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A PHOENICS MODEL FOR THE TRIAXIAL LOADING OF AN  
INITIALLY CYLINDRICAL MASS OF RATE-TYPE MATERIAL  
WITH PROVISIONS FOR BULGING AND YIELD

|                            |  |
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ABSTRACT

This work traces the response of a granular material via the Ten Coefficient Truesdell rate-type constitutive model into the simplest meaningful loading: the triaxial test configuration. A functional relation has been posed for computing the rather peculiar relation between average applied stress and average porosity. Using that relation an attack has been mounted on the dilemma that exists between dynamic and constitutive use of the pressure variable; that is relating dynamic pressure, thermodynamic pressure, stress deviator and higher stress invariants. The resolution was as a linear superposition with a one-way feedback, in that while the dynamic component could not effect the constitutive component, the converse was not true since density appears in the momentum transport relation.

There were two stages of preparation for this effort. The first was to use the PHOENICS Satellite to create the boundary conditions simulating the triaxial test. Two opposed parallel disks are joined with a thin cylindrical membrane with the granular material inside and a confining pressure outside. The membrane condition is modeled as a circular tension ring active on slices parallel to the disks; ie "slabs". Use was made of the PHOENICS provision for porosity greater than unity to represent bulging. The second preparation was for the granular constitution, which was coded into the Ground portion of the PHOENICS package. Provision was made by an ancillary computation to test for yield condition attempt to locate shear banding. Several non-granular cases were built to test the boundary conditions independent of the constitution and the low velocity response of PHOENICS and to disarm inherent simple fluid prejudice in the code. Some results were obtained and a covey of suggestions for parameter and computational control were generated.

## ACKNOWLEDGEMENT

The efforts of my counterpart, Dr. N.C. Costes have insured that this second interaction with NASA at MSFC has been as productive and mutually rewarding as the first. All the members of the Fluid Dynamics Branch from Dr. Fichtl to the USRA and NRC Fellows have shown real enthusiasm for cooperative work and have lent me physical insight and CFD consultation.

I would like to acknowledge the ASEE/ SFF management team as well. Dr. Gerald Karr has administered the program with great dispatch, having held to a minimum the required paperwork and shown a considerable flexibility in dealing with the special requirements of university faculty on short term assignment. And on the NASA side my compliments to Dr. Dozier and especially to Leroy Osborn for hospitality and an excellent series of overview presentations on the status and future of NASA.

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## INTRODUCTION

This study has been directed at supporting an established and ongoing preparation for a triaxial test of spherical glass beads to be operated in the low-gravity environment of the Space Shuttle on orbit. The acronym MGM, mechanics of granular media, has been coined by the principal investigators, Dr. N.C. Costes and Prof. S. Sture, as the descriptor of this activity, and it is into that general arena that this effort has been cast.

The first phases of this work were performed in the summer of 1984 and reported in the image of this volume as Section XIV; "The Compatibility of an Existing CFD Code with a Broader Class of Constitutions". The results and nomenclature of that report are to some extent assumed as background, especially the portions directed at the flexibilities built into the PHOENICS CFD Code. Reference will be made on occasion to groupings of variables unique to that report and to the general forms of transport density and pressure.

To some extent the preliminary planning has been preoccupied with some of the aspects of getting the experiment into position for loading. These are all fairly extensive problems in their own right. Some of them are: 1) consolidation by launch acceleration with vibration, 2) membrane leakage during drawdown during confinement, 3) consequences of granule friction on the end platens during preload cycling, 4) the visibility of shear-bands with non-orientable granules ( spheres ), and 5) the prejudice of a failure surface to a noncylindrical initial boundary.

Granular mechanics with its more complicated constitutive behaviour has had to wait longer for analytic treatment than simple fluids and simple solids. The general realm of constitutive types is shown as Figure 1. In looking to associate a complicated behaviour with "fluid" or "solid", the back and forth coupling lines on the figure are very illuminating. The point being that a step up the tree toward generality from either simple fluid or simple solid branches is coupled to both. Hence it is perhaps not surprising that a flexible scheme for CFD might be amenable to computational MGM.

The governing parameter is the extent of intergranular stresses. In gravity currents of dust laden air or silt laden water, the granular material supplements the density but can be ignored in the constitution. In a fluidized bed the granular matrix remains stationary or slowly circulating, but once levitated the intergranular stresses are low, likewise in the liquid counterpart percolation. Alternately in the very similar "pore-water" problem the active element is the granular matrix with the liquid adding a deviator hence the conditions; drained/ undrained. Recently there has been a flurry of activity using kinetic

theory to model high-speed granular flows, obviously a class of flows in which the effects of the environmental fluid are ignored. A CFD package like PHOENICS includes features for multiphase flows to make modeling of granular-laden fluids possible, but of course allow no automatic mechanism for intergranular stress computation. Further Phoenics provides a constant named DARCY which builds a linear pressure gradient on volume flow rate. Herein quite a different approach is taken in which the constitution is coded into the exchange coefficients and functionally related to gradient of velocity and the density as well as a "pressure" adjustment which adds intergranular stresses to the dynamic component.

This study is then at the intersection of a contemporary CFD package taken to the outer limit of its flexibility and numerical MGM taken as its most comprehensive constitutive description. The emphasis is on matching feature for feature rather than creating a massive supplement to the PHOENICS code.

