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DENSITY AND PRESSURE VARIABILITY IN THE MESOSPHERE AND THERMOSPHERE

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by

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ABSTRACT

In an effort to isolate the essential physics of the mesosphere and the thermosphere, a steady one-dimensional density and pressure model has been developed in support of related NASA activities, i.e., projects such as the AOTV and the Space Station. The model incorporates a zeroth order basic state including both the three-dimensional wind field and its associated shear structure, etc. A first order wave field is also incorporated in period bands ranging from about one second to one day. Both basic state and perturbation quantities satisfy the combined constraints of mass, linear momentum and energy conservation on the midlatitude beta plane. A numerical (iterative) technique is used to solve for the vertical wind which is coupled to the density and pressure fields. The temperature structure from 1 to 1000 km and the lower boundary conditions are specified using the U. S. Standard Atmosphere 1976. Vertical winds are initialized at the top of the Planetary Boundary Layer using Ekman pumping values over flat terrain. The model also allows for the generation of waves during the geostrophic adjustment process and incorporates wave nonlinearity effects.

Preliminary results indicate that lower atmosphere wave processes can account for much of the observed variability of the density and pressure in the mesosphere and lower thermosphere. Basic state processes, especially vertical gradients of vertical winds are also influential under certain conditions. At very high heights (greater than 150 km) the model predicts that air density and pressure can deviate by as much as 20 % from Standard Atmosphere conditions in the presence of strong updrafts. This kind of effect would be likely to occur in the high latitude thermosphere where Joule heating is a factor leading to the development of strong vertical winds.

ACKNOWLEDGEMENTS

I am extremely honored by being chosen by Dr. Robert Smith and Dr. Gerald Karr to participate a second time in the Summer Faculty Fellowship Program. I acknowledge with sincere gratitude the "re-boost" that working with Dr. Smith has given to my career as a professor and researcher.

Dr. Douglas ReVelle has been a continual source of enlightenment. It has been a pleasure to work with him on his atmospheric model. Dr. ReVelle is an outstanding researcher, possessing a quite impressive understanding of so many areas of research. I know that our talks took a lot of valuable time from him this summer; I thank him for his kindness and patience.

LIST OF FIGURES

LIST OF FIGURES

(concluded)

INTRODUCTION

The mesosphere extends from an altitude of approximately 50 km to approximately 90 km. The temperature in this region drops from about 270 K to around 180 K, and the density ranges from 10^{-3} kg/m^3 to 10^{-5} kg/m^3. The mesopause separates the mesosphere from the thermosphere, where the temperature rises to more than 1000 K at altitudes of hundreds of kilometers. The density of the thermosphere decreases to values near 10^{-13} kg/m^3.

It is important for scientists and engineers working on the Space Shuttle, Space Station, and Aeroassisted Orbital Transfer Vehicle (AOTV) projects to be able to predict the air densities which these vehicles would encounter in the upper atmosphere, since large changes in the density can drastically affect their status while on orbit or during (Shuttle and AOTV) re-entry. In an effort to improve understanding of conditions in the upper atmosphere which can result in these changes, Dr. Douglas ReVelle has developed a model (ReVelle, 1985) which contains some of the atmospheric drivers of these density variations: winds and waves. The computer model, called "AIR", has performed admirably at generating reasonable values of physical quantities.

OBJECTIVES

The objectives of this work were to:

1) transfer the program AIR from HP-75 BASIC to Applesoft BASIC so that

a) editing would be easy,
b) variables could designated with letters and numbers more closly resembling their names (HP-75 allows at most one letter and one number),
c) graphing could be done,
d) copies of the program could be distributed to others,

2) run the program for a large number of different situations to

a) determine whether or not the predictions of the model are reasonable,
b) narrow choices of input parameters,

3) modify the program to allow the input of latitude, making the model more "global" (calculations are now done only at 40^{o} latitude),

4) rewrite the code, separating the various modes and repeated blocks of calculations into subroutines to make the program run faster,

5) translate the program into FORTRAN.

THEORY

Consider an atmosphere which is assumed to obey the following relationships:

1) perfect stratification, i.e., density and pressure are functions of altitude only,

2) steady state, i.e., $\partial/\partial t$ of any variable vanishes,

3) inviscid flow,

4) ideal gas law.

The mass conservation equation is

$$\vec{\nabla}\cdot(\rho\vec{v}) = 0 \quad , \tag{1}$$

and the momentum equations are

$$(\vec{v}\cdot\vec{\nabla})\vec{v} + 2\vec{\Omega}\times\vec{v} = -(1/\rho)\vec{\nabla}p - g\hat{k} \quad , \tag{2}$$

where ρ is the density, $\vec{v}$ is the air flow velocity, $\vec{\Omega}$ is the angular velocity of the earth, p is the pressure, and g is the acceleration due to gravity. The unit vector in the vertical direction is $\hat{k}$; $\hat{j}$ points north, and $\hat{i}$ points east. The wind velocity components are

$v_x = u$ = zonal wind,

$v_y = v$ = meridional wind,

$v_z = w$ = vertical wind.

The theoretical model developed by ReVelle is along the lines of the work done by Ghosh (Ghosh, 1970). Ghosh considered a steady state atmosphere with a vertical wind increasing in strength with height, z, and no zonal or meridional wind. Effects of viscosity and rotation of the earth were neglected. The solution of the continuity and momentum equations in this case yields a density of

$$\rho = \rho_o \exp\{-\int\left(mg/kT + (1/T)\partial T/\partial z + (m/kT)(mg/kT + (1/T)\partial T/\partial z)w^2\right)dz\} \quad , \qquad (3)$$

where m is the mass of a molecule, k is Boltzmann's constant, and T is the temperature. The first term in the integral represents the gravity dependence of the density, the second term the temperature variation, and the third term the contribution of the vertical wind.

Substitution of typical values at 100 km yields (Ghosh, 1970)

$$\text{g-term} \sim 1.6(10)^{-4}\ m^{-1} \ ,$$

$$\text{T-term} \sim 1.4(10)^{-5}\ m^{-1} \ ,$$

$$\text{total of non-wind terms} \sim 1.7(10)^{-4}\ m^{-1} \ ,$$

$$\text{w-term} \sim 2.8(10)^{-9} w^2\ m^{-1} \ .$$

Therefore, if the vertical wind w = 100 m/sec,

$$\frac{\text{w-terms}}{\text{non w-terms}} \sim 10\%.$$

ReVelle considers a rotating earth with non-zero zonal wind and meridional winds and non-zero spatial derivatives of the winds. The continuity equation becomes

$$\rho\partial u/\partial x + \rho\partial v/\partial y + \rho\partial w/\partial z + w\partial\rho/\partial z = 0 \quad , \qquad (4)$$

or

$$\rho D_h + \rho\partial w/\partial z + w\partial\rho/\partial z = 0 \quad , \qquad (5)$$

where D_h, the horizontal divergence, is assumed to be constant for all z.

