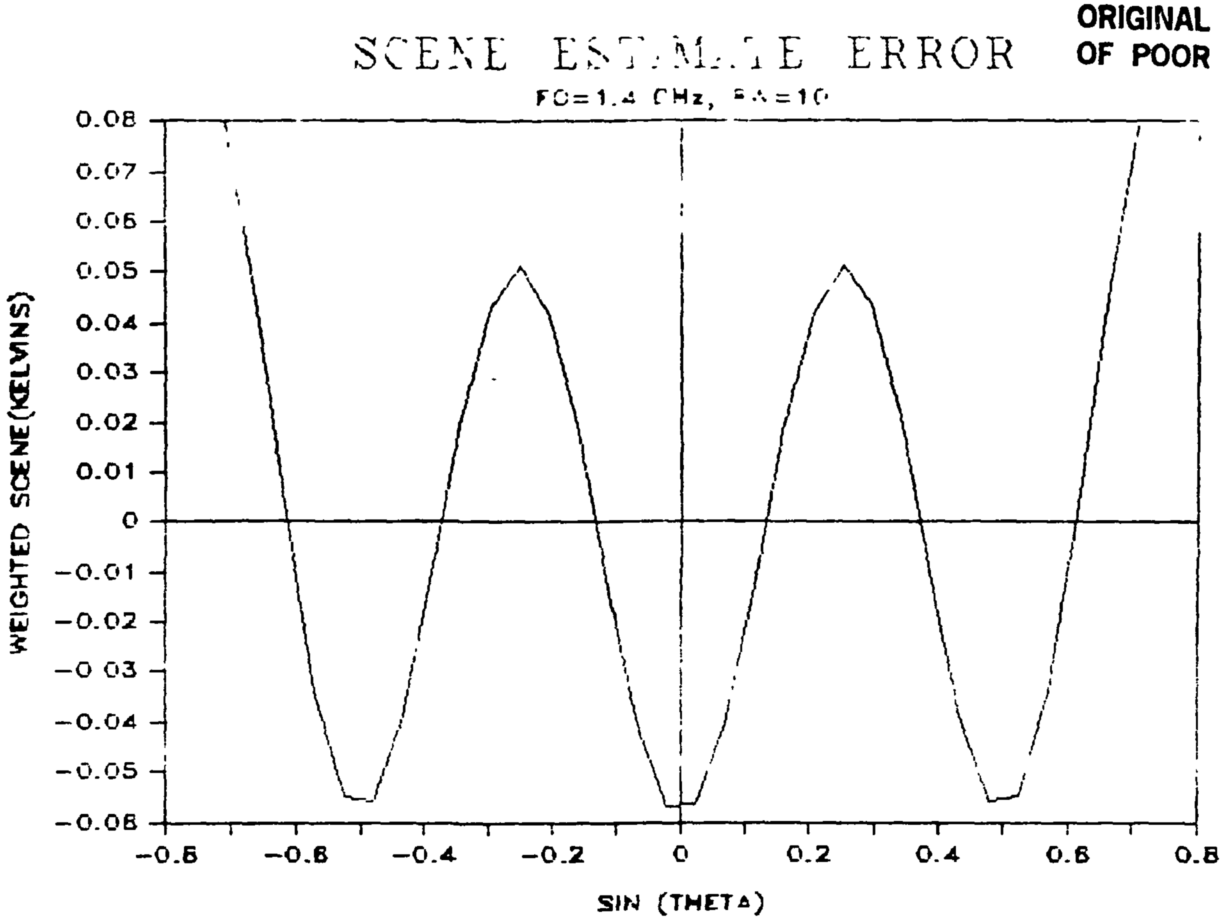

Figure VII-3. Scene Estimate Error, BW = 10 MHz.

