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"APPLICATION OF WEIBULL ANALYSIS TO SSME HARDWARE"

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A STUDY OF WEIBULL ANALYSIS TECHNIQUES AND ITS UTILIZATION TO STUDY PARTS/ASSEMBLIES FAILURE OF THE SPACE SHUTTLE MAIN ENGINE

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ABSTRACT

Generally, it has been documented that the wearing of engine parts forms a failure distribution which can be approximated by a function developed by Weibull. The purpose of the above mentioned study is to examine to what extent does the Weibull distribution approximate failure data for designated engine parts of the Space Shuttle Main Engine (SSME). The current testing certification requirements will be examined inorder to establish confidence levels. An examination of the failure history of SSME parts/assemblies (turbine blades, main combustion chamber, or high pressure fuel pump first stage impellers) which are limited in usage by time or starts will be done by using updated Weibull techniques. Efforts will be made by the investigator to predict failure trends by using Weibull techniques for SSME parts (turbine temperature sensors, chamber pressure transducers, actuators, and controllers) which are not severely limited by time or starts.

INTRODUCTION

During the early thirties, Fisher and other statisticians developed a new probability distribution. The distribution developed was not utilized significantly in application until the late forties by Waloddi Weibull. In 1951, Weibull presented his argument internationally for the extensive usage of the new distribution to describe mathematically the behavior of fatigue data. As time progressed, Weibull distributions were found to be very good mathematical approximations of failure data related to engine parts analysis, thermo cycles analysis (large numbers of thermo cycles), and electronics analysis. The Weibull distribution is currently being applied extensively to predict the reliability of Space Shuttle Main Engine (SSME) parts or units. The mathematical model for the Weibull distribution is:

$$F(t) = 1 - e^{-\left(\frac{t-t_o}{\eta}\right)^{\beta}}$$ where

$F(t)$ = Fraction Failing Up to Time t

t = Failure time

*t_o = Starting Point of the Distribution

η = Characteristic Life

β = Slope on Weibull Paper

e = Exponential

$$R(t) = e^{-\left(\frac{t-t_o}{\eta}\right)^{\beta}}$$ where

$R(t)$ = Reliability Up to Time t

$F(t) = 1 - R(t)$

*t_o is generally assumed to be zero.

The Weibull distribution's shape is determined by the value of beta. Beta is the slope of the line graphed on 1 - to -1 Weibull paper. The probability density function is: $\frac{dF(t)}{dt} = f(t)$

$$f(t) = \frac{\beta t^{\beta-1}}{\eta^{\beta}} e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

The hazard function describes the instantaneous failure rate. The hazard function is:

$$h(t) = \frac{f(t)}{R(t)}$$

$$h(t) = \frac{\beta t^{\beta-1}}{\eta^{\beta}}$$

For the engineer, beta gives a cue as to what type of failure conditions exist, i.e. rapid-wear, green-run etc. Eta does not provide extensive information. Eta is mathematically derived as always being the time where approximately 63.2% of the sample fails. Eta is used in calculations related to risk analysis and substantiation work. The engineer should examine B life points, i.e. B1 is the time where 1% of the sample fails, B.05 is the time where 0.05% of the sample fails etc. For the eta, which is called the characteristic life, can be large but due to the slope the B1 or B10 life may be small. This paradox occurred in a study by Rheinfurth (1985) related to chamfered and unchamfered blades. The unchamfered blade had eta equal to 1,142,538 seconds and a B1-life of 413 seconds. The chamfered blades had an eta equal to 69,324 seconds and a B1-life of 5,429 seconds. The difference in B1-life was created by the difference in the slope, beta. The usage of the Weibull distribution can also help to unmask multiple failure modes. A significant increase in the slope can indicate lot (batch) problems.

GENERAL FINDINGS

In most instances, there is an assumption made pertaining to the distribution type. Carter, Bompas-Smith, and Nelson describes the Weibull distribution as being one of the most accomodating distribution available. For, the distribution can represent increasing and decreasing failure rate. Particularly, skewed distributions can be represented. Skewed distributions can not be represented by log normal distributions. Brook asserts that the

Weibull distribution allows the making of simple statistical judgements with very little labour or efforts. Documentation by reliability engineers and statisticians in industry and government exist with respect to the high level of predictability of Weibull distributions. The U.S. Air Force and Pratt-Whitney in particular use Weibull Analysis extensively to predict the failure of jet engine parts. Breneman of Pratt-Whitney Company has 15 years of documentation that the physics of wear for a particular part of a jet engine does not significantly alter with modifications. This means the slope or failure mode doe not significantly change. This finding can be used in a process to predict failure on new parts with material changes or design modifications where the new parts have not experienced failure. The methodology used is Weibayes analysis. The confidence intervals for β or the Weibull line can assist the engineer in reaching a decision as to whether or not a given part's failure mode has been modified.

WEIBULL ANALYSIS FOR SSME PARTS

The establishment of a slope (beta) file for SSME parts should be the initial task of engineers. Due to the small number of failures allowed by NASA engineers for most SSME parts, documentation of the initially derived slopes over an extended period of time will not be possible. Yet, Breneman asserts that 3 failures in a sample are sufficient to give a good approximation of the failure mode.

The likelihood function for the Weibull distribution is a useful tool. The likelihood function is more mathematical in nature since reliability is a part of the computational process. Waloddi Weibull (1969) described the mathematical traits of the likelihood function for three different conditions. The models are:

Type 1: r units fail (L_1) for some t_i failure time

$$L_1 = \prod_{i=1}^{r} f(t_i) \text{ where } f(t_i) \text{ is the density function}$$

Type 2: S units still running (L_2) for some Z_j time

$$L_2 = \prod_{j=1}^{S} [1 - F(Zj)]$$

or

$$L_2 = \prod_{j=1}^{S} R(Zj) \text{ where } R(Zj) \text{ is the reliability}$$

Type 3: d_K units fail for some K number of inspection intervals, where the interval size is W_K - W_{K-1} of time.