The atmosphere is broken up into layers of thickness Δz ($\sim$.1 km), and the conservation equation and momentum equations are numerically integrated from a lower boundary of altitude z = 1 km. The initial value of the vertical wind is taken from Ekman pumping theory in which

$$w_o \sim \zeta_z h_{PBL}/6 \quad , \tag{6}$$

where ζ_z is the vorticity (in sec^{-1}) of some rotating body of air at the surface and h_{PBL} is the height of the planetary boundary layer, 1 km.

Note that if w is assumed a constant, w_o, over some layer of thickness Δz, then equation (5) can be written as

$$\rho w = \rho_o w_o \exp\{-(D_h/w_o)\Delta z\} \quad . \tag{7}$$

The case in which $D_h = 0$ and the case in which w is very large should both act similarly with $\rho w \sim$ constant. Also note that if w becomes negative (a downdraft), the exponent becomes positive, causing the density to increase with height. This sign in the exponent prevents a downdraft from becoming an updraft again; therefore, the solutions which were taken to be acceptable in this study were the ones in which the vertical wind was always upward.

It would be desirable to also utilize the conservation of energy equation in the model. However, due to the complexity of the equation and the uncertainty in values of some variables, it was decided to stick with only the mechanical relationships. A rough calculation does reveal that the winds that would be expected from an energy point of view are compatible with those being calculated by the model. The heating due to vertical lifting is calculated in the program (variable name Q7), and the numbers obtained are consistent with calculations which show heating and cooling rates of 1 - 10 K/day in the lower 100 km of atmosphere and up to 1000's K/day in the thermosphere.

The details of using the program are given in the next section. The rest of this section is devoted to a discussion of a few items essential to the understanding of the operation of the program.

There are a number of program input requirements; the first is that of mode. The program operates in one of four modes:

1) barotropic atmosphere with vertical winds (the only mode run in this study)

2) sub/super geostrophic with either constant or height variable vertical winds)

3) sub/super geostrophic with waves (includes nonlinear saturation limit)

4) hydrostatic with waves (includes nonlinear saturation limit).

A response of "yes" to the input concerning heating constraint forces the vertical wind to become constant when the heating due to vertical lifting becomes greater than 100 K/day. A response of "no" allows the vertical wind and its associated heating to increase without bound.

The value of vorticity input is used to calculate the initial vertical wind in equation (6). The number most often used in this study is $0.606433553(10)^{-5}$ sec^{-1}; it generates an initial vertical wind of 1.00687771 mm/sec. This vorticity was found by ReVelle prior to this study to yield results (winds, densities, etc.) which are realistic and well behaved.

The DEL:H DOT V and DELH.V terms in the program refer to the horizontal divergence, $D_h = \partial u/\partial x + \partial v/\partial y$, in equation (5). A response of "yes" to the "NOMINAL DELH.V CASE?" input causes the value of D_h to be set at $1.21915(10)^{-7}$ sec^{-1}, which was also previously determined by ReVelle to yield reasonable results. An input of "no" prompts a request for the desired value of D_h. Whatever the value of D_h used in the program, it is held constant everywhere.

The zonal wind can be either constant with altitude or variable according to the season. A constant zonal wind is obtained by answering "no" to the "VARIABLE CORIOLIS TERM?" input and then typing in the desired wind. Otherwise, the zonal wind is variable according to whether winter or summer is chosen. In this case the wind is assumed geostrophic such that

$$u = -(R/fM)\partial T/\partial y \quad , \qquad (8)$$

where f is the Coriolis term ($2\Omega \sin\Theta$), R is the gas constant, M is molecular mass per mole, and $\partial T/\partial y$ is the horizontal temperature gradient. The zonal wind may in addition be made subgeostrophic,

geostrophic, or supergeostrophic via the u geostrophic factor (<1, 1, >1, respectively) which simply multiplies the value calculated in equation (8).

OPERATING INSTRUCTIONS

Step-by-step instructions on how to use the Apple II+ or IIe (or III with II emulator) to run AIR and its derivatives are given in this section.

1) Insert the disk into the disk drive and close it.

2) Turn on the computer (switch is on your left, at rear of computer) and the monitor.

3) The menu of the disk contents will appear on the screen.

4) Type RUN AIR (or RUN AIR PRINT or RUN AIR LARGE SCALE) and press the "return" key.

5) Respond to each input request by typing the number and pressing "return".

6) When the run is complete, remove the disk and turn off the computer and monitor.

Notes on AIR:

* Input

  MODE (1, 2, 3, 4) =
  (Mode 1 was used for all runs.)

  HEATING CONSTRAINT: 1(Y), 0(N) =
  (0 used for all runs.)

  PBL Z VORTICITY: MEAN (1*E-5/SEC)
  (0.606433553 used for most runs.)

  NOMINAL DELH.V CASE? (1/YES, 0/NO)
  (YES chooses 1.21915.)

  VARIABLE CORIOLIS TERM? (1/YES, 0/NO)
  (Variable or constant zonal wind.)

ZONAL WIND =
(Appears only if Variable Coriolis Term? = NO.)

WINTER/SUMMER CASE? (1/WIN, 0/SUM)
(Appears only if Variable Coriolis Term? = YES.)

U GEOSTROPHIC FACTOR =
(1 used for all runs.)

DEL:H DOT V(1*E-7/SEC) =
Horizontal divergence in units of $(10)^{-7}$ sec^{-1}; usually in the range of 1.2. (Appears only if Nominal DEL.V Case? = NO.)

* Output

Any number of variables can be listed on the screen. Which variables are to be listed is set in line number 18700 (and 18710 if the list is very long). The altitude, L8, is already part of the PRINT statement. If, for example, the variable Y4 is also needed, then this sequence of steps should be followed:

1) Instead of typing RUN AIR, type LOAD AIR.

2) Type LIST 18700-18800 (press "return") to see what the PRINT statement looks like.

3) Retype line 18700. In this example it would be
```
18700 PRINT:PRINT "L8 = ";LR:PRINT "Y4 = ";Y4
```

4) Type RUN and press "return".

