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A METHOD FOR LINEARIZING A NONLINEAR SYSTEM WITH SIX STATE VARIABLES AND THREE CONTROL VARIABLES

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A METHOD FOR LINEARIZING A NONLINEAR SYSTEM WITH SIX STATE VARIABLES AND THREE CONTROL VARIABLES

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ABSTRACT

A nonlinear system governed by $\dot{x} = f(x,u)$ with six state variables and three control variables is considered in this project. A set of transformation from (x,u) - space to (z,v) - space is defined such that the linear tangent model is independent of the operating point in the z-space. Therefore, it is possible to design a control law satisfying all operating points in the transformed space. An algorithm to construct the above transformations and to obtain the associated linearized system is described in this report.

This method is applied to a rigid body using pole placement for the control law. Results are verified by numerical simulation. Closed loop poles in x-space using traditional local linearization are compared with those pole placements in the z-space.

LIST OF FIGURES

INTRODUCTION

In general, there are two commonly used approaches to linearize a nonlinear control system: one is to linearize locally about some operating points, the other is to linearize globally. However, difficulties will arise when these approaches are applied. For the former one, it is hard to design a unified control law over the entire set of operating points; on the other hand, for the later approach, usually, the control does not permit an acceptable closed-loop system behavior over a wide operating range. But, if a nonlinear system has a tangent model independent of the operating point, the above stated difficulties disappear. Using this concept, Reboulet and Champetier [1] developed a procedure which transforms a nonlinear system with one control variable to another space such that the desired independent property is satisfied. In this project, we generalize their procedure to the case where a nonlinear system consists of six state variables and three control variables. This method is then applied to a rigid body dynamic model using pole placement for the control law.

OBJECTIVES

The objectives of this project are:

1. Develop a pseudo-linearization technique to analyze a nonlinear control system with six state variables and three control variables.

2. Apply this technique to a rigid body dynamic model.

3. Verify the feasibility of this technique by computer simulation.

BODY OF REPORT

Concept

Consider the class of 6-state and 3-input nonlinear systems governed by

$$\dot{x} = f(x,u) \tag{1}$$

where $x \in R^6$, $u \in R^3$, and $f = (f_1, \ldots, f_6)^t : R^9 \to R^6$ of class C^1. The set of operating points is defined by

$$M_{x,u} \triangleq \{(x_o,u_o): f(x_o,u_o) = 0\}$$

and its projection in the state space is defined by

$$M_x \triangleq \{x_o : \exists\, u_o \text{ s.t. } f(x_o,u_o) = 0\}.$$

In the neighborhood of an operating point (x_o,u_o), the dynamic behaviour of the system may be considered as linear. It is then described by

$$\delta\dot{x} = F(x_o,u_o)\delta x + G(x_o,u_o)\delta u \tag{2}$$

where $\delta x \in R^6$, $\delta u \in R^3$ and $F \triangleq \partial f/\partial x$, $G \triangleq \partial f/\partial u$.

Our aim is to find mappings

$$\begin{aligned} z_i &= T_i(x) \quad &, i = 1, \ldots, 6 \\ v_j &= T_{6+j}(x,u) \quad &, j = 1, 2, 3 \end{aligned} \tag{3}$$

($T_1, \ldots, T_6$ being functionally independent and $\partial T_{6+j}/\partial u \neq 0$, $j = 1, 2, 3$) such that, in the z-state space, the linear tangent model is independent of the operating point and can be written under the controllability canonical form

$$\begin{aligned} \delta\dot{z}_1 &= \delta z_4 \\ \delta\dot{z}_2 &= \delta z_5 \\ \delta\dot{z}_3 &= \delta z_6 \\ \delta\dot{z}_4 &= \delta v_1 \\ \delta\dot{z}_5 &= \delta v_2 \\ \delta\dot{z}_6 &= \delta v_3 \end{aligned} \tag{4}$$

Note that the controllability of the linear tangent model is invariant under the mappings (3). Hence, to obtain (4) the following condition must hold:

For any $(x_o,u_o) \in M_{x,u}$ the pair $(F(x_o,u_o),\ G(x_o,u_o))$ is controllable. (5)

In practice, the solution provided by our method is effective along $M_{x,u}$, except at the points (x_o,u_o) for which (5) is not satisfied.

Now, the variations in the z-state space δz will be rewritten in terms of δx and δu. From eqn. (1) and (3) we have

$$\dot{z}_i = (\partial T_i/\partial x) f(x,u) \qquad , i = 1, \ldots, 6.$$

Hence, at any points (x_o,u_o) of $M_{x,u}$

$$\delta\dot{z}_i = \alpha_i F(x_o,u_o)\delta x + \alpha_i G(x_o,u_o)\delta u, \qquad i = 1, \ldots, 6$$

where α_i are the 1-forms over M_x

$$\alpha_i \triangleq d\, T_i\Big|_{M_x} \qquad , i = 1, \ldots, 6. \qquad (6)$$

For convenience, reference to the operating point will be omitted. To obtain eqn. (4) it is easy to show the following equations must be satisfied on $M_{x,u}$

$$\alpha_i G = 0 \qquad , i = 1, 2, 3 \qquad (7a)$$

$$\alpha_i F = \alpha_{3+i} \qquad , i = 1, 2, 3 \qquad (7b)$$

$$\alpha_{3+i} = \alpha_i (F\ G) \qquad , i = 4, 5, 6 \qquad (7c)$$

with $\alpha_j \triangleq d\, T_j\Big|_{M_{x,u}} \qquad , j = 7, 8, 9.$

With this result, the problem then becomes that of finding 1-forms $\alpha_1, \ldots, \alpha_6$ (resp. $\alpha_7, \alpha_8, \alpha_9$) satisfying eqns. (7) at any point of M_x (resp. $M_{x,u}$) and such that there exist mappings $T_i(x)$ $(i = 1, \ldots, 6)$ and $T_j(x,u)$, $(j = 7, 8, 9)$ such that

$$\alpha_i = d\,T_i\big|_{M_x} \qquad , i = 1, \ldots, 6$$

$$\alpha_j = d\,T_j\big|_{M_{x,u}} \qquad , j = 7, 8, 9.$$

Analysis

Recall that $M_{x,u}$ is a set of points described by six nonlinear equations with nine unknowns.

