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RELIABILITY MODELS APPLICABLE TO  
SPACE TELESCOPE  
SOLAR ARRAY ASSEMBLY SYSTEM

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SPACE TELESCOPE SOLAR ARRAY ASSEMBLY SYSTEM

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ABSTRACT

A complex system may consist of a number of sub-systems with several components in series, parallel, or combination of both series and parallel. In order to predict how well the system will perform, it is necessary to know the reliabilities of the subsystems and the reliability of the whole system. The objective of the present study is to develop mathematical models of the reliability which are applicable to complex systems. The models are determined by assuming  $k$  failures out of  $n$  components in a subsystem. By taking  $k = 1$  and  $k = n$ , these models reduce to parallel and series models; hence, the models can be specialized to parallel, series combination systems. The models are developed by assuming the failure rates of the components as functions of time and as such, can be applied to processes with or without aging affects. The reliability models are further specialized to Space Telescope Solar Array (STSA) System. The STSA consists of 20 identical solar panel assemblies (SPA's). The reliabilities of the SPA's are determined by the reliabilities of solar cell strings, interconnects, and diodes. The estimates of the reliability of the system for one to five years are calculated by using the reliability estimates of solar cells and interconnects given in ESA documents. Aging effects in relation to breaks in interconnects are discussed.

## 1. Introduction

If events occur in time and the outcomes are countable numbers, the outcomes of the process can be described by a continuous time discrete stochastic process  $N(t)$ . The failures of various components in a complex system are countable events occurring in time, and hence, form a continuous time discrete stochastic process. Let  $N_1(t)$ ,  $N_2(t)$  ...,  $N_n(t)$  be the outcomes of various failure event processes of various components in a system, and  $T_i$  be the random variable denoting the failure time of the  $i$ th component, then the reliability of this component is denoted by  $R_i(t)$  and is defined as the probability that the component has not failed during time  $t$ . The reliability of the overall system  $R(t)$  is the probability that the system has not failed during time  $t$ . This reliability  $R(t)$  can be determined from the various mathematical models for the reliabilities of the components  $R_i(t)$  and possible assembly of the components in parallel and series combination.

In this report, we shall develop mathematical models for systems which form continuous time discrete stochastic processes. We shall obtain the reliability models for the components of subsystems in which the failures occur randomly. Using these models, the reliability model for the system is obtained. The models are obtained for the failure rate  $\lambda(t)$ , which is a function of time  $t$ , and therefore, can be applied to a process in which failure rate is constant as well as time dependent failure rate which takes into account any aging effect. The models are formulated for series and parallel combination of components for which the reliabilities may be independent or nested.

The mathematical models developed will be applied in determining the reliability of the Space Telescope Solar Array (STSA) System. STSA System consists of two wings with two blankets in each wing. Each blanket is made up of five Solar Panel Assemblies (SPA's). The SPA consists of several strings. Each string consists of an array of solar cells in series and parallel, which are connected by interconnects and diodes. Assuming a reliability model for the solar cells and diodes, a reliability model will be built up for the whole STSA System. The reliability estimates for the STSA will be calculated for some models using the failure rates of solar cells and diodes given in European Space Agency (ESA)

documents. The estimates for the reliability of STSA System are obtained under different power losses in the STSA System. Also, the effects of breaks in solar cell interconnects will be discussed.

## 2. Stochastic Process, Failure Time and Reliability

A stochastic process is defined in Karlin and Taylor (1975) as a family of random variables determined by a process. A realization of a stochastic process is denoted by  $N(t)$ ,  $t \in (0, \infty)$ . If  $t$  denotes the time and  $N(t)$  corresponds to some outcome of the process, then  $N(t)$  is called the time dependent stochastic process. If the random variable takes on countable values,  $0, 1, 2, 3, \dots$  the  $N(t)$  process is called a discrete stochastic process. Many physical processes are time dependent discrete stochastic processes. If a complex system is working in time, then the failures of the components of the system occur randomly and the number of failures form a time dependent stochastic process  $N(t)$ . If  $N(t_1), N(t_2), \dots$  denote the outcomes of a time dependent stochastic process  $N(t)$ , corresponding to a complex system at times  $t_1, t_2, \dots, t_n, \dots$  then  $T = t_n - t_{n-1}$ , is a random variable and  $T$  is called the interevent or interarrival time of the process. If the outcomes of the stochastic process  $N(t)$  corresponds to failure events of the process then  $T$  is called the failure time of the process and is a random measure of the time between two successive failures. Since  $T$  is random, it has a distribution function denoted by  $F(t)$  and is defined by

$$F(t) = \text{Pr} \{ T \leq t \} . \quad (1)$$

Here  $\text{pr} \{ \}$  is abbreviated for probability. The reliability of the system is the probability that the system will not fail at least until time  $t$ , and is denoted by  $R(t)$  and can be expressed by

$$R(t) = \text{Pr} \{ T > t \} = 1 - F(t) . (2)$$

Since time  $T$  is a continuous random variable, its distribution function  $F(t)$  is continuous and also, differentiable. The derivative of  $F(t)$  is called the probability density function (p.d.f.) of  $T$  and is denoted by  $f(t)$ . The p.d.f.  $f(t)$  can be written as

$$f(t) = \frac{d}{dt} F(t) \quad (3)$$

For a given distribution of a process  $N(t)$ , there is an unique p.d.f.  $f(t)$ , of  $T$ . Hence, the process  $N(t)$  can be characterized by the p.d.f. of  $T$ .