$$L_3 = \prod_{k=1}^{K} [F(W_K) - F(W_{K-1})]^{d_K}$$

where: w_k = the end of the interval

w_{k-1} = the beginning of the interval

or

$$L_3 = \prod_{k=1}^{K} [R(W_{K-1}) - R(W_K)]^{d_K}$$

So for each given condition, the final likelihood function for a total sample is

$$L = L_1 \cdot L_2 \cdot L_3$$

$$\ln L = \ln L_1 + \ln L_2 + \ln L_3$$

For the Weibull distribution in logarithmic form

$$\ln L = \sum_{i=1}^{r} \ln \left[\frac{\beta t_i^{\beta-1}}{\eta^\beta} e^{-\left(\frac{t_i}{\eta}\right)^\beta} \right] + \sum_{j=1}^{s} \ln \left[e^{-\left(\frac{t_i}{\eta}\right)^\beta} \right] + \sum_{k=1}^{K} d_k \ln \left[e^{-\left(\frac{w_{k-1}}{\eta}\right)^\beta} - e^{-\left(\frac{w_k}{\eta}\right)^\beta} \right]$$

The Weibull likelihood function can be further expanded by the properties of logarithims. Other models can be generated when all three conditions do not exist.

$L_{12} = L_1 \cdot L_2$ Model which gives the description of η, Weibayes analysis

$$L_{12} = \prod_{i=1}^{r} f(t_i) \prod_{j=1}^{s} R(z_j)$$

$$L_{12} = \prod_{i=1}^{r} \frac{\beta t^{\beta-1}}{\eta^{\beta}} e^{-\left(\frac{t_i}{\eta}\right)^{\beta}} \prod_{j=1}^{s} e^{-\left(\frac{z_j}{\eta}\right)^{\beta}}$$

r = number of units run to failure

s = number of unfailed units

t_i = known failure time

z_j = operating time on unit

Take the logarithm of both sides and differentiate. We get:

$$\frac{\partial \ln L}{\partial \beta} = 0, \quad \frac{\partial \ln L}{\partial \eta} = 0 \qquad \text{solve simultaneously}$$

Generally the Newton-Raphson method can be used to solve the equations. But, caution is needed when using this technique since under certain conditions the process may not converge (Campbell, 1985). Initializing a beta value is best done by obtaining some estimation from a Weibull plot.

The model L_{23} should be utilized when the failure data is interval of inspection.

$$L_{23} = L_2 \cdot L_3$$

$$L_{23} = \prod_{k=1}^{K} \left[R(w_{k-1}) - R(w_k)\right]^{d_k} \prod_{j=1}^{s} R(z_j)$$

$$\ln L_{23} = \sum_{k=1}^{K} d_k \ln\left[e^{-\left(\frac{w_{k-1}}{\eta}\right)^{\beta}} - e^{-\left(\frac{w_k}{\eta}\right)^{\beta}}\right] + \sum_{j=1}^{s} \ln\left[e^{-\left(\frac{z_i}{\eta}\right)^{\beta}}\right]$$

The L_{23} model can be modified to coincide descriptively with test runs. Rheinfurth (1985) at MSFC, ED01, developed a Weibull likelihood for the observation interval data base on N trial runs. Using the L_{23} model

$$\ln L = \sum_{i=1}^{N} d_k \ln\left[e^{-\left(\frac{w_{k-1}}{\eta}\right)^{\beta}} - e^{-\left(\frac{w_k}{\eta}\right)}\right] - \left(\frac{1}{\eta^{\beta}}\right) \sum_{i=1}^{N} (M - d_k) w_k^{\beta}$$

d_k = number failing in the interval test

$m-d_k$ = number still working at the end of test

n = number of test runs

It should be noted that the first term goes to zero when there are no failures for all test runs. The likelihood function is best for interval data since it makes no assumption as to the time of failure. For Weibayes analysis where some units run to failure and others donot, we get a derived mathematical description of eta which is

$$\eta = \left(\frac{\sum_{i=1}^{N} t_i^{\beta}}{r} \right)^{\beta}$$

where N is the number of suspensions or unfailed units

t_i is the operating time accumulated

r is the number of failures (no failures, set r=1)

β is the assumed value based on previous experience (records, etc.)

Mathematically, the assumption of at least one failure ensures that η is definable. When there are no failures, the formula becomes

$$\eta = \left[\sum_{i=1}^{N} (t_i^{\beta}) \right]^{1/\beta}$$

Since Weibayes analysis estimates the possible value of the slope, engineers may wish to try a variety of slopes over some range. Weibayes tests will not indicate the existance of a new failure mode. If the parts being analyzed are nonserialized, then assumptions pertaining to success or failure times must be made. Mathematically, the confidence level of the Weibayes lower bound is unknown because it depends on the actual times to the first failure.

SUMMARY

The mathematical flexibility of the Weibull distribution makes it the best choice for failure analysis. The extension of the fundamental concepts of the Weibull distribution allows it to be highly applicable to perform risk and substantiation analysis. Not previously mentioned is the problem with identifying outliers. An engineer most decide when such points exists and why. For sometimes outliers at the beginning of Weibull line may actually be the results of a t_0 which is not initially zero. There is a basic technique for determining t described by Abernathy (1983). A procedure developed at General Motors also describes solving for t_0 (Bompas-Smith, 1973). This value is subtracted or added (depending on conditions) to each point of the Weibull line. Brook suggests a trial and error process to determine t_0 . The t_0 which has a none zero value may be the results of storage deteriation, testing procedures etc. When using the Weibayes techniques one must continuously keep in mind the underlying assumptions of the procedure. The most important caution for the engineer when using Weibayes is that new failure mode must not exist. With the fundamental likelihoods developed by Weibull, the engineer or statistician can by algebraic means describe their particular data situation i.e. trial runs, interval data, no failures. Confidence bands allow the engineer to visually examine the possibility of two or more slopes being produced by the same failure mode. Mathematically Weibull distributions are very pliable. Yet, one must keep in mind real life does not generate perfect solutions. Weibull distributions provide a descriptive idea as to failure behavior for a wide variety of failure rates.

Currently, a problem is the form of the failure data. With the actuators in particular, failure by cycles or hot-firings time can occur. Also, failure is partly a decision of the engineer. Modifications to data collection procedure is recommended to reflect the actual point of such failure. The cycles must be sufficiently large numbers in order to be represented by Weibull distributions. Also, multiple failure times on each machine is a problem. For my analysis, I used only the first time to failure.

