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RADIATIVELY DRIVEN WINDS FROM MAGNETIC, FAST-ROTATING STARS

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BY

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ABSTRACT

An analytical procedure is developed to solve the magnetohydrodynamic equations for the stellar wind problem in the strong-magnetic field, optically thick limit for hot stars. The slow-mode, Alfven, and fast-mode critical points are modified by the radiation terms in the force equation but in a manner that can be treated relatively easily. Once the velocities at the critical points and the distances to the points are known, the streamline constants are determined in a straight-forward manner. This allows the structure of the wind to be elucidated without recourse to complicated computational schemes.

I. INTRODUCTION

Since the study of Weber and Davis (1967, WD) showed that the sun could be spun down by the solar wind over its main sequence lifetime, work has continued on angular momentum loss in the solar wind and in stellar winds (Pizzo, et al., 1983; Nerney and Suess, 1975; Nerney, 1980; Goldreich and Julian, 1970; and Friend and MacGregor, 1984).

Early-type stars present additional problems in understanding angular momentum loss as stellar winds from O and B stars are primarily driven by their intense flux of ultraviolet radiation. The radiation has an associated outward force due to Thomson scattering by photospheric electrons as well as a line radiation force that acts on metals in the wind. Unfortunately, analytical solutions are not possible as the basic stream-line constants are not known until a full numerical solution is generated. The complexity arises because the force due to line radiation depends nonlinearly on the acceleration in the wind. The critical point analysis shows that there is a focus of singular points which the numerical solution must be tangent to. This is accomplished through an iteration scheme that produces an acceptable physical solution.

The purpose of this study is to attempt to analytically model radiatively driven stellar winds with strong magnetic fields from fast-rotating stars in the optically-thick limit. This will allow a simplification of the problem so that it becomes readily apparent how the streamline constants determine the solution. Also, the modified magnetohydrodynamic critical points are treated and a procedure is developed for constructing wind models which does not require a complicated interaction scheme.

II. EQUATIONS OF MOTION

The steady, spherically-symmetric, dissipationless, magnetohydrodynamic (MHD) equations, together with terms that give rise to radiative bulk forces may be written as follows:

(1) $4\pi r^2 \rho v_r = \dot{M}$, which is the continuity equation where $\dot{M}$ is the mass-loss rate in kg/sec, r is radial distance from the center of the star in the usual r, θ, ϕ spherical polar coordinate system, ρ is the mass density in kg/m^3, and v_r is the radial velocity of the wind.

$$(2) \quad v_r \frac{dv_r}{dr} = -\frac{1}{\rho}\frac{dP}{dr} - \frac{GM\ (1-\Gamma)}{r^2} + \frac{v_\phi^2}{r} - \frac{1}{8\pi\rho r^2}\frac{d}{dr}(rB_\phi)^2 + f_L$$

which is the radial momentum equation where P is the pressure, G is the gravitational constant, M is the mass of the star, Γ is a constant (see below) related to the radiative force due to Thomson scattering of continuum photons by electrons, v_ϕ is the rotational (azumuthal) velocity of the wind, B_ϕ is the azimuthal component of the magnetic field, and f_L is the force/kg due to line radiation. The term in B_ϕ is the radial component of the $\vec{j} \times \vec{B}$ force.

(3) $\Gamma = \frac{\sigma e\ L}{4\pi GMC}$ where L is the stellar luminosity, c is the speed of light, and σ_e is the electron scattering opacity per unit mass.

$$(4) \quad f_L = \frac{\Gamma GM}{r^2} kt^{-\alpha}$$

which is an approximate law derived in the original theory of Castor, Abbott, and Klein (1975, CAK) where k is a constant which is a measure of the mixture of the number of strong radiation lines in the wind, t is an optical depth parameter, and α is a constant which depends on the importance of optically thick and thin radiation lines and is model-dependent.

(5) $t = \sigma_e \rho v_{th} \left(\frac{dv_r}{dr}\right)^{-1}$ and v_{th} is the thermal velocity of the ions which absorb and scatter radiation. CAK used values of $k = 1/30$ and $\alpha = .7$, but I will use $\alpha = 1$ and choose various values for the constant that k appears in. This Sobolev high-velocity gradient approximation to the line radiation force is discussed in Mihalas, 1978, p.561.