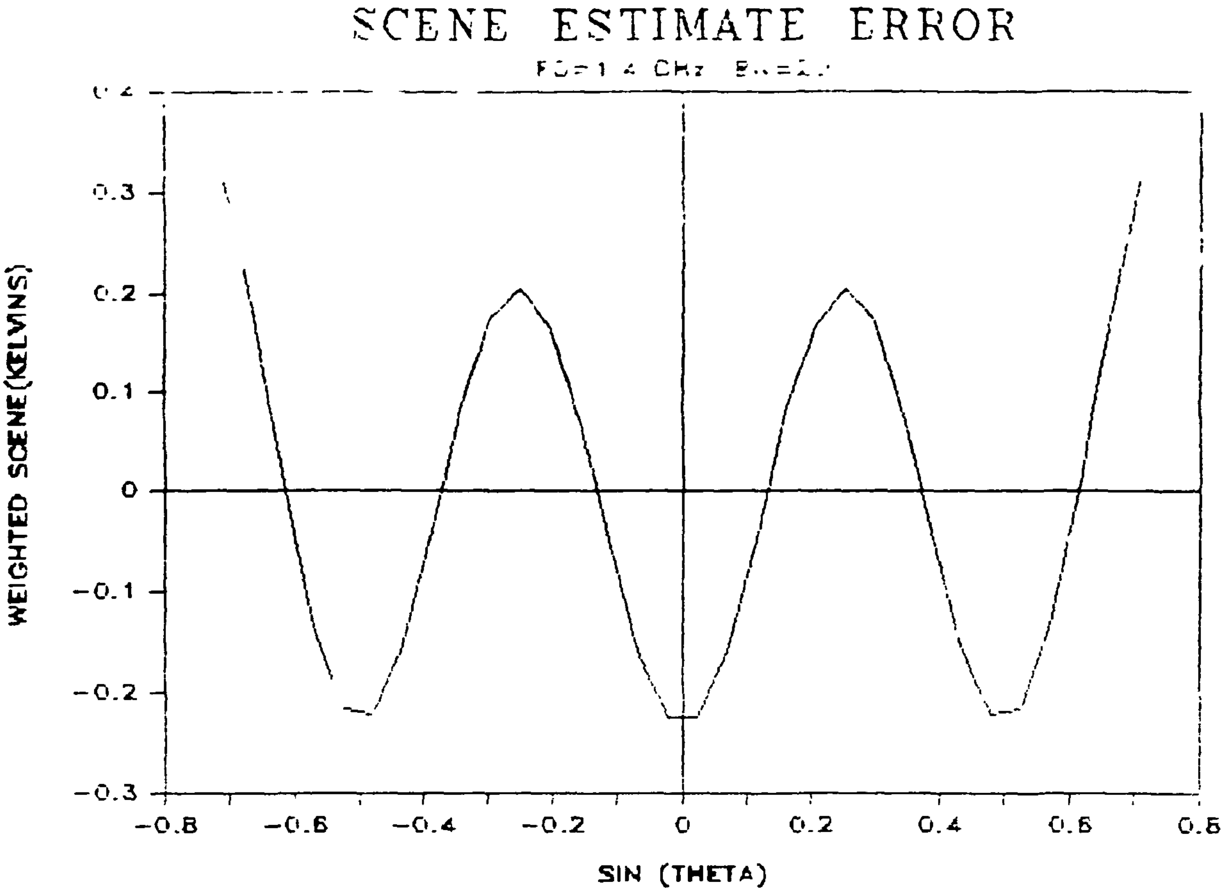

Figure VII-4. Scene Estimate Error, BW = 20 MHz.