Let $\dot{M}_{x,u}$ be the set of operating points in the neighborhood of which the linear tangent model of system (1) is controllable. Since the uncontrollability of the tangent model (2) is equivalent to the nullity of all 5-dimension minors (Kalman rank condition). Therefore, the complement of $\dot{M}_{x,u}$ is a closed subset in $M_{x,u}$ which implies $\dot{M}_{x,u}$ is an open subset in $M_{x,u}$.

Furthermore, another controllability condition is given by

$$\text{rank}(F-\lambda I \quad G) = 6 \qquad ,\forall \lambda .$$

In particular, for $\lambda = 0$

$$\text{rank}(F \quad G) = 6.$$

This means that the mappings f_i, $i = 1, \ldots, 6$ are functionally independent at the point of interest. Hence, $M_{x,u}$ is a 3-dimensional submanifold of R^9. Which also implies that $\dot{M}_{x,u}$ is a 3-dimensional submanifold of R^9 since $\dim M_{x,u} = \dim \dot{M}_{x,u}$ if $\dot{M}_{x,u} \neq \phi$.

Let $\dot{M}_x$ be the projection of $\dot{M}_{x,u}$ on the x-state space. In order to develop global results we will suppose that the following condition holds:

There exists a C^1-diffeomorphism $\phi = (\phi_1, \ldots, \phi_6)^t$ from R^6 to R^6 such that $\dot{M}_x$ is given by

$$\phi_4(x) = \phi_5(x) = \phi_6(x) = 0 \qquad (8)$$

Under this assumption, if we define

$$z_i = \phi_i(x) \qquad , i = 1, \ldots, 6$$

then the surface M_x is transformed in the z-space into a 3-dimensional manifold $D_z : z_4 = z_5 = z_6 = 0$. Let $\bar{\alpha}$ be the image of a 1-form $\alpha = d\,T\big|_{M_x}$ under ϕ, then

$$\bar{\alpha}(z) \triangleq \alpha(\phi^{-1}(z))(\partial x/\partial z)_{D_z}, \quad z \in D_z. \tag{9}$$

Note that if we post-multiply $\alpha = d\,T\big|_{M_x}$ by $(\partial x/\partial x)\big|_{D_z}$,

we obtain

$$\alpha(\phi^{-1}(z))(\partial x/\partial z)\Big|_{D_z} = d\,T(\Phi^{-1}(z))\Big|_{D_z}(\partial x/\partial z)_{D_z}, \; \forall\, z \in D_z, \tag{10}$$

i.e.,

$$\bar{\alpha} = d\,\bar{T}\Big|_{D_z} \tag{11}$$

with

$$\bar{T} = T \circ \phi^{-1} \tag{12}$$

Therefore, the integrability of α over M_x in the x-space can be transposed in terms of integrability of $\bar{\alpha}$ over D_z.

Note that $\bar{\alpha}$ only depends on z_1, z_2, z_3 over D_z, i.e.,

$$\bar{\alpha}(z) = \sum_{i=1}^{6} \bar{\alpha}_i(z_1, z_2, z_3)dz_i \qquad , \quad z \in D_z.$$

If we choose the 1-form α such that $\bar{\alpha}_1$, $\bar{\alpha}_2$, and $\bar{\alpha}_3$ depend on z_1, z_2, and z_3 only, respectively. We can define

$$\bar{T}(z) = \sum_{i=1}^{6} \int \bar{\alpha}_i(z_1, z_2, z_3)dz_i \tag{13}$$

and we have

$$d\bar{T}(z) = \sum_{i=1}^{6} \bar{\alpha}_i(z_i, z_2, z_3)dz_i + \sum_{i=4}^{6} z_i\Big[\sum_{j=1}^{3} (\partial\bar{\alpha}_i/\partial z_j)dz_j\Big]$$

$$= \sum_{i=1}^{6} \bar{\alpha}_i(z_i, z_2, z_3)dz_i$$

$$= \bar{\alpha}$$

Since the second term of the right hand side vanishes over D_z. Therefore, $\bar{\alpha}$ is integrable over D_z and so is α over M_x.

Method

The procedure for searching for these transformations can be decomposed into three steps.

Step 1: Find three independent directions α_1, α_2, α_3 perpendicular to G.

Step 2: From eqns. (7b) (resp. (7c)), α_4, α_5, α_6 (resp. α_7, α_8, α_9) can be computed.

Step 3: Integration of $\alpha_1, \ldots, \alpha_6$ (resp. α_7, α_8, α_9) along M_x (resp. $M_{x,u}$) provides the desired mapping $T_1, \ldots, T_9$.

Application

Consider a rigid body in which the angular velocities are measured along a body fixed axis, and with an Euler angle sequence (space - three 1-2-3) defining the attitudes in inertial space:

$$\dot{\omega} + \omega \times I\omega = u$$

$$\begin{aligned} \dot{\theta}_1 &= \omega_x + (\omega_y s_1 + \omega_z c_1) s_2/c_2 \\ \dot{\theta}_2 &= \omega_y c_1 - \omega_z s_1 \\ \dot{\theta}_3 &= (\omega_y s_1 + \omega_z c_1)/c_2 \end{aligned} \tag{14}$$

where $\omega = (\omega_x, \omega_y, \omega_z)$ is the angular velocity vector

$\theta = (\theta_1, \theta_2, \theta_3)$ is the Euler angle vector

$u = (u_x, u_y, u_z)$ is the external torque acting on the rigid body

$s_1 = \sin\theta_1$, $c_1 = \cos\theta_1$, $s_2 = \sin\theta_2$, $c_2 = \cos\theta_2$

and I, a 3x3 matrix, is the inertial tensor.