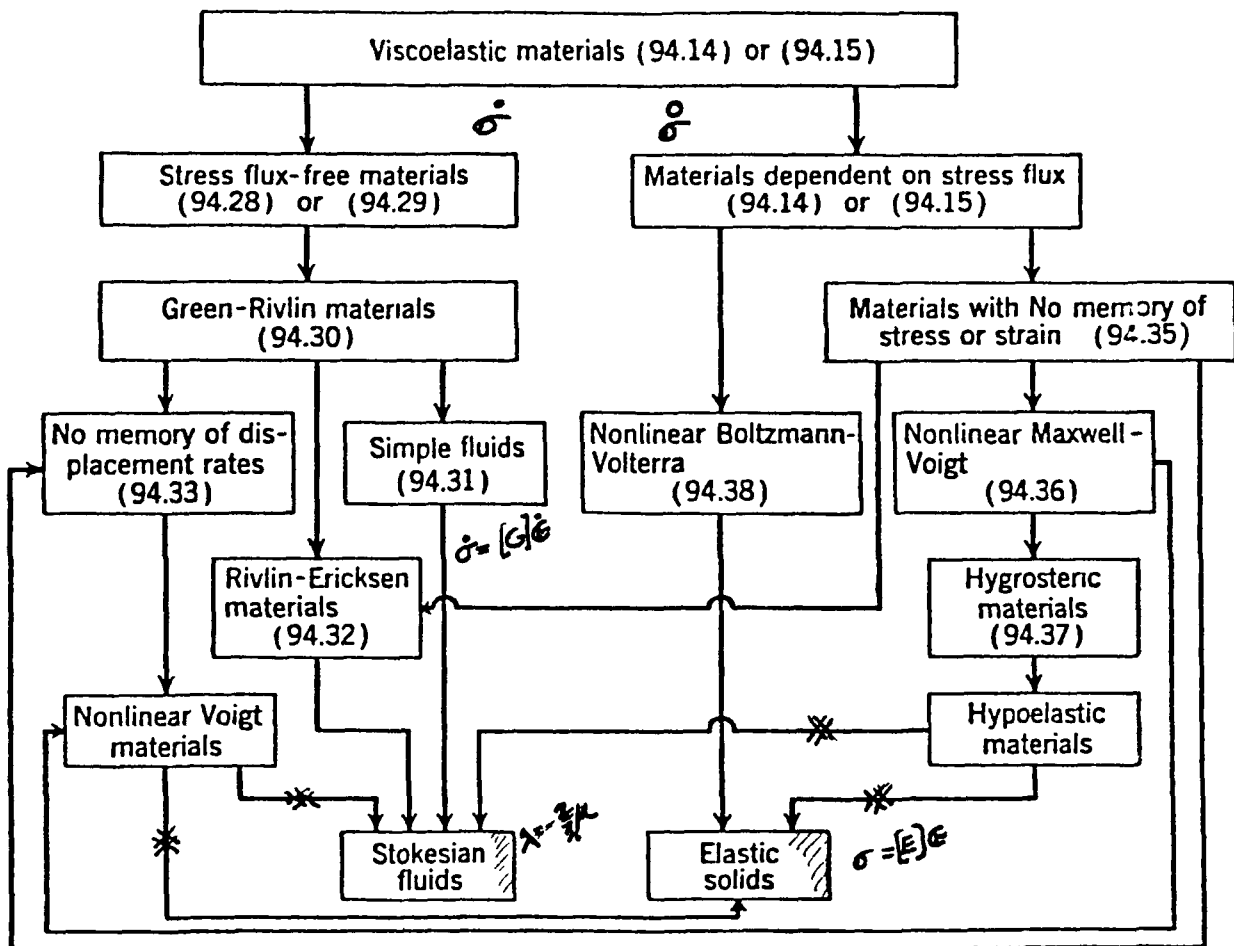


FIGURE 1 Constitutive Realms

## OBJECTIVES

The primary objective of this study was to produce a cataloging of constitutive and boundary features that must be included in numerical models whether the destination code be PHOENICS or some alternative. While "PHOENICS thought" has been used to give some explicit definitions to the features, the features themselves are generic. Hence the object sought was the simplest model that retained the full features of the rationally derived MSD procedure while applying only to the triaxial configuration. Perhaps the greatest accomplishment in this regard is to arrive at specific conclusions for handling the boundary conditions in a way that is amenable to the PHOENICS code, MICROFEM, or a custom code and more importantly to include the discoveries of preliminary testing. Further objectivity was sought by making explicit the offset terms in the Jaumann stress flux for this geometry and loading.

Specifically the intention herein was to approach the goal of predicting specific loading responses from the rational direction, carefully highlighting the simplifications to permit backtracking should comparison of the model with experimental results ultimately show it to be inadequate due to over simplification. Since this study is preliminary to extensive testing it was not desirable to over simplify just to obtain "instant gratification" as analytic solutions.

Another objective has been to emphasize the need for preparation of a numerical approach that explicitly generates fields of "signed" deviatoric stress power and makes them available to the evolving calculation. It is also desirable to determine the inhibition or enhancement parameter for shear bands based on granule size and shape, viz large spherical granules.

THE PRESSURE-DENSITY DILEMMA

The goal is to relate pressure in the PHOENICS sense with the elements of  $\text{tr } \sigma$  and to trace the flow of influence of density into the PHOENICS pressure both inherently and user coded. The goal is extended to include the realm of the triaxial test and to show what measures are necessary to circumvent PHOENICS provisions and to implement the full constitution of a granular media.

Sevenness to the ancient Hebrews indicated completion of a cycle and so it is with PHOENICS; six adjacent neighbors in space and one in time. Figure 2 shows PHOENICS pressure as the pressure component with a dynamic origin,  $P_{\text{DYN}}$ , and in obedience to Pascal's Principle it has

equal components and lies along the hydrostatic line. The source term for momentum transport equations ( $\phi_i = V_i$ ) at dynamic equilibrium is the gradient of pressure; hence:

$$S_{\text{VP}} = -\rho\beta \frac{D\phi}{Dt} + \frac{1}{R\beta} \nabla \cdot (X \frac{\nabla \phi}{\rho}) + \frac{\phi X}{R\beta} \text{tr } D$$