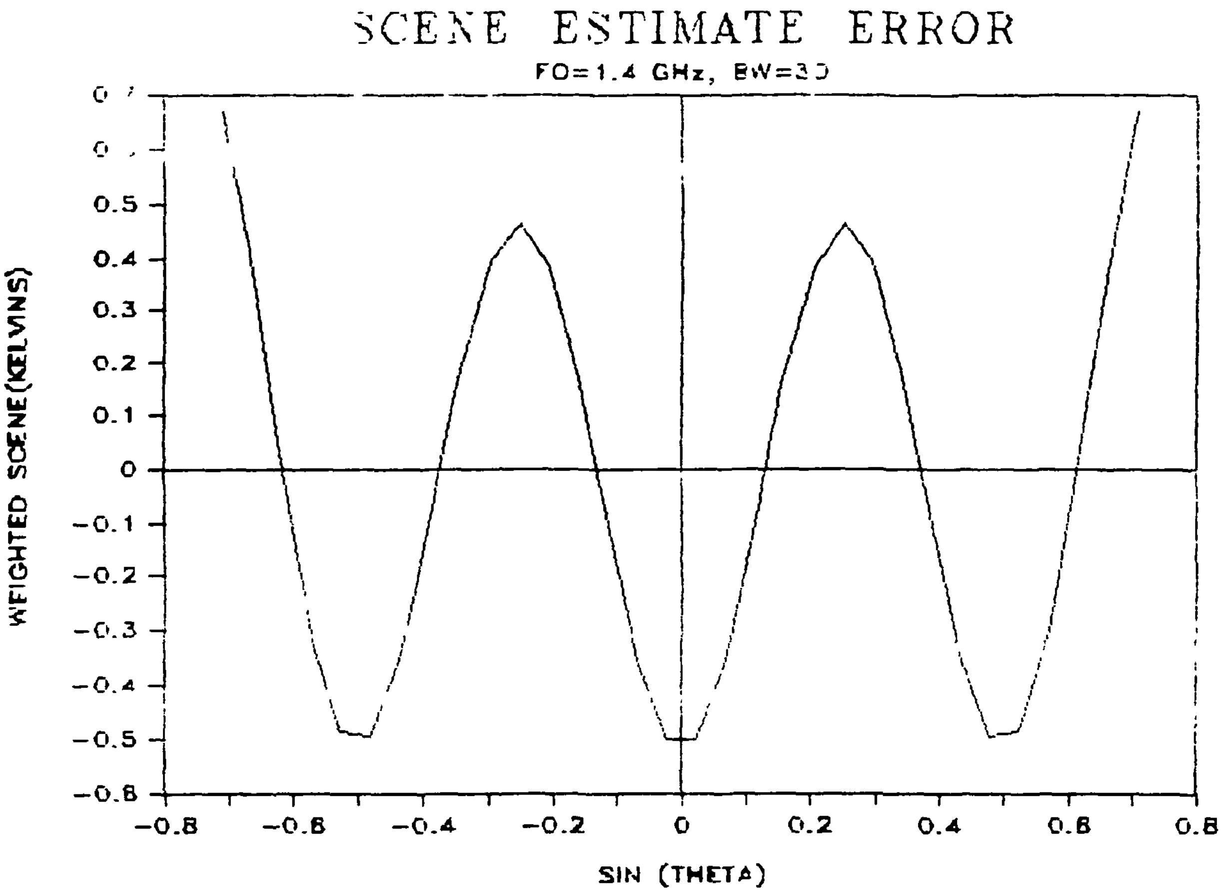

Figure VII-5. Scene Estimate Error, BW = 30 MHz.

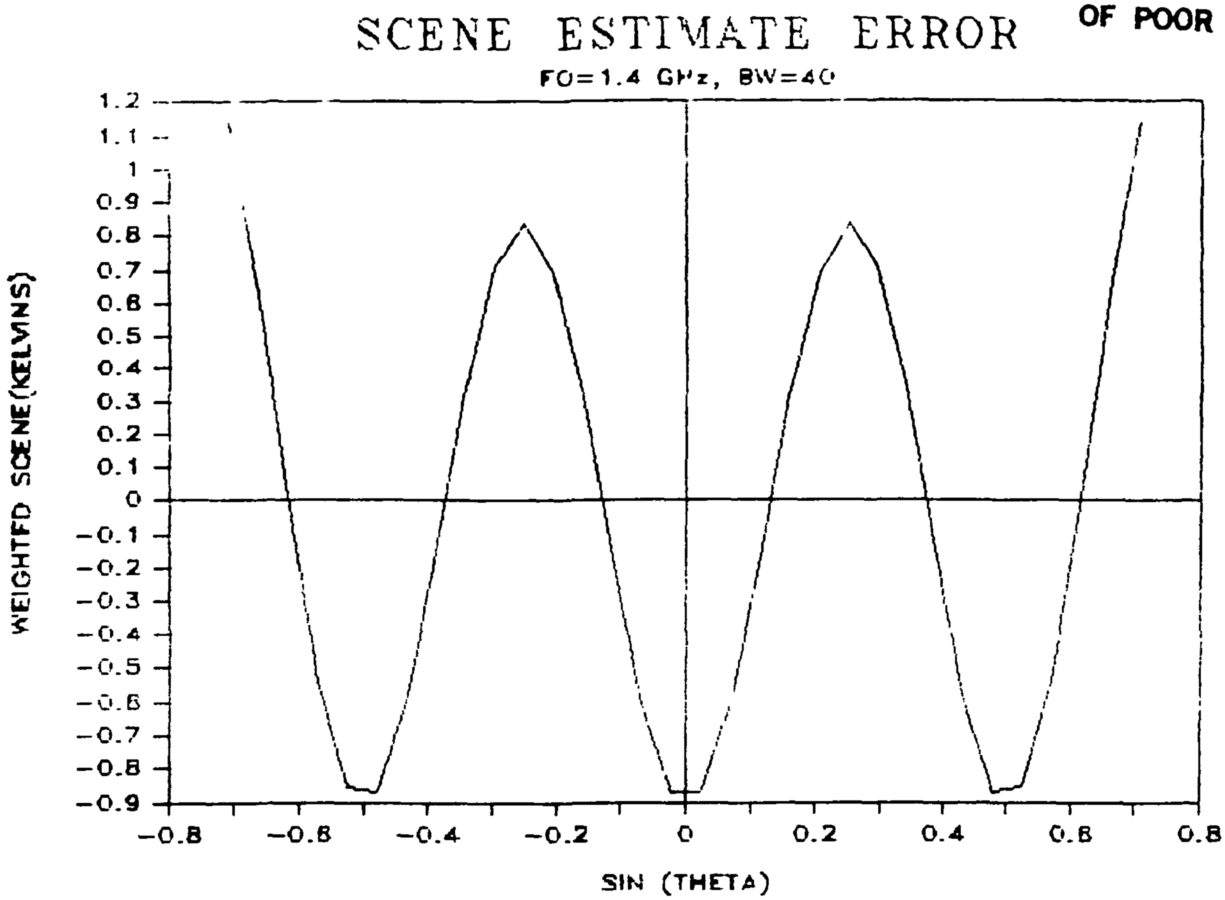

Figure VII-6. Scene Estimate Error, BW = 40 MHz.