Without loss of generality, principle axes are used to describe this system. Therefore, we can assume the matrix I is a diagonal matrix, i.e.,

$$I = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}$$

Using principle axes, system (14) become:

$$\begin{aligned}
\dot{\omega}_x &= [u_2 - (I_z - I_y)\omega_y\omega_z] \frac{1}{I_x} \\
\dot{\omega}_y &= [u_y - (I_x - I_z)\omega_x\omega_z] \frac{1}{I_y} \\
\dot{\omega}_z &= [u_z - (I_y - I_x)\omega_x\omega_y] \frac{1}{I_z} \\
\dot{\theta}_1 &= \omega_x + (\omega_y s_1 + \omega_z c_1) s_z / c_z \\
\dot{\theta}_2 &= \omega_y c_1 - \omega_z s_1 \\
\dot{\theta}_3 &= (\omega_y s_1 + \omega_z c_1)/c_2
\end{aligned} \tag{15}$$

The set of operating points is

$$M_{x,u} = \{(0,\ 0,\ 0,\ \theta_1,\ \theta_2,\ \theta_3,\ 0,\ 0,\ 0) \mid \tfrac{-\pi}{2} < \theta_2 < \tfrac{\pi}{2}\} \tag{16}$$

and its projection in the state space is

$$M_x = \{(0,\ 0,\ 0,\ \theta_1,\ \theta_2,\ \theta_3) \mid \tfrac{-\pi}{2} < \theta_2 < \tfrac{\pi}{2}\} \tag{17}$$

On $M_{x,u}$,

$$F(x_o, u_o) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & s_1 s_2/c_2 & c_1 s_2/c_2 & 0 & 0 & 0 \\ 0 & s_1 & -s_1 & 0 & 0 & 0 \\ 0 & s_1/c_2 & c_1/c_2 & 0 & 0 & 0 \end{bmatrix}$$

and

$$G(x_o,u_o) = \begin{pmatrix} 1/I_x & 0 & 0 \\ 0 & 1/I_y & 0 \\ 0 & 0 & 1/I_z \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

It is clear the system is controllable at every point $(x_o,u_o) \in M_{x,u}$.

Now, we shall choose three independent vectors α_1, α_2, α_3 all orthogonal to G. Let $\alpha_1 = (0, 0, 0, 1, 0, 0)$, $\alpha_2 = (0, 0, 0, 0, 1, 0)$ and $\alpha_3 = (0, 0, 0, 0, 0, 1)$.

From (7b) and (7c), we obtain

$\alpha_4 = (1, s_1s_2/c_2, c_1s_2/c_2, 0, 0, 0)$.

$\alpha_5 = (0, c_1, - s_1, 0, 0, 0)$

$\alpha_6 = (0, s_1/c_2, c_1/c_2, 0, 0, 0)$.

$\alpha_7 = (0, 0, 0, 0, 0, 0, 1/I_x, s_1s_2/I_yc_2, c_1s_2/I_zc_2)$,

$\alpha_8 = (0, 0, 0, 0, 0, 0, 0, c_1/I_y, - s_1/I_z)$,

$\alpha_9 = (0, 0, 0, 0, 0, 0, 0, s_1/I_yc_2, c_1/I_zc_2)$.

After integrating α_i, $i = 1, \ldots, 6$, (resp. α_7, α_8, α_9) over M_x (resp. $M_{x,u}$), the desired transformations are:

$$
\begin{aligned}
z_1 &= \Theta_1 \\
z_2 &= \Theta_2 \\
z_3 &= \Theta_3 \\
z_4 &= \omega_x + (\omega_y s_1 + \omega_z c_1) s_2/c_2 = \dot{\Theta}_1 \\
z_5 &= (\omega_y c_1 - \omega_z s_1) = \dot{\Theta}_2 \\
z_6 &= (\omega_y s_1 + \omega_z c_1)/c_2 = \dot{\Theta}_3 \\
v_1 &= v_{1a} + v_{1b} \\
v_2 &= v_{2a} + v_{2b} \\
v_3 &= v_{3a} + v_{3b}
\end{aligned}
\tag{18}
$$

where

$$v_{1a} = u_z/I_x + (u_y s_1/I_y + u_z c_1/I_z) s_2/c_2,$$

$$v_{1b} = \{[(I_z - I_x)\omega_z s_1/I_y + (I_x - I_y)\omega_y c_1/I_z]\omega_x s_2/c_2 + (I_y - I_z)\omega_y\omega_z/I_x\} + (\omega_y c_1 - \omega_z s_1)[\omega_x s_2/c_2 + (\omega_y s_1 + \omega_z c_1) \times (1 + 2s_2^2/c_2^2) ,$$

$$v_{2a} = u_y c_1/I_y - u_z s_1/I_z,$$

$$v_{2b} = [(I_z - I_x)\omega_z c_1/I_y + (I_x - I_y)\omega_y s_1/I_z]\omega_x - (\omega_y s_1 + \omega_z c_1)[\omega_x + (\omega_y s_1 + \omega_z c_1) s_2/c_2],$$

$$v_{3a} = (u_y s_1/I_y + u_z c_1/I_z)/c_1,$$

$$v_{3b} = [(I_z - I_x)\omega_z s_1/I_y + (I_x - I_y)\omega_y c_1/I_{zz}]\omega_x/c_z + (\omega_y c_1 - \omega_z s_1)[\omega_x + (\omega_y s_1 + \omega_z c_1)(1 + 1/c_2) s_2/c_2]/c_2$$

Note that each transformed input v_i (i_1 = 1, 2, 3) is partitioned into v_{ia} and v_{ib} terms. The v_{ia} term arises directly from integration of the corresponding α_{6+i} vector. The v_{ib} term can be identified with the nonlinear terms which appear in the expression of $\dot{z}_{3+i}$. According to [1], these high order terms may be neglected in the neighborhood of the operating point, in which case the equations of motion are said to be psuedo-linearized. However, in this particular case, since the operating points require $\omega = 0$, the transformation from v_i can be augmented to include the v_{ib} term without affecting the requirement that $dv_{ib}\Big|_{M_{x,u}} = \alpha_j$, j = i+6. Therefore, as shown before, the equation of motion for a rigid body become extirely linear when expressed in terms of Euler angles and Euler angular rates.