The appearance of density,  $\rho$ , and transport density  $X = \rho\beta R$ , are the coupling between the constitutive equation and the computation of pressure. Figure 4 illustrates a "Ring Model" of computation to highlight the numerical symbiosis of pressure, momentum and mass. If thermodynamic pressure is defined in terms of the stress tensor as;

$$P_{\text{TH}} = 1/3 \text{tr } \sigma$$

and further if thermodynamic pressure is considered to be a linear superposition of a constitutive and a dynamic part;

$$P_{\text{TH}} = P_{\text{CON}} + P_{\text{DYN}}$$

which would still appear along the hydrostatic axis for some types of constitutions viz; ideal gas. The linear assumption is of course not correct in general since the components have separate functional dependencies on density. In fact with a granular media behaviour changes during loading must be included and so tests have to be made for yield or rupture to determine the local functional relation between pressure and density. As a further complication, granular materials require a rate-type constitutive model, ( all the references but #2 deal with this necessity ). Otherwise the time dependency in the thermodynamic pressure could be isolated in the dynamic part, which would be very convenient in the CFD environment. The lure of incremental models is that time can be "eliminated" by multiplying the constitutive equation by  $Dt$ . But at best the incremental formulation introduces coordinate



system problems more difficult for CFD than the gradient of velocity components which are necessary for rate-type models with stress flux.

$$\begin{array}{l}
 \text{"rate - type"} \\
 \dot{\sigma} = [ ] \dot{\epsilon} + [ ] D + [ ] \sigma \\
 \downarrow \\
 \sigma W = W \sigma \\
 \downarrow \\
 \dot{\sigma} = [ ] \dot{\epsilon} \dots \text{"incremental type"} \\
 \sigma = [ ] \epsilon \dots \text{"Hooke's Law"}
 \end{array}$$

$\overset{\text{"Newton's Law"}}{\curvearrowright}$

To see what reductions can be made with a triaxial configuration, a glance at Figure 2 shows the profile of the triaxial test in normal stress space ( $\sigma_2 = \sigma_3$ ), the hydrostatic condition ( $\sigma_1 = \sigma_2 = \sigma_3$ ) which is a subset, and the stress deviator ( $\sigma_1 + \sigma_2 + \sigma_3 = C$ ) which has the same direction, but less restriction. Consider first the normal stress-normal strain relationship as shown in Figure 5; an accepted shape in soils testing. The very important "knee region" shows the difficult dependence on initial density; double-valued for initially dense conditions and for large axial strain approaching the critical state asymptotically; for which it is impossible to distinguish between initially loose and initially dense. It is analytically convenient to introduce without any rational justification (and apparently without precedence) the three parameter relation:

$$SR / SR_C = \tanh(ae) + b(1 - [\tanh(ae - c)])$$

The triaxial case gives:  $p = 1/3 \text{ tr } \sigma = 1/3 \sigma_3 (SR + 2)$

with:  $SR = \sigma_1 / \sigma_3$ . The slope of the lower portion of Figure 5 is a sort of 3-D Poisson's Ratio:  $\frac{\partial \epsilon}{\partial \nu} (\int d \ln v)$  it is notable that the zero of the ordinate represents:  $\text{tr } D = 0$ .

While the loose extreme compresses monotonically, the dense extreme exhibits an initial compression followed by expansion to critical state. This change of behaviour can prove to be very destabilizing to a numerical scheme.

The classical description of the critical state is given as;

$$P_c / P_0 = \exp\left(\frac{v_0 - v_c}{K}\right) \quad \text{in which specific}$$

volume, the inverse of density, has been used and  $P = \sigma_1 - \sigma_3$

so that  $\text{tr } \sigma = P - 1/3 \sigma_3 (1 + 2^*SR) \dots \sigma_3$  - confining pressure.  
 In the more comprehensive rate-type model, this behaviour is nested in the first term of the denominator of the pressure expression:

$$P = - \frac{DP}{Dt} / (\alpha \dot{v} + \frac{2b\mu}{M^2} [(\frac{v}{v_c})^\delta - 1] \text{tr } \sigma^* D^*)$$

and if all the thermodynamic time dependence were in the dynamic component of pressure;

$$P = - \dot{P}_{TH} / ( \dots )$$

The exponent  $\delta$  represents the nonclassical coupling between pressure and density. One of the difficulties of using this pressure model is that the deviatoric stress power can change sign in some regions of the test and the resulting loading/unloading behaviour must be traced by a supplementary computation in a CFD environment.

Isochoric plastic flow is one ostensibly at constant volume in which  $\text{tr } D = 0$  hence  $\nabla \cdot V = 0$ , a flow with identically constant density. For this case the full constitutive relation reduces to:

$$\dot{\sigma}^* = \sigma^* - W \sigma^* + \sigma^* W$$

and by definition the deviatoric stress rate tensor is:

$$\dot{\sigma}^* = \frac{2\mu}{M^2} (P_c \mathbb{I} - \sigma^*) \text{tr } \sigma^* D + M^2 D$$

The velocity vector has its gradient distributed in the standard way into deformation rate and rotation tensors:

$$\underline{\nabla V} = D + W$$

the assistance of the latter has already been invoked in making an objective quantity from the substantial derivative of stress. The deformation rate is inherent in the rate-type constitution and of course appears there in the stress power with deviatoric and normal components and in the yield condition also in the stress power.