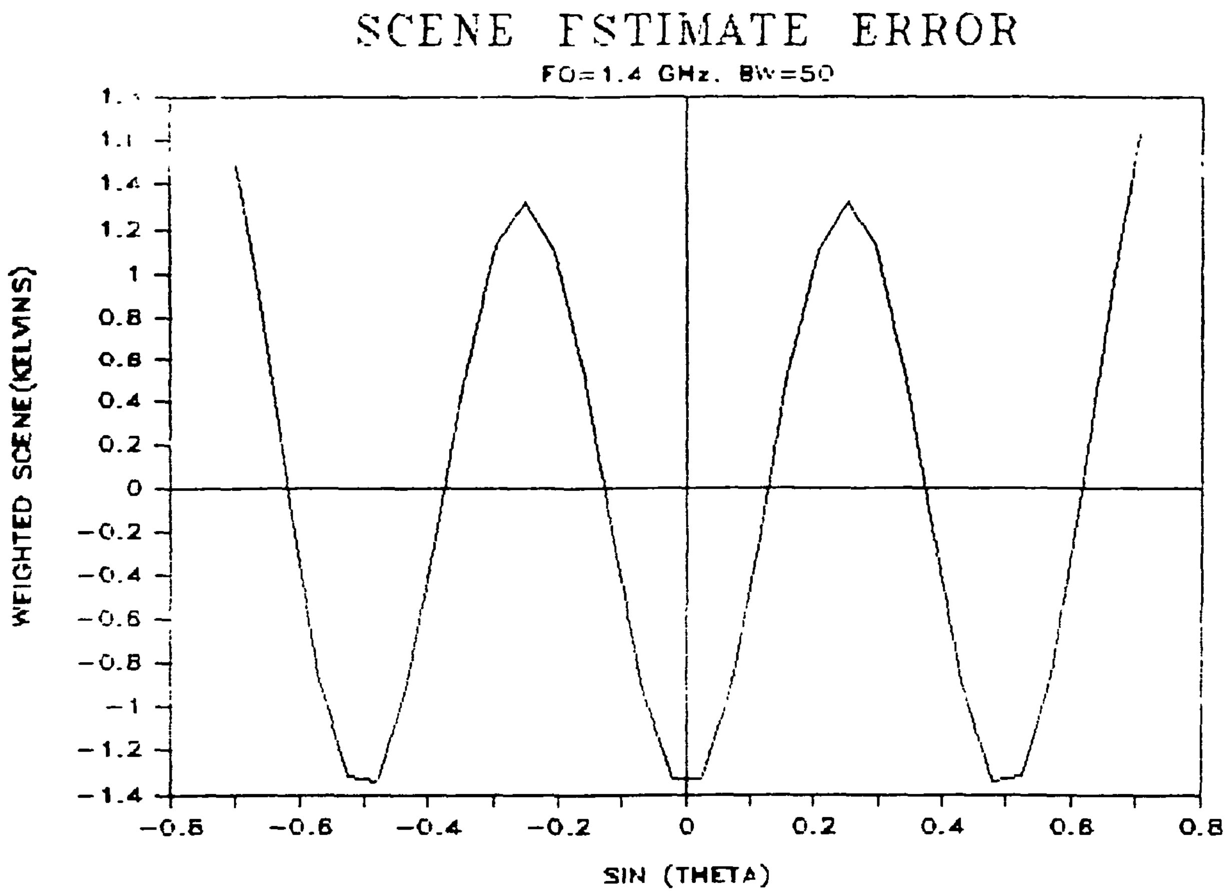

Figure VII-7. Scene Estimate Error, BW = 50 MHz.