The azimuthal equation of motion and the induction equation (a form of Faraday's law) are the same as in Weber and Davis (1967, WD) and Friend and MacGregor (1984), namely:

(6) $$\frac{d}{dr}(rv_\phi) = \frac{B_r}{4\pi\rho v_r}\frac{d}{dr}(rB_\phi)$$

(7) $$(\nabla \times E)_\phi = 0 = \frac{1}{r}\frac{d}{dr}[r(v_r B_\phi - v_\phi B_r)]$$

as $\nabla \cdot \vec{B} = 0$ gives

(8) $B_r = f_b/r^2$ for spherical symmetry and as $B_r/\rho v_r$ is a constant, (6) integrates to

(9) $rv_\phi - \frac{B_r}{4\pi\rho v_r} rB_\phi = L$ where L is the stream constant total angular momentum per unit mass which is the sum of the angular momentum carried in particles, rv_ϕ, as well as that in the electromagnetic field. Faraday's law integrates to

(10) $r(v_r B_\phi - v_\phi B_r) = \text{constant} = -\Omega^2 f_b$ and (9) and (10) together can be solved for v_ϕ and B_ϕ as in Weber and Davis (1967).

(11) $$v_\phi = \frac{\Omega r}{v_{rA}} \frac{v_{rA} - v_r}{1 - M_A^2{}_r}$$

(12) $$B_\phi = - B_r \frac{\Omega r}{v_{rA}} \frac{r_A^2 - r^2}{r_A^2 (1 - M_{Ar}^2)}$$

where Ω is the rotation rate of the region where the field lines are rooted (usually taken to be the photospheric value), f_b is the conserved radial magnetic flux, v_{rA} is the radial velocity at the Alfven radius (r_A) where v_r equals the speed of an Alfven wave in the medium, and M_{Ar} is the radial Alfven mach number.

(13) $$M_{Ar}^2 = 4\pi\rho \frac{v_r^2}{B_r^2} = \rho_a/\rho \ .$$

Closure is obtained by using the polytropic approximation.

(14) $P = P_o \ (\rho/\rho_o)^\gamma$ where P_o and ρ_o refer to values at the base of the wind (whether the base is in a corona or photosphere), and γ is the polytropic index.

This approximation is often used when the heating sources in the wind are not well-known. Generally, modellers replace this with an isothermal approximation for hot stars and use the stellar effective temperature. This is formally incorrect as it leads to inappropriate asymptotic states of the wind, Hundhausen (1972, p.9). However, the use of effective temperatures of even 50,000 K as for 0 stars does not lead to significant problems as it would for stars with coronae. Most early-type stars show x-ray emission. Therefore, I will examine a hot-wind solution and the polytropic approximation is appropriate.

The radial momentum equation, (2), can be simplified using $a^2 = \gamma p/\rho$ (a is the sound speed) together with

(15) $f_L = \frac{\Lambda}{\rho^{\alpha-1}} \left(\frac{udu}{dr}\right)^\alpha$. Using $\alpha = 1$, (2) eventually reduces to

$$(16)\quad \frac{r}{v_r}\frac{dv_r}{dr} = \frac{(v_r^2 - A_r^2)\left(2a^2 + v_\phi^2 - \frac{v_e^2}{2}(1-\Gamma)\right) + 2\, v_r\, v_\phi\, A_r\, A_\phi}{(v_r^2 - A_r^2)\,[v_r^2\,(1-\Lambda) - a^2] - v_r^2\, A_\phi^2}$$

where $A_r^2 = B_r^2/4\pi\rho$, $A_\phi^2 = B_\phi^2/4\pi\rho$, , the radial and azimuthal Alfven speeds, $v_e^2 = \frac{2 \cdot GM}{r^2}$, the escape velocity at r, and

$$(17)\quad \Lambda = \left(\frac{4\pi}{\dot{M}\,\sigma_e\, v_{th}}\right)^{\alpha} \Gamma\ GMk \text{ or, for } \alpha = 1,$$

$$(18)\quad \Lambda = \frac{kL}{c\dot{M}\, v_{th}}$$

Λ is the essential radiation parameter in this wind model and can be estimated for a particular star. For instance, Friend and MacGregor (1984) model the 06ef star λ Cephei with $\alpha = .7$, $\dot{M} \approx 5.2 \times 10^{-6} M_\odot$/year,