The reliability  $R(t)$  can be expressed in terms of  $f(t)$  as

$$R(t) = \int_t^{\infty} f(t) \, dt \quad . \quad (4)$$

and conversely,

$$f(t) = - \frac{d R(t)}{d t} \quad . \quad (5)$$

From (4) the reliability can be obtained from the p.d.f of  $T$ . The theory of reliability and historical prospective is discussed by Barlow (1984), also in the paper, a large list of references on the topic is given.

### 3. Failure Rates

The failure rate  $\lambda(t)$  is the rate of failure at which the components fail and the failure rate can be defined by

$$\begin{aligned} \lambda(t) &= \lim_{\Delta t \rightarrow 0} \Pr(t < T < t + \Delta t \mid T > t) \quad . \\ &= \lim_{\Delta t \rightarrow 0} P\{(t < T < t + \Delta t, T > t)\} / P(T > t) \\ &= f(t) / (1 - F(t)) = - \frac{d R(t)}{d t} \quad (6) \end{aligned}$$

From the relation (6), the failure rate can also be expressed in terms of  $R(t)$  as

$$\int_0^t \lambda(u) \, du = -\ln R(t) \quad (7)$$

and

$$R(t) = e^{-\int_0^t \lambda(u) du} \quad (8)$$

The failure rate is also called the hazard function. The hazard function may be increasing function, constant function or decreasing function of t.

### 3.1 Constant Failure Rate

If the events occur in time randomly and in a short interval of time, at most one event can occur and the events in nonoverlapping intervals occur independently, then it can be shown that the resulting process is a poisson process and the number of events occurring in the process in given time, t can be shown to be

$$\Pr \{ N(t) \leq n \} = \sum_{j=0}^n \frac{e^{-\lambda t} (\lambda t)^j}{j!}, \quad n=0,1,2,\dots \quad (9)$$

In this case, it can be shown that the waiting time for the process has a gamma distribution and the distribution of interevent time, T has the exponential distribution with p.d.f. f(t) given by

$$f(t) = \lambda e^{-\lambda t} \quad t > 0, \lambda > 0 \quad (10)$$

The corresponding reliability function R(t) is given by

$$R(t) = e^{-\lambda t} \quad (11)$$

and the failure rate function  $\lambda(t)$  reduces to

$$\lambda(t) = \frac{d}{dt} \ln \{R(t)\} = \lambda e^{-\lambda t} / e^{-\lambda t} = \lambda.$$

In this case, the failure rate  $\lambda(t)$  is a constant function.

### 3.2 Variable Failure Rate

The rate of failures in a small interval may not be constant, and the failure rate may be either increasing or decreasing. In the case of system components, the



components may have an aging effect and the hazard may increase with time, in this case  $\lambda(t)$  is an increasing function of time and failure may occur more often such a system is called by Asher (1983) as a "sad" system. On the other hand, the process may be a learning process, in this case the hazard rate  $\lambda(t)$  is a decreasing function of  $t$ , and the system in this case is called a "happy" system. There can be many different hazard functions associated with different phenomena. The failure rate which varies directly with a power of  $t$  can be defined by

$$\lambda(t) = \lambda \beta t^{\beta - 1}, \quad t > 0. \quad (12)$$

For  $\beta=1$  the hazard rate is constant, for  $\beta<1$  the hazard rate is decreasing and for  $\beta>1$  the hazard rate is increasing. The model with the failure rate given in (12) is called Weibull model. Using equation (8) the reliability can be written as

$$R(t) = e^{-\lambda t^\beta}. \quad (13)$$

and the p.d.f. of  $T$  is given by

$$f(t) = \lambda \beta t^{\beta - 1} e^{-\lambda t^\beta}, \quad t > 0, \lambda > 0. \quad (14)$$

It should be noted that the failure rate contains two parameters  $\lambda, \beta$ . Often  $\beta=2$  is used, in this case failure rate is linear function of  $t$ .

Other failure rate models could be used in different situations. If the failure rate decreases sharply with time, then

$$\lambda(t) = e^{-\lambda t}. \quad (15)$$

is suitable.

If the failure rate decreases initially, then increases, the failure rate in such situation is discussed in Shooman (1968) is given by

$$\lambda(t) = \begin{cases} K_0 - K_1 t, & 0 < t \leq \frac{K_0}{K_1} \\ 0, & \frac{K_0}{K_1} < t \leq t_0 \\ K(t-t_0), & t_0 < t < \infty \end{cases} \quad (16)$$

#### 4. Reliability Models

Suppose a system S consists of number of subsystems and each subsystem consists of number of components. The components may work in series, parallel or combination, also components may work independently or dependently. Different models would be used for different situations. Two types of model are discussed below.