DATA RESULTS

For the FPOVA, POPVA, and CCVA, the beta is approximately .75 when the OPOVA with 1.5 seconds is ignored. The beta of .75 indicates infant mortality. The Weibull of the CCVA indicates the possibility of two failure modes. The beta of 1 indicates a failure mode of random failure (green- run, etc.) The beta of 3.5 indicates normal wear. The initial graph of FPOVA failures indicate a beta of .75. A modified graph indicates a beta of .375. The modified graph approximates a straight line better. The engineer needs to examine all aspects of usage of the FPOVA prior to hot-firing. For the MFVA and MOVA, two failure modes seem to be indicated. Upon further examination, two failure modes for the MFVA in particular seems to exist. The MOVA has an infant mortality failure mode. Modification of the time does not alter the value of beta.

TABLE 1

Actuator	Number	Time (Sec.)	Rank	Median Rank
OPOVA	086-524	1.5	1	.53
CCVA	019-003	622	2	1.29
FPOVA	050-501	622	3	2.05
FPOVA	032-323	805	4	2.86
CCVA	016-003	1222	5	3.58
FPOVA	078-552	1566	6	4.34
FPOVA	045-588	1848	7	5.10
CCVA	034-573	2059	8	5,86
CCVA	028-255	2596	9	6.62
CCVA	027-154	2597	10	7.38
CCVA	020-001	2864	11	8.14
FPOVA	020-005	4056	12	8.90
CCVA	023-003	5133	13	9.67
FPOVA	022-006	5460	14	10.43
CCVA	013-003	6170	15	11.19
CCVA	021-004	6521	16	11.95
CCVA	015-002	3553	17	12.71
OPOVA	027-004	12569	18	13.47

Sample Size = 131 Failures = 18

FPOVA, OPOVA, CCVA
Weibull

OPOVA Point 1.5 sec
Not graphed

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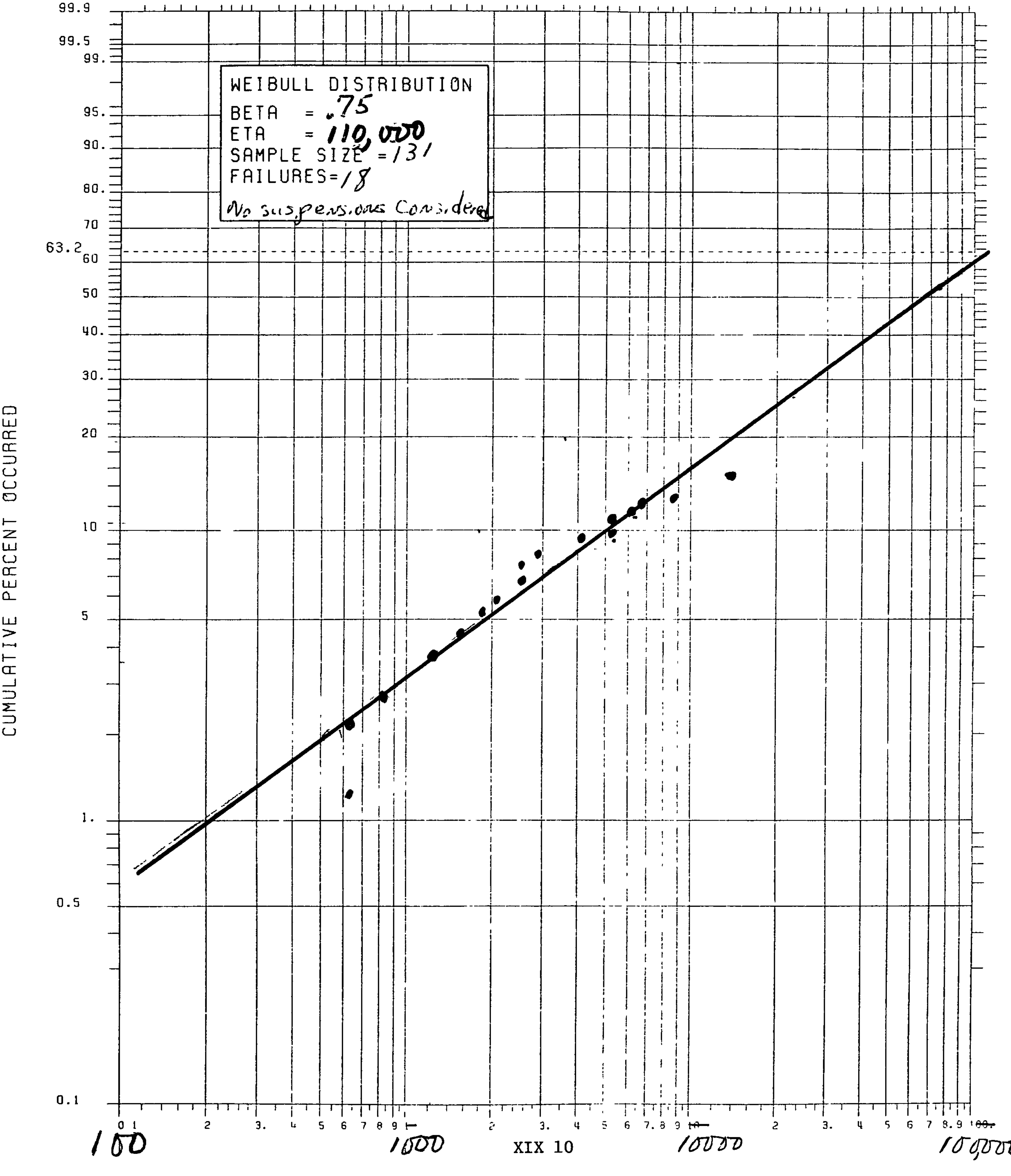


TABLE 2

Actuator	Number	Time (Sec.)	Rank	Median Rank
CCVA	019-003	622	1	1.5
CCVA	016-003	1222	2	3.8
CCVA	034-573	2059	3	6.1
CCVA	028-255	2596	4	8.4
CCVA	027-154	2597	5	10.7
CCVA	020-001	2864	6	13
CCVA	023-003	5133	7	15.3
CCVA	013-003	6170	8	17.6
CCVA	021-004	6521	9	20
CCVA	015-002	8553	10	22.3