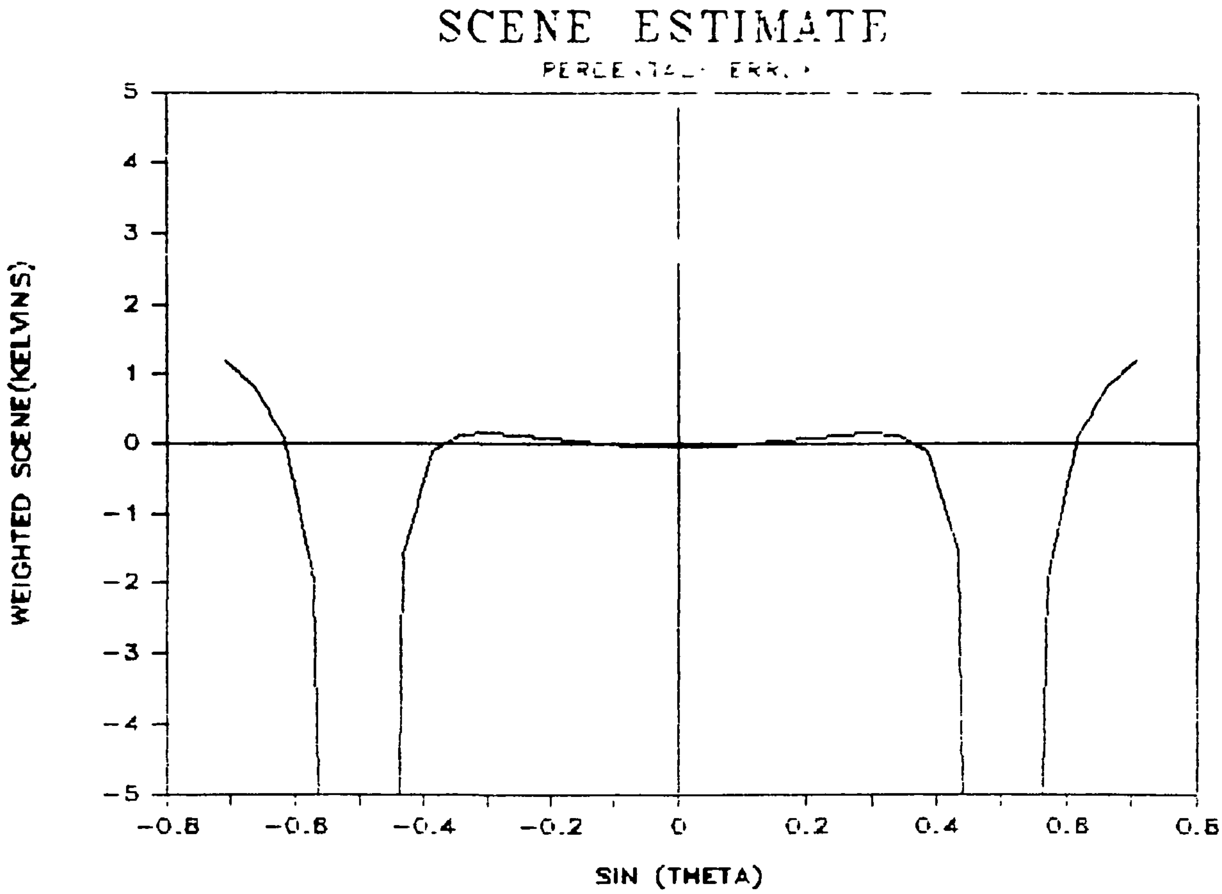

Figure VII-8. Radiometric Error, BW = 10 MHz.

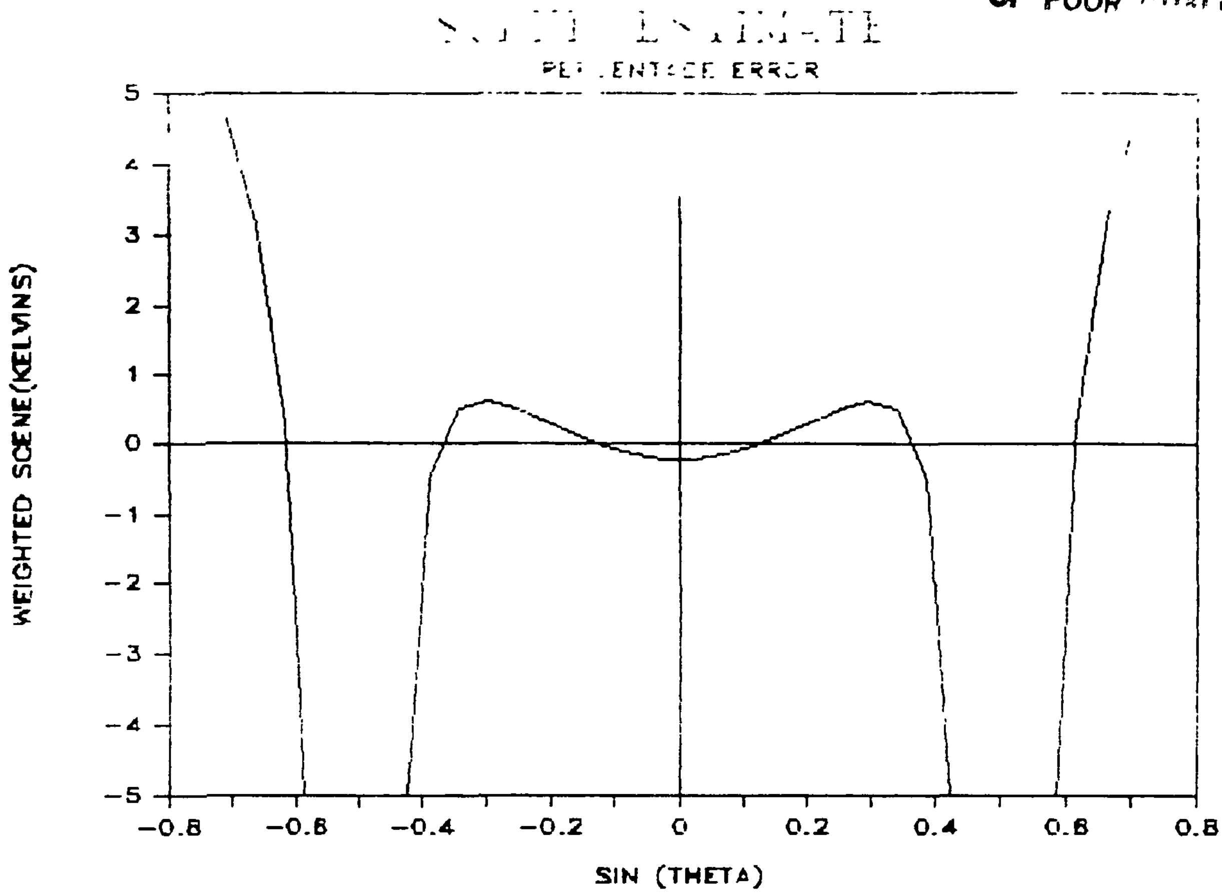

Figure VII-9. Radiometric Error, BW = 20 MHz.