$$\text{tr } \sigma D = \text{tr } \sigma^* D^* + P \text{tr } D$$

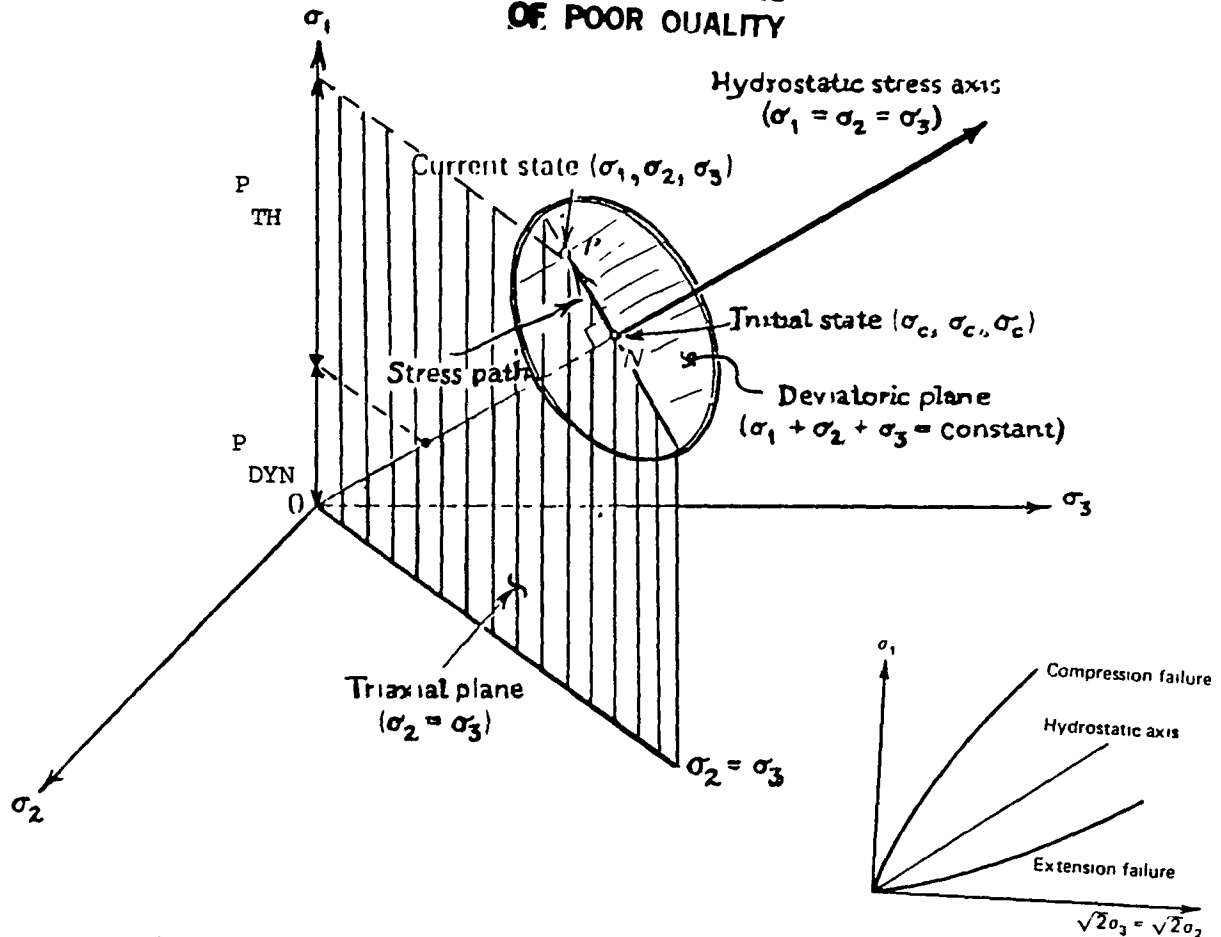
Macroscopically thermodynamics will place a condition on the integral

$$\int \left[ \frac{P}{Dt} \int [ \int \sigma \cdot dE ] dv \right] dt$$

which represents the total energy expenditure on the test material.

$$D = D^* + \left( \frac{1}{3} \text{tr } D \right) \mathbb{I}$$

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Triaxial Apparatus

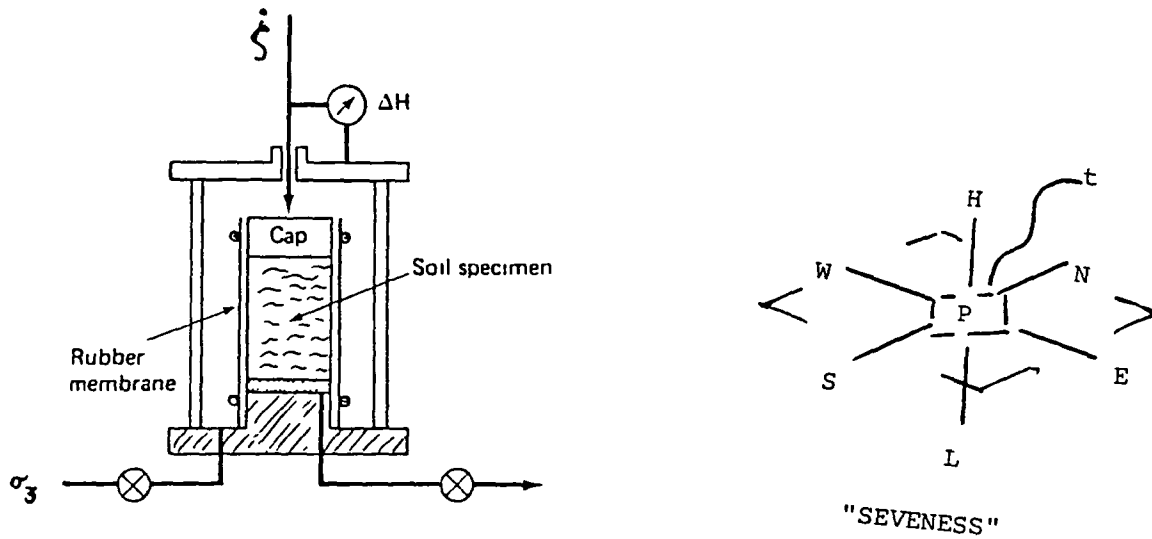


FIGURE 2 NORMAL STRESS SPACE

Failure comes in the forms of rupture and yield. Figure 2 shows on a triaxial plane the lack of symmetry of the rupture boundaries about the hydrostatic axis ( "compression/ extension failure" ). The other surface forming the failure boundary is given by the yield cap. Following the procedure of MSD ( 78 - 84 ) the yield surface is established by a set of Q vectors which render the array G, singular

$$\dot{Q} = [G(Q, \rho)] \mathbb{P} \dots Q = \begin{bmatrix} p^2 \\ q^2 \end{bmatrix}$$

p - stress deviator...1/3 tr O

q - square root of the second invariant  $\sqrt{\sigma^* \sigma^*}$   $q_c = M_0 p_c$