Sample Size = 43 Number of Failures = 10

CCVA
Weibull

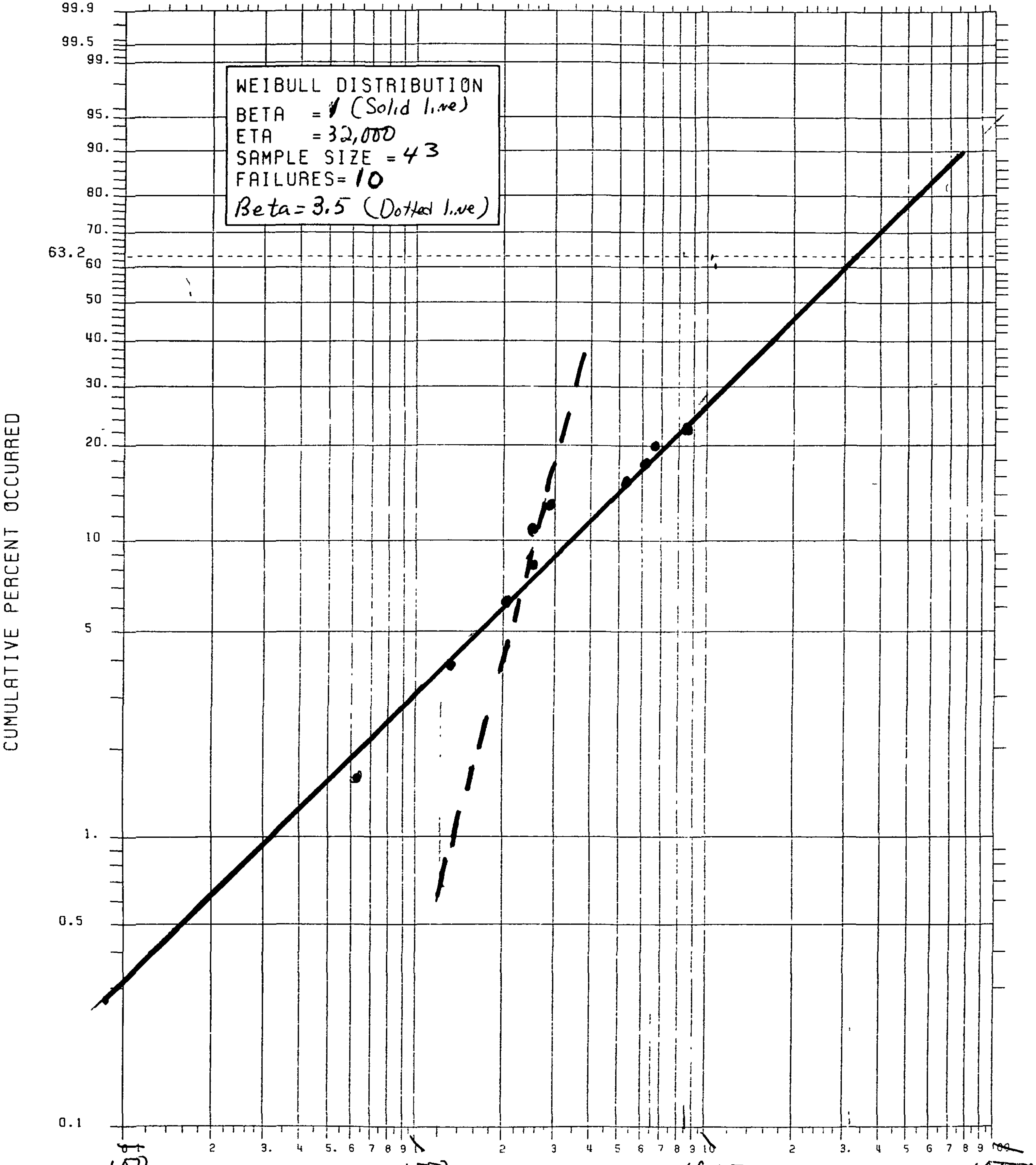


TABLE 3

Actuator	Number	Time (Sec.)	Rank	Median Rank
FPOVA	050-501	622	1	1.4
FPOVA	032-323	805	2	3.5
FPOVA	078-552	1566	3	5.6
FPOVA	045-588	1848	4	7.7
FPOVA	020-005	4056	5	9.8
FPOVA	022-006	5460	6	11.9

Sample size = 47 Number of Failures = 6

Using the formula for t_o (Abernethy, 1983),we t_o = 610 (subtract)

Time Changes: 12, 195, 956, 1238, 3446, 4850

$$t_o = t_2 - \frac{(t_3 - t_2)(t_2 - t_1)}{(t_3 - t_2) - (t_2 - t_1)}$$

where each t is determined by measuring or its location.