$T = 6.5\times10^4$K, $L = 6.76\times10^5 L_\odot$, and k was specified by using the CAK value of 1/30. These numbers give $\Lambda = .66$ in the current model. It is difficult to estimate Λ because k is model dependent, v_{th} depends on both the base temperature as well as the mass of the ions that absorb and scatter radiation, and mass-loss rates are not well-known for many stars.

Therefore Λ will be specified for a range of values and a variety of models will be calculated.

The general results of this study are based on the strong magnetic field limit of the MHD equations. This allows the explicit calculation of the position of the critical points based upon a similar prescription for non-radiative strong-field winds in the study of Hartmann and MacGregor (1982, HM).

III. THE CRITICAL POINTS

A. Critical Velocities

The WD theory gives rise to three critical points in the radial momentum equation. These points correspond to distances from the star where the speed of the stellar wind equals the speed of the characteristic MHD disturbances that occur in the fluid approximation; namely, the slow-mode wave, Alfven wave, and fast-mode wave. Setting $\Lambda = \Gamma = 0$ for the moment, the denominator of equation (16) goes to zero for three different wind speeds and, hence, the numerator must be zero at these critical points. These boundary conditions partially specify the physical characteristics of the wind. In the current study, radiation changes the nature of the critical points in a manner that can be explicitly shown.

The strong-field limit requires that $v_r^2 << A_r^2$ at the modified slow-mode point, r_s. This is equivalent to $m_{A\,r}^2 << 1$ at r_s.

This will be true for sufficiently strong magnetic fields and is justified a posteriori. Setting the denominator of (16) equal to zero:

$$A_r^2 \{v_r^2 (1 - \Lambda) - a^2 - v_r^2 A_\phi^2/A_r^2\} = 0$$

Faraday's law can be written as

$$\frac{v_r^2 A_\phi^2}{A_r^2} = (v_\phi - \Omega r)^2$$

and as v_ϕ is equal to the corotation value at r_s to $O(M^2_{Ar})$ we can neglect $v_r^2 A_\phi^2/A_r^2$ in the strong-field limit. Therefore,

$$(19) \quad v_{rs} = \frac{a_s}{1 - \Lambda}$$

which reduces to a well-known result when $\Lambda = 0$, $v_{rs} = a_s$.

At the Alfven point, the radial velocity is assumed to be much greater than its value at r_s so that

$$v_{rA}^2 (1 - \Lambda) >> a^2_A$$

Setting the denominator of (16) equal to zero.

(20) $v_r^2 = A_r^2 + \frac{A_\phi^2}{1-\Lambda}$ at r_a at which reduces to the well-known

result $v_r = A$ at r_A when $\Lambda = 0$.

The modified fast-mode velocity is found by assumming $v_r^2 > A_r^2$ ($M^2_{Ar} > 1$) at r_f. In a similar manner,

(21) $$v_{rf}^2 = \frac{A^2_{\phi f}}{1 - \Lambda}$$

B. Optically-Thick Slow-Mode Point

Expanding Equation (11) for v_ϕ

(22) $$v_\phi = \Omega r(1 - M_{Ar} + \frac{r^2}{r_A^2} M_{Ar} + \ldots)$$

For clarity, let $v_\phi = \Omega r(1 - \delta)$ where $\delta < 1$. Setting the numerator equal to zero at r_s

$$2a^2 - v_e^2/2 + v_\phi (v_\phi - 2V_r A_\phi/A_r) = 0 .$$

Using equation (10) gives

(23) $$2a^2 - \frac{v_e^2}{2} (1-\Gamma) + \Omega^2 r^2 = 0 \text{ to } O(\delta^2)$$

as terms of $O(\delta)$ exactly cancel.