The program may be stopped at any time by pressing the CTRL ("control") and C keys simultaneously. The current value of a variable, for example R6, is listed by typing ?R6 (press "return"). The program operation is continued with CONT.

* Some useful variables and their code names:

LR - altitude (rounded to the nearest 0.01 km)
T1 - vertical temperature gradient
T2 - temperature
M - density
M9 - standard atmosphere density
P1 - pressure
P9 - standard atmosphere pressure
Y4 - % difference between M and M9
Y9 - % difference between P1 and P9

R6 - Reynolds number
G1 - density scale height
S2 - pressure scale height
W - vertical wind
M0 - Mach number
Q7 - heating due to vertical lifting
U2 - zonal wind
H0 - horizontal divergence

Notes on AIR LARGE SCALE:

This program was created to eliminate the need to modify AIR every time a run up to 300 km was desired. The main difference between AIR LARGE SCALE and AIR is in the definition of the x- and y-coordinates on the graph. The altitude scale goes to 300 km (as opposed to 150 km in AIR) and the wind scale goes to 14 mm/sec (2.8 mm/sec in AIR).

Other variables may be graphed by redefining the array variable W(Z) used in the graphics. W(Z) is dimensioned in line 50 and defined in line 18630.

Notes on AIR PRINT:

This program was created to give a paper printout of the variables being listed in line 18700. The printer is accessed through slot #1 (PR#1 in line 3310), and it must be turned on before the program is run. The program is set to run until the altitude reaches 500 km, but it can be stopped at any point with CTRL-C. Type PR#0 after this to return to use of the monitor.

RESULTS

This section comprises a summary of the results of the runs of AIR and their significance. All runs were made with the vorticity $\zeta_z = 0.606433553(10)^{-5}$ sec^{-1} unless otherwise stated.

1) u = 10 m/sec, D_h nominal

Figure 1 shows a typical behavior of the vertical wind. A later run of this case to higher heights shows that the vertical wind increases to 10 m/sec at 155 km, where the density is 0.2 % greater than the 1976 Standard Atmosphere (National Oceanic and Atmospheric Administration, 1976) density, to 100 m/sec at 214 km, where the density is 1.3 % less than standard, to almost 400 m/sec at 257 km, where the density drops to 16.6 % less than standard.

It is noted here that the shape of the curve in Figure 1 is very similar to the temperature profile, Figure 2. Because of this similarity, it was decided to attempt to find values of D_h and u which would cause the w vs. z graph to follow the temperature curve. The results of this search are given later in this section.

2) $u = 0$, D_h nominal

Figure 3 shows how the vertical wind profile changes with the zonal wind reduced to zero. A negative (toward the west) zonal wind drives the vertical wind to large values at even lower heights. See Figure 4.

3) $u = 10$ m/sec, $D_h = 1.2(10)^{-7}$ sec^{-1} and
$u = 10$ m/sec, $D_h = 1.24(10)^{-7}$ sec^{-1}

These graphs demonstrate how sensitive the w vs. z graphs are to the value of the horizontal divergence. A value of D_h only slightly smaller than $1.21915(10)^{-7}$ sec^{-1} causes the wind to increase very rapidly (Figure 5). In fact a zero divergence yields a vertical wind of 200 m/sec at only 70 km. A value of D_h slightly larger than nominal drives the wind negative very quickly. See Figure 6.

A comparison of the $u = 0$ and $D_h = 0$ case with Ghosh's results might be in order, although they will still differ somewhat due to Ghosh's assumption of a non-rotating earth. A run with $u = 0$ and $D_h = 0$ yields a vertical wind of 100 m/sec at 83 km ($\sim$ 100 km), which produces a density 9 % ($\sim$ 10 %) smaller than Standard Atmosphere density, which includes the gravity and temperature variations.

4) $u = 11.723013$ m/sec, D_h nominal

Figure 7 shows that this value of a constant zonal wind constrains the wind to follow the temperature profile for a good distance and limits it to a few cm/sec near 300 km.

Figure 8 demonstrates how the winds vary as a result of a change in only the last digit of the zonal wind. The vertical wind is just beginning to decrease at 300 km and will eventually go negative.

5) Winter and summer zonal winds

Figures 9 and 10 show the zonal (geostrophic) wind profile for winter and summer, respectively.

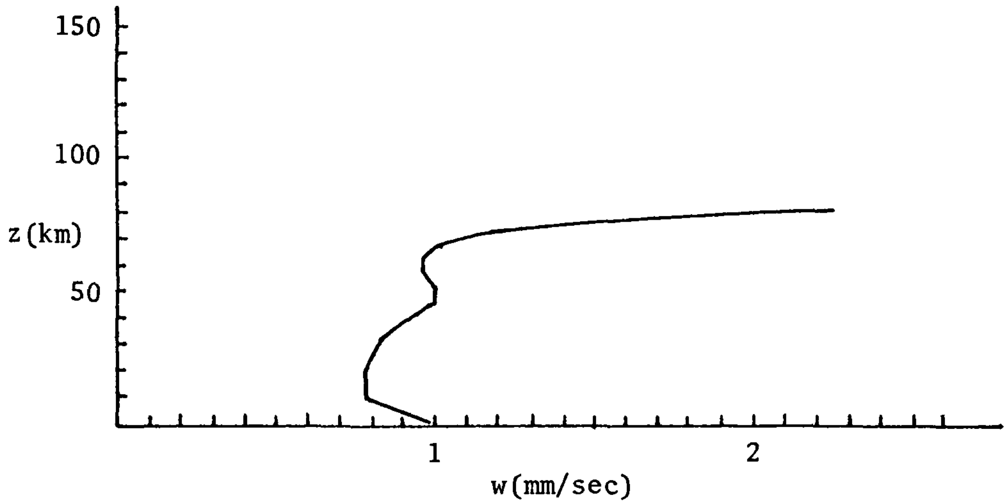

Figure 1. Vertical wind vs. altitude for u = 10 m/sec and $D_h = 1.21915(10)^{-7}$ sec^{-1}.