Taking account of (18), (15) can be rewritten as a linear systems:

$$\dot{z}_1 = z_4, \quad \dot{z}_2 = z_5, \quad \dot{z}_2 = z_6, \quad \dot{z}_4 = v_1, \quad \dot{z}_5 = v_2, \quad \dot{z}_6 = v_3,$$

or, in a matrix form:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \\ \dot{z}_5 \\ \dot{z}_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \qquad (19)$$

Furthermore, the inverse transformations also can be found as:

$$\Theta_1 = z_1$$

$$\Theta_2 = z_2$$

$$\Theta_3 = z_3$$

$$\omega_y = - z_5 s_1 + z_6 c_1 c_2$$

$$\omega_y = (z_5 + \omega_z s_1)/c_2 \quad , \quad -\frac{\pi}{2} < \Theta_1 < \frac{\pi}{2} \tag{20}$$

$$\omega_x = z_4 - (\omega_y s_1 + \omega_z c_1) s_2/c_2 \quad , \quad -\frac{\pi}{2} < \Theta_2 < \frac{\pi}{2}$$

$$u_x = I_x[(v_1 - v_{1b}) - (v_3 - v_{3b}) s_2]$$

$$u_y = I_y[(v_2 - v_{2b}) c_1 + (v_3 = v_{3b}) c_2 s_1]$$

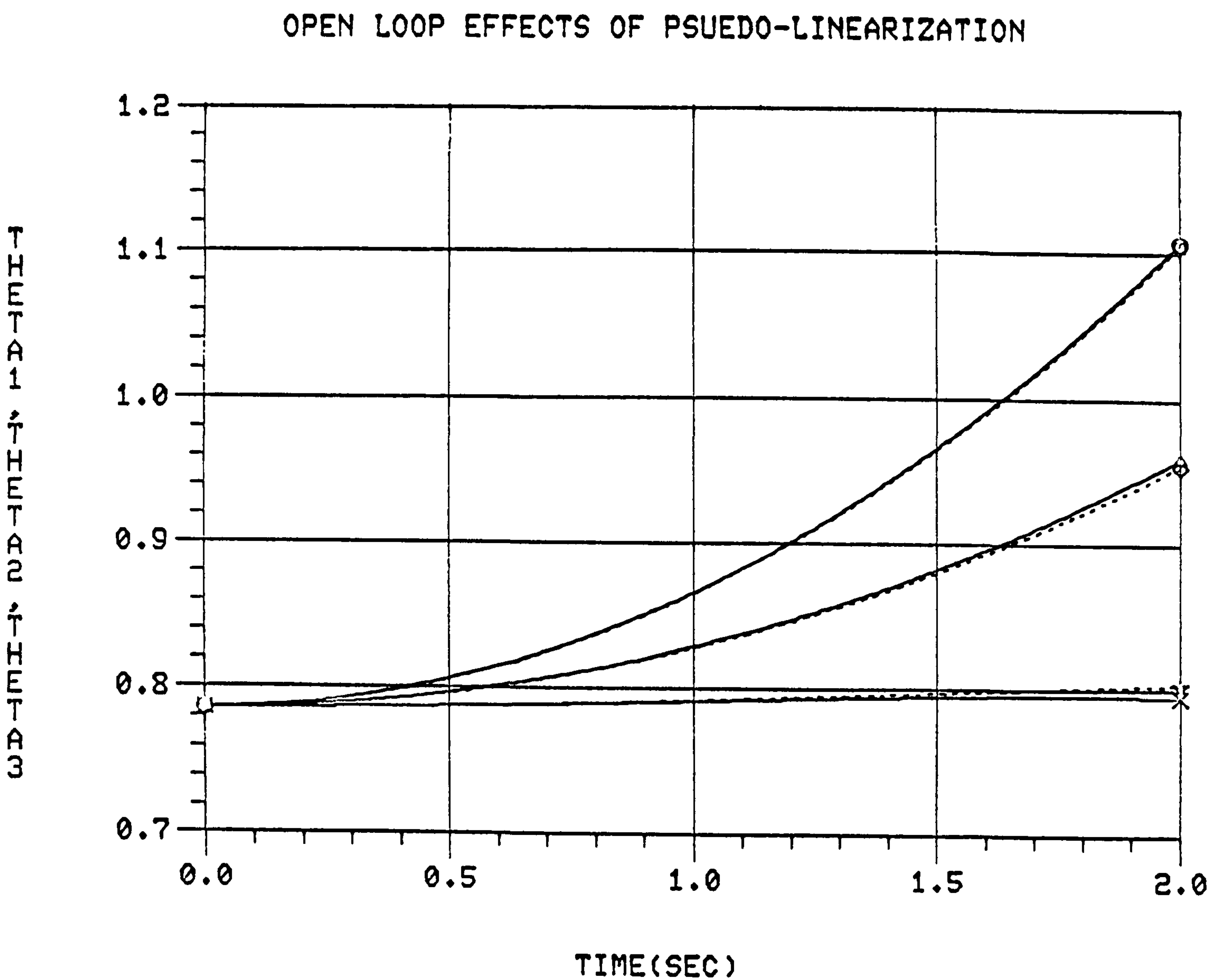
$$u_z = I_z[c_1 c_2 (v_3 - v_{3b}) - s_1 (v_2 - v_{2b})]$$

The above transformations were verified by numerical integration in both the x-space and z-space for a prescribed input u. Trajectories obtained by integrating in x-space were virtually identical to those obtained by integrating in z-space then transforming to x-space via equation (20). Figure 1 shows the effect of psuedo-linearization at an operating point of $\Theta_1 = \Theta_2 = \Theta_3 = \pi/4$ with a unit step input for u. The solid lines are trajectories obtained by directly integrating equation (15), while the dashed curves represent the transformation from z-space to x-space neglecting the v_{ib} terms.