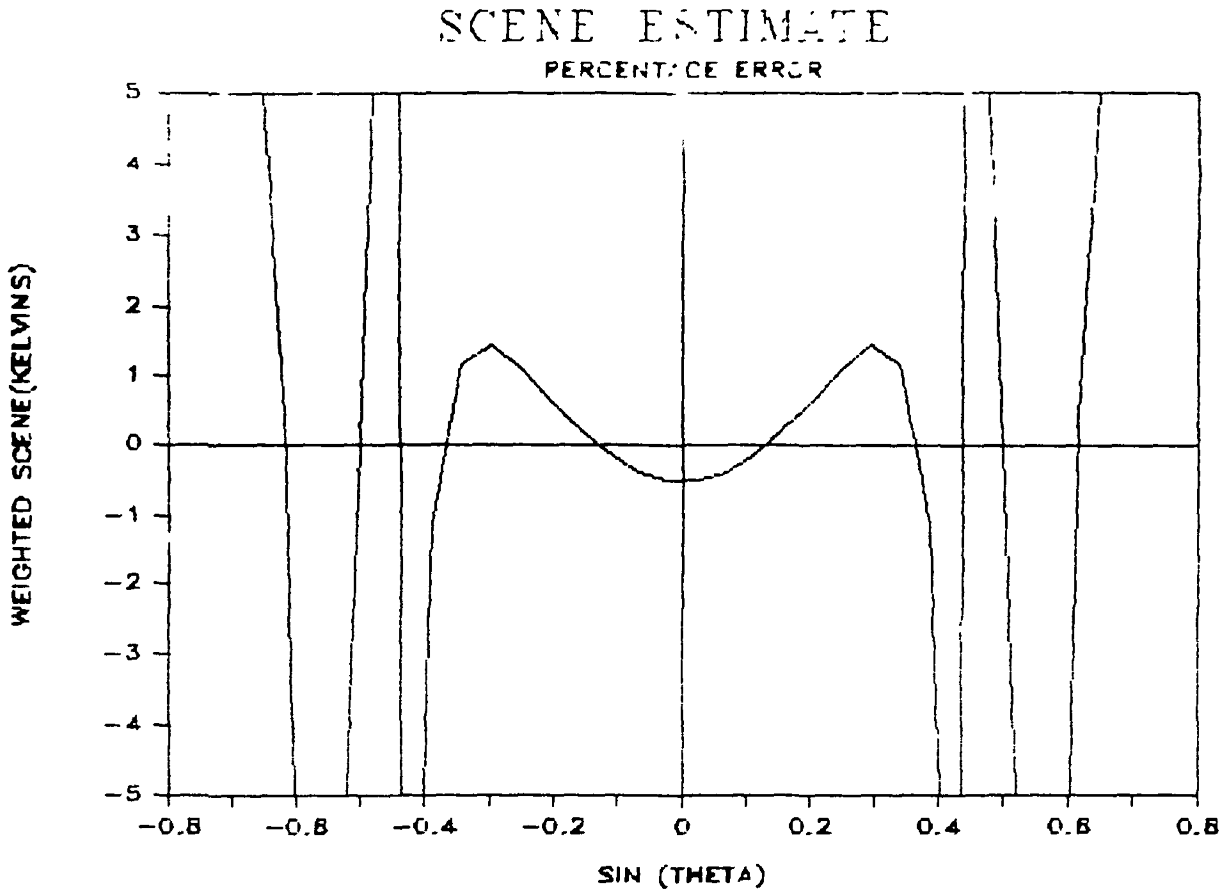

Figure VII-10. Radiometric Error, BW = 30 MHz.