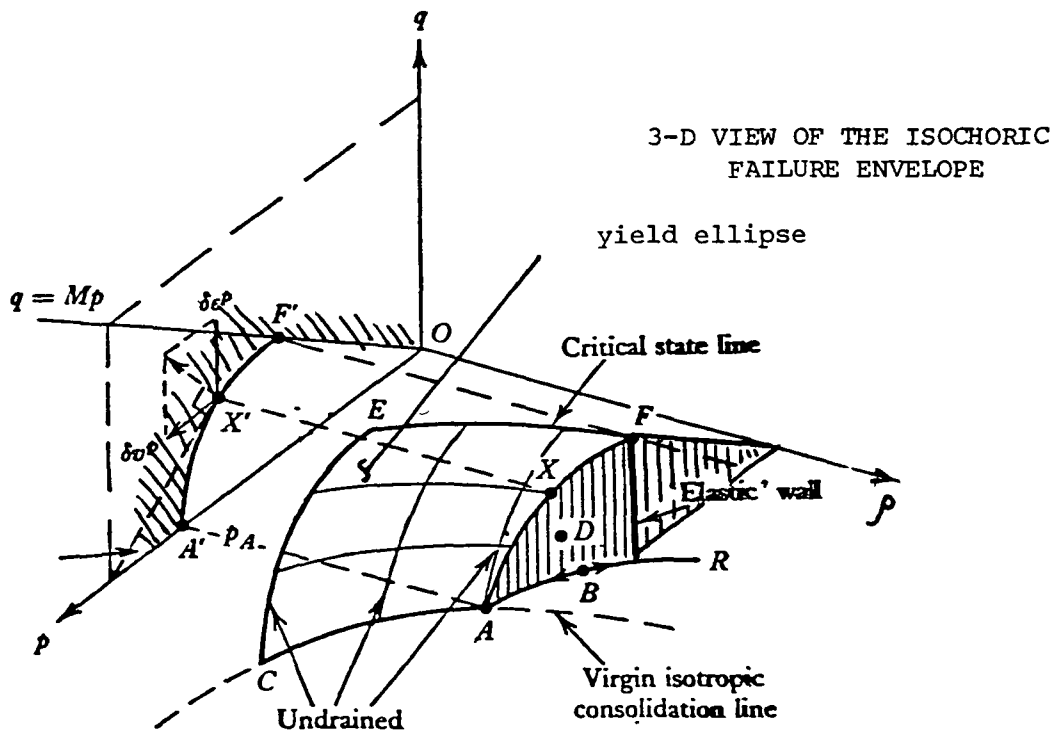
p tr D

$\mathbb{P}$  - power tensor -  $\begin{bmatrix} p & \text{tr } D \\ \text{tr } \sigma^* & D^* \end{bmatrix}$

The density variation coming from the critical pressure-specific volume relation given above. An overly simple verbal description of this equation might run; the stress invariants are related to exponentially decaying transients scaled in the components of stress power. Aside from that description an algebraic result can be obtained if the reduced constitution ( isochoric) above is used;

$$q^2 = M_0^2 ( 2 p_c p + p^2 )$$

$M_0$  and  $p_c$  are the classical parameters of critical state. This relation plots as an ellipse in q-p space and hence the failure envelop looks like a somewhat distorted icecream cone; as it appears in sections of the q-p- $\rho$  space at constant density.



Digressing to the specifics of a triaxial configuration with full  $\theta$  symmetry; the stress deviator is:

$$1/3 \text{tr } \sigma = \sigma_1 + 2\sigma_3$$

and hence the deviatoric stress is:

$$\sigma^* = \begin{bmatrix} 2/3(\sigma_1 - \sigma_3) & & \\ & 1/3(\sigma_3 - \sigma_1) & \\ & & 1/3(\sigma_3 - \sigma_1) \end{bmatrix}$$

and the deformation rate is:

$$D = \frac{1}{2} \begin{vmatrix} 2 \frac{\partial v_z}{\partial z} & \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} & 0 \\ \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} & 2 \frac{\partial v_r}{\partial r} & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

and the deformation rate deviator is ;

$$\frac{1}{3} \text{tr } D = \frac{1}{3} \nabla \cdot \underline{v} = \frac{1}{3} \left( \frac{\partial v_z}{\partial z} + \frac{\partial v_r}{\partial r} \right)$$

$$D^* = \begin{vmatrix} \frac{2}{3} \frac{\partial v_z}{\partial z} - \frac{1}{3} \frac{\partial v_r}{\partial r} & - & - & 0 \\ - & \frac{2}{3} \frac{\partial v_r}{\partial r} - \frac{1}{3} \frac{\partial v_z}{\partial z} & - & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

and the spin rate is:

$$W = \begin{vmatrix} 0 & \frac{\partial v_z}{\partial r} - \frac{\partial v_r}{\partial z} & 0 & 0 \\ \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

All these components then can be used with the exchange coefficients for the PHOENICS momentum transport equations; for the  $i$ th :

$$\sigma_{*i}^0 \rightarrow \begin{vmatrix} 2/3 \dot{\sigma}_i & & \\ & -1/3 \dot{\sigma}_i & \\ & & -1/3 \dot{\sigma}_i \end{vmatrix}$$

$$(\dot{\sigma}_3 = 0) \& (W \sigma^* = \sigma^* W)$$

$$\Gamma_{\text{mom}, i} = \left( \frac{\sigma_{*i}^0 - 2\mu D^*}{b \left( \frac{\rho c}{\rho} \right)^{\delta} 2\mu \frac{\tau \sigma_{*i} D^*}{M^2}} \right)$$