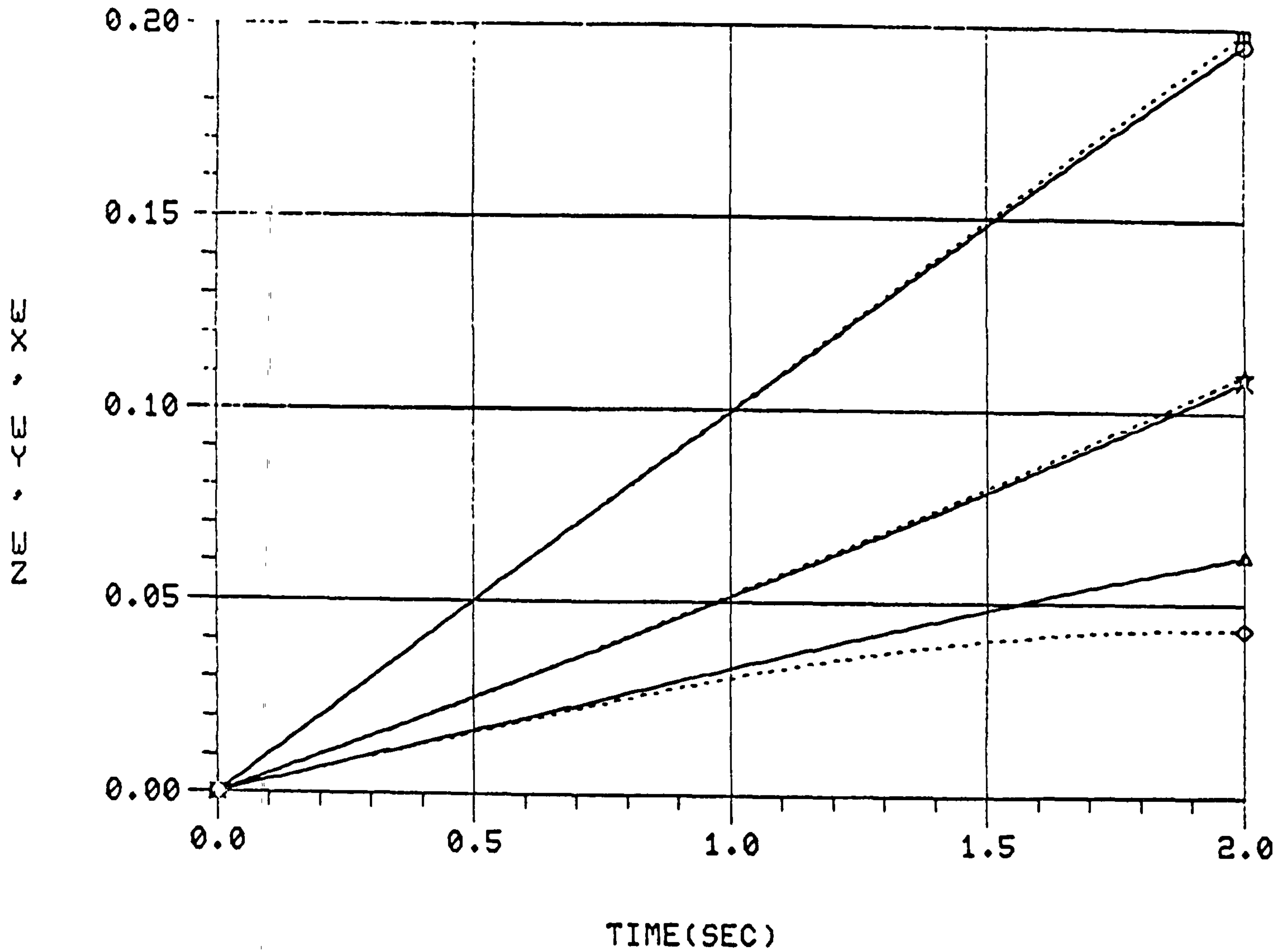
For control studies, a constant gain, full state feedback control $v = -Kz$ was employed. Working in the z-space with $\dot{z} = Az + Bv$, the following gain matrix K allows for arbitrary, complex conjugate pole placement,

$$K = \begin{bmatrix} \omega_1^2 & 0 & 0 & 2\zeta_1\omega_1 & 0 & 0 \\ 0 & \omega_2^2 & 0 & 0 & 2\zeta_2\omega_2 & 0 \\ 0 & 0 & \omega_3^2 & 0 & 0 & 2\zeta_3\omega_3 \end{bmatrix} \qquad (21)$$