##### 4.1 Independent Components

Suppose a subsystem has m components and the components work independently and the system works satisfactory if at least k of the m components work. If all the components are different then from Basu and Mawaziny (1978) if  $R_{\alpha_i}(t)$  is the reliability of  $\alpha_i$ th component then reliability of the subsystem  $R_{Si}(t)$  can be written as

$$R_{Si}(t) = \sum_{j=k}^m \sum_{\alpha_i} \prod_{i=1}^j (R_{\alpha_i}(t)) \prod_{i=j+1}^m (1-R_{\alpha_i}(t)) \quad (17)$$

Where  $\sum_{\alpha_i}$  is taken over  $\binom{m}{j}$  distinct values of  $\alpha_i$ .

If all the components are identical then R(t) reduces to

$$R_{Si}(t) = \sum_{j=k}^m \binom{m}{j} (R(t))^j (1-R(t))^{m-j} \quad (18)$$

The system could be called k redundant system, for k=1 the system is called completely redundant or parallel system, for k=n the system is completely nonredundant and system works if each component works and the system reduces to a series system.

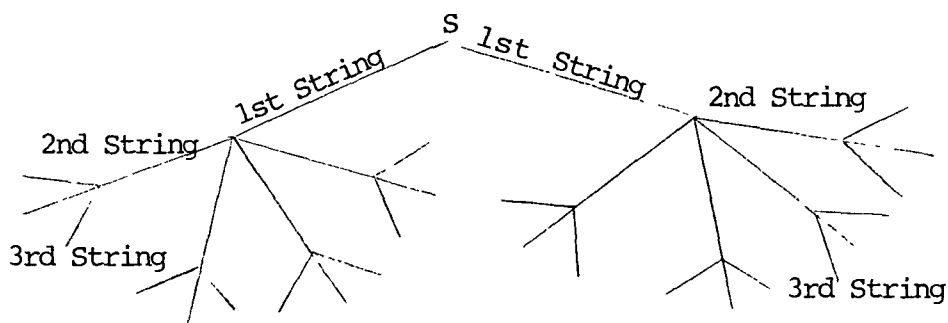
If there are  $s$  subsystems in a total system and the  $R_i(t)$  is the reliability of components in  $i$ th subsystem and the subsystems are independent then the reliability of the system using (18) can be obtained as

$$R_S = \prod_{i=1}^s R_{Si} = \prod_{i=1}^s \left\{ \sum_{j=k}^m \binom{m}{j} (R_i(t))^j (1-R_i(t))^{m-j} \right\}. \quad (19)$$

The subsystems in a system may not be completely independent and the system might work with one or more subsystems working, in this case, the nested model may be useful.

#### 4.2 Nested Model

In the nested models we consider the subsystems which work independently and the system may work with one or more subsystems, further, the probability of success of a subsystem may depend on sub-subsystems and so on. Suppose there are  $y$  stages of a subsystem connected in strings, the system could be represented with a tree diagram. As an example, we consider a system with 3 stages with 2 components in 1st string, 4 components in 2nd string and 3 components in 3rd string. The system can be represented by the tree diagram as follows:



Suppose the components in each string are identical and the reliabilities of the upper string depend on the reliabilities of the lower branches. Suppose the system works if at least 2 components work in 3rd string, at least 2 in the 2nd string, and at least 1 in 1st string.

Then the reliability of the system is determined as

$$R_S = \sum_{j=1}^2 \binom{2}{j} R_1^j (1-R_1)^{2-j} \quad (20)$$

$$R_1 = \sum_{j=2}^4 \binom{4}{j} R_{12}^j (1-R_{12})^{4-j} \quad (21)$$

$$R_{12} = \sum_{j=2}^3 \binom{3}{j} R_{123}^j (1-R_{123})^{3-j} \quad (22)$$

The reliability  $R_{123}$  is either estimated or given then  $R_S$  can be obtained successive using (22), (21), and (20).

More generally, if the system consists of  $y$  stages and there are  $n_i$  identical components in  $i$ th stage and  $i$ th stage works if at least  $k_i$  components work and the system is completely nested then

$$R_S = \sum_{j=k_1}^{n_1} \binom{n_1}{j} R_1^j (1-R_1)^{n_1-j} \quad (23)$$

$$R_1 = \sum_{j=k_2}^{n_2} \binom{n_2}{j} (R_{12})^j (1-R_{12})^{n_2-j} \quad (24)$$

$$R_{12\dots y-1} = \sum_{j=k_y}^{n_y} \binom{n_y}{j} (R_{12\dots y})^j (1-R_{12\dots y})^{n_y-j} \quad (25)$$

The reliability of the system can be completely determined knowing the reliability of  $R_{12\dots y}$  and using equations (25), (24), and (23). Such models are used often in complex systems. The Space Telescope Solar Array could be based on such a model. Also, it is noted that if  $k_i = n_i$ ,  $i=1, 2, y$  then each subsystem becomes series model and the whole system reduces model with no loss allowance.

## 5. Estimation of Parameters

In this section we discuss estimation of parameters which appear in the reliability expressions. We shall restrict to exponential and Weibull models as these are the models which are used often and we have given expressions for the reliabilities involving these models. We consider the maximum likelihood estimators (m.l.e), these are most widely used and have good statistical properties like asymptotic normality and consistency.