Using the non-dimensional parameters in HM, $Z_s = r_s/r_o$, $\Omega^2 = \alpha^2\, GM/r_o^{\,3}$, and the non-dimensional Parker critical point $Z_p \equiv \dfrac{GM}{2r_o\, a_o^{\,2}}$, we find

$$(24)\quad Z_s \frac{a^2}{a_o^{\,2}} + \alpha^2\, Z_p\, Z_s^{\,3} - Z_p\,(1-\Gamma) = 0$$

which reduces to equation (9) in HM for the isothermal limit. Equation (24) does not explicity depend on the radiation parameter, Λ. The distance to the slow-mode point is the same in the radiative wind (except for the hottest stars where Γ is not negligible) as in the non-radiative MHD wind, except that the radial velocity in the radiative case is greater by the factor $1/\sqrt{1-\Lambda}$.

Another point is that the calculation of Z_s assumes v_ϕ is given by the corolation value at r_s. This is accurate to $O(M_{Ar})$, yet due to a fortuitous cancellation, the equation for Z_s is accurate to $O(M^2_{Ar})$ so that these results and HM's are more accurate than anticipated.

To solve (24) for Z_s, we must know $a^2/a_o^{\,2}$ which depends on v_{ro}. In the isothermal limit, $a^2/a_o^{\,2} = 1$ and this problem does not arise. However,

$$(25)\quad \frac{a^2}{a_o^{\,2}} = \left(\frac{v_{ro}\sqrt{1-\Lambda}}{a_o\, Z_s^{\,2}}\right)^{\beta} \quad \text{where}$$

$$(26)\quad \beta = \frac{2\,(\gamma-1)}{\gamma+1}$$

A second equation in Z_s and v_{ro} can be found by examining the energy/kg in the flow.

$$(27)\quad E = \frac{1}{2}\,(1-\Lambda)\, v_r^{\,2} + \frac{1}{2}\, v_\phi^{\,2} + \frac{a^2}{\gamma-1} - \frac{v_e^{\,2}}{2}\,(1-\Gamma) - \frac{\Omega r\, A_r\, A_\phi}{v_r}$$

The Poynting flux term can be calculated from equation (9) so that

$$(28)\quad E = \frac{1}{2}(1 - \Lambda)\, v^2 + \frac{1}{2} v_\phi^2 + \frac{a^2}{\gamma-1} - \frac{v_e^2}{2} + \Omega L - \Omega r\, v_\phi$$

Consider the two terms in v_ϕ using $v_\phi = \Omega r(1 - \delta)$

$$\frac{1}{2} v_\phi^2 - \Omega r\, v_\phi = -\frac{\Omega^2 r^2}{2}(1 - \delta^2)$$

so that terms of $O(M_{Ar})$ have again cancelled.

Setting $E_o = E_s$ we find after some work

$$(29)\quad \left(\frac{1-\Lambda}{2}\right)\frac{v_{ro}^2}{a_o^2} - \left(\frac{1}{2} + \frac{1}{\gamma-1}\right)\left(\frac{v_{ro}\sqrt{1-\Lambda}}{a_o\, z_s^2}\right)^{\beta} =$$

$$2z_p \left\{\left(1 - \frac{1}{z_s}\right)(1-\Gamma) - \frac{\alpha^2}{2}(z_s^2 - 1)\right\} - \frac{1}{\gamma-1}$$

This reduces to equation (13) in HM with $\gamma = 1$ and $\Lambda = 0$ provided we replace $\frac{a^2}{\gamma-1}$ by the isothermal term $a^2 \ell n\rho$.

Equations (24) and (29) can be self-consistently iterated between to find v_{ro} and z_s using HM's initial guess, modified for radiation

$$z_s,\ z_s = \frac{3z_p(1-\Gamma)}{1+3\,\alpha^{2/3}\, z_p\,(1-\Gamma)^{2/3}}$$

which assumes $a^2/ao^2 \approx 1$. This value for z_s allows the calculation of v_{ro} in (29). The iteration procedure rapidly converges to a solution.

C. Optically Thick Fast-Mode Point

We have shown that for $M^2_{ar} >> 1$ $v_{rf} \approx - \frac{A_\phi}{1-\Lambda}$ which together with equation (12) gives

$$v_{rf} = \frac{v_m^{3/2}}{\sqrt{1-\Lambda}\ v_{rf}^{1/2}} \left(1 - \frac{r_A^2}{r_f^2}\right)$$

where v_m is the usual Michel velocity, Michel (1969).