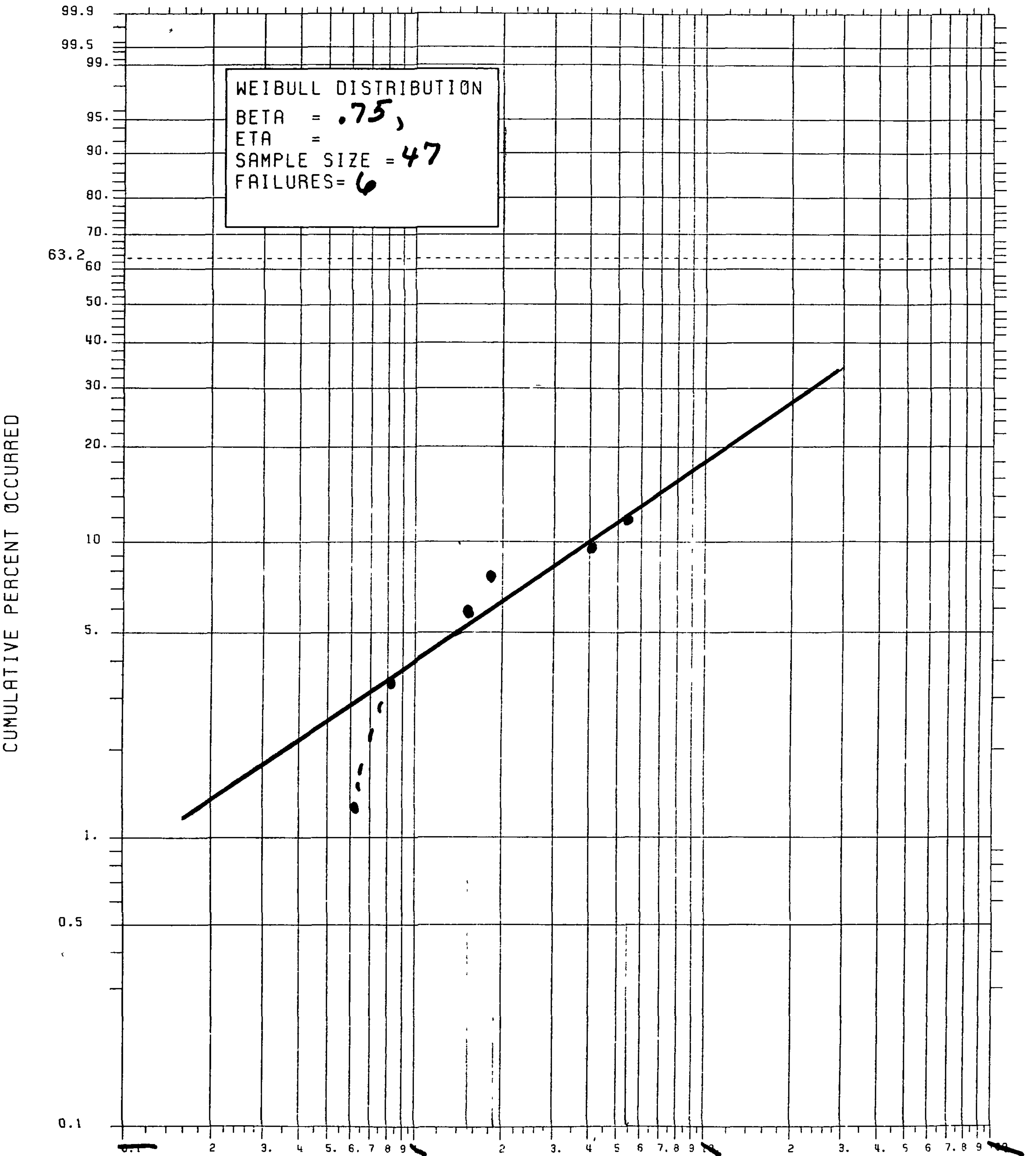
t_1 = 622

t_2 = 850 (by measuring)