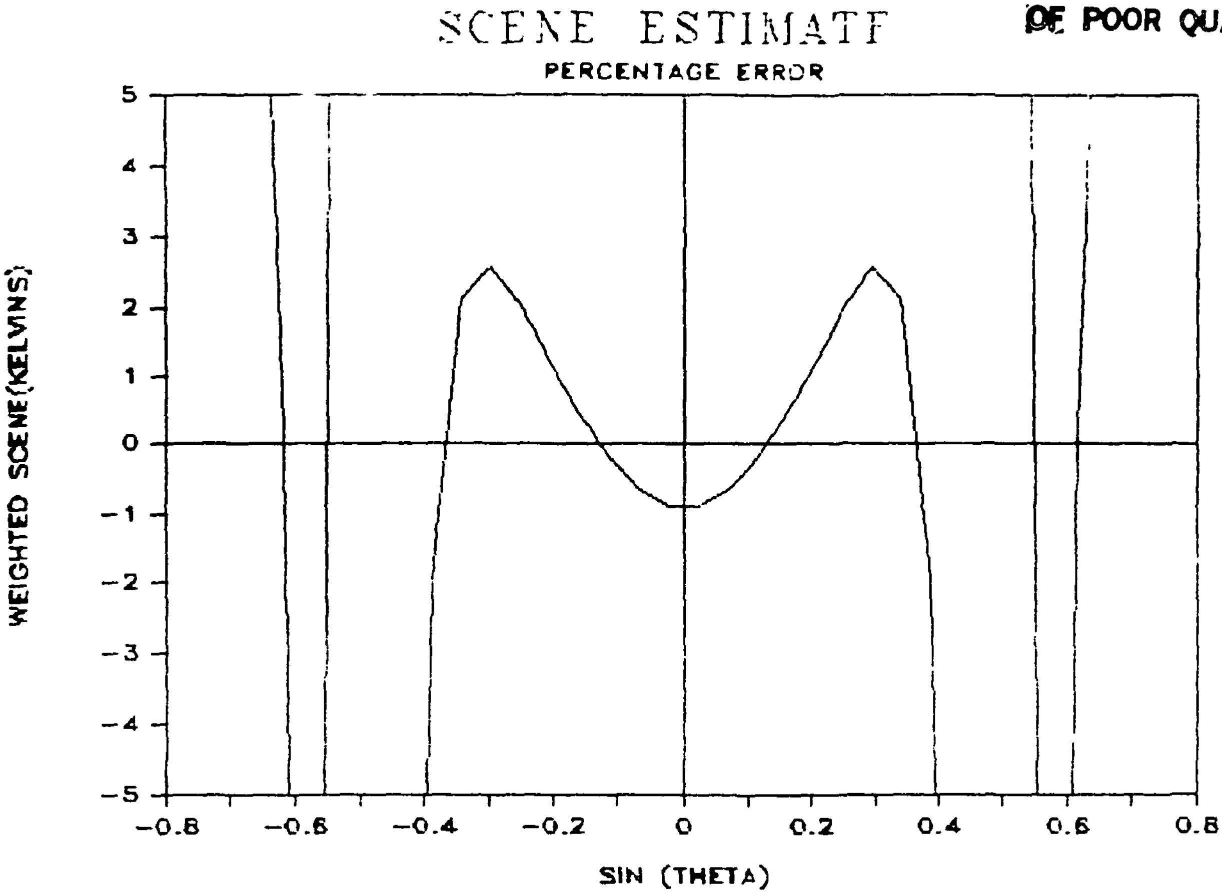

Figure VII-11. Radiometric Error, BW = 40 MHz.

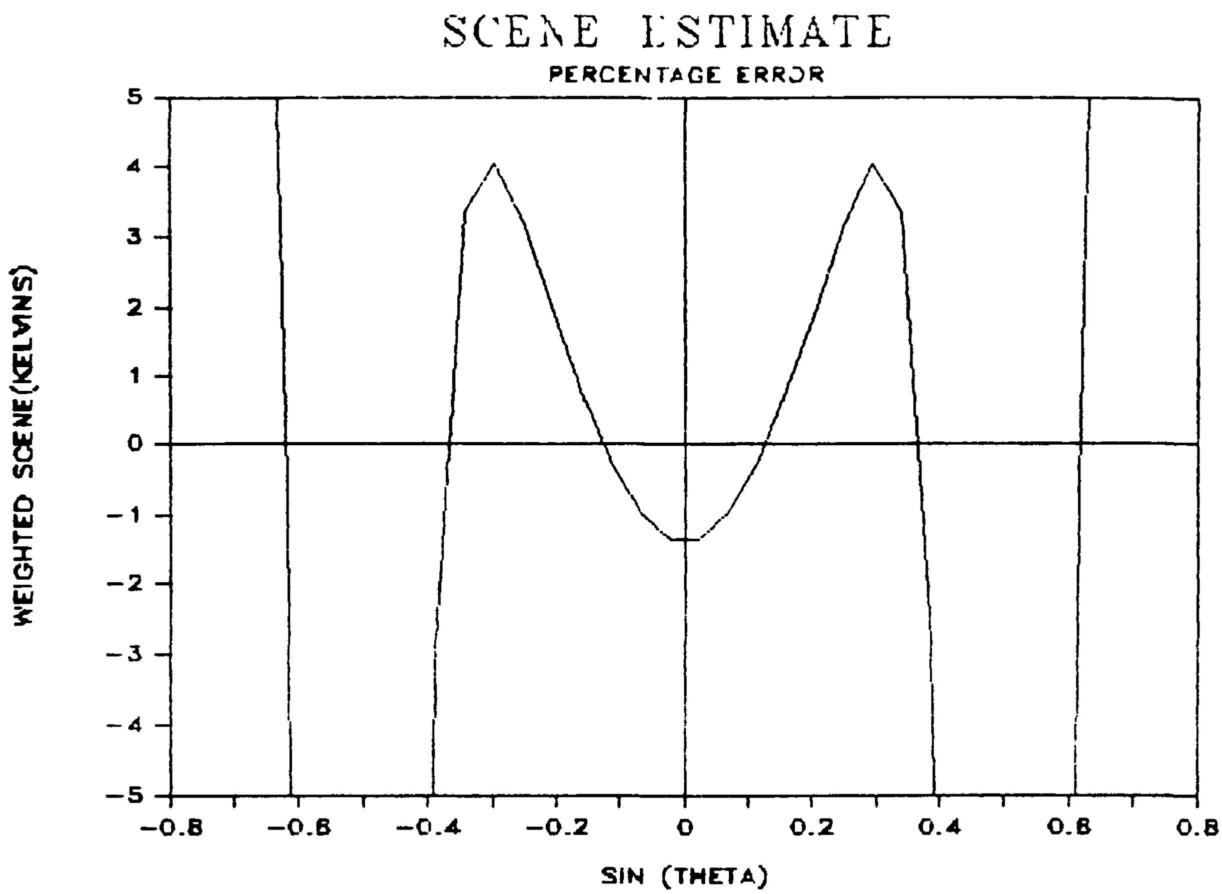

Figure VII-12. Radiometric Error, BW = 50 MHz.