$$\begin{array}{c} \nabla V_i \\ \curvearrowright \\ D_i + W_i \end{array}$$

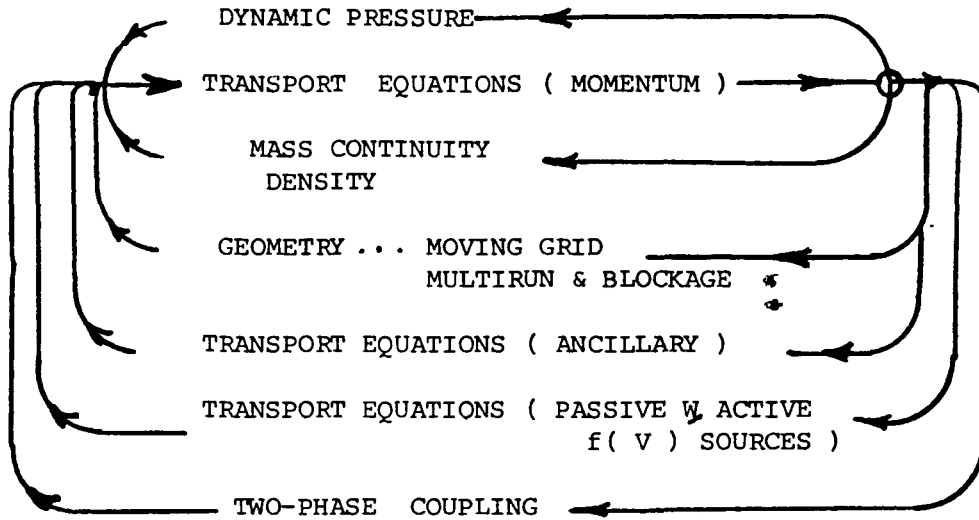
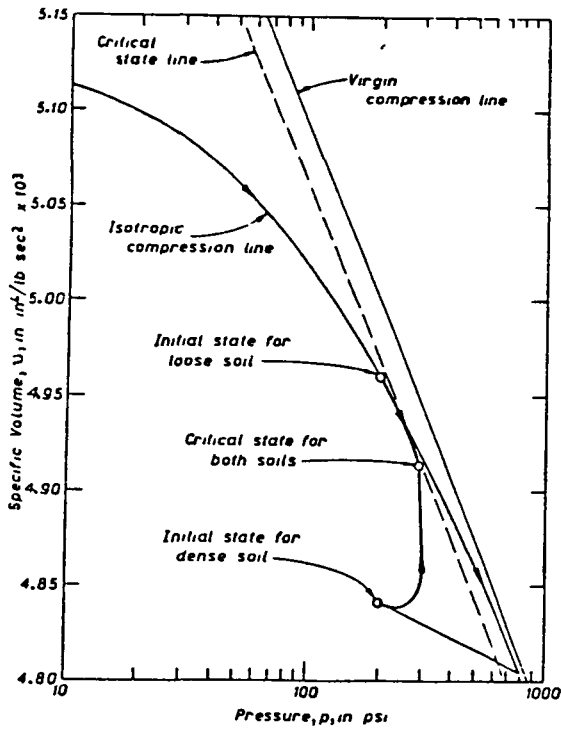
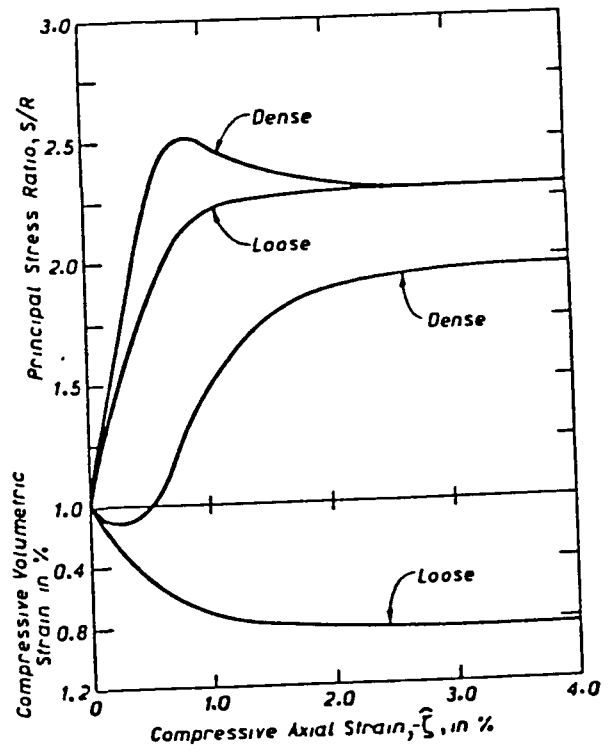


FIGURE 5 A RING MODEL FOR PHOENICS



PRESSURE-DENSITY STATES

FIGURE 3



GRANULAR STRESS-STRAIN & POISSON

FIGURE 4

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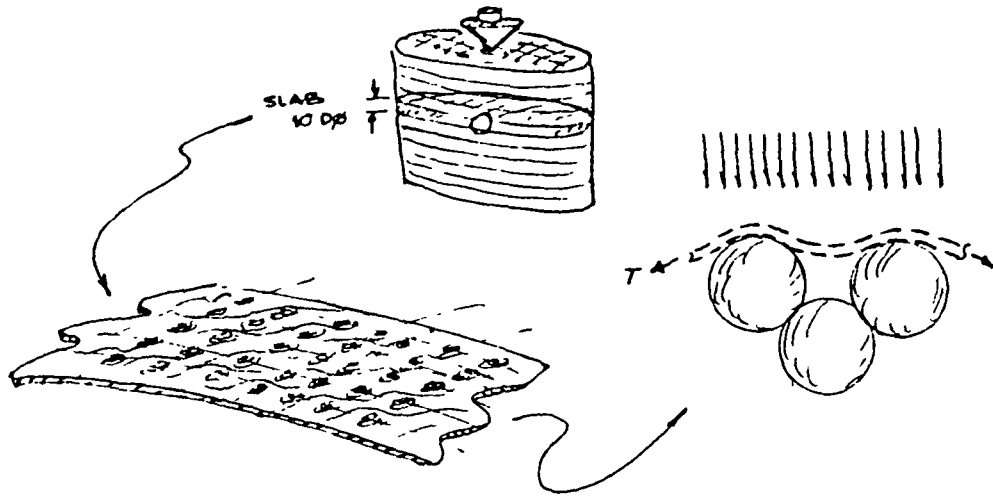