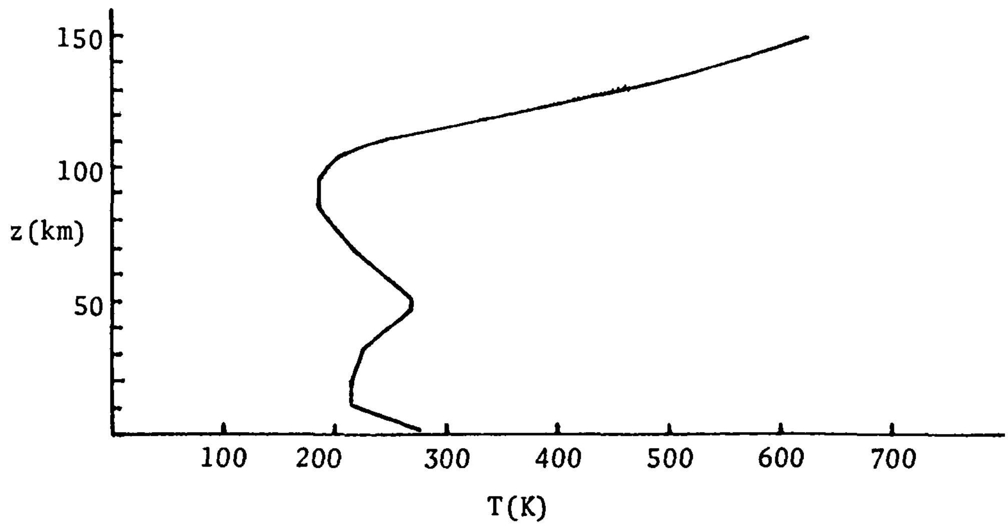

Figure 2. Temperature profile.

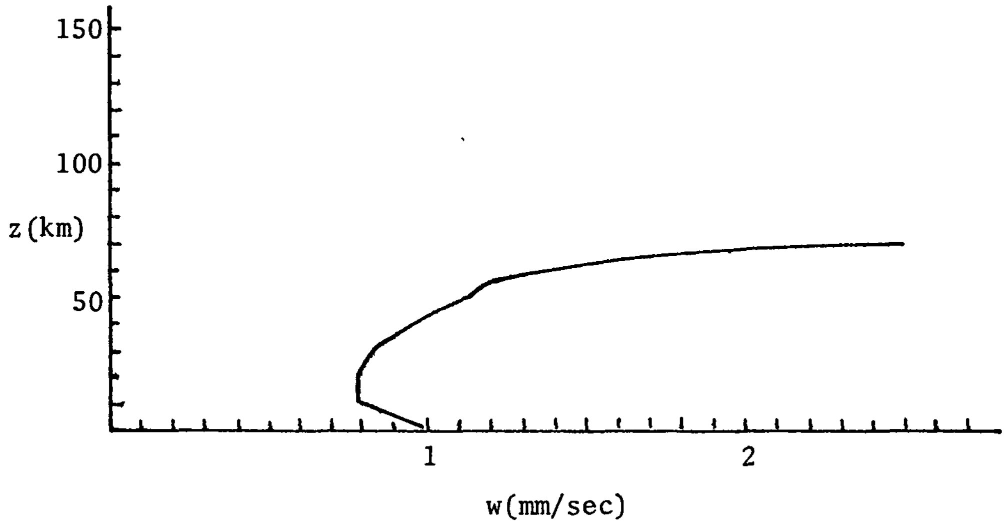

Figure 3. Vertical wind vs. altitude for u = 0 m/sec and $D_h = 1.21915(10)^{-7}$ sec^{-1}.

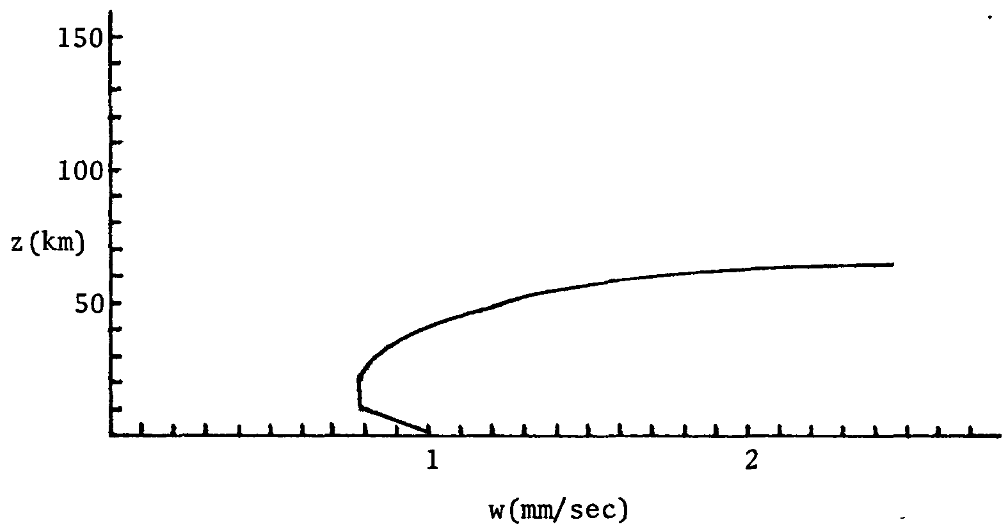

Figure 4. Vertical wind vs. altitude for u = -10 m/sec and $D_h = 1.21915(10)^{-7}$ sec^{-1}.

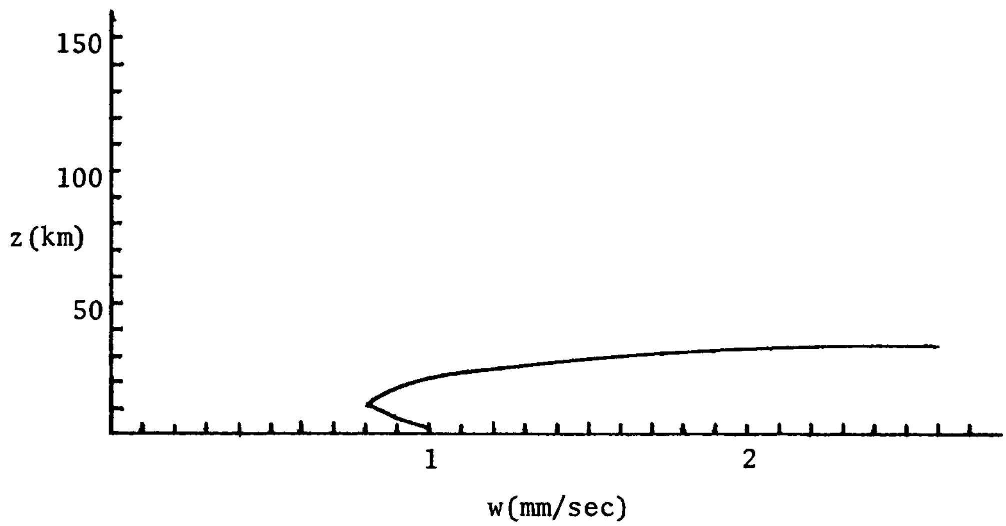

Figure 5. Vertical wind vs. altitude for u = 10 m/sec and $D_h = 1.2(10)^{-7}$ sec^{-1}.