### 5.1 The Exponential Model

If  $t_1, t_2, \dots, t_n$  is a sample from the exponential distribution with p.d.f. in (10). The m.l.e. for  $\lambda$  is denoted by  $\hat{\lambda}_T$  and is given by

$$\hat{\lambda}_T = n / \sum_{i=1}^n t_i \quad . \quad (26)$$

The estimator is biased but is consistent.

However, if the observations are made on the counting process, if  $n_1, \dots, n_k$  are failures in  $k$  independent trials in given time  $t$  from the homogenous poisson distribution, then the maximum likelihood estimator for  $\lambda$  can be written as

$$\hat{\lambda}_N = \sum_{i=1}^k n_i / kt \quad . \quad (27)$$

This estimator is the minimum variance unbiased estimator for  $\lambda$  and its variance can be found. Both estimators can be used to find interval estimators for  $\lambda$ .

### 5.2 Weibull Model

If  $t_1, t_2, \dots, t_n$  is a sample from the distribution with p.d.f given in (14), if  $\beta$  is known, then taking the likelihood of the sample the m.l.e of  $\lambda$  can be found as

$$\hat{\lambda}_T = n / \sum_{i=1}^n t_i^\beta \quad . \quad (28)$$

If the observations are made on the number of failures, for given time  $t, n_1, n_2 \dots n_k$  are failures in  $k$  independent trials from the nonhomogenous poisson process with intensity function  $\lambda(t) = \lambda \beta t^{\beta-1}$  then the m.l.e of  $\lambda$  can be obtained as

$$\hat{\lambda}_N = \sum_{i=1}^k n_i / (kt^\beta). \quad (29)$$

If the shape parameter  $\beta$  is not known, then the maximum likelihood estimators of  $\lambda$  and  $\beta$  can not be obtained explicitly, however, the estimating equations for  $\lambda$  and  $\beta$  can be obtained. If  $t_1, t_2 \dots t_n$  are  $n$  independent failure time observations from the Weibull p.d.f in (14) then the m.l.e for  $\hat{\beta}$  is obtained by solving

$$1/\hat{\beta} - \left( \sum_{i=1}^n t_i^{\hat{\beta}} \ln t_i \right) / \left( \sum_{i=1}^n t_i^{\hat{\beta}} \right) + \left( \sum_{i=1}^n \ln t_i \right) / n = 0. \quad (30)$$

and the m.l.e of  $\lambda$  is obtained as

$$\hat{\lambda} = n / \left( \sum_{i=1}^n t_i^{\hat{\beta}} \right). \quad (31)$$

The m.l.e's of  $\hat{\lambda}, \hat{\beta}$  and their properties are discussed by Cohen (1965).

The confidence interval estimators of  $\lambda$  can be obtained by using distributions of  $\hat{\lambda}$ . Large sample confidence intervals are discussed by Abernethy et al. (1983).

Crow (1974) has obtained explicit expressions for m.l.e's of  $\beta$  and  $\lambda$  based ordered observations for a repairable system.

## 6. Application to Space Telescope Solar Array System

The methodology developed in section 2-5 is now applied to Space Telescope Solar Array (STSA) System. First, we describe the STSA System.

## 6.1 Description of STSA System

Space Telescope is made up two identical wings. Each wing consists of two identical blankets. Each blanket is made up of five identical Solar Panel Assembly (SPA). There are 20 identical SPA's in STSA System. These SPA's are connected by 40 connecting diodes with 2 on each SPA. Each SPA is built up of three strings of solar cells. Each string has a length of 106 solar cells. Two end strings have eight cells in parallel and the middle string has seven cells in parallel. Strings 1 and 3 are built by 848 cells each whereas the middle string is made up of 742 cells. Each string is further broken into 7 substrings of 14, 15, or 16 cells so the substring consists of either 14, 15, and 17 long and 8 or 7 cells wide. There are seven shunting diodes. Each SPA has 21 shunting diodes and 2,438 solar cells. The cells are connected by Cell Interconnects (CIC's). There are the same number of CIC's as the cells. The STSA System consists of 48,760 cells, same number of CIC's, 420 shunting diodes, 40 connecting diodes and same number of solders. The description of STSA System is given in a number of ESA documents. Two of these are GL-SA-B002 and AN-1367-108.

Each individual cell is made up of silicon and 20 mms. wide, 40 mms. long, and generates .349 volts and carries current .300 amp. at 55°. However, total voltage at SA/SSM interface is approximately 34 volts and working current is .27 amps. Approximate power without any loss of cells diodes and CIC's is 4.18 watts per string. Since 460 parallel strings in STSA System, STSA produces approximately 4,222.8 watts. The losses due to solder and other connectors in current and voltages are not clear. The estimates mentioned are taken from GL-SA-B002.