FIGURE 9 SLABS OF SPHERES

- \* PLATEN FRICTION  
(pseudo-bulging)
- \* LOCAL CURVATURE
- \* SHEAR BAND  
LOCATION/ DETECTION
- \* CONSOLIDATION  
HISTORY
- \* CONFINEMENT  
vs  
MEMBRANE ELASTICITY

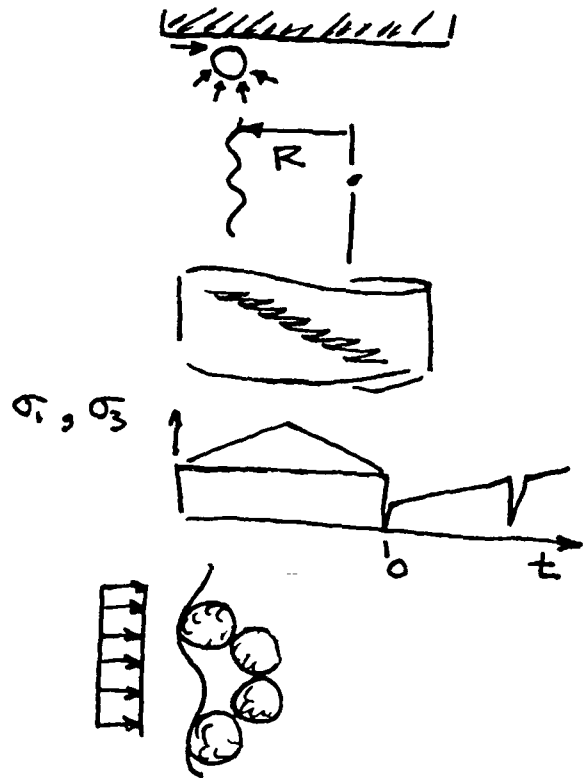


FIGURE 10 SPECIAL CONSIDERATIONS

FIGURE 7

.....List of PHOENICS FILES (KENFILES.TXT).....xxxSAT.FTN

| #  | xxx        | Implementations  |
|----|------------|--|
| 1  | KEN        | 2D POLAR PORUS PLUG 10X20X1<br>UNIFORM INFLOW INTO ANNULUS<br>FREE REGION AROUND PLUG                              |
| 2  | FRE        | 3D CARTESIAN DIAMOND FIELD<br>20X20X20 OBSTRUCTIONS<br>UNIFORM INFLOW  |
| 3  | KWF        | DEL SQR T = DT/Dt..CONDUCTION<br>TRANSIENT 20X20X1 2D <u>NOFLOW</u>  |
| 4  | NCH        | DEL SQR H1 = KC1 & DEL SQR H2 = KC2  |
| 5  | NET        | BUILD FILE..LOW V IN KWF   |
| 6  | TAL        | TRIAxIAL LOADING   |
| 7  | TAG        | TRIAxIAL GEOMETRY WITH IDEAL GAS   |
| 8  | TAG1       | ENCHROACHING END CAP   |
| 9  | TAG2       | DRAINING UNDER VACUUM  |
| 10 | TAG3       | MASS REDISTRIBUTED OVER MULTIRUN   |
| 11 | TAG4       | MEMBRANE CONDITION/ LATERAL  |
| 12 | TRI        | TRIAxIAL LOADING: HYPERBOLIC TANH<br>PREYIELD: "EXCO":GRANULAR   |
| 13 | TRY        | (TRI AND TRY AGAIN ! ) YIELD<br>DET G SINGULAR PORTION OF FIELD  |
| 14 | BEN        | BENARD: LOW VELOCITY TEST WITH ONSET<br>OF CONVECTION:PLANE LAYER 10X10X10<br>BENFLD INITIAL FIELD FOR INSTABILITY |
| 15 | KEN (.JOB) | USER NAME KWF  |
| 16 | TAL (.JOB) | USER NAME KWF..RUNS TALSAT.FTN, .EAR,.LOG  |