t_3 = 5460

FPOVA

needs to correction



FPOVA
to Correction
Subtract 610

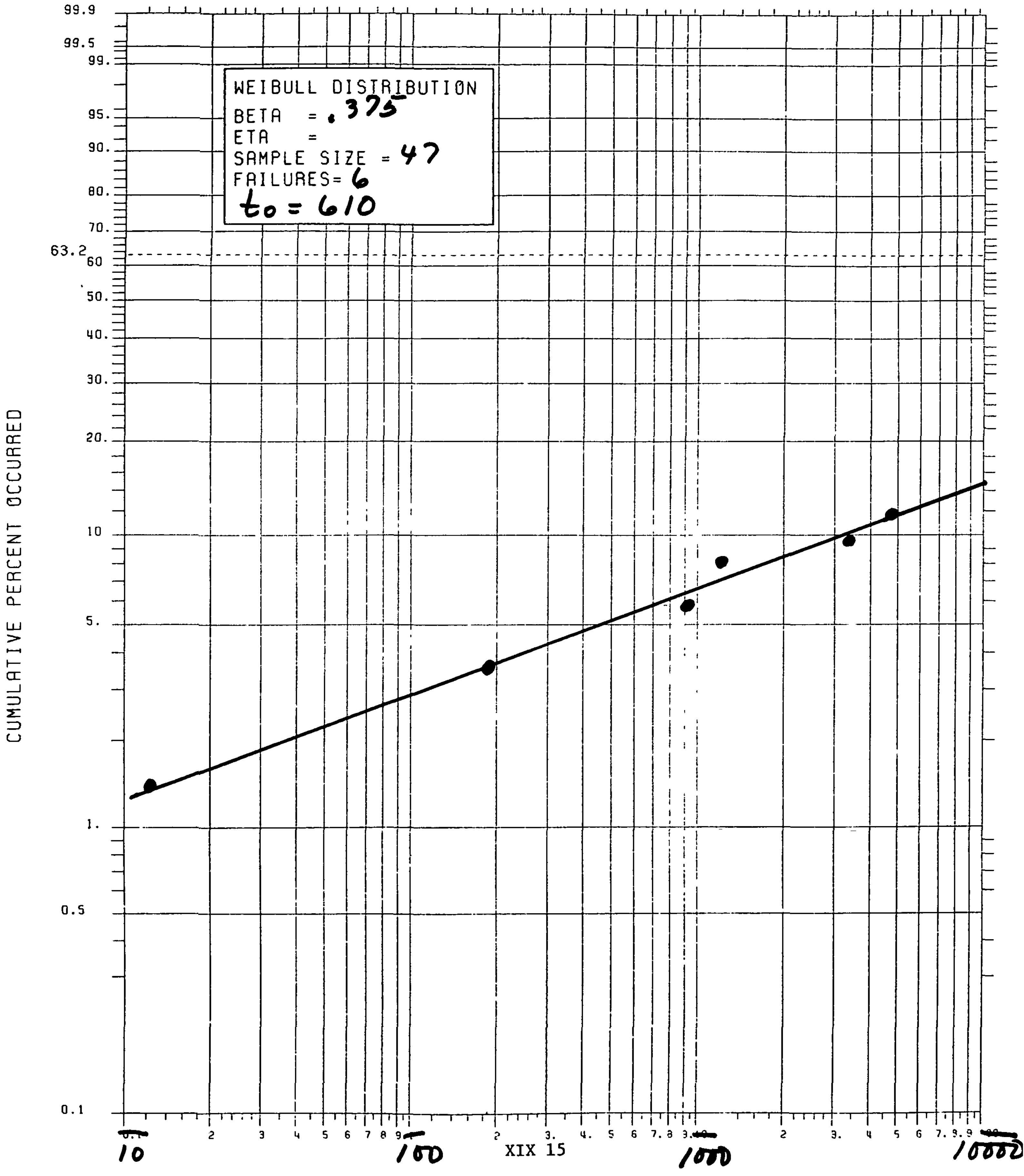


TABLE 4

Actuator	Number	Time(Sec)	Rank	Median Rank
MFVA	068-452	20	1	.774
MFVA	046-002	622	2	1.88
MFVA	070-490	932	3	2.99
MFVA	088-591	1012	4	4.90
MFVA	036-272	1593	5	5.20
MOVA	075-252	1778	6	6.31
MFVA	011-007	2035	7	7.41
MFVA	050-445	2698	8	8.52
MFVA	064-049	2946	9	9.62
MFVA	030-006	2968	10	10.73
MFVA	032-001	3325	11	11.84
MOVA	029-001	3703	12	12.94
MOVA	022-002	6725	13	14.05
MFVA	034-004	9866	14	15.15
MFVA	054-114	9882	15	16.26
MOVA	063-079	11590	16	17.37
MFVA	066-142	11590	17	18.47
MOVA	051-415	16643	18	19.58
MOVA	027-002	23834	19	20.69
MFVA	028-004	28094	20	21.79

Sample Size = 90 Number of Failures = 20

MFVA
MOVA
Together

20 sec not graphed
Two failure modes

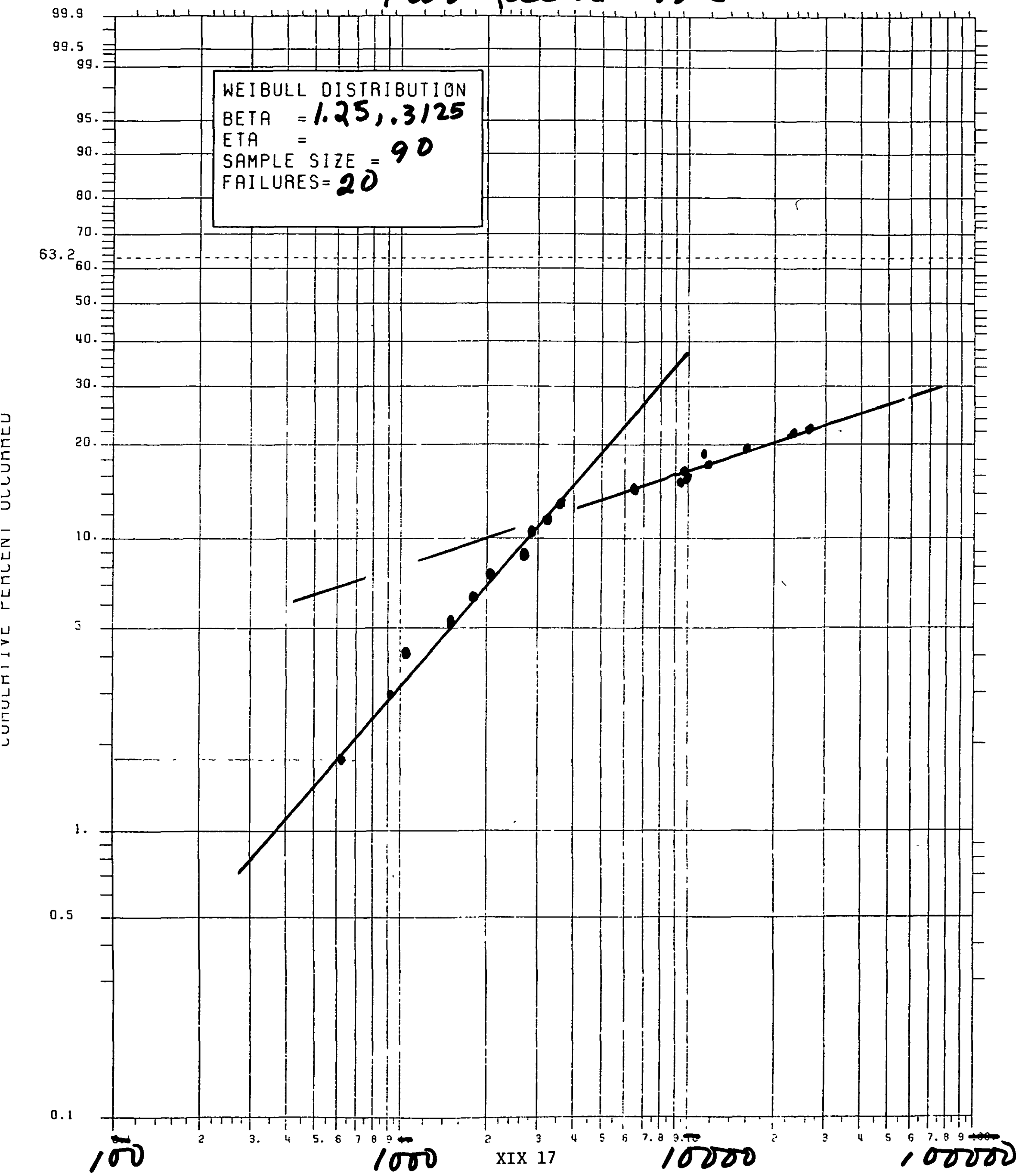


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TABLE 5

Actuator	Number	Time (Sec)	Rank	Median Rank
MFVA	068-453	20	1	1.6
MFVA	046-002	622	2	3.9
MFVA	070-490	932	3	6.3
MFVA	088-591	1012	4	8.6
MFVA	036-272	1593	5	11.0
MFVA	011-007	2035	6	13.3
MFVA	050-445	2698	7	15.7
MFVA	064-049	2946	8	18.1
MFVA	030-006	2968	9	20.4
MFVA	032-001	3325	10	22.8
MFVA	034-004	9866	11	25.7
MFVA	054-114	9882	12	27.5
MFVA	066-142	11590	13	29.9
MFVA	028-004	28094	14	32.2