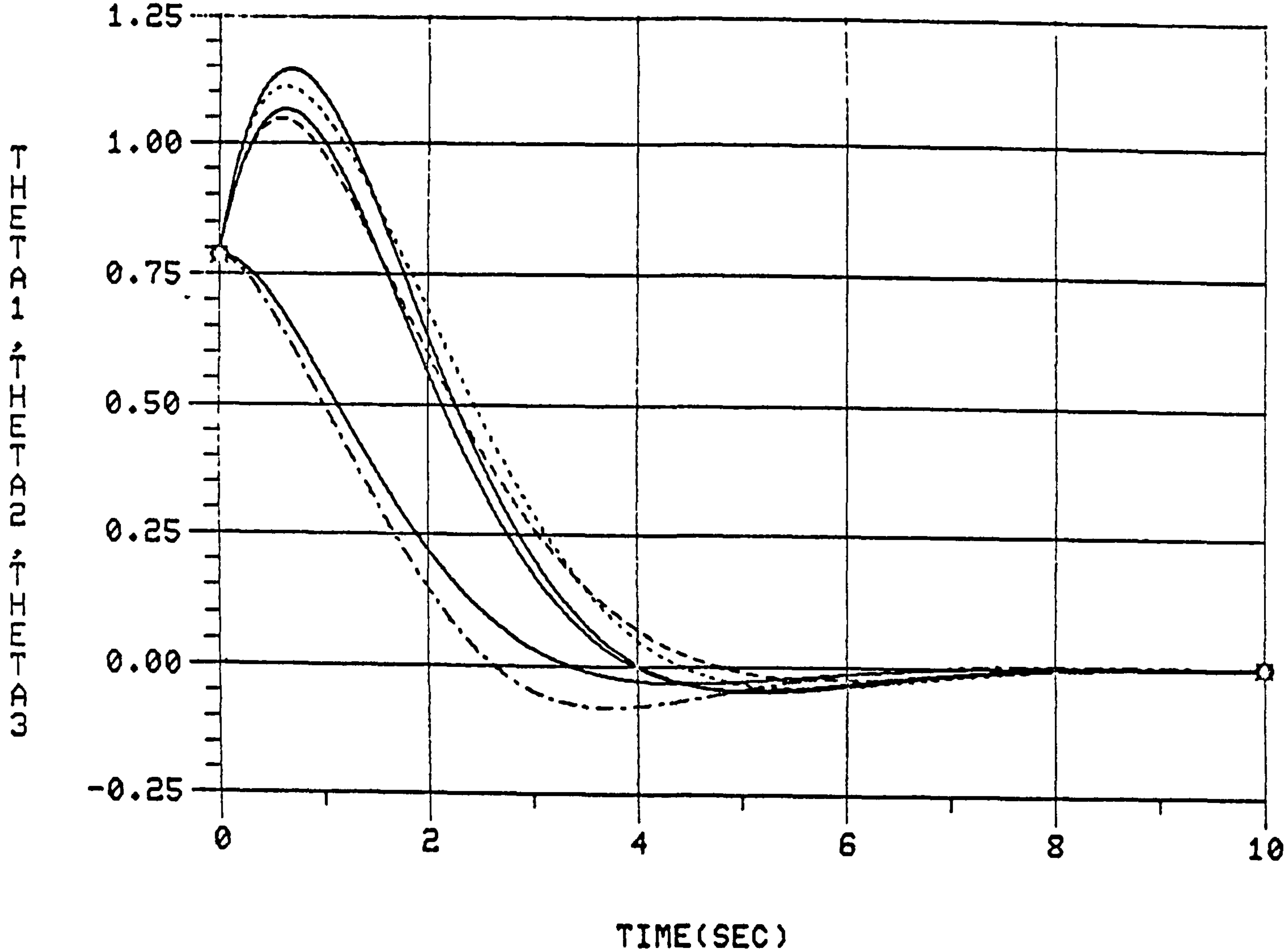
where ω_i and ζ_i prescribe the closed loop frequency and damping ratio of the ith axis. The actual control to be used in x-space comes from equations (18), (20) and (21), where both ω and Θ are assumed to be available as plant measurements. Figures 2, 3 show the system response to the arbitrary initial conditions $\Theta_1 = \Theta_2 = \Theta_3 = \pi/4$ and $\omega_x = \omega_y = \omega_z = 0.5$ using control frequencies and dampings of $\omega_1 = \omega_2 = \omega_3 = 1.0$ and $\zeta_1 = \zeta_2 = \zeta_3 = 0.707$. Once again, the solid lines represent the response using the exact transformation of equation (20) while the dashed lines show the effect of psuedo-linearization.