## 6.2 The Reliability of STSA System

The reliabilities of photovoltaic devices and system are discussed in a special issue of IEEE Transaction (1982) Vol. R-31. The papers of some interest on the topic are, "Photovoltaic Module Reliability Improvement Through Application Testing and Failure Analysis" by Dumas and Shumk (1982). "Reliability Terminology and Formulae for Photovoltaic Power System" by Lauffenburger and Anderson (1982), and "A Methodology for Photovoltaic System Reliability and Economic Analysis" by Stember Huss and Bridgman (1982).

The reliability of STSA System depends on the reliabilities of the various components in the system, the design of the components, the definition of the failure,

and the mathematical models used in determining the reliabilities of the components. Most of the estimates for failure rate are based on the constant failure rate, which arises out of the exponential models as these models have been used by ESA. However, estimates based on Weibull model could be obtained if some estimates or knowledge of shape parameter is assumed.

First, we discuss the reliabilities of blocking and shunted diodes. For connecting diodes, since there are 40 blocking diodes, if all of them have to work, then we find

$$R_0 = (R_{BD})^{40}, \quad (32)$$

$R_1$  is the reliability of individual connecting diode.

If the success is defined with  $k$  ( $k$  is an integer close to 40) diodes working the  $R_1$  reduces to

$$R_1 = \sum_{j=k}^{40} \binom{40}{j} R_{BD}^j (1-R_{BD})^{40-j}. \quad (33)$$

For shunting diodes, the reliability, when each diode is working, is

$$R_0 = (R_{SD})^{420}. \quad (34)$$

For  $k$  diodes working, the reliability reduces to

$$R_1 = \sum_{j=k}^{420} \binom{420}{j} (R_{SD})^j (1-R_{SD})^{420-j}. \quad (35)$$

The reliabilities  $R_{SD}$  for the exponential for given time  $t$  can be evaluated by

$$R_{SD} = e^{-\lambda_1 t},$$

Where  $\lambda_1$  is the failure rate of the diodes to be evaluated from the data.

Next, we consider the reliabilities associated with CIC. Since there are 20 SPA's and each SPA has 3 strings with width 8 and 1 with width 7. Here, for calculating



reliability, we assume width 8. Each string has 106 CIC's in series. The reliabilities of CIC's by nested design can be written as

$$R_{CIC} = \sum_{j=k_1}^{60} \binom{60}{j} R_1^j (1-R_1)^{60-j}, \quad (36)$$

$$R_1 = \sum_{j=k_2}^8 \binom{8}{j} R_{12}^j (1-R_{12})^{8-j}, \quad (37)$$

and

$$R_{12} = R_{123}^{106}. \quad (38)$$

Here  $k_1, k_2$  are integers close to upper limits and  $R_{123}$  is the reliability of interconnect can be obtained from

$$R_{123} = e^{-\lambda_c t},$$

$\lambda_c$  being failure rate of an interconnect.

We now discuss the reliability models associated with the solar cell arrays. In the first model, we assume that a substring consists of an average of 15 cells in series and each string has 7 substrings and there are 8 parallel strings associated with each string. Since there are 3 strings per SPA, out of 21 subgroups any number of subgroups might fail, a nested model could be used. Let  $R_c = R_{1234}$  be the reliability of a cell, then

$$R_{123} = R_{1234}^{15} \text{ is the reliability of a substring, and}$$

$$R_{12} = \sum_{j=k_3}^{21} \binom{21}{j} (R_{123})^j (1-R_{123})^{21-j}. \quad (39)$$

$R_{12}$  gives the reliability with  $k_3$  substrings working. Since there are 8 strings in parallel, if some of these fail, then the power of the system would be affected. Hence, using  $k_2$  parallel strings working the reliability of SPA is determined by

$$R_1 = R_{SPA} = \sum_{j=k_2}^8 \binom{8}{j} (R_{12})^j (1-R_{12})^{8-j}. \quad (40)$$

Since there are 20 SPA's, the reliability of STSA System from cell failures can be reduced to

$$R_S = \sum_{j=k_1}^{20} \binom{20}{j} R_1^j (1-R_1)^{20-j} . \quad (41)$$

If there is no loss of power due to any cells in the entire system, then the reliability of the system reduces to

$$R_S = (R_C)^{48760} . \quad (42)$$

There are other models considered for the reliability of solar cell system. In AN-1367-108, it is suggested that each string should be considered as one unit. Since there are 460 strings of 106 cells long in the STSA System, the reliability  $R_S$  can be written as

$$R_S = \sum_{j=k}^{460} \binom{460}{j} R_1^j (1-R_1)^{460-j} , \quad (43)$$

where  $R_1$  is the reliability of the string and is given by

$$R_1 = R_C^{106} . \quad (44)$$

$R_C$  is the reliability of a single cell and is determined from

$$R_C = e^{-\lambda_C t}$$

in the exponential case and for Weibull case  $R_C$  is given by

$$R_C = e^{-\lambda_C t^\beta} ,$$

$\lambda_C t^\beta$  could be determined from a set of data.

## 7. Estimates of Reliabilities

In this section, we calculate the reliabilities associated with diodes, CIC's and solar panels. These reliabilities are then used to calculate the reliabilities of STSA System. In calculating the reliabilities, we are using various models proposed in section 6. The estimates are based on failure rate estimates given in ESA documents.