(30) $v_m^3 = \frac{\Omega^2\ f_b^2}{\dot{m}}$ so that

(31) $v_{rf} \approx \frac{v_m}{(1-\Lambda)^{1/3}}$

In order to calcuate z_f, we need to know $v_{\phi f}$ which can be calcuated from (11):

$$v_{\phi f}\ \frac{L}{r}\ \{1 - \varepsilon\} \approx \frac{L}{r}\left(1 - \frac{v_{rA}}{v_{rf}}\right\} \approx \varepsilon < 1$$

which is formally correct with or without radiation.

Setting the numerator of (16) equal to zero at Z_f:

(32) $2a^2 - v_e^2/2\ (1-\Gamma) + v_\phi\ (v_\phi + 2\ (v_\phi - L/r)) = 0$

The term in v_ϕ reduces to $\frac{L^2}{2}$ (1 - 4δ) to $O(M_{Ar}^{-1})$. to Equation (32) now reduces to

$$(33)\quad \frac{a_f^{\,2}}{a_o^{\,2}} Z_f^{\,2} - Z_p Z_f (1-\Gamma) + \alpha^2 Z_p Z_A^{\,4} - 2Z_A^{\,2} \frac{v_m^{\,2}}{a_o^{\,2}} (1-\Lambda)^{1/3} = 0$$

where

$$(34)\quad \frac{a_f^{\,2}}{a_o^{\,2}} = \left(\frac{v_{ro}(1-\Lambda)^{1/3}}{Z_f^{\,2}\; v_m}\right)^{\beta}$$

Equation (33) reduces to (17) in HM in the isothermal limit with $\Lambda = 0$ except for the term in $\frac{v_m^{\,4}}{a_o^{\,4}}$. This is $O(M_{Ar}^{\;-2})$ and should not appear as other terms of the same size have been neglected in calculating Z_f.

As Z_A appears in the calculation of Z_f we must generate another equation for Z_A and Z_f. The Alfven radius is not important except insofar as it is necessary in the calculation of Z_f. If the wind passes through Z_s and Z_f, it must of necessity pass through Z_A, Goldreich and Julian (1970).

Setting $E_o = E_f$

$$(35)\quad Z_A^{\,2} = \frac{1}{2\alpha^2 Z_p} \left\{\eta \frac{v_m^{\,2}}{a_o^{\,2}} + \frac{a_f^{\,2}/a_o^{\,2} - 1}{\gamma - 1} - \frac{v_{ro}^{\,2}}{2a_o^{\,2}}\right\}$$

$$+ \frac{1}{\gamma^2}\left(1 - \frac{1}{Z_f}\right)(1-\Gamma) + \frac{1}{2}$$

with

$$(36)\quad \eta \equiv \frac{1}{2}(1-\Lambda)^{1/3} + 1$$

A good first approximation to Z_A neglects $1/Z_f$ and sets $a_f^2/a_o^2 \approx 0$. This allows the calculation of Z_f in (33) and the iteration proceeds as usual.

III. CONCLUSIONS

While the results of this study are preliminary, several important conclusions can be drawn from the analytical techniques that have been developed. The strong-magnetic field, optically thick limit of the MHD equations produces physical solutions through a tractable technique that shows both how the critical points are to be treated and gives the values of the velocities at these points as well as the distances to the critical points.

The streamline constants are determined once the solution has been generated at the critical points. In particular, the distance to the slow-mode point and the radial velocity at the base must be iterated on to produce physical solutions. Once this is done, the mass-loss rate can be determined from the value of the density in the photosphere together with the radius of the star. Additionally, the total energy/kg is also determined. The angular momentum/kg is determined after the iteration procedure to determine the Alfven radius and fast-mode radius is completed.

Preliminary numerical solutions have been generated and are being compared to the work of Friend and MacGregor, 1984, although it is still too early to report these results. These solutions have been found by using an IBM-PC and, in principle, could have been done on a hand calculator.

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