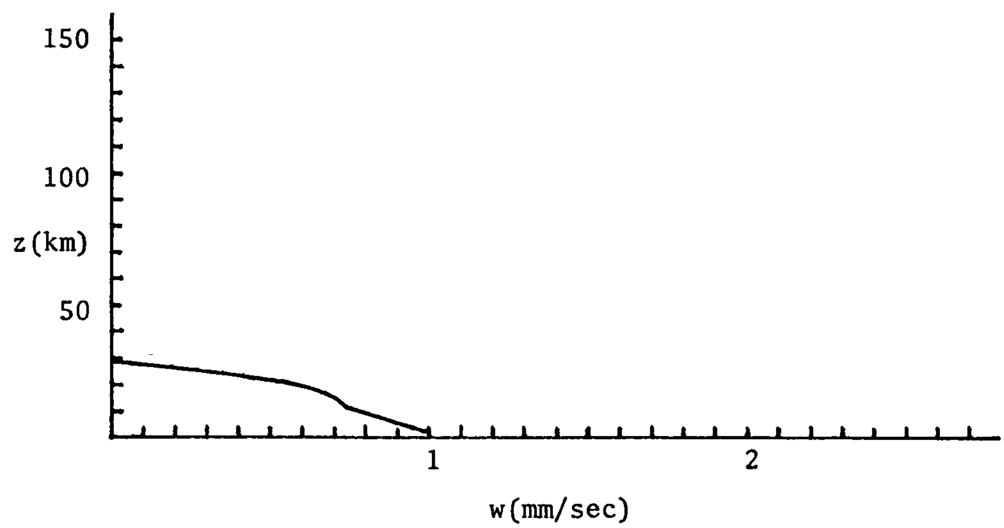

Figure 6. Vertical wind vs. altitude for u = 10 m/sec and $D_h = 1.24(10)^{-7}$ sec^{-1}.

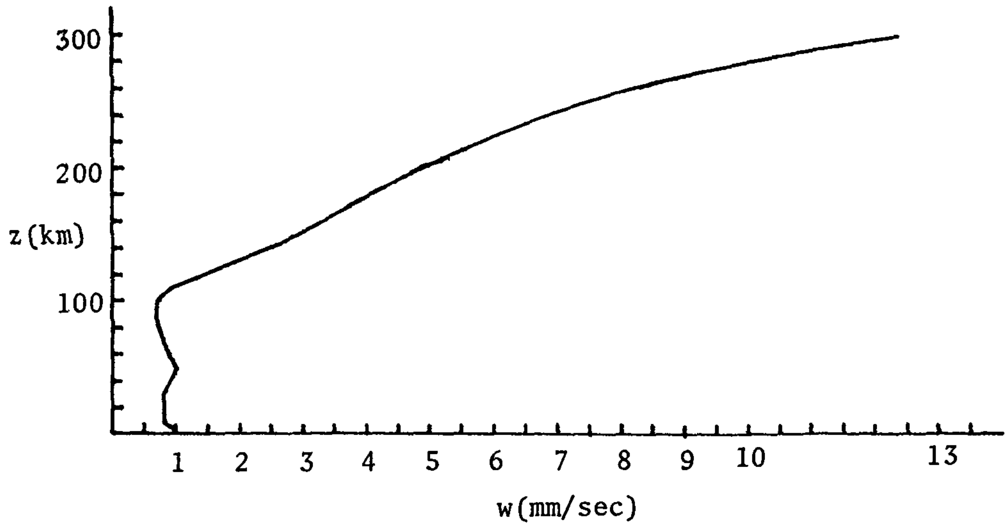

Figure 7. Vertical wind vs. altitude for u = 11.723013 m/sec and $D_h = 1.21915(10)^{-7}$ sec^{-1}.

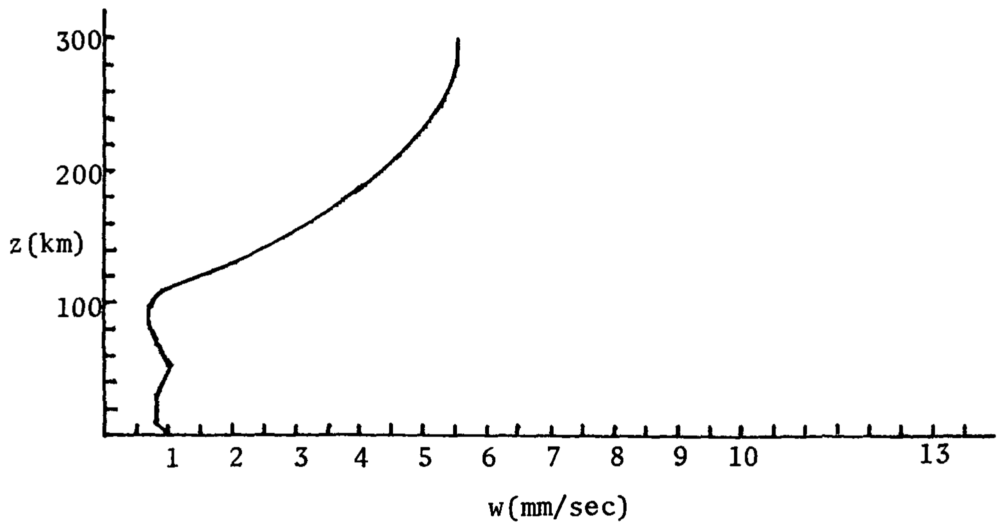

Figure 8. Vertical wind vs. altitude for u = 11.723014 m/sec and $D_h = 1.21915(10)^{-7}$ sec^{-1}.