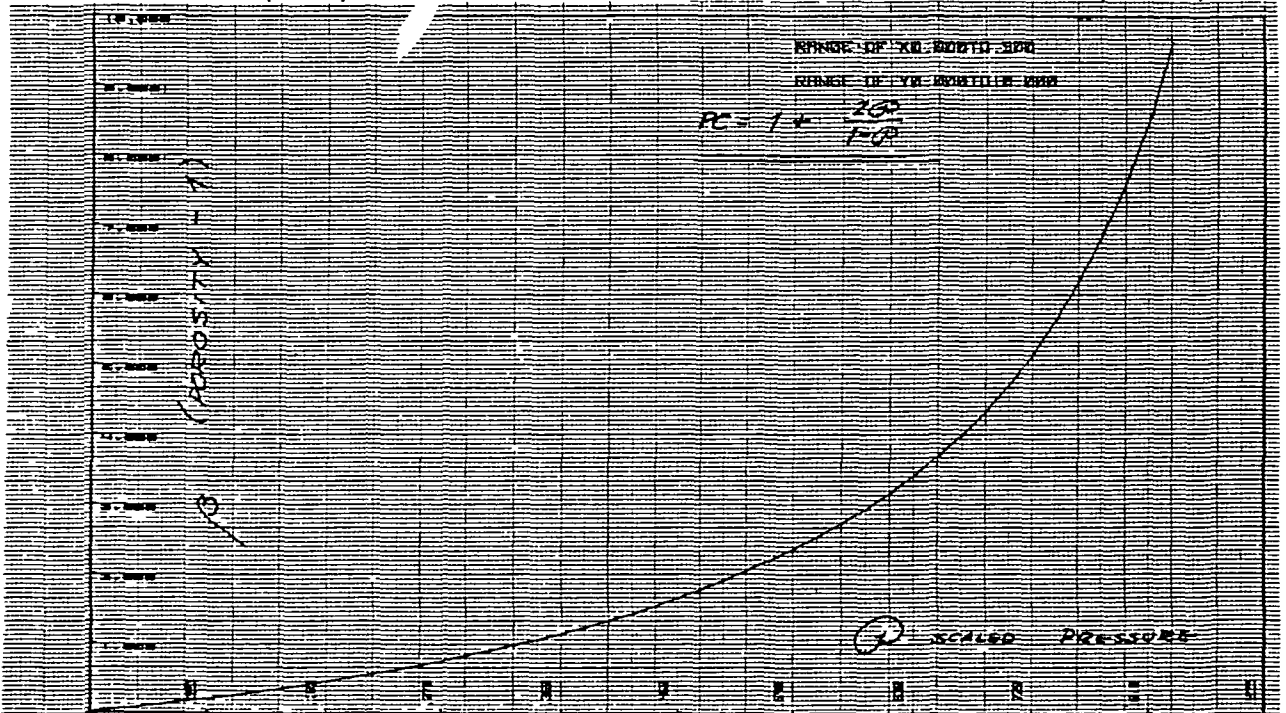


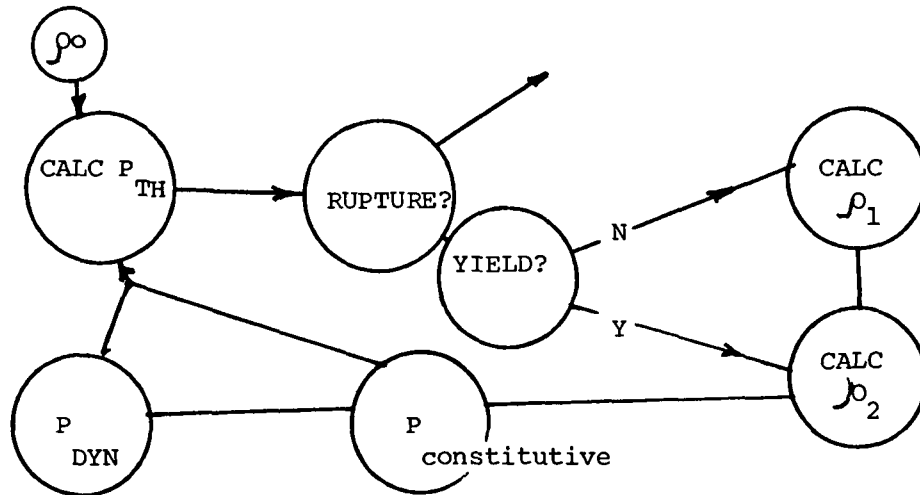
FIGURE 8 POROSITY VARYING WITH SCALED PRESSURE



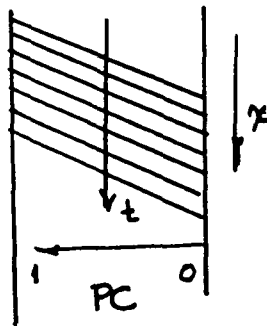
LATERAL & LONGITUDINAL BOUNDARIES

Aside from dealing with a very difficult constitution internal to the cylindrical test region, the boundary conditions present formidable difficulties to a CFD package; Figures 2,9&10. The end platens are rigid and nominally frictionless; however preliminary testing has shown that if the granular material is glass spheres( a necessity for a SS based experiment ), the friction is very tenuous. The consequences of friction to the boundary conditions and to numerical computation are that an additional confinement is present on the upper and lower regions ( 1/3 platen diameter ? ) of the cylinder of test material. Typically this over-confinement causes premature, asymmetrical bulging. Further there will certainly be a disruption to the proper growth of the failure surfaces; viz. shear bands.

The steadily advancing ( constant  $c$  Figure 4 ) upper platen must be treated with caution numerically, since a step displacement might cause a jump in local density and hence an irreversible catastrophic change in the local constitution. That is the parallel running computations for testing realms of behaviour might be triggered, before the density wave could diffuse...."wave and diffuse referring to the numerical and not physical uses of the terms".



This advancing platen problem has been tackled in the same way as the confining membrane condition. In both a use of the porosity in the transport density,  $\rho \beta R$ , was implemented. The platen is replaced by an advancing wave of increasing porosity; ie the "fuzzy platen".



For the bulging of the membrane it will be treated by allowing the circumferential cells to have porosity in excess of one. Figure 8 shows a simple model of the change of porosity with which is a pressure scaled to membrane stress,  $P/2MK$ . Since friction over the spherical surfaces changes the membrane tension exponentially (  $T/T_0 = \exp(-f\theta)$  ), the lateral membrane effects

are small and a narrow slab ( say  $10 D_0$  ) can be isolated for analysis. This will give an artificial "aliasing" as the resolution jumps from  $D_0$  to  $10 * D_0$  and since shear bands will be suppressed to vertical orientation on each slab.

## RESULTS & CONCLUSIONS

The preliminary specifics for the numerical treatment of triaxial loading of a granular material in the PHOENICS environment have been laid to rest; constitution, boundary conditions and loading simulation; with the details given herein for slabwise, 0 independent modeling. Figure 6 shows some of the forays into the numerical realm to test various schemes for handling this configuration and loading. Special interest was placed on the treatment of dynamic and thermodynamic pressure and the correct decomposition of stress tensor into that pressure and the momentum exchange coefficient. The 0 dependent terms in  $\text{Grad } V$ ,  $O$ , etc, if the shear bands are to be predicted. That inclusion will be necessary but advisable only after the much simpler version is compared with test results. Those 0 dependent terms make a considerable change in the accounting and in the "objectivity offset", but the changes are straightforward in the current framework.

A continuing doubt of the validity of PHOENICS at very low velocities and especially at small displacements, (  $V dt$  ) persists. However the preparations and the lessons in coding will be useful in general and to any numerical approach to the triaxial loading of a granular material. A considerable portion of the summer's effort was spent pursuing minor avenues ( draining, percolation, preloading, etc ) that are not included in this report but which provide support to the project as a whole and all dealt with various aspects of MGM.

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