Sample Size = 42 Number of Failures = 14

MFVA
Sample Size 42
Two failure modes

20 sec is not graphed
Maybe outlier

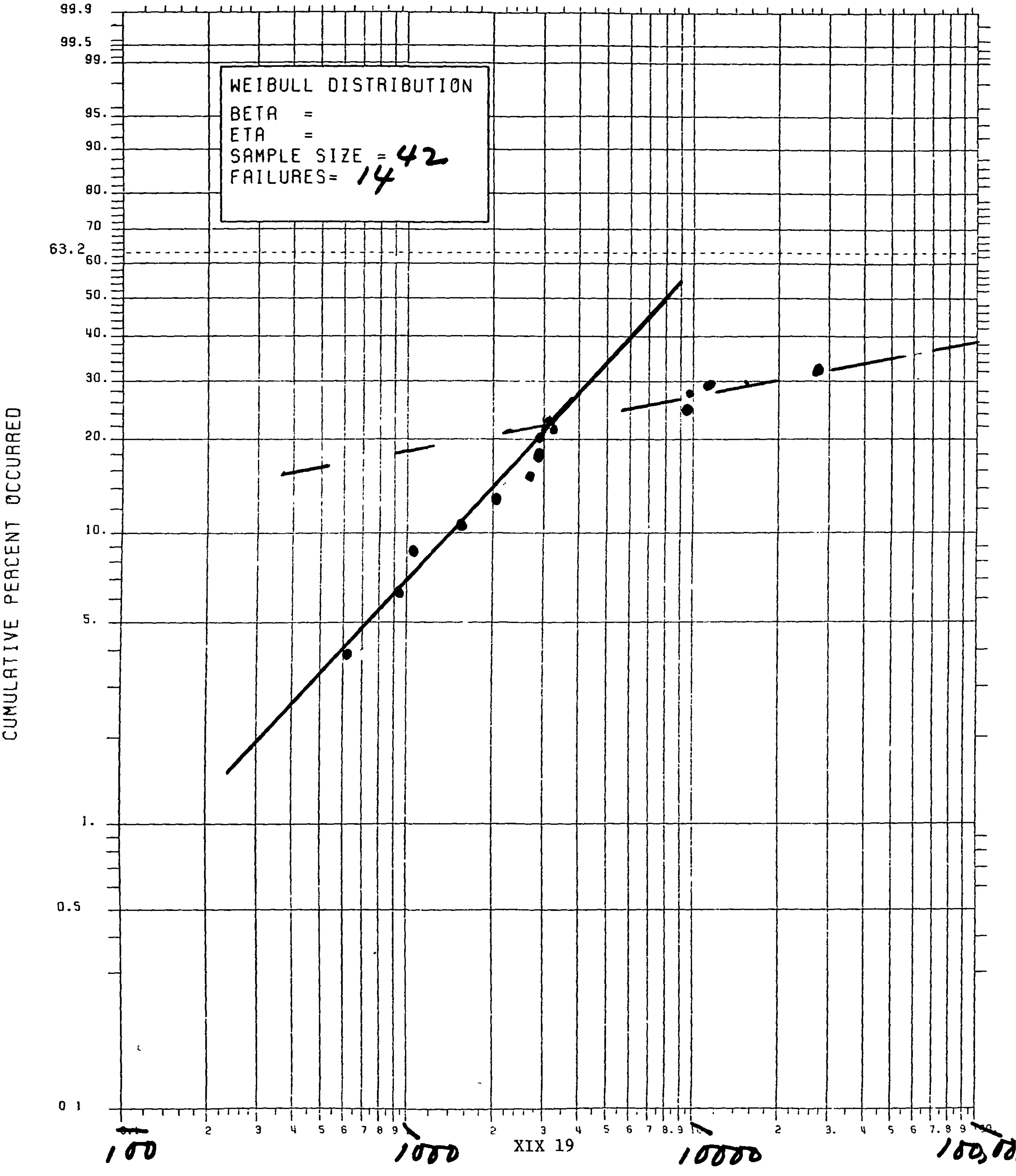

TABLE 6

Actuator	Number	Time (Sec)	Rank	Median Rank
MOVA	075-252	1778	1	1.4
MOVA	029-001	3703	2	3.4
MOVA	022-002	6725	3	5.5
MOVA	063-079	11590	4	7.5
MOVA	051-415	16643	5	9.6
MOVA	027-002	23834	6	11.7

Sample Size = 48 Number of Failures = 6

For Correction of t_o :