Since these estimates are based on the exponential models, we have to resort to these models. The reliabilities are calculated from one to five years and presented in a tabular form as well as in graphical form. First, we calculate the reliabilities of the diodes.

### 7.1 Reliability Estimates of Diodes

For blocking diodes, the failure rate given in TN-SA-B147 is  $\lambda = 1.2 \times 10^{-9}/\text{hr.}$ , hence, the reliabilities are calculated from  $R_{BD}(t) = e^{-\lambda t}$ . Since there are 40 diodes, if all of them have to work reliabilities are calculated from

$$R_0(t) = (R_{BD})^{40} . \quad (45)$$

If we allow one of them to fail then

$$R_1(t) = (R_{BD})^{40} + 40 (R_{BD})^{39} (1-R_{BD}) \quad (46)$$

Using these formulae, we find the reliabilities assuming one year is 8,760 hours. The results are presented in table I.

Table I. Reliabilities of Blocking Diodes

Time t (in years)	$R_{BD}(t)$	$R_0(t)$	$R_1(t)$
1	.99998948	.99957961	.99999991
2	.99997897	.99915939	.99999965
3	.99996846	.99873935	.99999922
4	.99995795	.99831949	.95999862
5	.99994744	.99789981	.99999785

Next, we find the reliabilities for the shunting diodes. Since there are 420 shunting diodes and failure rate of each diode is  $\lambda = 1 \times 10^{-9}$ , the reliabilities for all diodes working are obtained from

$$R_D(t) = e^{-\lambda t}$$

$$R_0(t) = (R_{SD})^{420} . \quad (47)$$

If we allow in our success criteria one diode to fail, then the reliabilities are calculated from

$$R_1(t) = (R_{SD})^{420} + 420 (R_{SD})^{419} (1-R_{SD}). \quad (48)$$

The reliabilities for different years are given in table II.

Table II. Reliabilities of Shunting Diode

Time t (years)	$R_{SD}(t)$	$R_O(t)$	$R_1(t)$
1	.99999124	.99632760	.99999328
2	.99998248	.99266860	.99997314
3	.99997372	.98902309	.99993969
4	.99996496	.98539096	.99989302
5	.99995620	.98177217	.99983327

## 7.2 Reliability Estimates of Solar Cells

We now discuss the reliabilities of solar cells. First, we use the nested model discussed in section 6. For the series model

$$R_O(t) = R_S = (R_C)^{48760}. \quad (49)$$

Also, taking  $K_3 = 21$ ,  $K_2 = 8$  and  $K_1 = 20$  in equations for solar cells in section 6, we find

$$R_{123} = R_C^{15}. \quad (50)$$

$$R_{12} = R_{123}^{21} + (21) (R_{123})^{20} (1-R_{123}). \quad (51)$$

$$R_{SPA} = R_{12}^8 + 8 R_{12}^7 (1-R_{12}). \quad (52)$$

$$R_1(t) = R_S = R_1^{20}. \quad (53)$$

This model allows a loss of up to 1 substring and up to 1 parallel string to fail in each of the 20 SPA's. This amounts to 4.9% power loss in STSA System. The loss figure is taken from TN-SA-B151.

Alternative model with .22% of power loss is considered. This is based on 460 total strings (counting parallel strings) in the STSA System. The loss corresponds to a loss of one string of 106 cells in series. The reliabilities are obtained by taking  $K = 459$  in equation (43). The reliability of STSA System is calculated from

$$R_1 = R_C^{106}. \quad (54)$$

$$R_1'(t) = R_1^{460} + 460 R_1^{459} (1-R_1) \quad (55)$$

For the cell, the failure rate from TR-STSA-42 is  $1 \times 10^{-9}$ /hr. Using this estimate and the above equations for various t's, the reliabilities  $R_0(t)$ ,  $R_1(t)$ , and  $R_1'(t)$  are calculated and presented in table III.

Table III. Reliabilities of Solar Cells in STSA System

Time t	$R_C(t)$	$R_0(t)$ (Zero Loss)	$R_1(t)$ (1 String Loss)	$R_1'(t)$ (.22% Loss)
1	.99999124	.65237696	= .99999999	.93115794
2	.99998248	.42559154	= .99999989	.78950169
3	.99997372	.27764476	= .99999941	.63391835
4	.99996496	.18112816	= .95999815	.49117021
5	.99995620	.11816326	= .99999550	.37110988

The reliabilities of the CIC's could be calculated from the formulae for CIC's. However, the reliabilities for a CIC as functions of time are not available. Since there are the same number of CIC's as the cells and their configuration is similar to those of the cells, it is possible to combine the reliabilities of CIC's and cell and find the reliabilities of CIC's with cells in STSA System. The estimate of reliability for a CIC from AN-1367-108 is  $R = .9999976714$ . The reliability of CIC and cell is given by

$$R_C'(t) = (.9999976714) e^{-\lambda t} \quad (56)$$

We use this formula to calculate  $R_C$  and the formulae  $R_0(t)$ ,  $R_1(t)$ , and  $R_1'(t)$  given in equation (49) to equation (55). The reliabilities for different t are given in table IV.