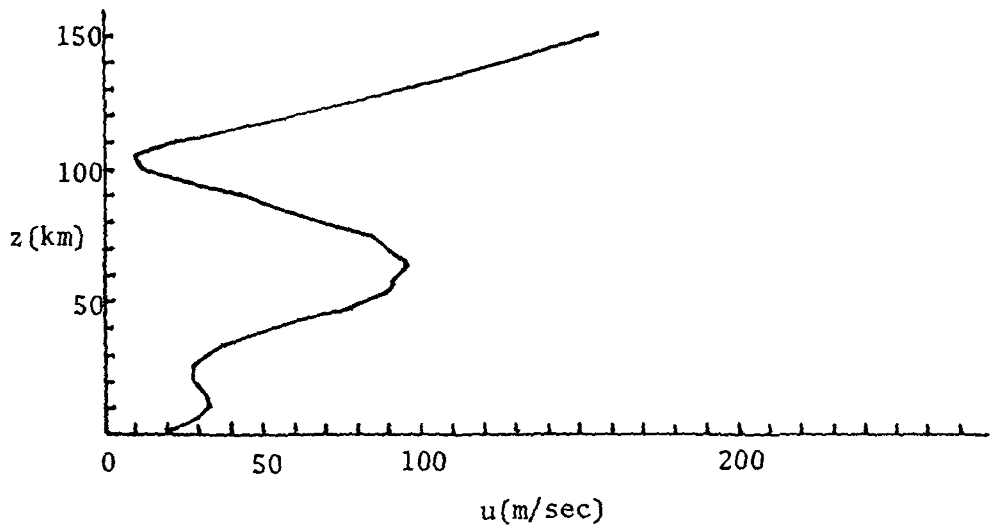

Figure 9. Winter zonal wind profile.

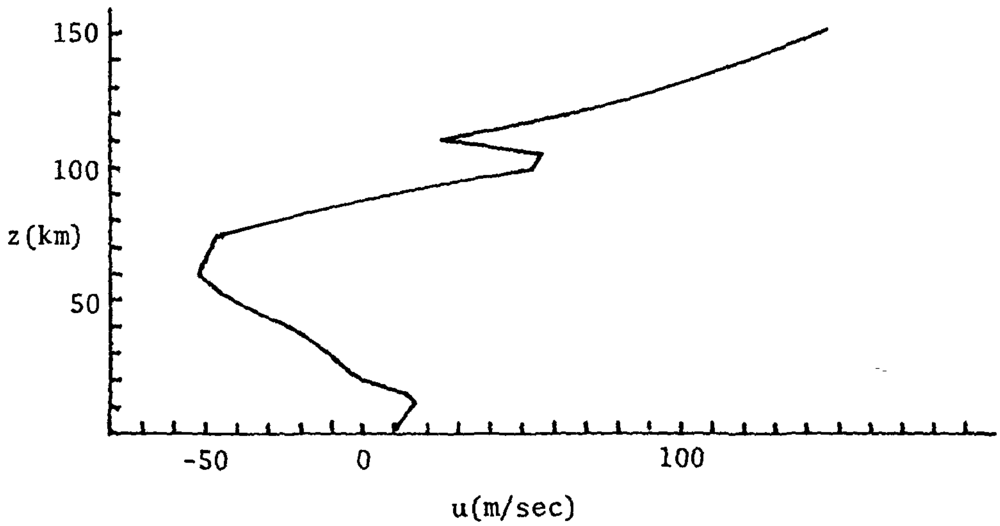

Figure 10. Summer zonal wind profile.

6) u = summer profile, $D_h = 1.21917253(10)^{-7}\ sec^{-1}$

This value of the horizontal divergence was found to be the one which constrained the vertical wind to follow the temperature profile as closely as possible in the case of summer zonal winds. The vertical wind (see Figure 11) is only about 1 cm/sec at 170 km and rises to 10.4 m/sec at 500 km, where the density is 3.2 % above the Standard Atmosphere density.

7) u = winter profile, $D_h = 1.21892828(10)^{-7}\ sec^{-1}$

Similarly, this constraining value of D_h was found for the winter case. See Figure 12. The vertical wind increases to 1 cm/sec at 155 km, where the density is 1.3 % greater than standard, to 21.5 m/sec at 500 km, where the density is 4.9 % greater than standard.

8) u = 10.105 m/sec, $D_h = 1.21917253(10)^{-7}\ sec^{-1}$

It was noted early in the study that for a nominal D_h the w vs. z graphs for u = 10 m/sec and u = summer profile were almost identical and that the u = 25 m/sec and u = winter profile were almost identical. The ratio of these two constant zonal winds is 2.5, which is roughly equal to the ratio of the altitude-averaged zonal winds for winter and summer. Therefore, once the constraining values of the horizontal divergence were found for winter and summer, the corresponding constant zonal winds for these divergences were sought. Figure 13 shows how the vertical wind follows the temperature profile for u = 10.105 m/sec.

9) u = 27.6 m/sec, $D_h = 1.21892828(10)^{-7}\ sec^{-1}$

The constrained vertical wind for the case of a constant zonal wind and the winter divergence is graphed in Figure 14. Again, note that the ratio of these two constant zonal winds in the winter and summer cases is 2.73 ($\sim$ 2.5). Time did not permit the determination of the zonal winds to more decimal places.

10) Note on heating

As discussed earlier, if the theory is to be reasonable, the heating due to the vertical winds must be within the limits mentioned. A run of the constrained summer case produced vertical wind heating which a) followed the temperature profile, b) was less than or approximately equal to 1 K/day up to 108 km, c) hit 100 K/day at 279 km, and d) rose to 3700 K/day at 500 km.

The heating due to vertical lifting in the constrained winter case behaved much like the summer case except that the heating hit 100 K/day at 242 km and 7700 K/day at 500 km.

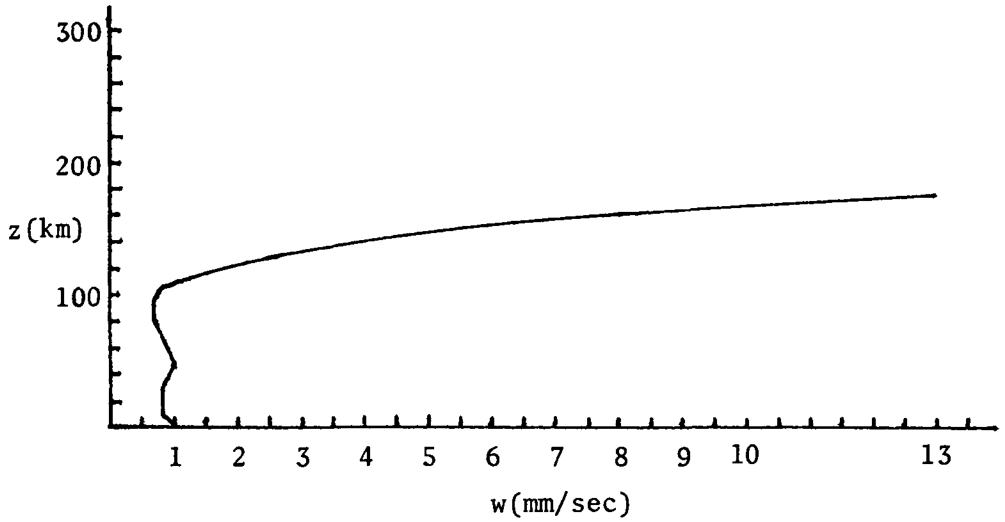

Figure 11. Vertical wind vs. altitude for u = summer and $D_h = 1.21917253(10)^{-7}$ sec^{-1}.