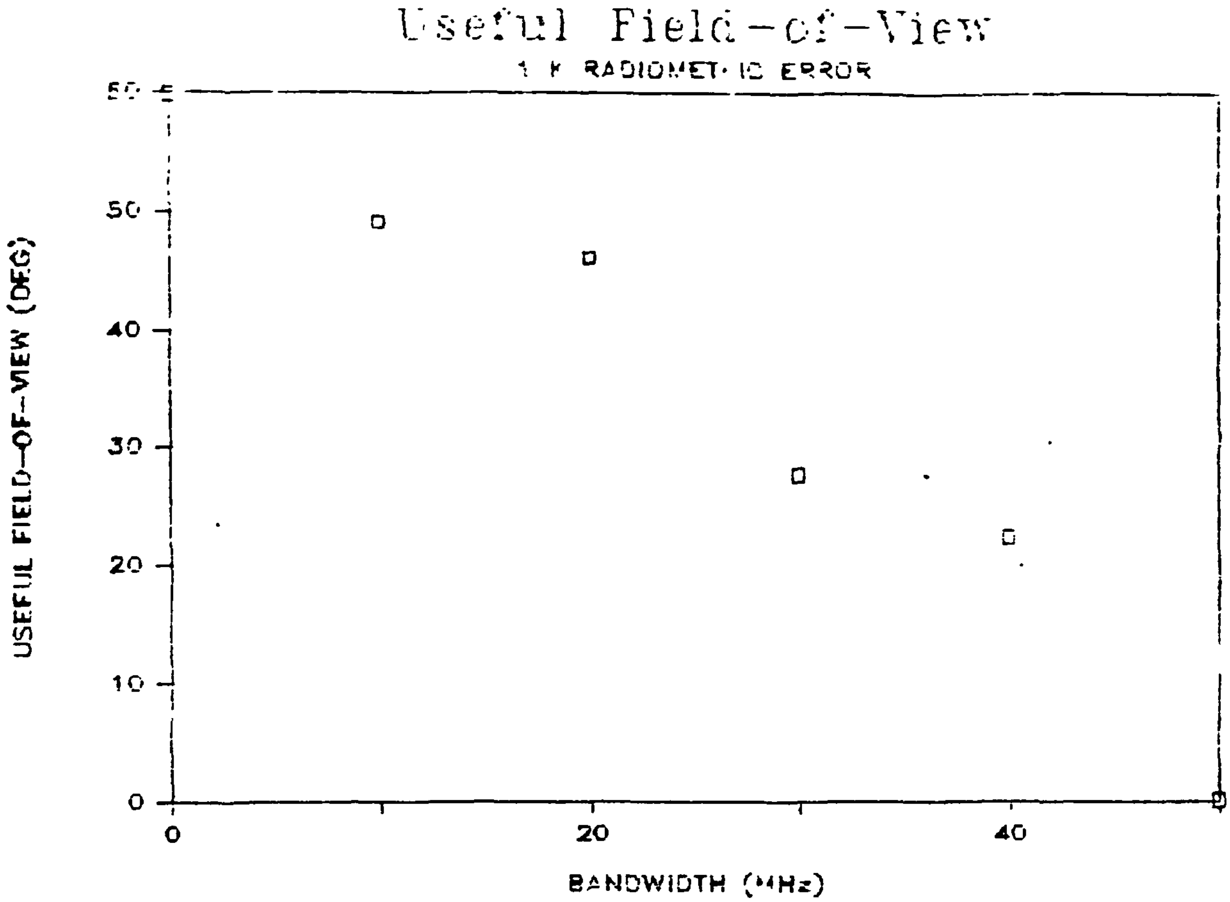

Figure VII-13. Useful Field-of-View.

V. CONCLUSIONS AND AREAS FOR FURTHER STUDY

Aperture synthesis may be an attractive approach to perform the soil moisture measurement mission. It can have both a resolution and a sensitivity advantage over real aperture systems. Further study is needed especially in the areas of:

1) Technologies

    a) antenna design
    b) signal distribution
    c) digital correlator design

2) Calibration
3) Antenna and array structural tolerances
4) Trade-offs between pure and hybrid aperture synthesis techniques
5) Trade-offs between real and synthetic aperture systems (i.e., when does the cost of electronics exceed the savings from antenna area).
6) Failure modes and reconfigurability
7) Adapting of the spacecraft concepts to aircraft prototypes.