Table IV. Reliabilities of Solar Cells With CIC in STSA

Time t (in yrs.)	$R_C'(t)$	$R_0(t)$ (Zero Loss)	$R_1(t)$ (1 String Loss)	$R_1'(t)$ (.22% Loss)
1 yr.	.99998891	.58234352	.99999998	.89739874
2 yrs.	.99998015	.37990379	.99999982	.74797591
3 yrs.	.99997139	.24784048	.99999917	.59409527
4 yrs.	.99996263	.16168463	.99999760	.45687567
5 yrs.	.99995387	.10547829	.99999445	.34330636

### 7.3 The Reliabilities of STSA System

The STSA System consists of diodes, CIC's and solar cells. The reliability of STSA System  $R_S$  is calculated from

$$R_S(t) = R_D(t) R_{Cell}(t) \quad (57)$$

For zero loss  $R_{S_0}(t)$  is calculated by multiplying  $R_0(t)$  from tables I, II, and IV. For .22% power loss  $R_{S_1}(t)$  is tabulated from  $R_1(t)$  in tables I and II and  $R_1$  in table IV, and these are tabulated in table V. Also, at the beginning there is no loss, hence, at time 0, the reliability is 1.

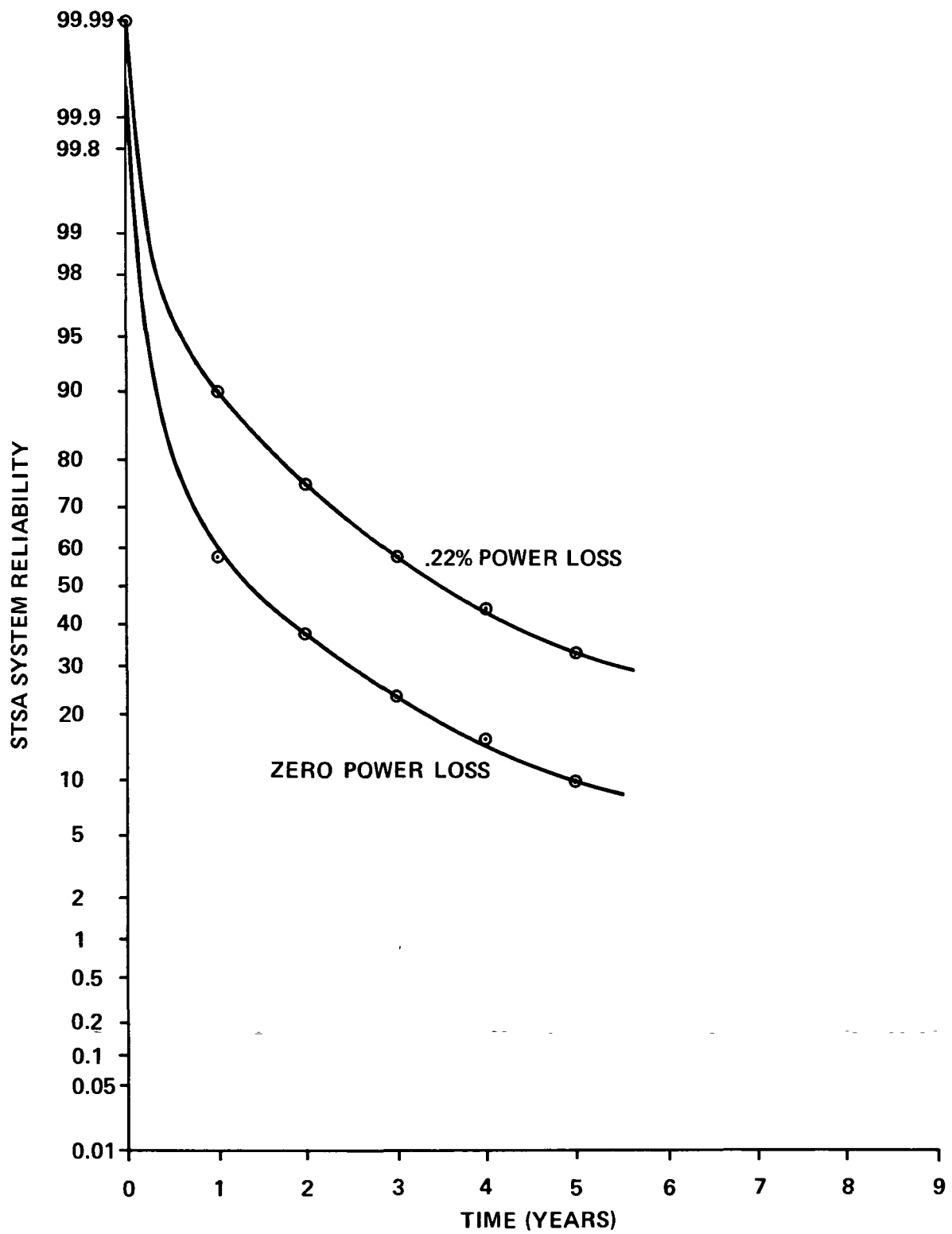
Table V. STSA System Reliabilities

Time t	$R_{S_0}(t)$ (No Power Loss)	$R_{S_1}(t)$ (.22% Power Loss)
0	1	1
1	.57996100	.89739874
2	.37680155	.74795555
3	.24481094	.59405897
4	.15905482	.45682516
5	.10333816	.34324838

The graphs of the reliabilities of STSA System are presented for zero power loss and .22% power loss.