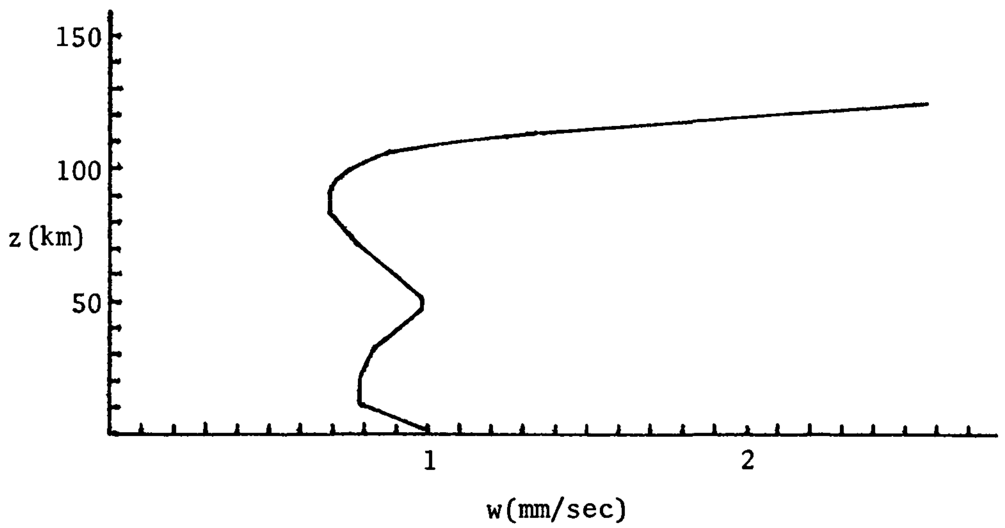

Figure 12. Vertical wind vs. altitude for u = winter and $D_h = 1.21892828(10)^{-7}$ sec^{-1}.

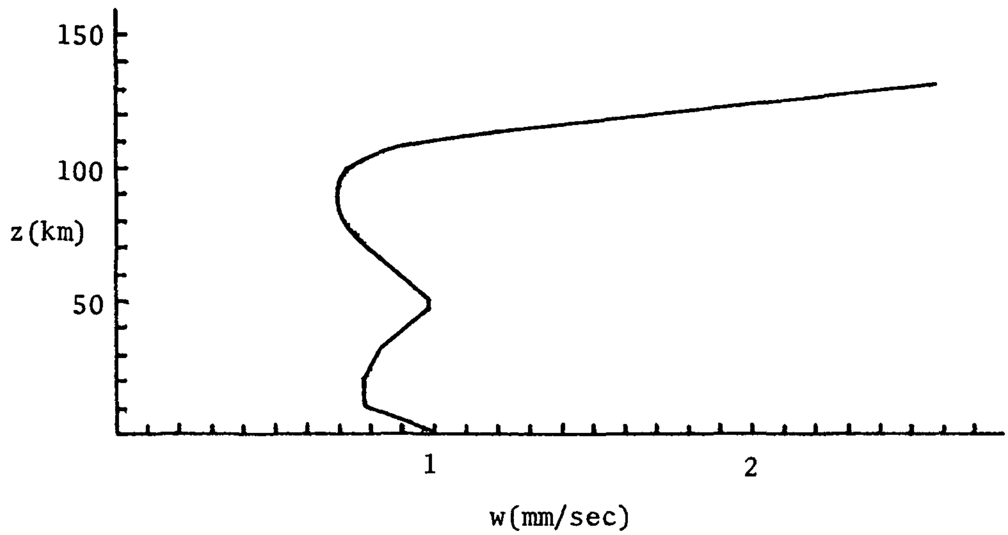

Figure 13. Vertical wind vs. altitude for u = 10.105 m/sec and D_h = 1.21917253$(10)^{-7}$ sec^{-1}.

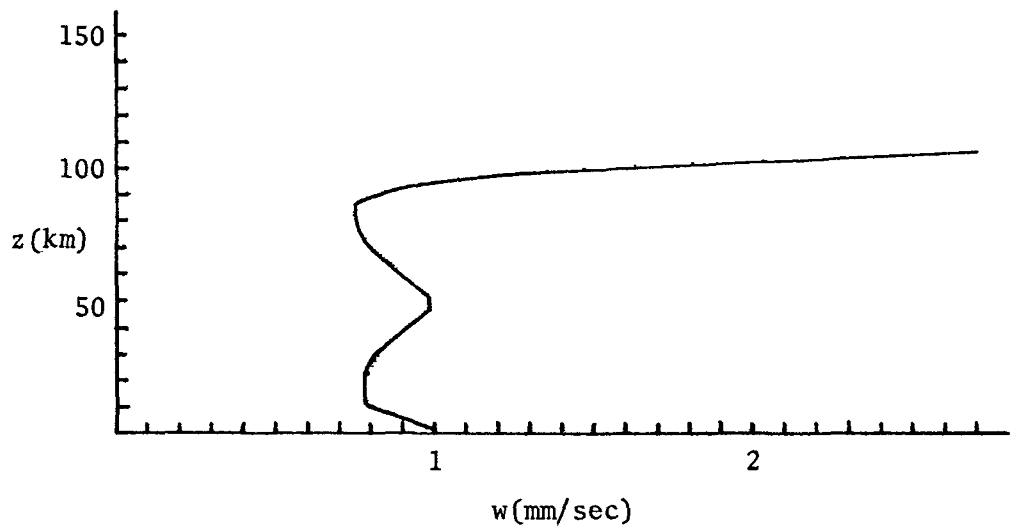

Figure 14. Vertical wind vs. altitude for u = 27.6 m/sec and D_h = 1.21892828$(10)^{-7}$ sec^{-1}.

11) Note on mass flux

A run was made to test the earlier assertion that as w gets very big, ρw, the mass flux in $kg \cdot m^{-2} \cdot sec^{-1}$, should become constant. Figures 15 and 16 give the wind vs. altitude and mass flux vs. altitude graphs, respectively. The vertical wind is 435 m/sec at 263 km, and the density is 21 % smaller than standard. The mass flux reaches a constant value of $1.5(10)^{-8}$ $kg \cdot m^{-2} \cdot sec^{-1}$ at around 120 km.