$t_1 = 1778$ $t_2 = 4500$ $t_3 = 19334$

Subtract $t_o = 1332$

Time: 466, 2371, 5393, 10258, 15311, 22502

MOVA

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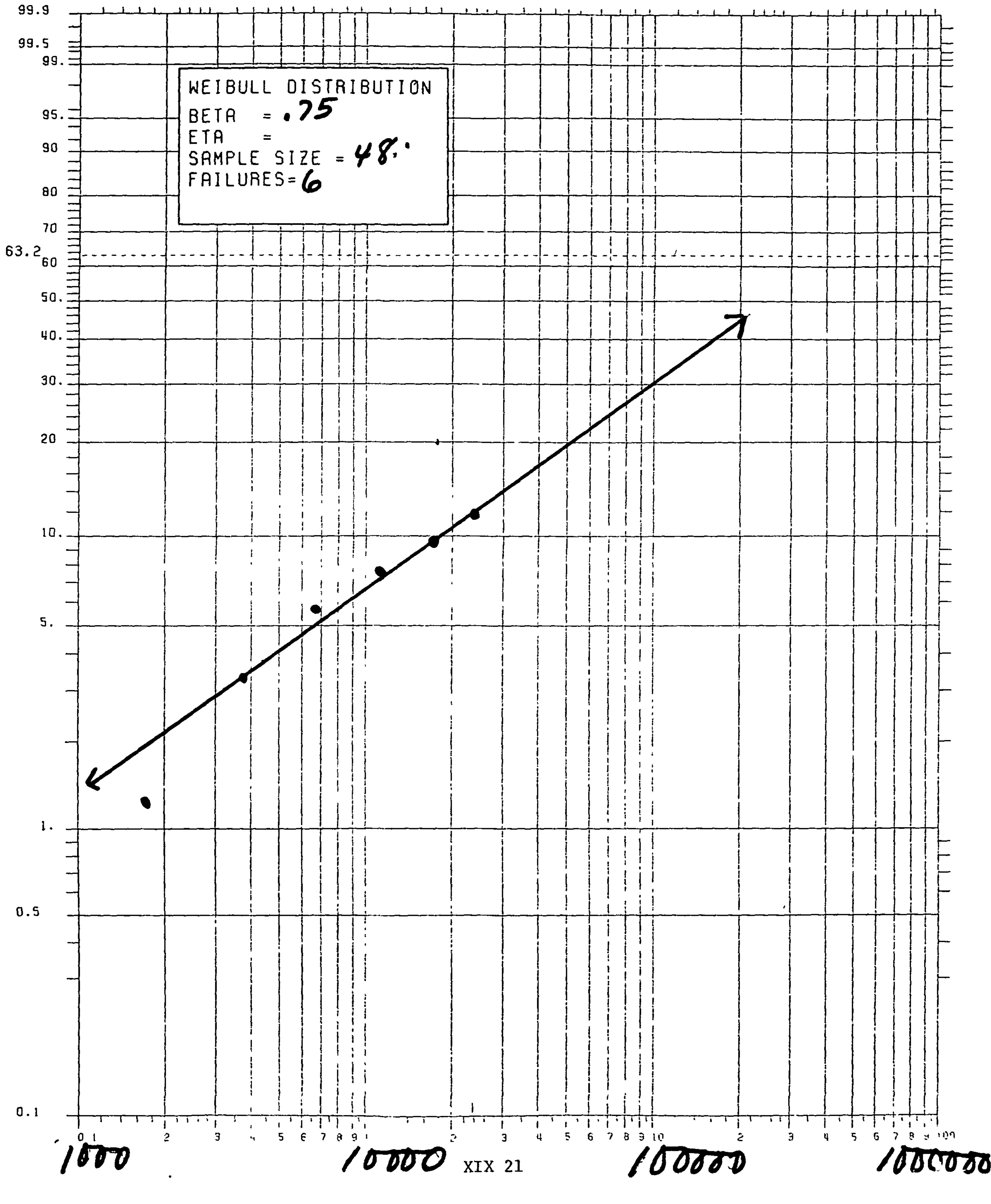
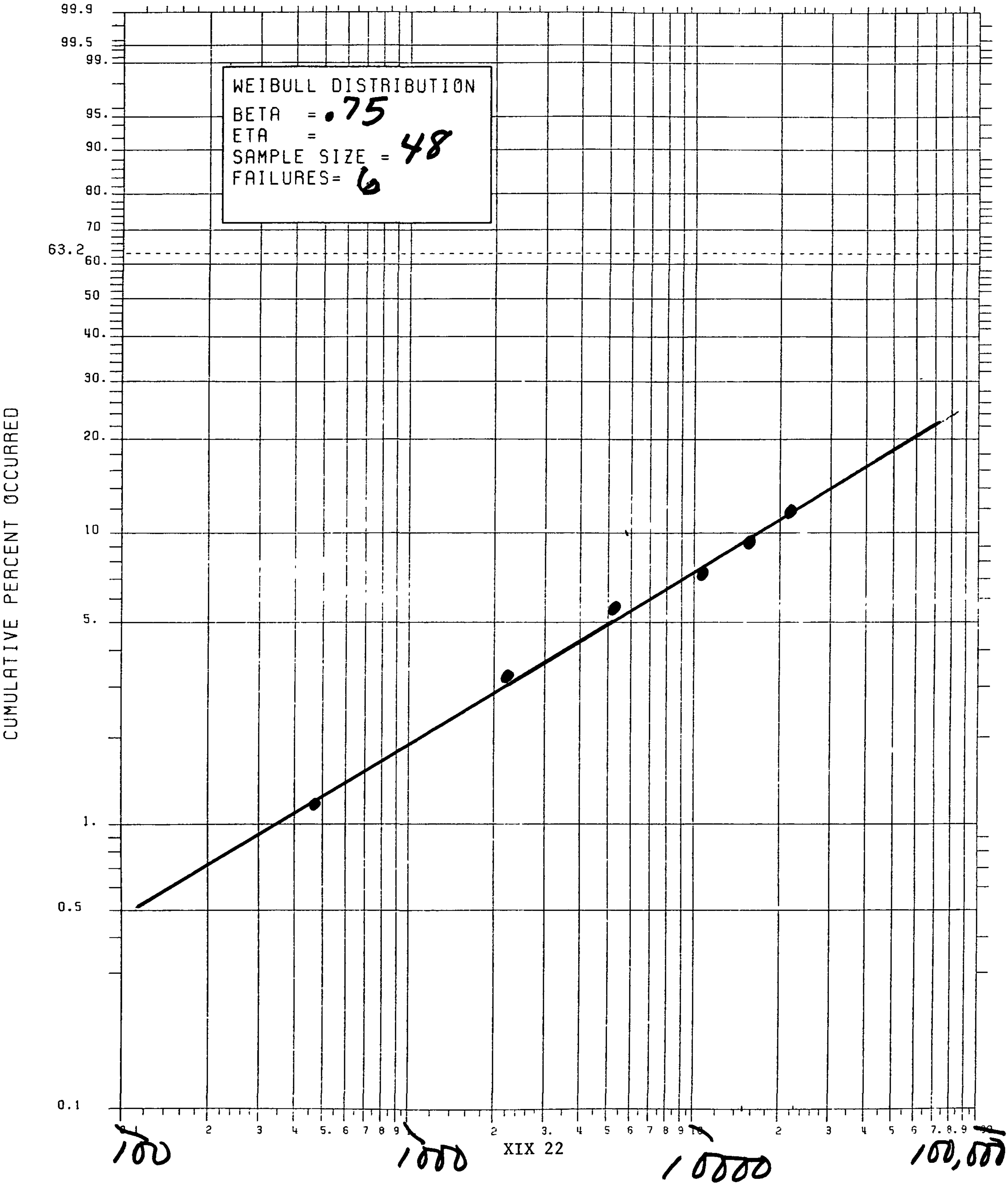


MOVA
to correction

$t_0 = 1332$
Subtract



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