### 8. Discussion and Conclusion

After a search of literature on the failure rate estimates and reliabilities of solar cells, most of the identifiable sources on the subject were found in the ESA documents. However, none of the documents contained any original data. The failure rate of the solar cells was determined from the failure rate of the solar cell diodes. The failure rate of solar cells is taken as the same as that of the diode (STSA B151) because both have the same base material, doping, contact system, cover slides, and adhesive. The data on failure rate of diodes is also not available. It appears that the failure rate of diode is determined from the estimates of the components in the diode. The failure rate on CIC's is estimated from a confidence procedure on the binomial distribution, which is independent of time.



There are no explicit results available on the effect on the failure rate due to the temperature change of the solar cells, although the cells go through a change of temperature from  $-80^{\circ}$  to  $180^{\circ}$  centigrade. The effect of temperature on the cell voltage is discussed by Rajeswaren et al. (1982). Also, Anderson and Kim (1978) give the relation between open voltage and temperature. There may not be an immediate impact on the performance of the solar cell, but long term effect is not known. In the (1978) paper Anderson and Kim state that there was a degradation after the crack appeared in the cell. This suggests that the constant failure rate models may not be suitable for systems working for a long time.

Also recently, Alexander (1985) has run some tests on the solar cells. The tests were run on only 12 cells. In the study, he found some breaks in the strands on interconnects. The total number of unbroken strands on the interconnects is not clear. Also, there were not any complete breaks in interconnects. It is difficult to determine failure rate on the interconnects.

Although general models applicable to STSA System are developed, the models are not tested for any other model except the constant failure rate model. There is a need to test aging models. From the available failure rate estimates, the reliability estimate of STSA System is obtained based on the maximum likelihood estimators of  $\lambda$ . The maximum likelihood estimators of the reliability are biased, hence, the biases and variances need to be investigated. And the confidence intervals on the reliabilities need to be developed to gain further information on the reliability of STSA System.



## REFERENCES

1. Abernethy, R. B; Breneman, J. E.; Medlin, C. H.; and Reinman, G. L.; Weibull Analysis Handbook (1983), Pratt Whitney Aircraft, Govt. Products Division.
2. Alexander, D., "Space Telescope Solar Cell Module Thermal Cycle Test," (1985), Marshall Space Flight Center NASA Report.
3. Anderson, W. A. and Kim, J. K., "Reliability Studies on MIS Solar Cells," (1978), Applied Physics, Vol. 17, p. 401-404.
4. Asher, H., "Discussion," On Statistical Methods in Reliability, (1983), Technometrics, Vol. 25, p. 320-326.
5. Barlow, R., "Mathematical Theory of Reliability: A Historical Prospective," (1984), IEEE Tran., Vol. R-33, p. 16-20.
6. Basu, A. P. and Mawaziny, A. H. FL, "Estimates of Reliability of K-Out-Of-M Structures in the Independent Exponential Case," (1978), Jour. Amer. Stat. Asso., Vol. 73, p. 850-854.
7. Cohen, A. C., "Maximum Likelihood Estimation in the Weibull Distribution Based on Complete and on Censored Samples," (1965), Technometric, Vol. 7, p. 579-588.
8. Crow, H. L., "Reliability Analysis For Complex Repairable System," (1974), Reliability and Biometry Statistical Analysis of Life Length SIAM Philadelphia.
9. Dumas, L. N. and Shuma, A., "Photovoltaic Module Reliability Improvement Through Application Testing and Failure Analysis," (1982), IEEE Trans., Vol. R-31, p. 228-233.
10. Garlach, "Reliability Prediction For STSA-BSFR Solar Array Blanket," (1984), AN-1367-108 (ESA Document).
11. Garner, P. D., "Solar Array Power Generation Reliability Analysis," (1981), TN-SA-B151 (ESA Document).

12. Hazell, H. H. C., "Solar Array System Description Handbook," GL-SA-B002 (ESA Document).
13. Karlin, S. and Taylor, H. M., A First Course in Stochastic Processes (1975), Academic Press, New York.
14. Lauffenburger, H. A. and Anderson, R. T., "Reliability Terminology and Formulae For Photovoltaic Power System," (1982), IEEE Trans., Vol. R-31, p. 289-295.
15. Rajeswaren, G.; Anderson, W. A.; Thayer, M.; and Lee, B. W.; "Statistical Analysis of Cr-Mis Solar Cells," (1982), IEEE Tran., Vol. R-31, p. 276-280.
16. Shooman, M. L., Probabilistic Reliability: An Engineering Approach (1968), McGraw-Hill
17. Stember, L. H.; Huss, W. R.; and Bridgeman, M. S., "A Methodology For Photovoltaic System Reliability and Economic Analysis," (1982) IEEE Tran., Vol. R-31, p. 296-303.