At this rate of mass flux the earth's atmosphere would be lost in

$$t \sim \frac{5.3(10)^{18}\ kg}{\{1.5(10)^{-8}\ kg/m^2 \cdot sec\}\{4\pi(6.7(10)^6 m)^2\}} \sim 5.2(10)^{11}\ sec$$

or about 20,000 years. Even if it is only the upper 1 % that is lost, the time is reduced to 200 years. These numbers are not meant to show that the atmosphere is actually dissipating at this rate, rather that the quantities produced by AIR are reasonable. That is, the time is not as short as 2 seconds, nor is it as long as 2 billion years.

12) Note on pressure variations

All runs yielded a pressure variation (variable Y9) very close to the density variation (variable Y4). So, when the density was a certain percent above (below) the standard, the pressure was also that percent above (below) the standard.

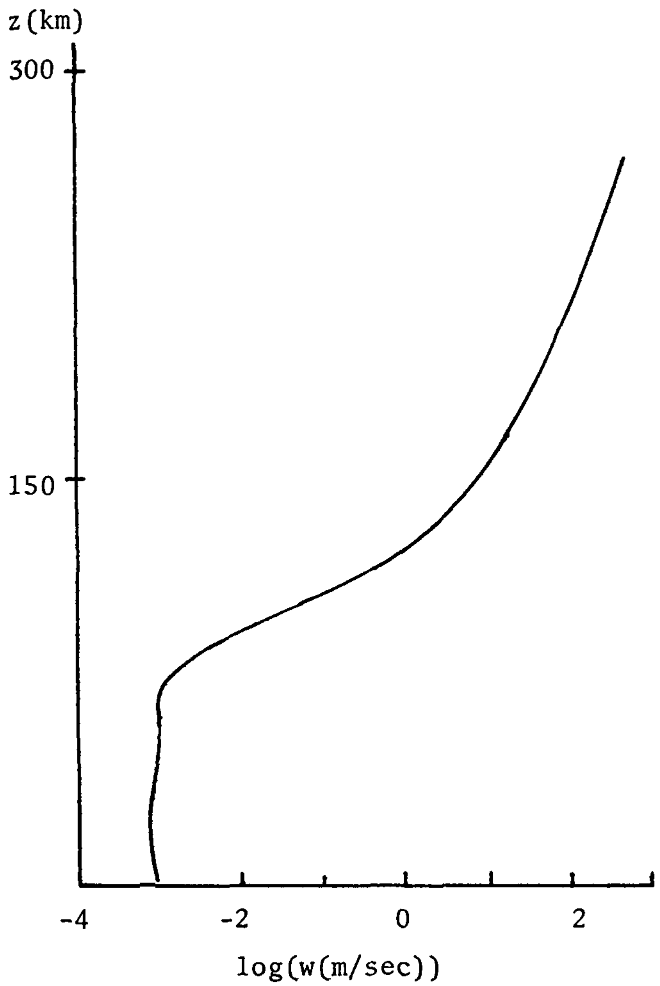

Figure 15. Vertical wind vs. altitude for $\zeta_z = 0.57(10)^{-5}$ sec^{-1}, u = 10 m/sec, $D_h = 1.21915(10)^{-7}$ sec^{-1}.

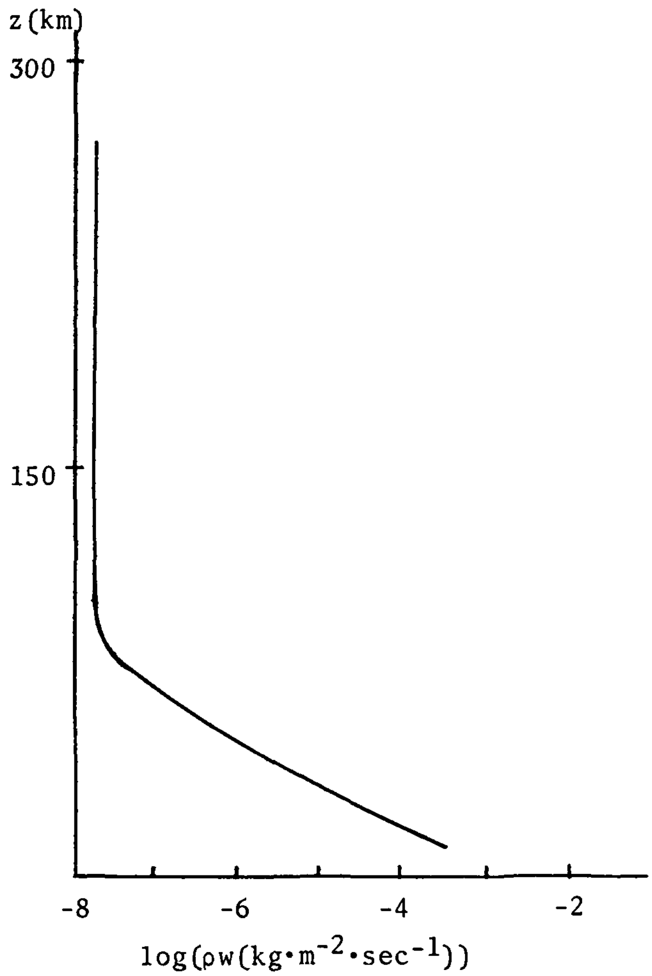

Figure 16. Mass flux vs. altitude for $\zeta_z = 0.57(10)^{-5}$ sec^{-1}, u = 10 m/sec, $D_h = 1.21915(10)^{-7}$ sec^{-1}.

CONCLUSIONS AND RECOMMENDATIONS

Objectives 1) and 2) were achieved with the possible exception of 1b); some variables have been renamed, but the changes have not been made in the code. Due to the fact that the code was still in a state of flux throughout most of the summer, time was short and objectives 3) and 4) were not achieved. They will remain as long term objectives. Objective 5) was never really attempted because a) the Apple microcomputer is so convenient in terms of accessibility and editing and graphing capabilities, b) time on the bigger computers is very limited, c) a decrease in run time by a factor of four on the Hewlett-Pachard 1000F is, although desirable, not significant, and d) time, as mentioned above, was limited. Perhaps, once the present code is brought to its final form, the translation to FORTRAN can take place.

As a result of this study, the following recommendations are put forth:

1) continue work on the code to achieve objectives 3) and 4),

2) extend the work done on objective 2) to the other modes in the program,

3) study the possibility of allowing for a variable horizontal divergence, rather than keeping it constant for all altitudes.

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