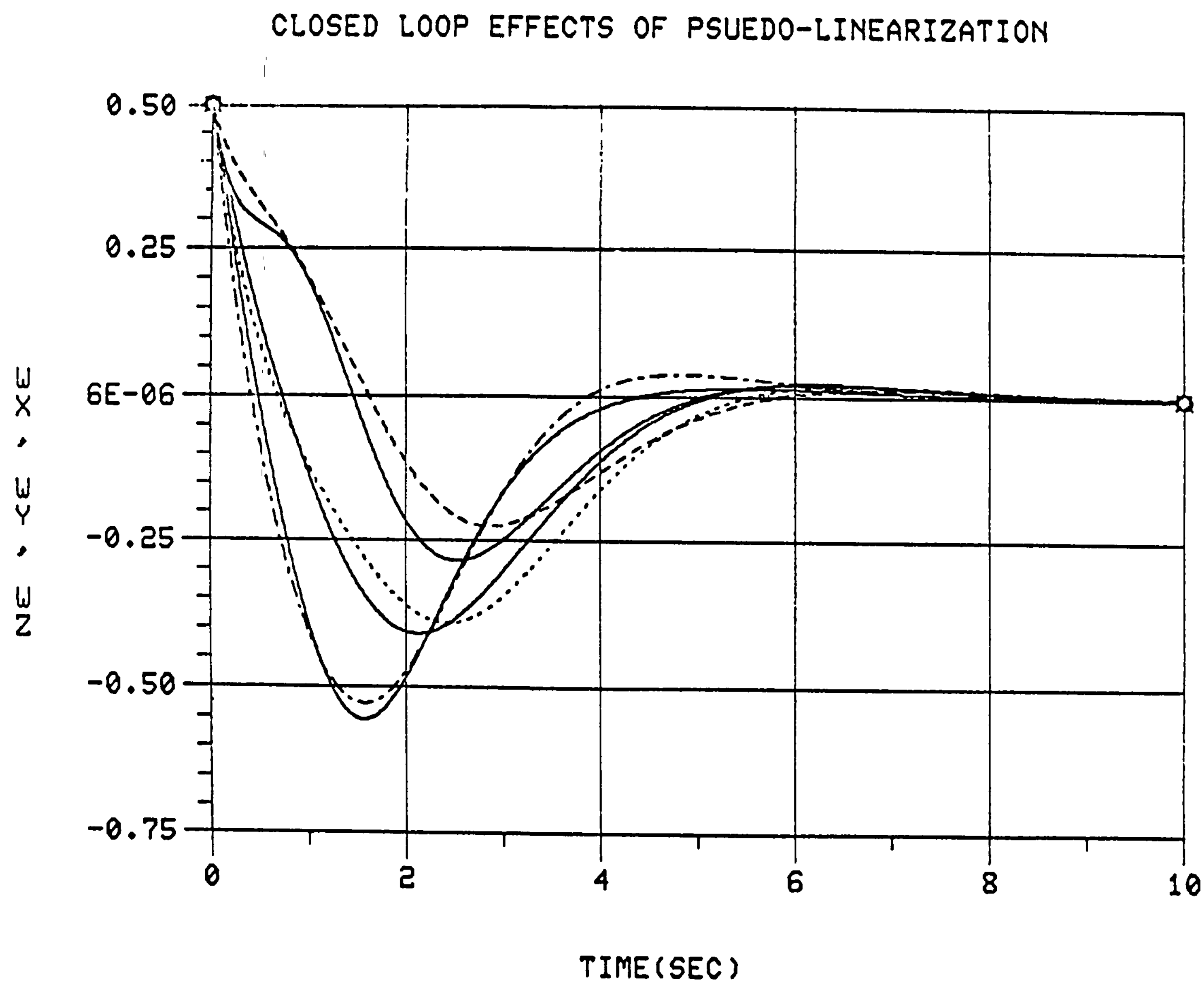

OPEN LOOP EFFECTS OF PSUEDO-LINEARIZATION
THETA1, THETA2, THETA3
1.2
1.1
1.0
0.9
0.8
0.7
0.0
0.5
1.0
1.5
2.0
TIME(SEC)

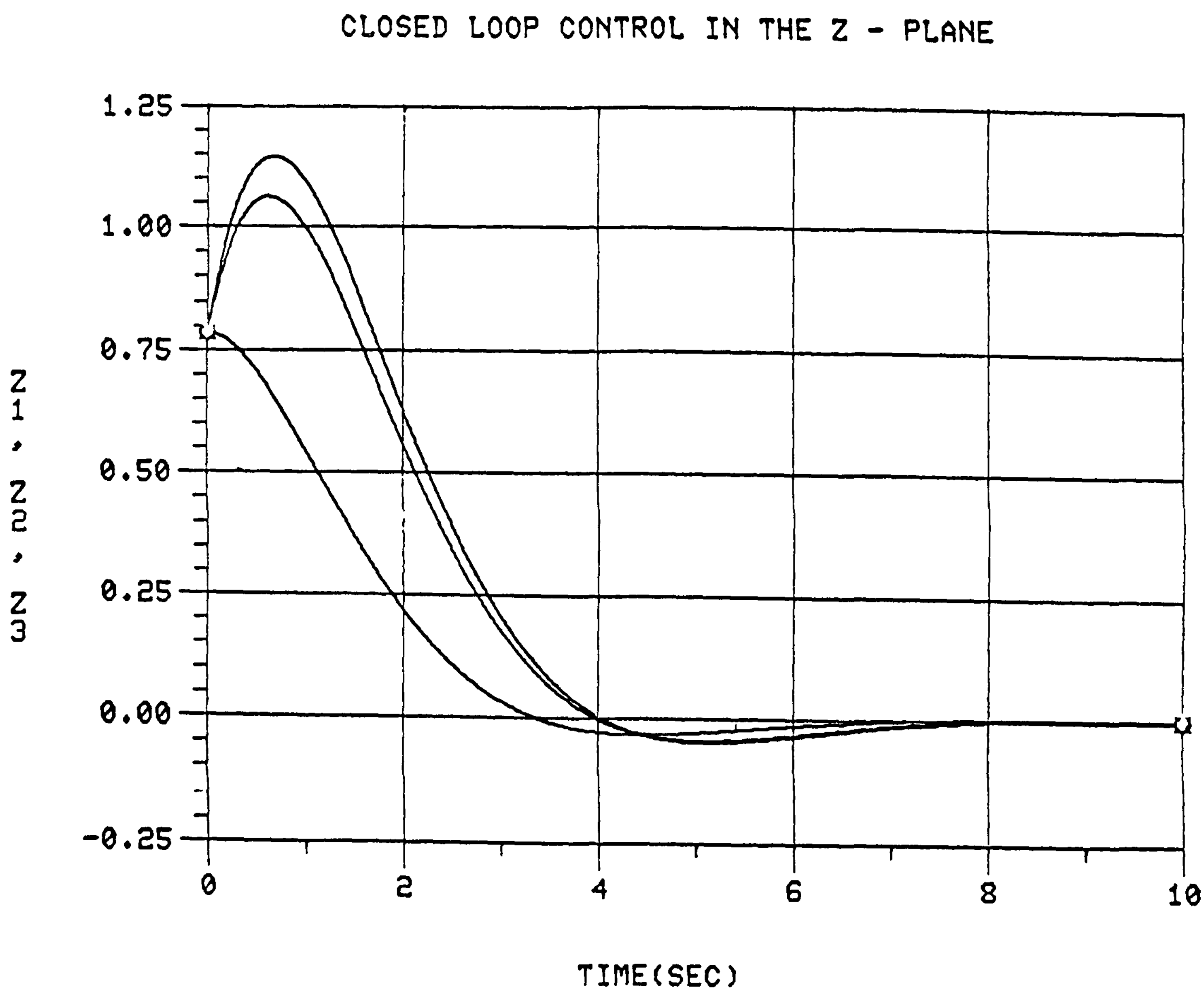
OPEN LOOP EFFECTS OF PSUEDO-LINEARIZATION



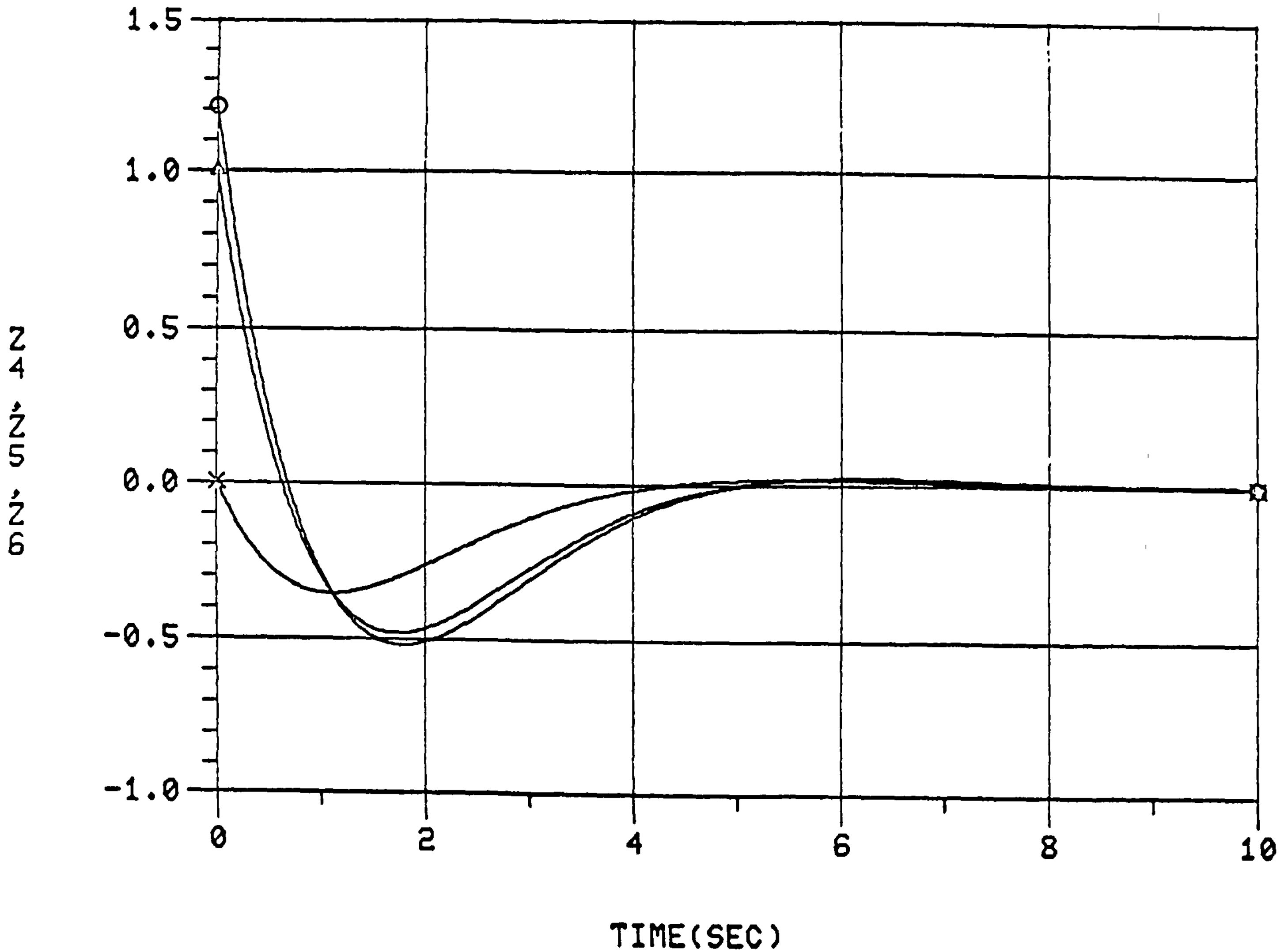
CLOSED LOOP EFFECTS OF PSUEDO-LINEARIZATION




CLOSED LOOP EFFECTS OF PSUEDO-LINEARIZATION
0.50
0.25
6E-06
-0.25
-0.50
-0.75
0
2
4
6
8
10
TIME(SEC)
WZ , WY , WX


CLOSED LOOP CONTROL IN THE Z - PLANE
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
Z1, Z2, Z3
0
2
4
6
8
10
TIME(SEC)

CLOSED LOOP CONTROL IN THE Z - PLANE



CONCLUSION AND RECOMMENDATION

A global linearization method for a nonlinear control system with 6 states and 3 inputs is successfully developed in this project. This method is an extension of a pseudo-linearization technique proposed by Reboulet and Champetier [1] in which only one input is considered. After applying this method to a rigid body in which the angular velocities are measured along a body fixed axis, and with an Euler angle sequence defining the altitudes in inertial space, the original non-linear system is described by a linear system in a transformed space. Therefore, a global control law can be established accordingly.

For further application of this method, we will turn our attention to the case of control systems of multiple bodies. Applying this method, Mr. Sharkey has obtained very positive numerical results on a two-body model using pole placement for a control law. We suggest a mathematical research to support this new application which is necessary and should start immediately after this program to pave the road for the next summer's project.

REFERENCE

1. Reboulet, A. and Champetier, C., "A New Method for Linearizing Non-linear Systems: The Pseudo-Linearization," International Journal of Control, 1984, Vol. 40, No. 4, 631-638.